St George Girls High School

Trial Higher School Certificate Examination

2011

Mathematics

General Instructions

- Reading time – 5 minutes
- Working time – 3 hours
- Write using blue or black pen
- Begin each question on a new booklet
- Write your student number on each page
- All necessary working must be shown.
- Diagrams are not to scale.
- Board-approved calculators may be used.
- The mark allocated for each question is listed at the side of the question.

Total Marks – 120

- Attempt ALL questions.
- All questions are of equal value.

Students are advised that this is a Trial Examination only and does not necessarily reflect the content or format of the Higher School Certificate Examination.

Question 1 – (12 marks)

Marks

a) Simplify $\sqrt{75} - 2\sqrt{12}$ 2

b) Solve $5x^2 = 2x + 3$ 2

c) Factorise $y^3 - 8$ 1

d) If $0^\circ \leq \theta \leq 360^\circ$ and $\cos\theta = -\frac{\sqrt{3}}{2}$ find all possible values of θ 2

e) If $y = xe^x$ find $\frac{dy}{dx}$ 2

f) Convert 225° to radians. 1

g) Show on a neat diagram (at least $\frac{1}{3}$ of a page) the region defined by the intersection of: 2

$$x - y \leq -1 \text{ and } 5x + 3y \leq 19$$

Question 2 – (12 marks) **Marks**

a) (i) Find the equations of the tangents to $y = 4x - x^2$ at the points where the curve crosses the x axis. 4

(ii) Calculate the area bounded by the tangents and the x axis. 2

b) Find the sum of the series 2

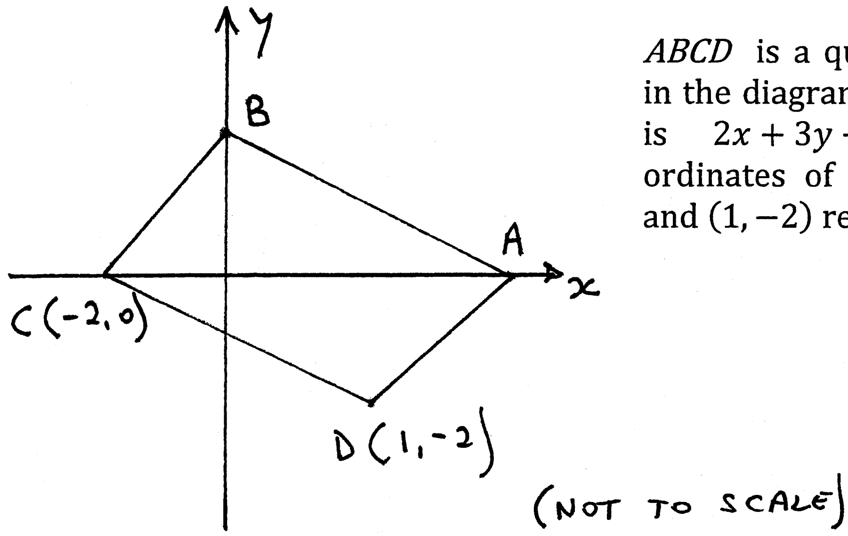
$$1 - \frac{1}{4} + \frac{1}{16} - \frac{1}{64} + \cdots$$

c) If $f(x) = x(x-1)(x+2)$ find the area bounded by $f(x)$ and the x axis between $x = -1$ and $x = 1$. 3

d) Write as a logarithm $2^x = 7$ 1

Question 3 – (12 marks) **Marks**

a)



$ABCD$ is a quadrilateral as shown in the diagram. The equation of AB is $2x + 3y - 12 = 0$. The co-ordinates of C and D are $(-2, 0)$ and $(1, -2)$ respectively.

(i) What is the size (to the nearest minute) of the acute angle between AB and the x axis. **2**

(ii) Show $AB \| CD$ **1**

(iii) Show the equation of CD is $2x + 3y + 4 = 0$ **2**

(iv) If M is the midpoint of AB, find the distance from M to CD **2**

(v) Hence, or otherwise, write down the equation of the circle centre M, having CD as a tangent. **1**

b) Find:

(i) $\int \frac{3x}{x^2-1}\, dx$ **2**

(ii) $\int_{\pi}^{2\pi} \cos\frac{x}{3}\, dx$ **2**

Question 4 – (12 marks)

Marks

a) Express $\frac{\sqrt{3}-1}{2\sqrt{3}-1}$ with a rational denominator. 2

b) If $\frac{ds^2}{dt^2} = 3t - 4$ find s, given that when $t = 0,\ s = 6$ and $\frac{ds}{dt} = 0$ 2

c) In ΔABC $\sin B = \frac{2}{3}$, $a = 4$ and $b = 7$. Find $\sin A$ as a simple fraction. 2

d) Find $\int \frac{x^2+x-1}{x^2}\,dx$ 2

e) Draw a neat sketch of $y = 3\sin 2x$ for $0^\circ \leq x \leq 360^\circ$ 2

f) Solve $3 \times 9^x - 28 \times 3^x + 9 = 0$ 2

Question 5 – (12 marks) **Marks**

a) Differentiate with respect to x: $y = \dfrac{\tan x}{x}$ **2**

b) For the curve $y = x^3 - 3x^2$

(i) Find the coordinates of any stationary points and determine their nature. **4**

(ii) Find the coordinates of any points of inflexion. **1**

(iii) Sketch the graph of this function, indicating clearly all relevant features. **3**

c) Find $\int \sqrt[3]{x} + \sin 3x \;\, dx$ **2**

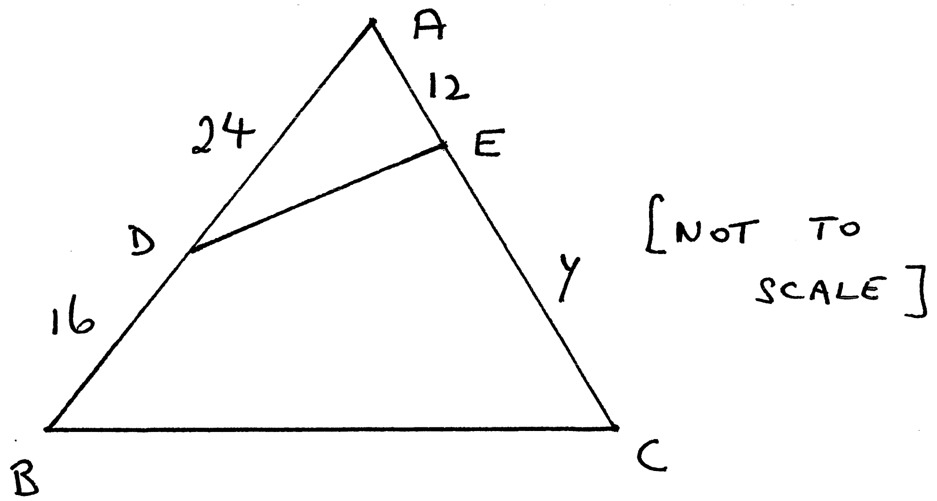
Question 6 – (12 marks)

Marks

a) Prove that 3

$$\frac{\sin^2 x}{1-\cos x}+\frac{\sin^2 x}{1+\cos x}=2$$

b)



In the diagram $\angle ADE = \angle ACB$. Prove that $\Delta AED \, ||| \, \Delta ABC$ 3

Hence find the value of y.

c) The equation of a parabola is given by $x^2 - 4x - 2y + 8 = 0$. Find the:

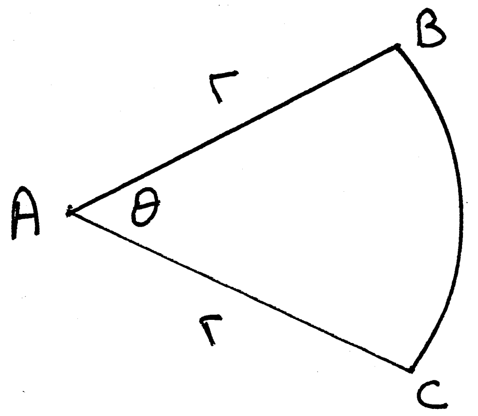
(i) coordinates of the vertex. 2

(ii) coordinates of the focus. 2

(iii) gradient of the normal to this parabola at the point $(0, 4)$ 2

Question 7 – (12 marks) **Marks**

a)



In the diagram AB and AC are radii of length r of a circle centre A. The arc BC subtends an angle of θ radians at A.

(i) Write down a formula for:

(α) the length of arc BC. 1

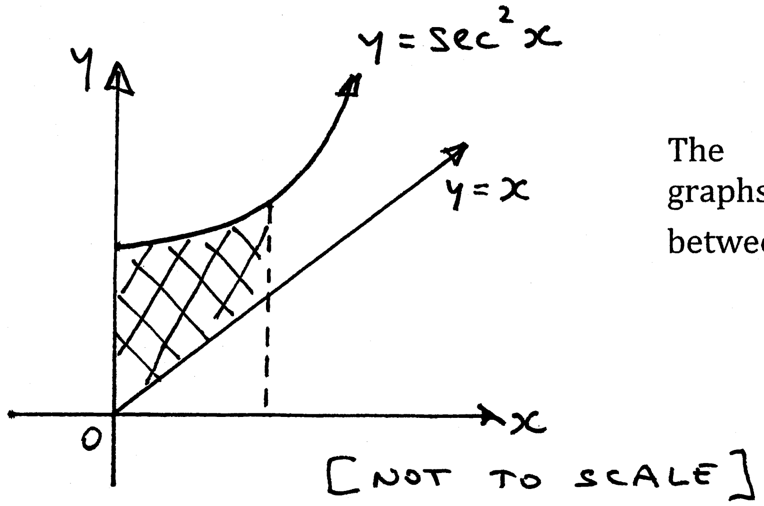
(β) the area of sector ABC. 1

(ii) The perimeter of ABC is 12m. Show that the area, y square metres, is given by 2

$$y = \frac{72\theta}{(\theta + 2)^2}$$

(iii) Show the maximum area is 9m^2. 3

b)



The diagram shows the graphs of $y = \sec^2 x$ and $y = x$ between $x = 0$ and $x = \frac{\pi}{4}$

Calculate the area of shaded region, correct to 2 decimal places. 3

c) Find the values of m for which the equation $4x^2 - mx + 9 = 0$ has real roots. 2

Question 8 – (12 marks)

Marks

a) The second term of a geometric series is $\frac{3}{8}$. and the seventh term is 12. Find the 14th term. **2**

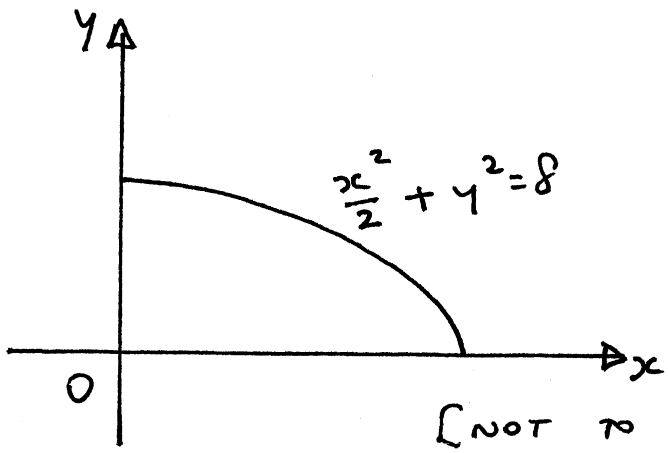
b) Let α and β be the roots of $3x^2 + 4x - 3 = 0$ evaluate

(i) $\alpha + \beta$ **1**

(ii) $\alpha\beta$ **1**

(iii) $2\alpha^2 + 2\beta^2$ **2**

c)



[NOT TO SCALE]

The part of the curve $\frac{x^2}{2} + y^2 = 8$ that lies in the first quadrant is rotated about the x axis. Find the volume of revolution. **3**

d) Grace invests \$10 000 at 6% p.a., interest calculated and added monthly, in a savings account.

(i) What is the value of her investment after 2 years? **1**

(ii) What rate of simple interest would produce the same value over the 2 years (answer to 2 decimal places). **2**

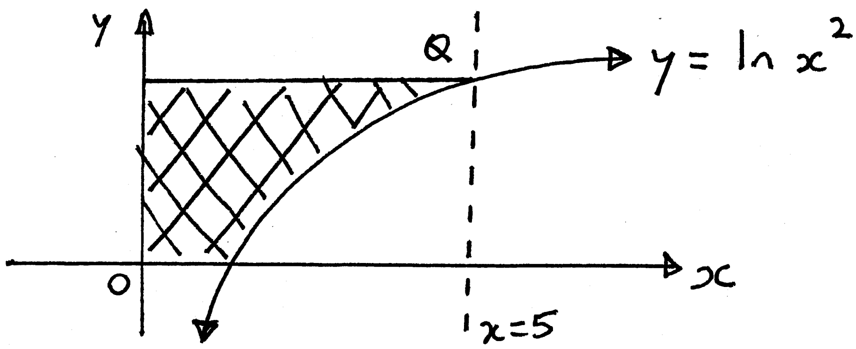
Question 9 – (12 marks) **Marks**

a) (i) Show that $\dfrac{d}{dx}(x\ln x - x) = \ln x$ 1

(ii) Using the relationship $\ln x^2 = 2\ln x$, or otherwise, find 1

$$\int \ln x^2 \, dx$$

(iii) 3



The graph shows the curve $y = \ln x^2$ which meets the line $x = 5$ at Q. Using your answers from (i) and (ii), or otherwise, find the area of the shaded region.

b) (i) Write down the domain of 1

$$y = \frac{1}{x} + \log x$$

(ii) Show that the first and second derivatives may be expressed as 2

$$\frac{dy}{dx} = \frac{x-1}{x^2} \quad \text{and} \quad \frac{d^2y}{dx^2} = \frac{2-x}{x^3}$$

(iii) Show that the curve has a minimum at $(1, 1)$ 2

c) Find k if 2

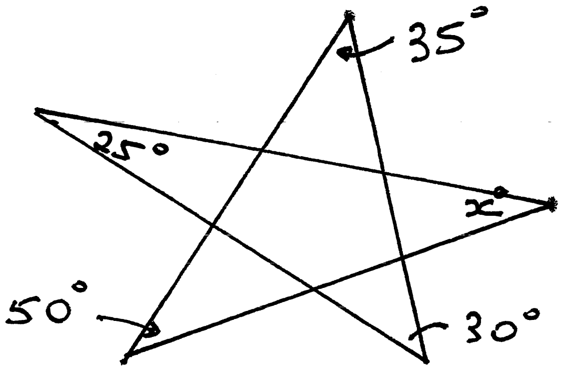
$$\int_1^k \frac{4}{x^2}\, dx = 3$$

Question 10 – (12 marks)

Marks

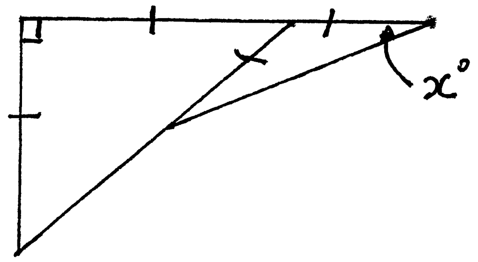
a) Find the size of x, no reason required.

(i) 1



[NOT TO SCALE]

(ii) 1



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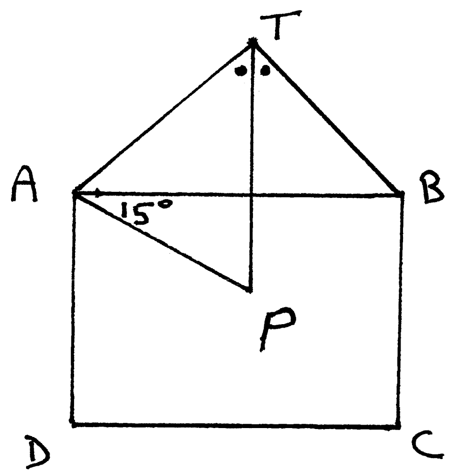
b)



[NOT TO SCALE]

In the diagram $ABCD$ is a square and ABT is an equilateral triangle. The line TP bisects $\angle ATB$ and $\angle PAB = 15°$

(i) Copy the diagram into your answer booklet and show why $\angle PAT = 75°$ 1

(ii) Prove $\Delta TAP \equiv \Delta DAP$ 3

Question 10 – (cont'd) **Marks**

c) A car company offers a loan of \$20 000 to purchase a new car for which it charges interest at 1% per month. As a special deal the company does not charge interest for the first 6 months, however, the monthly payments start at the end of the first month. Beth takes out a loan and agrees to repay the loan over 5 years by making 60 equal monthly repayments of $\$m$. Let A_n be the amount owing at the end of the n^{th} month (after a repayment and interest, if any).

(i) Find an expression for A_4 1

(ii) Show $A_8 = (20\,000 - 6m)(1.01)^2 - m(1 + 1.01)$ 1

(iii) Find an expression for A_{60} 1

(iv) Find the value of m 3

End of Paper

St George GHS Trial HSC Mathematics 2011

Question 1

(a) $5\sqrt{3} - 2\times 2\sqrt{3}$

$= \sqrt{3}$

(b) $5x^2 - 2x - 3 = 0$

$(5x+3)(x-1) = 0$

$x = -\frac{3}{5}, 1$

(c) $(y-2)(y^2+2y+4)$

(d) $\theta_{acute} = 30^\circ$

$\theta = 180^\circ - 30^\circ, 180^\circ + 30^\circ$

$= 150^\circ, 210^\circ$

(e) $y = xe^x$

$\frac{dy}{dx} = 1.e^x + x.e^x$

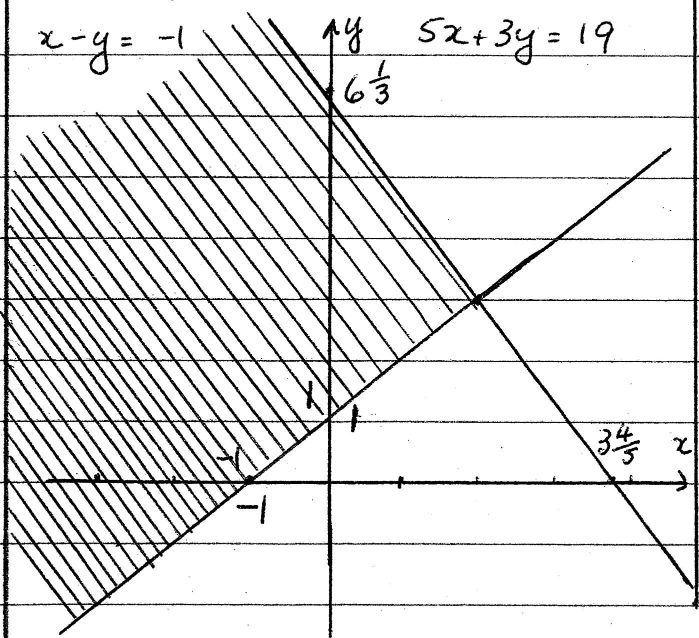
$= e^x(1+x)$

(f) $225^\circ = \frac{5\pi}{4}$ rad.

(g)

x	0	-1
y	1	0

x	0	$3\frac{4}{5}$
y	$6\frac{1}{3}$	0



Question 2

(a)(i) $y = 4x - x^2$

Cuts x axis when $y = 0$

$4x - x^2 = 0$

$x(4-x) = 0$

$x = 0, 4$

$\frac{dy}{dx} = 4 - 2x$

When $x = 0$ $\frac{dy}{dx} = 4$

$x = 4$ $\frac{dy}{dx} = 4 - 2\times 4 = -4$

Eq^n of tangent at $(0,0)$

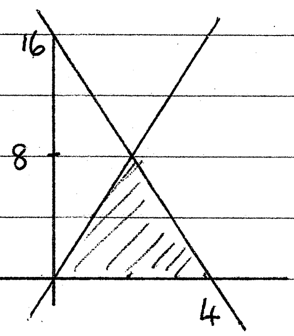
$y = 4x$

Eq^n of tangent at $(0,4)$

$y - 0 = -4(x-4)$

$y = -4x + 16$

(ii)



$y = 4x$ mee

$y = -4x + 16$

$4x = -4x + 16$

$8x = 16$

$x = 2$

$y = 8$

$A = \frac{1}{2}\times 4\times 8$

Area $= 16$ units^2

(b) $1 - \frac{1}{4} + \frac{1}{16} - \frac{1}{64} + \cdots$

Infinite geometric series

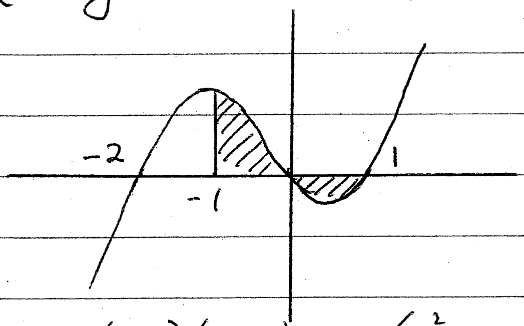
$a = 1$ $r = -\frac{1}{4}$

$S = \frac{a}{1-r}$

$= \frac{1}{1 - -\frac{1}{4}}$

$= \frac{4}{5}$

(c) $y = x(x-1)(x+2)$



$x(x-1)(x+2) = x(x^2+x-2)$
$= x^3+x^2-2x$

$A = \int_{-1}^{0} x^3+x^2-2x\,dx + \left|\int_0^1 x^3+x^2-2x\,dx\right|$

$= \left[\frac{x^4}{4}+\frac{x^3}{3}-x^2\right]_{-1}^{0} + \left|\left[\frac{x^4}{4}+\frac{x^3}{3}-x^2\right]_0^1\right|$

$= 0-\left(\frac{1}{4}-\frac{1}{3}-1\right) + \left|\left(\frac{1}{4}+\frac{1}{3}-1\right)-0\right|$

$= 1\frac{1}{12} + \left|-\frac{5}{12}\right|$

$= 1\frac{1}{2}$

Area $= 1\cdot 5$ units^2

(d) $2^x = 7$

$x = \log_2 7$

Question 3

(a) (i) AB: $2x+3y-12=0$

$3y = -2x+12$

$y = -\frac{2}{3}x+4$

Grad of AB $= -\frac{2}{3}$

Let θ be acute angle between AB and the x axis

$\tan\theta = \left|-\frac{2}{3}\right| = \frac{2}{3}$

$\theta = 33°41'$

(ii) Grad CD $= \frac{-2-0}{1--2}$

$= \frac{-2}{3}$

$=$ Grad AB

$\therefore$ AB ∥ CD

(iii) Eq^n of CD: $m=-\frac{2}{3}$ $C(-2,0)$

$y-0 = -\frac{2}{3}(x--2)$

$3y = -2x-4$

$2x+3y+4=0$

(iv) $2x+3y-12=0$

When $y=0$ $x=6$ $A(6,0)$

$x=0$ $y=4$ $B(0,4)$

M is $\left(\frac{6+0}{2}, \frac{0+4}{2}\right) = (3,2)$

$d = \frac{|2\times3+3\times2+4|}{\sqrt{2^2+3^2}}$

$= \frac{16}{\sqrt{13}}$

(v) Eq^n of circle is

$(x-3)^2+(y-2)^2 = \frac{256}{13}$

(b) $\int \frac{3x}{x^2-2}\,dx = \frac{3}{2}\int \frac{2x}{x^2-1}\,dx$

$= \frac{3}{2}\log_e(x^2-1)+c$

(c) $\int_{\pi}^{2\pi} \cos\left(\frac{x}{3}\right)dx = \left[3\sin\frac{x}{3}\right]_{\pi}^{2\pi}$

$= 3\sin\frac{2\pi}{3} - 3\sin\frac{\pi}{3}$

$= 3\left(\frac{\sqrt{3}}{2}-\frac{\sqrt{3}}{2}\right)$

$= 0$

Question 4

(a) $\frac{\sqrt{3}-1}{2\sqrt{3}-1} \times \frac{2\sqrt{3}+1}{2\sqrt{3}+1}$

$= \frac{2\times 3 - 2\sqrt{3} + \sqrt{3} - 1}{4\times 3 - 1}$

$= \frac{5-\sqrt{3}}{11}$

(b) $\frac{d^2s}{dt^2} = 3t - 4$

$\frac{ds}{dt} = \frac{3t^2}{2} - 4t + c_1$

When $t=0$ $\frac{ds}{dt} = 0$

$\therefore 0 = 0 + c_1$

$c_1 = 0$

$\therefore \frac{ds}{dt} = \frac{3t^2}{2} - 4t$

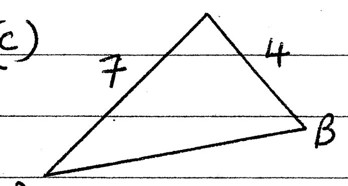
$\therefore s = \frac{t^3}{2} - 2t^2 + c_2$

When $t=0$ $s=6$

$6 = 0 + c_2$

$\therefore s = \frac{t^3}{2} - 2t^2 + 6$

(c)



$\frac{\sin A}{a} = \frac{\sin B}{b}$

$\frac{\sin A}{4} = \frac{\frac{2}{3}}{7}$

$\sin A = 4 \times \frac{2}{3} \times \frac{1}{7}$

$= \frac{8}{21}$

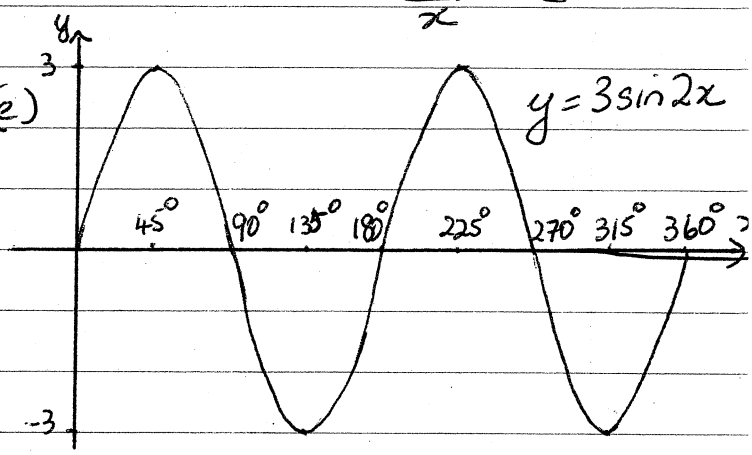
(d) $\int \frac{x^2 + x - 1}{x^2} \, dx$

$= \int 1 + \frac{1}{x} - x^{-2} \, dx$

$= x + \ln x - \frac{x^{-1}}{-1} + c$

$= x + \ln x + \frac{1}{x} + c$

(e)



(f) $3\times 9^x - 28\times 3^x + 9 = 0$

$3\times(3^x)^2 - 28\times 3^x + 9 = 0$

Let $m = 3^x$

$3m^2 - 28m + 9 = 0$

$(3m-1)(m-9) = 0$

$m = \frac{1}{3}, 9$

$3^x = 3^{-1}, 3^2$

$x = -1, 2$

Question 5

(a) $y = \frac{\tan x}{x}$

$\frac{dy}{dx} = \frac{x.\sec^2 x - \tan x \times 1}{x^2}$

$= \frac{x \sec^2 x - \tan x}{x^2}$

(b) $y = x^3 - 3x^2$

(i) $\frac{dy}{dx} = 3x^2 - 6x$

$\frac{d^2y}{dx^2} = 6x - 6$

Stationary points occur when $\frac{dy}{dx} = 0$

ie $3x^2 - 6x = 0$

$3x(x-2) = 0$

$x = 0, 2$

When $x = 0$ $y = 0$

$\frac{d^2y}{dx^2} = -6 < 0$

$\therefore (0,0)$ is a max tp

When $x = 2$ $y = 8 - 12 = -4$

$\frac{d^2y}{dx^2} = 6 > 0$

$\therefore (2,-4)$ is a min tp

(ii) Possible inflexions when

$\frac{d^2y}{dx^2} = 0$

$6x - 6 = 0$

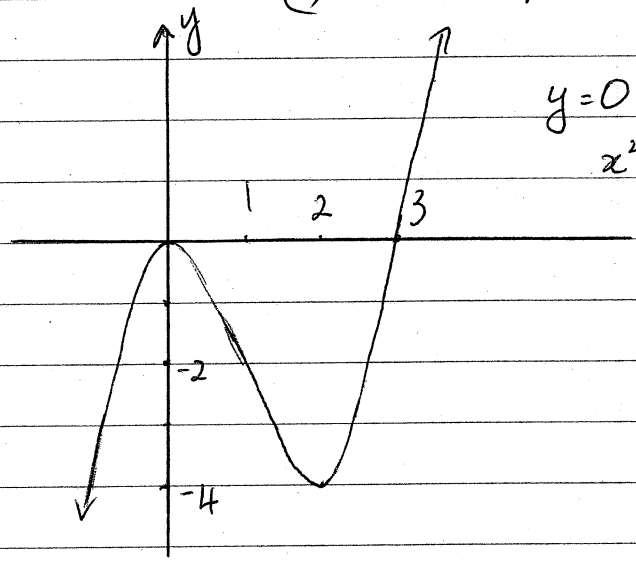
$x = 1$

$y = -2$

x	0	1	2
$\frac{d^2y}{dx^2}$	-6	0	6
	∩		∪

Change of concavity

$\therefore (1,-2)$ is a point of inflexion



$y = 0$ when

$x^2(x-3) = 0$

$x = 0, 3$

(c) $\int \sqrt[3]{x} + \sin 3x \, dx$

$= \int x^{1/3} + \sin 3x \, dx$

$= \frac{x^{4/3}}{4/3} + \frac{-\cos 3x}{3} + c$

$= \frac{3}{4}\sqrt[3]{x^4} - \frac{\cos 3x}{3} + c$

(OR $\frac{3}{4}x\sqrt[3]{x} - \frac{\cos 3x}{3} + c$)

Question 6

(a) $LHS = \frac{\sin^2 x}{1-\cos x} + \frac{\sin^2 x}{1+\cos x}$

$= \sin^2 x\left(\frac{1}{1-\cos x} + \frac{1}{1+\cos x}\right)$

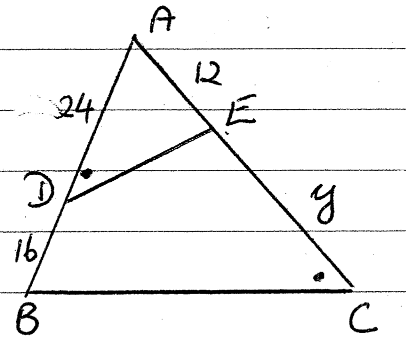
$= \sin^2 x \frac{(1+\cos x + 1 - \cos x)}{1-\cos^2 x}$

$= \sin^2 x \times \frac{2}{\sin^2 x}$

$= 2$

$= RHS$

(b)



in Δs AED, ABC

$\angle DAE$ is common

$\angle ADE = \angle ACB$ (given)

$\therefore \Delta ADE \mathbin{|||} \Delta ACB$ (equiangular)

$\frac{AD}{AC} = \frac{AE}{AB} = \frac{DE}{CB}$ (corresponding sides in similar triangles)

$\frac{24}{12+y} = \frac{12}{40}$

$\frac{2}{12+y} = \frac{1}{40}$

$12+y = 80$

$y = 68$

(c) $x^2 - 4x - 2y + 8 = 0$

$x^2 - 4x = 2y - 8$

$x^2 - 4x + 4 = 2y - 8 + 4$

$(x-2)^2 = 2y - 4$

$= 2(y-2)$

$= 4 \times \frac{1}{2}(y-2)$

(i) Vertex is $(2,2)$

(ii) Focal length $= \frac{1}{2}$

Focus is $(2, 2\frac{1}{2})$ or $(2, \frac{5}{2})$

(iii) $2y = x^2 - 4x + 8$

$y = \frac{1}{2}x^2 - 2x + 4$

$\frac{dy}{dx} = x - 2$

When $x = 0$ $\frac{dy}{dx} = -2$

$\therefore$ Grad of normal at $(0,4)$ is $\frac{1}{2}$

Question 7

(a) (i) (α) $l = r\theta$

(β) $A = \frac{1}{2}r^2\theta$

(ii) Perimeter $= 2r + r\theta$

$$2r + r\theta = 12$$
$$r(\theta+2) = 12$$
$$r = \frac{12}{\theta+2}$$
$$\therefore A = \frac{1}{2}\times\left(\frac{12}{\theta+2}\right)^2\times\theta$$
$$= \frac{72\theta}{(\theta+2)^2}$$

(iii) $$\frac{dA}{d\theta} = \frac{72(\theta+2)^2 - 72\theta . 2(\theta+2)}{(\theta+2)^4}$$
$$= \frac{72(\theta+2)(\theta+2-2\theta)}{(\theta+2)^4}$$
$$= \frac{72(2-\theta)}{(\theta+2)^3}$$

Stationary points occur when $\frac{dA}{d\theta} = 0$

$$\frac{72(2-\theta)}{(\theta+2)^3} = 0$$
$$\theta = 2$$

θ	1	2	3
$\frac{dA}{d\theta}$	$\frac{72}{27}$	0	$\frac{-72}{125}$

+ / ‾ \ -

$\therefore$ Max tp when $\theta = 2$

When $\theta = 2$ $A = \frac{72\times2}{(2+2)^2} = 9$

$\therefore$ Max area $= 9m^2$

(b) $$A = \int_0^{\frac{\pi}{4}} (\sec^2 x - x)\,dx$$
$$= \left[\tan x - \frac{x^2}{2}\right]_0^{\pi/4}$$
$$= \left(\tan\frac{\pi}{4} - \frac{1}{2}\cdot\frac{\pi^2}{16}\right) - (0-0)$$
$$= 1 - \frac{\pi^2}{32}$$

Area $= 0.69157\ldots$

$= 0.69$ unit^2 (2dp)

(c) $4x^2 - mx + 9 = 0$

has real roots when $\Delta \geq 0$

$$\Delta = (-m)^2 - 4\times4\times9$$
$$= m^2 - 144$$

$\therefore$ Real roots when

$(m-12)(m+12) \geq 0$

$m \leq -12$ or $m \geq 12$

Question 8

(a) Let a be 1st term and r the common ratio

$t_2 = ar = \frac{3}{8}$ ①

$t_7 = ar^6 = 12$ ②

② ÷ ① $\frac{ar^6}{ar} = 12 \div \frac{3}{8}$

$r^5 = 32$

$r = 2$

$a \times 2 = \frac{3}{8}$

$a = \frac{3}{16}$

$\therefore t_{14} = ar^{13}$

$= \frac{3}{16} \times 2^{13}$

$= 3 \times 2^9$

$= 1536$

(b) $3x^2 + 4x - 3 = 0$

(i) $\alpha + \beta = -\frac{4}{3}$

(ii) $\alpha\beta = -\frac{3}{3}$

$= -1$

(iii) $2(\alpha^2 + \beta^2) = 2\left[(\alpha+\beta)^2 - 2\alpha\beta\right]$

$= 2\left[\left(\frac{-4}{3}\right)^2 - 2 \times -1\right]$

$= 2\left[\frac{16}{9} + 2\right]$

$= 7\frac{5}{9} \left(= \frac{68}{9}\right)$

(c) $\frac{x^2}{2} + y^2 = 8 \quad x \geq 0$

When $y = 0$ $\frac{x^2}{2} = 8$

$x = 4 \ (x \geq 0)$

$V = \pi \int_0^4 y^2 \, dx$

$= \pi \int_0^4 8 - \frac{x^2}{2} \, dx$

$= \pi \left[8x - \frac{x^3}{6}\right]_0^4$

$= \pi\left[\left(32 - \frac{64}{6}\right) - (0 - 0)\right]$

Volume $= \frac{64\pi}{3}$ units^3

(d) Interest rate = 6% pa = 0.5% per month

(i) $V = 10000 \times 1.005^{24}$

$= 11271.59776\ldots$

Value is \$11271.60 (nearest cent)

(ii) Interest for 2yrs = \$1271.60

Interest for 1yr = \$635.80

Simple Interest rate $= \frac{635.8}{10000} \times 100\%$

$= 6.358\%$ p.a

$\approx 6.36\%$ pa

Question 9

(a)(i) $\frac{d}{dx}(x\ln x - x)$

$= 1.\ln x + x \times \frac{1}{x} - 1$

$= \ln x + 1 - 1$

$= \ln x$

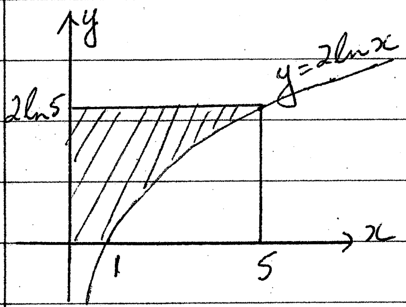
(ii) $\int \ln x^2 \, dx = 2\int \ln x \, dx$

$= 2(x\ln x - x) + c$

$= 2x\ln x - 2x + c$

(iii) When $x = 5$ $y = \ln 5^2$

$= 2\ln 5$



Shaded area

$= \text{Area of rectangle} - \int_1^5 2\ln x \, dx$

$= 5 \times 2\ln 5 - 2[x\ln x - x]_1^5$

$= 10\ln 5 - 2\{(5\ln 5 - 5) - (\ln 1 - 1)\}$

$= 10\ln 5 - 10\ln 5 + 10 - 2$

$= 8$

OR (Otherwise)

$y = \ln x^2$

$x^2 = e^y$

$x = e^{y/2}$

$A = \int_0^{2\ln 5} e^{y/2} dy$

$= 2\left[e^{y/2}\right]_0^{2\ln 5}$

$= 2(e^{\ln 5} - e^0)$

$= 2(5 - 1) = 8$

(b)(i) $y = \frac{1}{x} + \log x$

Domain: $x > 0$

(ii) $y = x^{-1} + \log x$

$\frac{dy}{dx} = -1x^{-2} + \frac{1}{x}$

$= -\frac{1}{x^2} + \frac{1}{x}$

$= \frac{-1 + x}{x^2} = \frac{x - 1}{x^2}$

$\frac{d^2y}{dx^2} = 2x^{-3} - \frac{1}{x^2}$

$= \frac{2}{x^3} - \frac{1}{x^2}$

$= \frac{2 - x}{x^3}$

(iii) Stationary points occur when $\frac{dy}{dx} = 0$

$\frac{x - 1}{x^2} = 0$

$x = 1$

$y = \frac{1}{1} + \log 1$

$= 1$

When $x = 1$ $\frac{d^2y}{dx^2} = \frac{2-1}{1^3} = 1 > 0$

$\therefore (1,1)$ is a maximum t.p.

(c) $\int_1^k \frac{4}{x^2} dx = \int_1^k 4x^{-2} dx$

$= \left[\frac{4x^{-1}}{-1}\right]_1^k$

$= \left[-\frac{4}{x}\right]_1^k$

$\therefore$ 3 $= -\frac{4}{k} - -4$

$= 4 - \frac{4}{k}$

$\frac{4}{k} = 1$

$k = 4$

Question 10

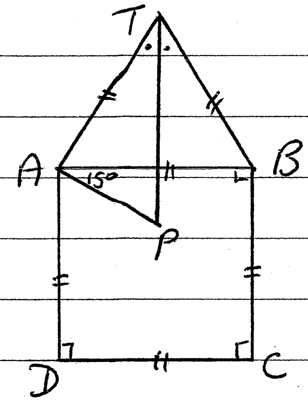
(a) (i) $x = 40$

(Use exterior $\angle$ of triangles, angle sum of Δ)

(ii) $x = 22.5$

(use equal $\angle$s in isosceles Δ and exterior $\angle$ of Δ)

(b)



(i) $\angle TAB = 60°$ ($\angle$ in equilateral Δ)

$\angle PAT = 60° + 15° = 75°$

(ii) $\angle BAD = 90°$ (angle in a square)

$\therefore \angle DAP = 90° - 15°$

$= 75°$

In Δs TAP, DAP

$AT = AB$ (equal sides in equilat Δ)

$AD = AB$ (equal sides of square)

$\therefore AT = AD \; (= AB)$

$\angle TAP = \angle DAP \; (= 75°)$

AP is common

$\therefore \Delta TAP \equiv \Delta DAP$ (SAS)

(c) (i) $A_1 = 20000 - m$

$A_2 = 20000 - m - m$

$= 20000 - 2m$

$A_4 = 20000 - 4m$

(ii) $A_6 = 20000 - 6m$

$A_7 = A_6 + \text{Interest} - m$

$= A_6 \times 1.01 - m$

$= (20000 - 6m) \times 1.01 - m$

$A_8 = A_7 \times 1.01 - m$

$= (20000 - 6m) \times 1.01^2 - m \times 1.01 - m$

$= (20000 - 6m) \times 1.01^2 - m(1 + 1.01)$

By the same pattern

$A_n = (20000 - 6m) \times 1.01^{n-6} - m(1 + 1.01 + \ldots + 1.01^{n-7})$

$\therefore A_{60} = (20000 - 6m) \times 1.01^{54} - m(1 + 1.01 + \ldots + 1.01^{53})$

$= (20000 - 6m) \times 1.01^{54} - m \cdot \dfrac{(1.01^{54} - 1)}{1.01 - 1}$

$= (20000 - 6m) \times 1.01^{54} - \dfrac{m(1.01^{54} - 1)}{0.01}$

Since loan is repaid after 5yrs

$A_{60} = 0$

$\therefore 0 = 20000 \times 1.01^{54} - 6m \times 1.01^{54} - 100m(1.01^{54} - 1)$

$6m \times 1.01^{54} + 100m(1.01^{54} - 1) = 20000 \times 1.01^{54}$

$m\left[6 \times 1.01^{54} + 100 \times 1.01^{54} - 100\right] = 20000 \times 1.01^{54}$

$m = \dfrac{20000 \times 1.01^{54}}{106 \times 1.01^{54} - 100}$

$= 420.4448\ldots$

$m = 420.44$ (2dp)