Name: ______________________________

St George Girls High School

Trial Higher School Certificate Examination

2014

Mathematics

General Instructions

- Reading time – 5 minutes
- Working time – 3 hours
- Write using blue or black pen.
- Begin each question in a new booklet
- Write your student number on each booklet.
- Board-approved calculators may be used.
- A table of standard integrals is provided at the back of this paper.
- Show all necessary working in Questions 11 – 16.
- Diagrams are not to scale.
- The mark allocated for each question is listed at the side of the question.

Total Marks – 100

Section I – Pages 2 – 4
10 marks

- Attempt Questions 1 – 10 using the answer sheet provided at the end of the paper
- Allow about 15 minutes for this section

Section II – Pages 5 – 12
90 marks

- Attempt Questions 11 – 16
- Allow about 2 hours 45 minutes for this section

Students are advised that this is a Trial Examination only and does not necessarily reflect the content or format of the Higher School Certificate Examination.

Section I

10 marks
Attempt Questions 1 to 10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1–10.

1. The angle of inclination of the straight line $x + 3y - 5 = 0$ to the positive direction of the x-axis is closest to:

   (A) 18°
   (B) 162°
   (C) 72°
   (D) 108°

2. The ratio of the lengths of the corresponding edges of two similar pyramids is 2: 3. If the volume of the larger pyramid is 243 cm^3, the volume of the smaller is:

   (A) 81 cm^3
   (B) 162 cm^3
   (C) 108 cm^3
   (D) 72 cm^3

3. $y = f(x)$ is an odd function. The value of $\int_{-a}^{a} f(x)dx$ is:

   (A) $f(a)$
   (B) $2\int_{0}^{a} f(x)dx$
   (C) 0
   (D) a

4. The x-coordinates of the two stationary points on the curve $y = 2x^3 - 3x^2 - 12x + 18$ are:

   (A) $x = -1,\ x = 2$
   (B) $x = 1,\ \ x = -2$
   (C) $x = 6,\ \ x = \frac{3}{2}$
   (D) $x = -6,\ x = -\frac{3}{2}$

Section I (cont'd) Marks

5. The area enclosed between the curves $y = 4 - x^2$ and $y = 4 - 2x$ is:

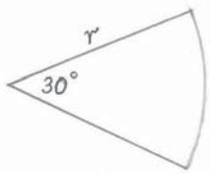
(A) $\frac{16}{3}$ square units

(B) $-\frac{16}{3}$ square units

(C) $\frac{4}{3}$ square units

(D) $\frac{2}{3}$ square units

6. The sector below has an area of 5π cm².



The value of r is:

(A) $\sqrt{30}\,\pi$ cm

(B) $2\sqrt{15}$ cm

(C) $\sqrt{\frac{\pi}{6}}$ cm

(D) $\frac{1}{\sqrt{6}}$ cm

7. The equation $8^x = 32$ can be rewritten as:

(A) $x = \log_8 5$

(B) $32 = \log_8 x$

(C) $8 = \log_x 32$

(D) $3x = \log_2 32$

Section I (cont'd) Marks

8. The derivative of e^{x^3} is:

(A) $x^2e^{x^3}(3 + x)$

(B) e^{x^3}

(C) $3x^2e^{x^3}$

(D) $\frac{e^{x^3}}{3x^2}$

9. A particle moves along a straight horizontal line with acceleration of $(2t - 1)$ m/s². Initially it is 3 metres to the right of the origin, moving with velocity of -2 m/s. The position of the particle after 3 seconds is:

(A) 1.5 metres to the right of the origin

(B) 1.5 metres to the left of the origin

(C) 11.5 metres to the right of the origin

(D) 11.5 metres to the left of the origin

10. After an Electrical Engineering course at the UNSW, Matilda starts on a salary of \$65 000 with annual increments of 2.75%, so her consecutive salaries are:

$65 000, $65 000 × 1.0275, $65 000 × 1.0275², . . .

What is total amount (to the nearest dollar) Matilda would earn during first 6 complete years of her employment?

(A) \$390 041

(B) \$417 816

(C) \$419 010

(D) \$479 238

End of Section I

Section II

90 Marks
Attempt Questions 11 – 16
Allow about 2 hours and 45 minutes for this section

Answer each question in the appropriate writing booklet. Extra writing booklets are available.

In Questions 11 – 16, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) Use a SEPARATE writing booklet **Marks**

a) If $t = 0.23$, evaluate $\frac{1-t^2}{1+t^2}$, correct to three significant figures. 1

b) Evaluate $\ln 5$, correct to two decimal places. 1

c) Convert $\frac{5\pi}{7}$ to degrees and minutes. 1

d) Find $\int \left(\frac{1}{\sqrt{x}} - 3\right) dx$. 2

e) Solve $|2x - 1| \leq 7$. 2

f) Differentiate $\frac{x^3}{e^x}$ and simplify the result fully. 2

g) Evaluate $\int_1^3 \frac{-x}{1+x^2}\, dx$. 3

h) Sketch the region defined by 3

$$(x+3)^2 + (y-3)^2 > 9 .$$

Question 12 (15 marks) Use a SEPARATE writing booklet **Marks**

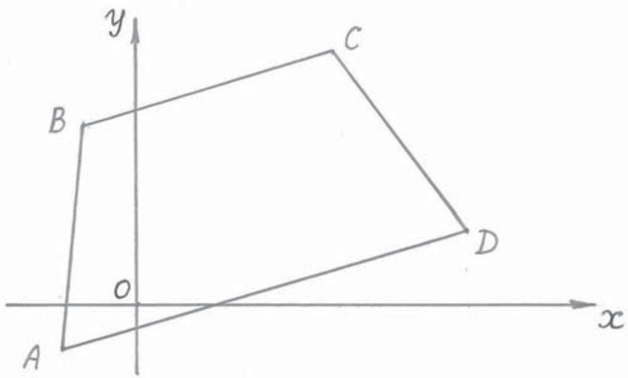
a) Differentiate

(i) $\tan 3x$. 1

(ii) $\log\left(\frac{x^2+5}{x-3}\right)$. 2

(iii) $x\, e^{\sin x}$. 2

b) The points $A(-3,-3)$, $B(-2,12)$, $C(8,17)$ and $D(13,5)$ form a trapezium, as shown on the diagram below (NOT TO SCALE).



(i) Show that the equation of the line through A and D is $x - 2y - 3 = 0$. 2

(ii) Show that the perpendicular distance from B to the line AD is $\frac{29}{\sqrt{5}}$ units. 2

(iii) Find the length of BC in exact form. 1

(iv) Show that $BC \parallel AD$. 1

(v) Point P (not shown) lies on the interval AD so that $AB \parallel PC$.
Find the coordinates of the point P. 2

(vi) What type of a quadrilateral is $ABCP$? 1

(vii) Hence find the area of $ABCP$. 1

Question 13 (15 marks) Use a SEPARATE writing booklet **Marks**

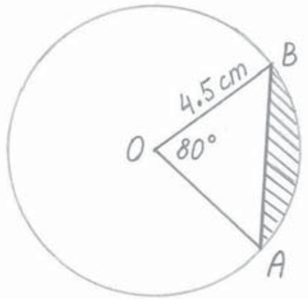
a) Consider the geometric series $2 + \frac{2}{3} + \frac{2}{9} + \frac{2}{27} + \cdots$

(i) Explain why the series has a limiting sum. 1

(ii) Find the limiting sum. 1

b) Find the equation of the tangent to the curve $y = \ln x$ at the point $(e^2, 2)$. 2

c)



In the diagram above O is the centre of the circle with radius 4.5 cm. The chord AB subtends an angle of $80°$ at the centre of the circle.

(i) Convert $80°$ into radians in terms of π. 1

(ii) Find the exact length of the minor arc AB. 1

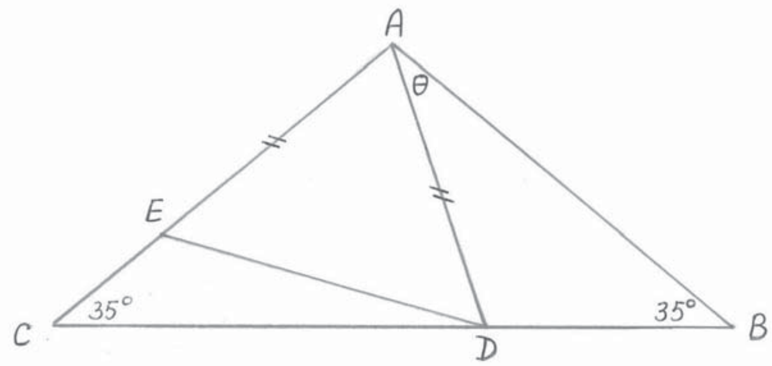
(iii) Evaluate the area of the minor segment that has been shaded, correct to three decimal places. 2

Question 13 (cont'd) **Marks**

d) The 7th term of an arithmetic sequence is 11 and the 21st term is 53. 2

Find the value of the common difference and the value of the first term of the sequence.

e)



In the diagram above ΔABC is isosceles with $\angle ABC = \angle ACB = 35°$, $AD = AE$ and $\angle DAB = \theta$.

(i) Explain why $\angle ADC = 35° + \theta$. 1

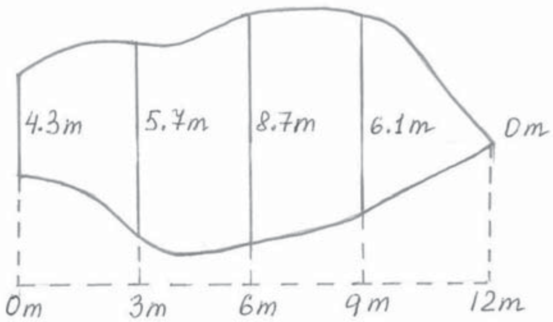
(ii) Find the expression for $\angle CAD$ in terms of θ. 1

(iii) Show that $\angle EDC = \frac{1}{2}\theta$. 3

Question 14 (15 marks) Use a SEPARATE writing booklet **Marks**

a) Solve $(\cos x - 3)(2\cos x - 1) = 0$ in the domain $0 \le x \le 2\pi$. 3

b)



The diagram above shows the shape and dimensions of one of the ponds of a fishing farm. 3

Use the Simpson's Rule with all five given values to estimate the surface area of the pond to the nearest square.

c) A ball is thrown vertically up from the edge of a cliff 100 m above the valley floor. The initial velocity of the ball is 30 m/s. Consider the valley floor as the origin of space and the direction up as positive. The acceleration is always -10 m/s^2.

(i) Show that the velocity is $v = 30 - 10t$. 1

(ii) Show that the displacement is $x = -5t^2 + 30t + 100$. 1

(iii) After how many seconds the ball is stationary? 1

(iv) How high is the ball above the valley floor at that instance? 1

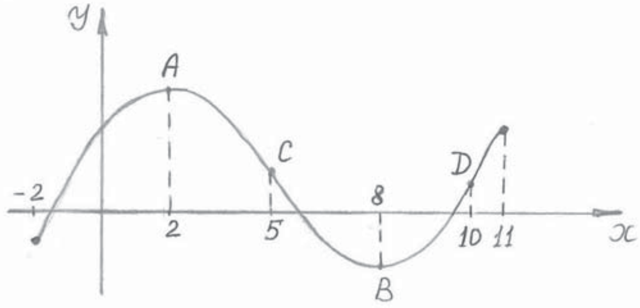
(v) When is the ball at the cliff level again? 2

(vi) Show that the ball will hit the valley floor when $t = 3 + \sqrt{29}$ seconds. 2

(vii) Find the speed of the impact, correct to the nearest metre per second. 1

Question 15 (15 marks) Use a SEPARATE writing booklet **Marks**

a)



The diagram above shows the graph of a function $y = f(x)$ over the domain $-2 \le x \le 11$. A and B are its two stationary points and C and D are its two points of inflection.

State all the intervals of x for which:

(i) $f'(x) > 0$. 1

(ii) $f''(x) > 0$. 1

(iii) $f'(x) \times f''(x) > 0$. 2

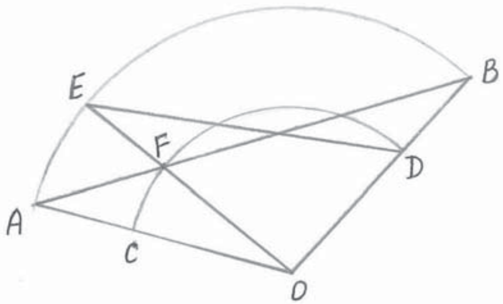
b) A particle moves in a straight line such that at time t seconds its distance x metres from a fixed point O on the line is given by $x = 2 + \cos 3t$.

(i) What is the period of the motion? 1

(ii) What is the furthest distance of the particle from point O? 1

Question 15 (cont'd) **Marks**

c)



In the diagram above O is the centre of two circular arcs AEB and CFD.

(i) Copy the diagram into your booklet. 1

(ii) Show that ΔOBF and ΔOED are congruent. 3

(iii) Let $OC = a$ and $CA = b$. Show that ΔOCD and ΔOAB are similar and state the ratio of corresponding sides. 3

(iv) The ratio of the area of ΔOCD to the area of ΔOAB is $49 : 144$. 2

Find the ratio of a to b.

Question 16 (15 marks) Use a SEPARATE writing booklet **Marks**

a) Find the stationary points on the curve $y = x^3(2 - x)$ and determine their nature. 4

b) At the beginning of every month, starting on the 1st of January 2015, Grace plans to deposit \$1500 into a superannuation account, paying 6% interest per annum, compounded monthly.

(i) Express the monthly interest rate as a decimal. 1

(ii) Show that at the end of n months the total amount, A_n, on her superannuation account can be expressed as 3

$$A_n = 301\,500\,(1.005^n - 1)\,.$$

(iii) If Grace continues with her superannuation scheme, what would be the total amount on her account on the 31st of December 2030? 2

(iv) Grace has estimated that she would be able to retire after the total amount on her superannuation account reaches \$400 000. In this case, what would be the date of the first day of her retirement? 3

(v) After a serious consideration Grace decided that she would retire on the 31st of December 2025 with the total of \$400 000, paying larger monthly instalments. Calculate her monthly instalments, correct to the nearest cent. 2

End of Paper

ST George Girls High School

Trial HSC 2014 SOLUTIONS

Section 1

1. $y = -\frac{1}{3}x + 5$

$\tan\theta = -\frac{1}{3}$

$\theta = 162^\circ$ (B)

2. volume ratio

$2^3 : 3^3$

$8 : 27$

$\frac{8}{27} = \frac{x}{243}$

$x = 72\,cm^3$ (D)

3. If $f(x)$ is odd

$\int_{-a}^{a} f(x)\,dx = 0$ (C)

4. $y = 2x^3 - 3x^2 - 12x + 18$

$\frac{dy}{dx} = 6x^2 - 6x - 12$

$= 6(x^2 - x - 2)$

$= 6(x-2)(x+1)$ (A)

$\frac{dy}{dx} = 0$, when $x = 2, x = -1$

5. To find x coordinates of the points of intersection.

$4 - x^2 = 4 - 2x$

$-x^2 + 2x = 0$

$-x(x-2) = 0$

$x = 0$, $x = 2$

Area $= \int_0^2 (4 - x^2) - (4 - 2x)\,dx$

$= \int_0^2 4 - x^2 - 4 + 2x\,dx$

$= \left[-\frac{x^3}{3} + x^2\right]_0^2$ (C)

$= -\frac{8}{3} + 4 = \frac{4}{3}\,u^2$

6. $\frac{1}{2} \times \frac{\pi}{6} \times r^2 = 5\pi$

$r^2 = 12 \times 5$ (B)

$r = \sqrt{60}$

$r = 2\sqrt{15}$

7. $8^x = 32$

$2^{3x} = 2^5$ (D)

$\log_2 2^{3x} = \log_2 2^5$

$3x = \log_2 32$

8. $\frac{d}{dx} e^{x^3} = 3x^2 \cdot e^{x^3}$ (C)

9. $a = (2t-1)\,m/s^2$

$v = \frac{2t^2}{2} - t + c$

when $t = 0$, $v = -2$ (given)

$-2 = 0^2 - 0 - c$

$c = -2$

$\therefore v = t^2 - t - 2$.

$s = \frac{t^3}{3} - \frac{t^2}{2} - 2t + c$

when $t = 0$, $s = 3$ (given)

$\therefore s = \frac{t^3}{3} - \frac{t^2}{2} - 2t + 3$

when $t = 3$,

$s = 9 - \frac{9}{2} - 6 + 3$ (A)

$= 6 - \frac{9}{2}$

$= 1.5\,m$ to the right of origin.

10. $S_n = \frac{a(r^n - 1)}{r - 1}$

$= \frac{65000(1.0275^6 - 1)}{0.0275}$ (B)

$= \$417816$

Section II

Question 11

(a) $\frac{1 - 0.23^2}{1 + 0.23^2} = 0.89951\ldots$

$= 0.900$ (3 sig. fig)

(b) $\ln 5 = 1.61$ (2 d.p)

(c) $\frac{5\pi}{7} = 128^\circ 34' 17.1''$

$= 128^\circ 34'$ (nearest minute)

(d) $\int \frac{1}{\sqrt{x}} - 3\,dx$

$= \int \frac{1}{x^{1/2}} - 3\,dx$

$= \int x^{-1/2} - 3\,dx$

$= \frac{x^{1/2}}{1/2} - 3x + C$

$= 2\sqrt{x} - 3x + C$

(e) $2x - 1 \le 7$ | $-(2x-1) \le 7$

$2x \le 8$ | $-2x + 1 \le 7$

$x \le 4$ | $-2x \le 6$

$x \ge -3$

$-3 \le x \le 4$

(f) $y = x^3 \cdot e^{-x}$

$y' = u'v + uv'$

$= 3x^2 e^{-x} + x^3 \cdot -e^{-x}$

$= 3x^2 e^{-x} - x^3 e^{-x}$

$= x^2 e^{-x}(3 - x)$

$u = x^3$, $u' = 3x^2$, $v = e^{-x}$, $v' = -e^{-x}$

Question 11

(g) $\int_1^3 \frac{-x}{1+x^2}\,dx$

$= -\frac{1}{2}\int_1^3 \frac{2x}{1+x^2}\,dx$.

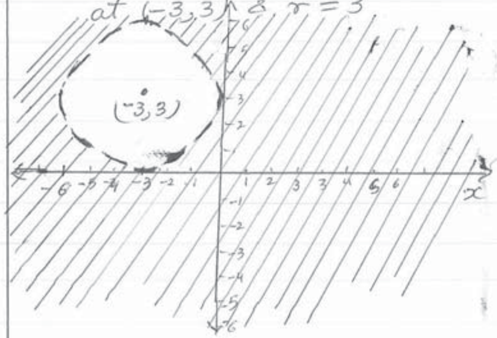
$= -\frac{1}{2}\left[\log_e(1+x^2)\right]_1^3$

$= -\frac{1}{2}\left[\log_e 10 - \log_e 2\right]$

$= -\frac{1}{2}\log_e 5$

(h) $(x+3)^2 + (y-3)^2 > 9$

the centre of the circle is at $(-3, 3)$ & $r = 3$



Question 12

(a) (i) $y = \tan 3x$

$\frac{dy}{dx} = 3\sec^2(3x)$

(ii) $y = \log\left(\frac{x^2+5}{x-3}\right)$

$y = \log(x^2+5) - \log(x-3)$

$\frac{dy}{dx} = \frac{2x}{x^2+5} - \frac{1}{x-3}$

(iii) $y = x \cdot e^{\sin x}$

$u = x$, $u' = 1$

$v = e^{\sin x}$, $v' = \cos x\, e^{\sin x}$

$y' = u'v + uv'$

$y' = e^{\sin x} + x\cos x\, e^{\sin x}$

(b)(i) $(y - y_1) = m(x - x_1)$ $\quad m = \frac{8}{16} = \frac{1}{2}$

$(y-5) = \frac{1}{2}(x-13)$

$2y - 10 = x - 13$

$x - 2y - 3 = 0$

(ii) for line $x - 2y - 3 = 0$ and point $B(-2, 12)$ (x_1, y_1).

$p = \frac{|ax_1 + by_1 + c|}{\sqrt{a^2+b^2}}$

$= \frac{|1\times -2 + -2\times 12 + -3|}{\sqrt{1+4}}$

$= \frac{29}{\sqrt{5}}$

(iii) $BC^2 = (8 - -2)^2 + (17-12)^2$

$= 100 + 25$

$BC = \sqrt{125}$

$= 5\sqrt{5}$ units.

(iv) gradient $(BC) = \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{5}{10}$

$= \frac{1}{2}$

gradient $(AD) = \frac{1}{2}$ [from Part (i)]

$m(BC) = m(AD) = \frac{1}{2}$

$\therefore BC \parallel AD$

(v) B to A translation is 15 down and 1 left. Translating C 15 down and 1 left will give us the point $P(7, 2)$

(vi) $S(AB) = \sqrt{15^2 + 1^2}$

$= \sqrt{226}$

$\therefore S(AB) \neq S(BC)$

$\therefore ABCP$ is not a rhombus. hence $ABCP$ is a parallelogram.

(vii) Area $ABCP$ = base × height

$= 5\sqrt{5} \times \frac{29}{\sqrt{5}}$

$= 145$ unit^2

Question 13

(a) (i) $2 + \frac{2}{3} + \frac{2}{9} + \frac{2}{27}$

$r = \frac{2/3}{2} = \frac{2/9}{2/3}$

$\frac{2}{6} = \frac{2}{9_3} \times \frac{3}{2}$

$\frac{1}{3} = \frac{1}{3}$

as $-1 < r\,(\frac{1}{3}) < 1$, the series will have a limiting sum.

(ii) $S_\infty = \frac{a}{1-r}$

$= \frac{2}{1 - 1/3}$

$= \frac{2}{2/3}$

$= 3$

(b) $y = \ln x$

$\frac{dy}{dx} = \frac{1}{x}$

at point $(e^2, 2)$ when $x = e^2$, $\frac{dy}{dx} = \frac{1}{e^2}$

equation of tangent.

$y - y_1 = m(x - x_1)$

$y - 2 = \frac{1}{e^2}(x - e^2)$

$y - 2 = \frac{x}{e^2} - 1$

$y = \frac{x}{e^2} + 1$ [OR] $x - e^2 y + e^2 = 0$

(c) (i) $180° = \pi$ radians

$80° = \frac{\pi}{180} \times 80$

$= \frac{4\pi}{9}$ radians.

(ii) Arc length $= r\theta$

$= 4.5 \times \frac{4\pi}{9}$

$= 2\pi$ cm.

(iii) Area $= \frac{1}{2}r^2(\theta - \sin\theta)$

$= \frac{1}{2} \times 4.5^2 \left(\frac{4\pi}{9} - \sin\frac{4\pi}{9}\right)$

$\doteqdot 10.125\,(1.39626... - 0.984807)$

$\doteqdot 4.166$ cm^2 (3 d.p)

(d) 7^{th} term $= 11$

21^{st} term $= 53$

$d = \frac{53 - 11}{21 - 7}$

$= \frac{42}{14}$

$= 3$.

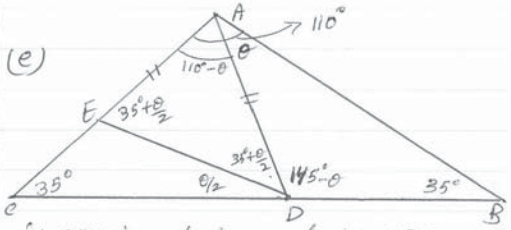
$T_n = a + (n-1)d$

$11 = a + (7-1)\times 3$

$11 = a + 18$

$a = -7$

(e)



(i) ADC is exterior angle of $\triangle ADB$.

$\therefore \angle ADC = \angle DAB + \angle DBA$

$= \theta + 35°$

(The exterior angle is equal to the sum of the two interior opposite angles)

(ii) $\angle CAD = 180 - 35° - (\theta + 35°)$ ($\angle ACD$; $\angle ADC$)

$= 110° - \theta$

(iii) $\angle ADE = \frac{180 - (110 - \theta)}{2}$ (base angle isosceles triangle)

$= \frac{70° + \theta}{2}$

$= 35° + \frac{\theta}{2}$

$\angle ADB = 180° - 35° - \theta$

$= 145° - \theta$.

$\therefore \angle EDC = 180° - \left(\frac{70° + \theta}{2}\right) - (145° - \theta)$

$= 180° - 35° - \frac{\theta}{2} - 145° + \theta$

$= \frac{\theta}{2}$

[OR]

$E\hat{D}C = A\hat{D}C - A\hat{D}E$

$= 35° + \theta - (35° - \frac{\theta}{2})$

$= 35° + \theta - 35° + \frac{\theta}{2}$

$= \frac{\theta}{2}$

Question 14

(a) $(\cos x - 3)(2\cos x - 1) = 0$

either $(\cos x - 3) = 0$ or $2\cos x - 1 = 0$

when $\cos x - 3 = 0$	when $2\cos x - 1 = 0$
$\cos x = 3$	$\cos x = \frac{1}{2}$
no valid solution	$x = 60°$ and
as $\cos x \neq 3$	$x = 300°$

(b) Area $\doteqdot \frac{b-a}{6}[(f_f + 4f_m + f_l) + (f_f + 4f_m + f_l)]$

$\doteqdot \frac{6}{6}[(4.3 + 4\times 5.7 + 8.7) + (8.7 + 4\times 6.1 + 0)]$

$\doteqdot 68.9\ m^2$

$\doteqdot 69\ m^2$

(c) $a = -10 m/s^2$

(i) $v = -10t + C$

$= -10t + 30$

$= 30 - 10t$ (when $t = 0$, $v = 30 m/s$)

(ii) $x = -\frac{10t^2}{2} + 30t + C$

$x = -5t^2 + 30t + 100$ (when $t = 0$, $x = 100 m$)

(iii) for the ball to be stationary

$v = 0$

$-10t + 30 = 0$

$t = 3$ seconds.

(iv) when $t = 3$

$x = -5\times 9 + 30\times 3 + 100$

$= -45 + 90 + 100$

$= 145 m$ above the valley floor.

(v) when the ball is at cliff level again, $x = 100 m$.

$-5t^2 + 30t + 100 = 100$

$-5t^2 + 30t = 0$

$-t^2 + 6t = 0$

$(t)(-5t + 6) = 0$

$t = 0$ and $t = 6$

initial position | after 6 seconds the ball is again at the cliff level.

(vi) when the ball hits valley floor

$x = 0$

$-5t^2 + 30t + 100 = 0$

$-t^2 + 6t + 20 = 0$

$-t^2 + 6t = -20$

$t^2 - 6t + 9 = 20 + 9$

$(t-3)^2 = 29$

$t - 3 = \pm\sqrt{29}$

$t = 3 \pm \sqrt{29}$.

as $\sqrt{29} > 3$

$3 - \sqrt{29} < 0$,

$\therefore 3 - \sqrt{29}$ is not a valid value for t.

$\therefore t = 3 + \sqrt{29}$ sec.

(vii) when $t = 3 + \sqrt{29}$

$v = -10(3 + \sqrt{29}) + 30$

$= -30 - 10\sqrt{29} + 30$

$= -10\sqrt{29}$.

speed $= |V|$

$= |-10\sqrt{29}|$

$= 10\sqrt{29}\ m/s$

$\doteqdot 54\ m/s$

Question 15

(a) (i) $f'(x) > 0$ when

$-2 \leq x < 2$ and

$8 < x \leq 11$

(ii) $f''(x) > 0$ when

$5 < x < 10$

(iii) $f'(x) \times f''(x) > 0$ when

$2 < x < 5$ and when

$8 < x < 10$

we find this by locating regions when $f'(x)$ & $f''(x)$ are either both positive or both negatives, as only only $- \times - = +$ & $+ \times + = +$.

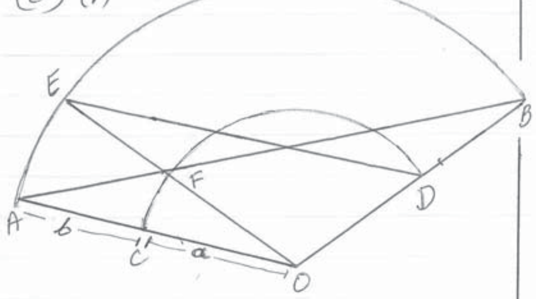
(b) $x = 2 + \cos 3t$

(i) $\frac{2\pi}{3}$

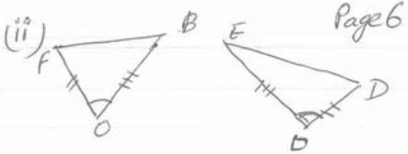
(ii) $2 + 1 = 3$ meters.

This is the furthest point in positive direction for $\cos 3t$.

(c) (i)



(c) (ii)



in $\triangle OBF$ and $\triangle OED$.

$OF = OD$ (equal radii of arc OFD)

$OE = OB$ (equal radii of arc AEB)

$\angle FOB = \angle EOB$ (angle subtended at centre by arc EB)

$\therefore \triangle OBF \equiv \triangle OED$ (SAS rule of congruency)

(iii) In $\triangle OCD$ an $\triangle OAB$.

$OC = OD = a$ (equal radii of arc CFD)

$OA = OB = a + b$ (equal radii of arc ACD)

$\therefore \frac{OC}{OA} = \frac{OD}{OB} = \frac{a}{a+b}$

$\angle COD = \angle AOB$ (common angle)

$\therefore \triangle OCD \parallel\!| \triangle OED$ (two sides are in the same ratio and the included angles are equal — SAS rule of similarity)

(iv) let $a + b = c$

ratio of area $= a^2 : c^2$

$\therefore$ ratio of side lengths $= \sqrt{a^2} : \sqrt{c^2}$

$= \sqrt{49} : \sqrt{144}$

$= 7 : 12$

$a = 7$

$c = 12$

$b = 12 - 7 = 5$

ratio of a to b = $7 : 5$

Question 16

(a) $y = x^3(2-x)$

$u = x^3$, $u' = 3x^2$

$v = 2-x$, $v' = -1$

$\frac{dy}{dx} = u'v + uv'$

$= 3x^2(2-x) - x^3$

$= 6x^2 - 3x^3 - x^3$

for stationary point

$\frac{dy}{dx} = 0$

$\therefore 6x^2 - 4x^3 = 0$

$x^2(6-4x) = 0$

$\therefore$ stationary points are at $x = 0$ & $x = \frac{3}{2}$

x	-1	0	1	$3/2$	2
y	-1	0	1	$27/16$	0
$\frac{dy}{dx}$	/	—	/	—	\

$\therefore$ at $(0,0)$ we have a horizontal point of inflexion and at $\left(\frac{3}{2}, \frac{27}{16}\right)$ we have a local maximum point.

(b) (i) Interest rate $= \frac{6}{12 \times 100}$

$= 0.005.$

Part (II) at the end of solution (last page)

(iii) 1st Jan 2015 to 31 Dec 2030 = 16 years

$n = 16 \times 12$

$= 192$

$A_n = 301500(1.005^{192} - 1)$

$= \$484045.19$

(iv) $301500(1.005^n - 1) = 400000$

$1.005^n = \frac{400000}{301500} + 1$

$\log 1.005^n = \log\left(\frac{701500}{301500}\right)$

$n = 169.312$

$\doteqdot 170$ months

we can say just before 170 month or somewhere in the 170th months.

Since accounts are usually settled at the end of the month.

$170 \div 2 = 14\frac{2}{12}$ years

or 14 years & 2 months.

$\therefore$ 1st March of 2029 or sometime in February 2029.

(v) 1st January 15 to 31st Dec 2025

= 11 years.

$400000 = \frac{M(1.005)(1.005^{132} - 1)}{0.005}$

$M(1.005)(1.005^{132} - 1) = 2000$

$M = \$2136.13$

Let A_n be value of investment after n months

II) $A_1 = 1500(1.005)$ 1

$A_2 = A_1(1.005) + 1500(1.005)$

$= 1500(1.005)^2 + 1500(1.005)$

$A_3 = A_2(1.005) + 1500(1.005)$

$= 1500(1.005)^3 + 1500(1.005)^2 + 1500(1.005)$

1

$A_n = 1500(1.005)\left[1 + 1.005 + 1.005^2 + \ldots 1.005^{n-1}\right]$

$= 1500(1.005)\left[\frac{1\,((1.005)^n - 1)}{1.005 - 1}\right]$

$= 1507.50\left[\frac{1.005^n - 1}{0.005}\right]$ 1

$= 301500\left[1.005^n - 1\right]$

$A_3 = 1500$