Student Number: _____ Class Teacher: _____

St George Girls High School

Trial Higher School Certificate Examination

2016

Mathematics

General Instructions

- Reading time – 5 minutes
- Working time – 3 hours
- Write using black pen
- Board-approved calculators may be used
- A reference sheet is provided
- In Questions 11 – 15, show relevant mathematical reasoning and/or calculations

Section I	/10
Section II	
Question 11	/18
Question 12	/18
Question 13	/18
Question 14	/18
Question 15	/18
Total	**/100**

Total Marks – 100

Section I Pages 2 – 6

10 marks

- Attempt Questions 1 – 10
- Allow about 15 minutes for this section
- Answer on the multiple choice answer sheet provided at the back of this paper

Section II Pages 7 – 16

90 marks

- Attempt Questions 11 – 15
- Allow about 2 hours and 45 minutes for this section
- Begin each question in a new writing booklet

Students are advised that this is a Trial Examination only and does not necessarily reflect the content or format of the Higher School Certificate Examination.

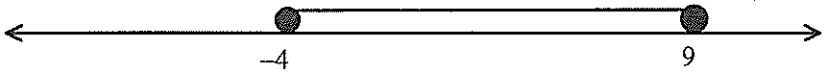
Section I

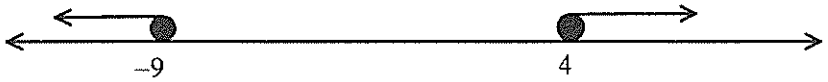
10 marks
Attempt Questions 1 - 10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1-10

1. Which graph shows the solution to $|2x - 5| \leq 13$?

(A)


(B)


(C)


(D)


2. What is the value of $\lim_{x \to 7} \dfrac{x^2 + 5x - 84}{x - 7}$?

(A) -5

(B) 0

(C) 12

(D) 19

Section I (continued)

3. What is the value of $\sum_{r=1}^{4} 2^{1-r}$?

(A) $\frac{1}{64}$

(B) $\frac{9}{8}$

(C) $\frac{15}{8}$

(D) $\frac{9}{4}$

4. Which of the following is equivalent to cosec $(\pi + \theta)$?

(A) $\frac{-1}{\sin\theta}$

(B) $\frac{-1}{\cos\theta}$

(C) $\frac{1}{\cos\theta}$

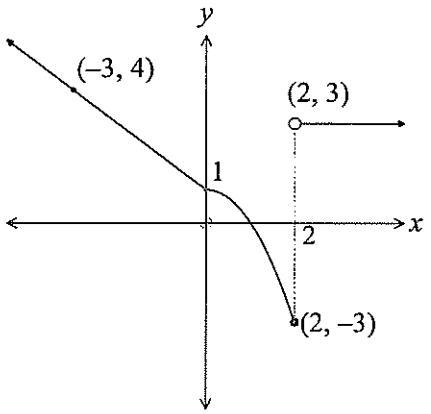
(D) $\frac{1}{\sin\theta}$

Section I (continued)

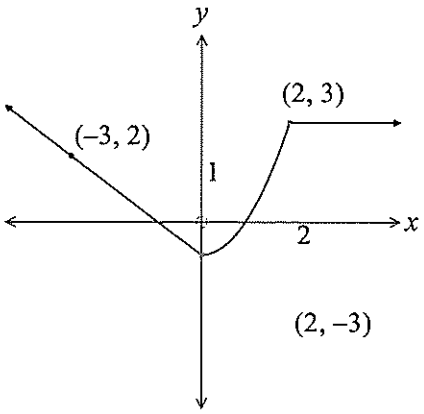
5. Which is the correct graph of the function below?

$$\begin{cases} y = 1 - x & x < 0 \\ y = 1 - x^2 & 0 \leq x \leq 2 \\ y = 3 & x > 2 \end{cases}$$

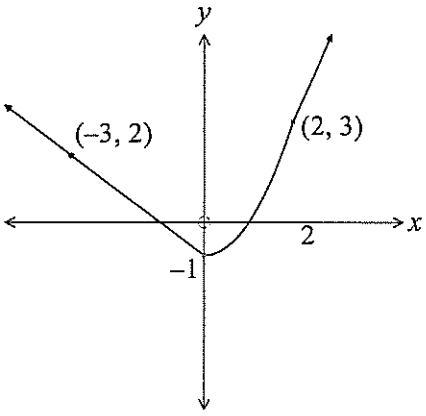
(A)



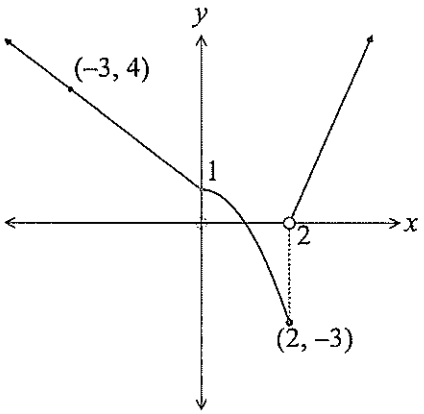
(B)



(C)



(D)



Section I (continued)

6. What is the solution to the equation $\log_e(x+2)-\log_e x=\log_e 4$?

(A) $\frac{2}{5}$ (B) $\frac{2}{3}$ (C) $\frac{3}{2}$ (D) $\frac{5}{2}$

7. The gradient function of a curve is $\frac{dy}{dx}=3-\frac{2}{x^2}$.

What is the equation of the curve if it passes through the point $(1,-2)$?

(A) $y=\frac{4}{x^3}$

(B) $y=\frac{2}{x}-4$

(C) $y=3x-\frac{2}{x}-3$

(D) $y=3x+\frac{2}{x}-7$

8. What is the domain and range of the function $y=\frac{1}{\sqrt{x-9}}$?

(A) $\{x: x\geq 9\}$ and $\{y: y>0\}$

(B) $\{x: x>9\}$ and $\{y: y>0\}$

(C) $\{x: -\infty\leq x\leq\infty\}$ and $\{y: -\infty\leq y\leq\infty\}$

(D) $\{x: -3\geq x\geq 3\}$ and $\{y: y<0\}$

9. What are the amplitude and period of the function $f(x)=2-\sin 2x$?

(A) Amplitude 1, period π

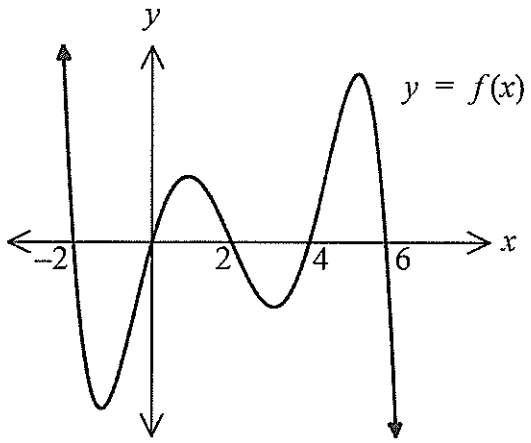
(B) Amplitude 1, period 2π

(C) Amplitude 2, period π

(D) Amplitude 2, period 2π

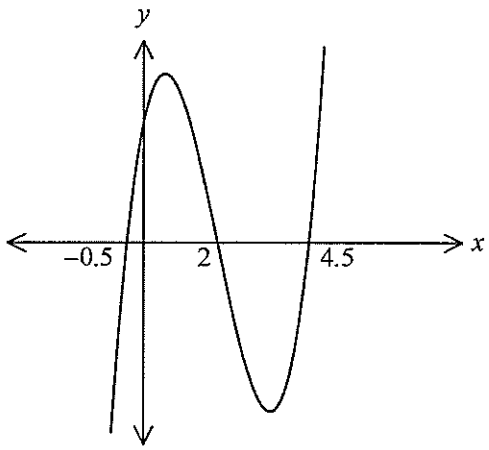
Section I (continued)

10. The graph of $y = f(x)$ is shown below.

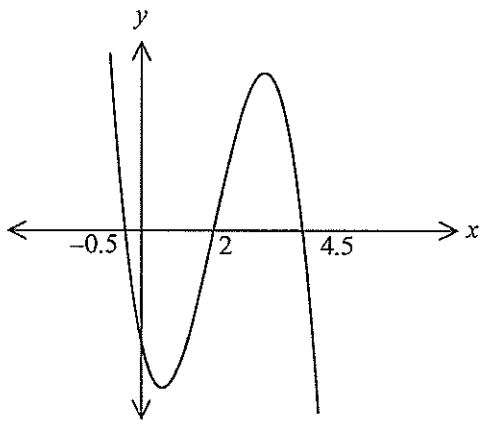


Which of these graphs represents $y = f'(x)$?

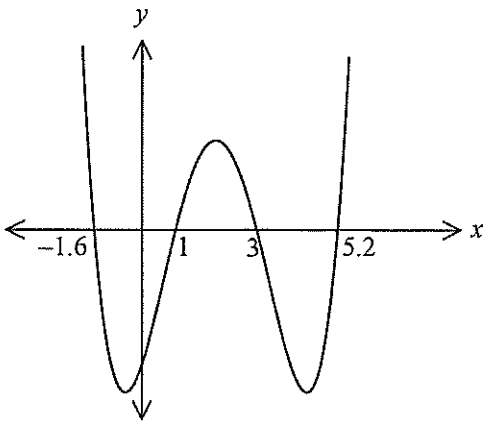
(A)



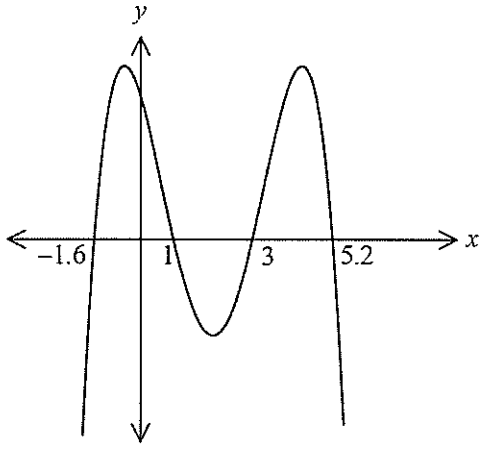
(B)



(C)



(D)



End of Section I

Section II

90 marks
Attempt Questions 11 – 15
Allow about 2 hours and 45 minutes for this section

Start each question in a new writing booklet.

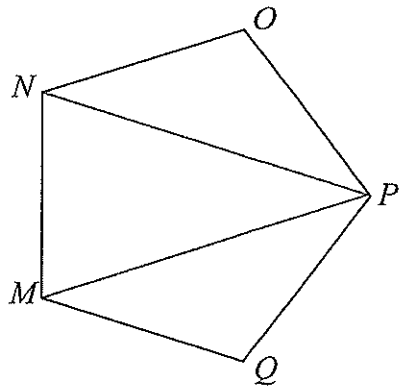
Your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (18 marks) Start a New Writing Booklet. **Marks**

a) Simplify $\dfrac{5}{x-2}-\dfrac{2}{x-3}$. 2

b) Express $\dfrac{1}{\sqrt{5}-2}$ with a rational denominator. 2

c)



The diagram shows a regular pentagon $MNOPQ$.

(i) Show that triangle NOP is isosceles and hence find $\angle ONP$. 3

(ii) Show that triangles NOP and PQM are congruent. 2

(iii) Find the size of $\angle MPN$. 1

Question 11 (continued) **Marks**

d) Boxes are stacked in layers, where each layer contains one box less than the layer below. There are six boxes in the top layer, seven boxes in the next layer, and so on. There are n layers altogether.

(i) Write down the number of boxes in the bottom layer. 2

(ii) Show that there are $\frac{1}{2}n(n+11)$ boxes. 2

e) A particle moves along the x-axis with acceleration $3t-2$. Initially it is 4 units to the right of the origin, with a velocity of 2 units per second. 3
What is the position of the particle after 5 seconds?

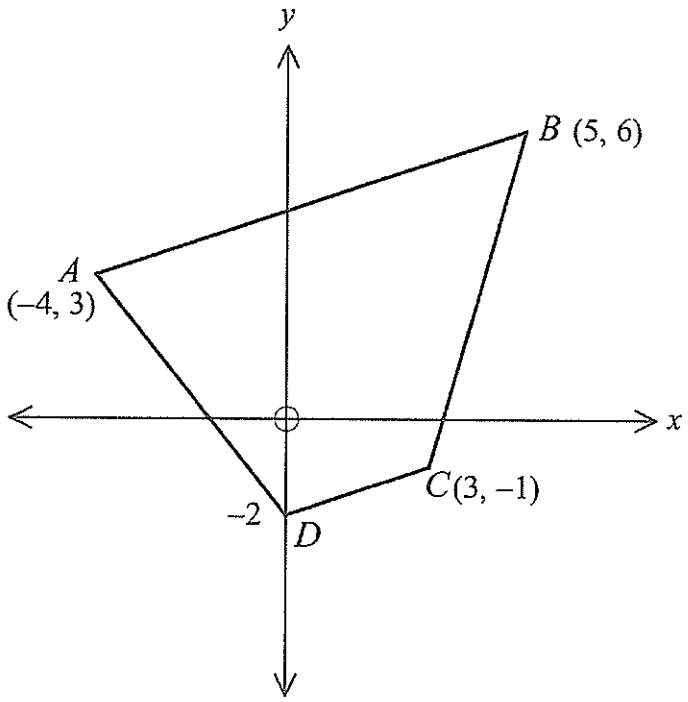
f) Find $\int \frac{x}{x^2+3}dx$. 1

Question 12 (18 marks) Start a New Writing Booklet. **Marks**

a) Consider the series $S_n = -18 + (-15) + (-12) + \ldots\ldots + u_n$ 2

For what value of n is $S_n = 0$?

b) A quadrilateral is formed by the points $A(-4, 3)$, $B(5, 6)$, $C(3, -1)$ and $D(0, -2)$ as shown in the diagram.



(i) Show that the quadrilateral is a trapezium, with $AB \parallel DC$. 2

(ii) Show that the equation of AB is $x - 3y + 13 = 0$. 1

(iii) Find the perpendicular distance from D to AB. 1

(iv) Find the area of the trapezium $ABCD$. 2

Question 12 (continued)

Marks

c) Evaluate $\int_0^{\frac{\pi}{3}} \cos 2x dx$. 2

d) Consider the curve $y = 2x^3 + 3x^2 - 36x + 4$ for $-5 \leq x \leq 5$.

(i) Find the stationary points and determine their nature. 3

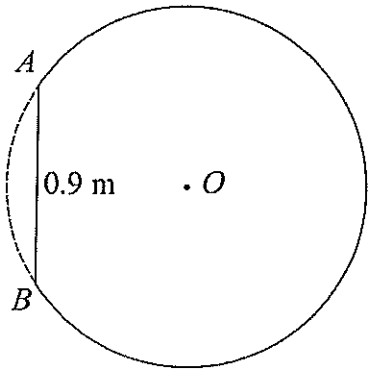
(ii) Find the point of inflexion. 2

(iii) Sketch the curve for $-5 \leq x \leq 5$. 2

(iv) Find the maximum value in the domain given. 1

Question 13 (18 marks) Start a New Writing Booklet. **Marks**

a) The circle has a centre O and radius 0.9 metres.
A small segment of this circle has been removed as shown below.
The length of the straight edge AB is also 0.9 metres.



Not to scale

(i) Explain why $\angle AOB = \frac{\pi}{3}$. 1

(ii) What is the shaded area to 3 significant figures? 3

b) Find the value of k if the sum of the roots of $x^2 - (k-1)x + 2k = 0$ is equal to the product of the roots. 2

c) Prove that $\dfrac{1}{\sec A - 1} + \dfrac{1}{\sec A + 1} = 2\cot A \operatorname{cosec} A$. 3

Question 13 (continued)

Marks

d) The second term of an arithmetic series is 37 and the sixth term is 17. What is the sum of the first ten terms? 3

e) Find the volume when $y = \log_e x$ is rotated about the y-axis between $y = 1$ and $y = 3$. Express your answer in exact form. 3

f) Differentiate with respect to x.

(i) $(e^x - 3)^4$ 1

(ii) $x \tan x$ 1

(iii) $\log_e(\cos x)$ 1

Question 14 (18 marks) Start a New Writing Booklet. **Marks**

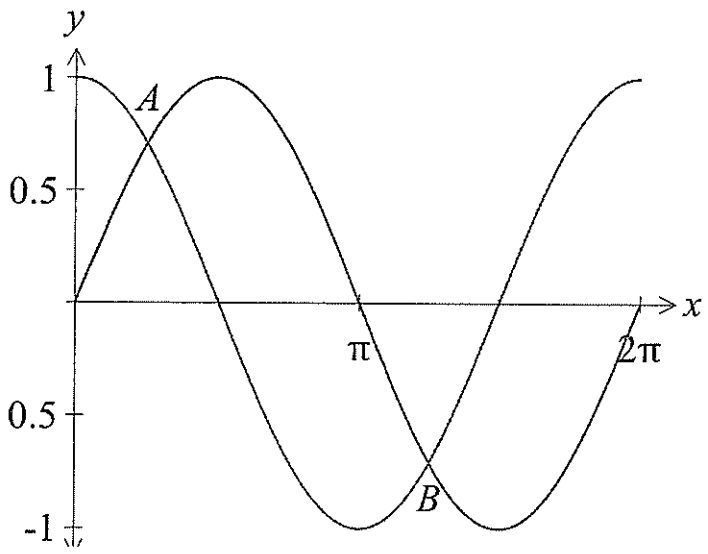
a) Find the equation of the normal to the curve $y = x\log_e x$ at the point on the curve where $x = 1$. 3

b) Twenty kilograms of sugar is placed in a container of water and begins to dissolve. After t hours the amount A kg of undissolved sugar is $A = 20e^{-kt}$.

(i) Calculate k, given that $A = 4.8$ when $t = 5$. 1

(ii) After how many hours does 1 kg of sugar remain undissolved? 2

c)



The diagram above shows the graphs $y = \sin x$ and $y = \cos x$ in the domain $0 \le x \le 2\pi$. The graphs intersect at points A and B.

(i) What are the coordinates of A and B? 2

(ii) Find the area enclosed by the two graphs between A and B. 2

Question 14 (continued)

Marks

d) On 1 June 2006, Patrick invested \$20,000 in a bank account that paid interest at a fixed rate of 7% per annum, compounded annually.

(i) How much would be in the account after the payment of interest on 1 June 2016 if no additional deposits were made? 1

(ii) Patrick decided to add \$2000 to his account on 1 June each year beginning on 1 June 2007. How much is in his account after Patrick makes a final deposit on 1 June 2016? 3

(iii) Patrick's friend Bella invested \$20,000 in an account at another bank on 1 June 2006 and made no further deposits. On 1 June 2016, the balance of Bella's account was \$49,565. What was the annual rate of compound interest paid on Bella's account? 2

e) Differentiate $\log_2 x^2$. 2

Question 15 (18 marks) Start a New Writing Booklet. **Marks**

a) A spokesman made the following statement about immigration:
"Over that period of time the number of immigrants was increasing at a decreasing rate".

(i) Given I is the level of immigration and t is time, what does the statement mean about

$$\frac{dI}{dt} \text{ and } \frac{d^2I}{dt^2} \text{ ?}$$ 1

(ii) Sketch a graph of immigration against time that fits the above information. 1

b) (i) Shade the intersection of these regions. 2

$$x \leq 2$$
$$y \leq 3$$
$$y \geq |x|$$

(ii) Hence, find the area of the region defining the intersection. 1

c) The speed of a train was recorded at intervals of one minute. The times, in minutes, and the corresponding speeds v, in kilometres per hour, are listed in the table below.

Time (min)	0	1	2	3	4
Speed (km/h)	0	24	35	28	50

Given that the distance x, in km, travelled by the train, may be found by calculating the area under a velocity/time graph, use Simpson's Rule with five function values to estimate the distance travelled by the train during the first four minutes. 3

Question 15 (continued) Marks

d) A parabola has the equation $16y = x^2 - 4x - 12$.

(i) Find the coordinates of the vertex. 2

(ii) Find the coordinates of the focus. 1

(iii) Find the equation of the directrix. 1

e) A can is in the shape of a closed cylinder with a height h cm and a radius r cm. The volume of the can is 200 cm^3.

(i) Find an expression for h in terms of r. 1

(ii) Show that the surface area $S\ cm^2$ of the can is given by the formula 1

$$S = 2\pi r^2 + \frac{400}{r} .$$

(iii) If the area of the metal used to make the can is to be minimized, find the area of the can to the nearest cm^2. 4

End of Examination

MATHEMATICS TRIAL HSC 2016 – SECTION I – MULTIPLE CHOICE

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS

(1) $|2x-5| \leq 13$

$+(2x-5) \leq 13$ $\qquad -(2x-5) \leq 13$

$2x \leq 18$ $\qquad 2x-5 \geq -13$

$x \leq 9$ $\qquad 2x \geq -8$

$x \geq -4$

$\therefore$ (B)

(2) $\lim_{x \to 7} \dfrac{x^2+5x-84}{x-7}$

$= \lim_{x \to 7} \dfrac{(x+12)(x-7)}{x-7}$

$= 7+12$

$= 19$ $\therefore$ (D)

MATHEMATICS – SECTION I – MULTIPLE CHOICE

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS

(3) $\sum_{r=1}^{4} 2^{1-r}$

$= 2^0 + 2^{-1} + 2^{-2} + 2^{-3}$

$= 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8}$

$= 1\frac{7}{8}$

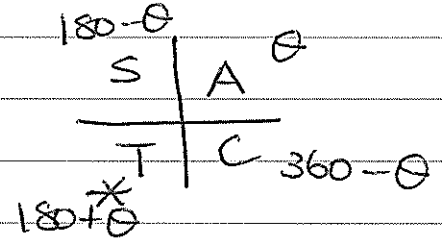
$= \frac{15}{8}$ $\therefore$ (C)

(4) $\operatorname{cosec}(\pi+\theta)$

$= \frac{1}{\sin(\pi+\theta)}$

$= \frac{1}{-\sin\theta}$

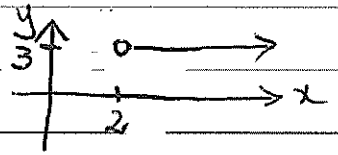
$= \frac{-1}{\sin\theta}$ $\therefore$ (A)



(5) $y = -x + 1$ ~~(B)~~ ~~(C)~~

(A) or (D)

$y = 3$



$\therefore$ (A)

MATHEMATICS – SECTION I – MULTIPLE CHOICE

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS

(6) $\log_e(x+2) - \log_e x = \log_e 4$

$$\frac{x+2}{x} = 4$$

$$x+2 = 4x$$

$$3x = 2$$

$$\therefore x = \frac{2}{3} \quad \therefore \text{(B)}$$

(7) $\frac{dy}{dx} = 3 - 2x^{-2}$

$$y = 3x - \frac{2x^{-1}}{-1} + C$$

$$y = 3x + \frac{2}{x} + C$$

subst. $(1,-2)$ to find C

$$-2 = 3 + 2 + C$$

$$-2 = 5 + C$$

$$C = -7$$

$$\therefore y = 3x + \frac{2}{x} - 7 \quad \therefore \text{(D)}$$

MATHEMATICS – SECTION I – MULTIPLE CHOICE

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
(8) $y = \frac{1}{\sqrt{x-9}}$		
$\sqrt{x-9} \neq 0$		
$x-9 > 0$		
$\therefore D : x > 9$		
$R : y > 0$ $\therefore$ (B)		
(9) $f(x) = 2 - \sin 2x$		
$= -\sin 2x + 2$		
amplitude $= 1$		
period $= \frac{2\pi}{2} = \pi$ $\therefore$ (A)		
(10) turning points become x-intercepts		
ie 4 x-intercepts (A) (B)		
(C) OR (D)		
between -1.6 & 1, $f'(x) > 0$		
$\therefore$ (D)		
	10	

MATHEMATICS - QUESTION NO: 11 continued

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
a) $\frac{5}{x-2} - \frac{2}{x-3}$		Generally very well done
$= \frac{5(x-3) - 2(x-2)}{(x-2)(x-3)}$		
$= \frac{5x-15-2x+4}{(x-2)(x-3)}$ (1 mark)	1	
$= \frac{3x-11}{(x-2)(x-3)}$ (1 mark)	1	
b) $\frac{1}{\sqrt{5}-2} \times \frac{\sqrt{5}+2}{\sqrt{5}+2} = \frac{\sqrt{5}+2}{(\sqrt{5})^2-4}$ (1 mark)	1	some mistakes where made by multiplying by $\sqrt{5}-2$ instead of $\sqrt{5}+2$
$= \frac{\sqrt{5}+2}{1}$		
$= \sqrt{5}+2$ (1 mark)	1	
c) i) In the regular pentagon, ΔNOP is isosceles [ON = OP] (1 mark)	1	generally well done
Interior angles of regular pentagon $= \frac{180 \times 3}{5}$		
$= 108^\circ$ (1 mark)	1	
$\angle ONP = \frac{180^\circ - 108^\circ}{2}$		
$= 36^\circ$ (1 mark)	1	

MATHEMATICS - QUESTION NO: 11 continued

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
ii) In ΔNOP and ΔPQN		Students should tidy
$ON = MQ$ [sides of regular pentagon] } 1 mark		up their proof
$\angle NOP = \angle PQM$ [equal interior angles] } 1 mark	1	and be more concise.
$OP = QP$ [sides of regular pentagon] } 1 mark		
$\therefore \Delta NOP \equiv \Delta PQN$ (SAS TEST) 1mark	1	well done.
iii) $O\hat{P}Q = O\hat{P}N + N\hat{P}N + M\hat{P}Q$		
$108^\circ = 36^\circ + N\hat{P}N + 36^\circ$		
$\angle NPN = 108^\circ - 72^\circ$		
$= 36^\circ$ (1mark)	1	
d) i) $T_n = a + (n-1)d$. where $a = 6$, $d = 1$ — (1mark)	1	well done.
$= 6 + (n-1) \cdot 1$		
$= n + 5$ (1 mark). bottom layer.	1	
ii) Total number of boxes		
$S_n = \frac{n}{2}\{2a + (n-1)d\}$ or $\frac{n}{2}\{a + l\}$		well done
$= \frac{n}{2}\{12 + (n-1)1\}$ $= \frac{n}{2}\{6 + n + 5\}$ (1mark)	1	
$= \frac{n}{2}\{n + 11\}$ $= \frac{n}{2}\{n + 11\}$ (1mark)	1	

MATHEMATICS - QUESTION NO: 11 continued

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
e) $a = 3t - 2$		ONLY MISTAKE MADE
$\dot{x} = \frac{3t^2}{2} - 2t + c$ when $t=0$, $\dot{x}=2$ $\therefore c=2$		IN THIS PART WAS
$\dot{x} = \frac{3t^2}{2} - 2t + 2$ (1 mark)	1	NEGLECTING THE
		CONSTANT.
$x = \frac{t^3}{2} - t^2 + 2t + c$ when $t=0$, $x=4$ $\therefore c=4$		SINCE IT WAS CRUCIAL
		MARKS WERE LOST EACH
$x = \frac{t^3}{2} - t^2 + 2t + 4$ (1 mark)	1	TIME!
when $t=5$		
$x = \frac{5^3}{2} - 5^2 + 10 + 4$		
$= 51.5$		
particle is 51.5 units to the right after 5 seconds. (1 mark)	1	MANY students left
f) $\frac{1}{2}\int \frac{2x}{x^2+3}\,dx = \frac{1}{2}\ln(x^2+3) + c$		off the constant.
		but no marks were
		taken off if they
		had included
		constant in part(e)

MATHEMATICS - QUESTION NO: 12

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
11 (a) $S_n = (-18)+(-15)+(-12)+\ldots+U_n$		
$a = -18 \quad d = 3$		
$S_n = \frac{n}{2}[2a+(n-1)d]$		
$0 = \frac{n}{2}[2\times-18+(n-1)\times3]$		
$0 = \frac{n}{2}(-36+3n-3)$		
$0 = n(3n-39)$	1	
$n = 0$ or $n = 13$		I only gave them 1½ if they did not mention $n=0$ as a soln to $n(3n-39)=0$
But $n \neq 0$ as $n > 0$, thus $n = 13$	1	
(b) $m_{DC} = \frac{-1--2}{3-0} = \frac{1}{3} = m_1$		NOTE: A lot of students, nearly all thought a trapezium had only one pair parallel sides
$m_{AB} = \frac{6-3}{5-4} = \frac{1}{3} = m_2$	1	
$\therefore AB \parallel DC$ since $m_1 = m_2$	½	I took ½ off for not writing the conclusion, first part, (last line)
$\therefore ABCD$ is a trapezium (at least one pair sides parallel)	½	
ii) $m = \frac{1}{3}$		
Either $y-6 = \frac{1}{3}(x-5)$ (OR) $y-3 = \frac{1}{3}(x+4)$		Most students got the mark for this
$3y-18 = x-5$ $\quad$ $3y-9 = x+4$		
$x-3y+13 = 0$ $\quad$ $x-3y+13 = 0$	1	

MATHEMATICS - QUESTION NO: 12 continued

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
(iii) $d = \left\| \frac{ax_1 + by_1 + c_1}{\sqrt{a^2+b^2}} \right\|$ $\quad \ell\ x - 3y + 13 = 0$, $D(0,-2)$		
$= \left\| \frac{1\times 0 - 3\times -2 + 13}{\sqrt{1^2+(-3)^2}} \right\|$		Some did $1\times 0 = 1$ and so got $\frac{20}{\sqrt{10}} = 2\sqrt{10}$
$= \frac{19}{\sqrt{10}}$	1	penalised $\frac{1}{2}$ a mark.
(iv) $AB = \sqrt{90}$ $\quad DC = \sqrt{10}$		
$= 3\sqrt{10}$	1	$\frac{1}{2}$ mark each
$A = \frac{1}{2}h(a+b)$		
$= \frac{1}{2}\times \frac{19}{\sqrt{10}}(3\sqrt{10}+\sqrt{10})$		Full marks given if incorrect answer to (iii) was carried forward correctly.
$= 38\ u^2$	1	
(c) $\int_0^{\pi/3} \cos 2x\, dx$		
$= \frac{1}{2}\left[\sin 2x\right]_0^{\pi/3}$		Most did this
$= \frac{1}{2}\left[\sin \frac{2\pi}{3} - \sin 0\right]$	1	correctly, some did not write sin 0 but I did not penalise them
$= \frac{1}{2}\left(\frac{\sqrt{3}}{2} - 0\right)$		
$= \frac{\sqrt{3}}{4}$	1	

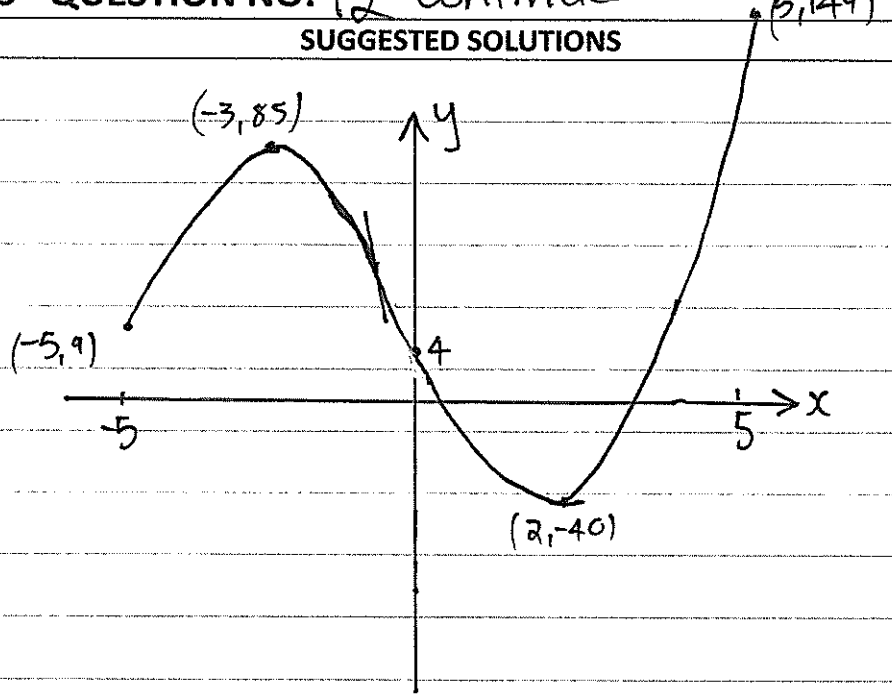
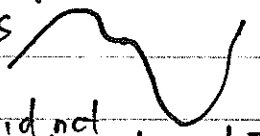
MATHEMATICS - QUESTION NO: 12 continued

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
(d)(i) $\frac{dy}{dx} = 6x^2 + 6x - 36$		
Stationary points occur when $\frac{dy}{dx} = 0$	1	
$6(x+3)(x-2) = 0$		
$x = -3 \qquad x = 2$		
$y = 85 \qquad y = -40.$	1	$\frac{1}{2}$ for each correct set of values.
$\frac{d^2y}{dx^2} = 12x + 6$		
When $x = -3$ $\frac{d^2y}{dx^2} = -30 < 0$ thus $(-3, 85)$ is a max. turning point	1	Some did not put values in for $\frac{d^2y}{dx^2}$, not many, and were penalised a mark.
When $x = 2$ $\frac{d^2y}{dx^2} = 30 > 0$ thus $(2, -40)$ is a min. turning point		
(OR) could do table of values for x and y'		
(ii) Possible point of inflexion when $\frac{d^2y}{dx^2} = 0$		
$12x + 6 = 0$		
$x = -\frac{1}{2}$, $y = 22\frac{1}{2}$	1	MANY did not show concavity change in the table and just assumed it was a point of inflexion

x	-1	$-\frac{1}{2}$	0
$\frac{d^2y}{dx^2}$	-6	0	6

Since change in concavity $\left(-\frac{1}{2}, 22\frac{1}{2}\right)$ is a point of inflexion | 1 |

MATHEMATICS - QUESTION NO: 12 continued

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
(III)  ½ shape + y intercept ½ end pts ½ turning points ½ inflexion point	2	Many thought a point of inflexion had to be horizontal and thus their graph looked like this  some did not show y intercept and I penalised them ½ (this was what was in the original guidelines)
(IV) Maximum value is 149 when $x = 5$		½ given for (5,149) ½ given for $x = 149$ (generous).

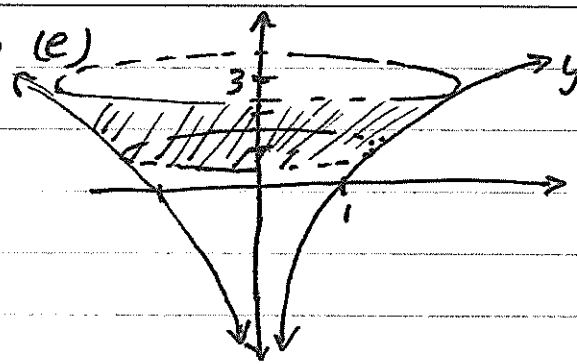
MATHEMATICS - QUESTION NO: 13

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
Q13 (a)(i) $OA = OB = AB = 0.9$		
$\therefore \triangle AOB$ is equilateral		
$\therefore \angle AOB = \frac{180^\circ}{3} = 60^\circ = \frac{\pi}{3}$	1	
(ii) Shaded area $= \pi \times 0.9^2 - \left[\frac{1}{2}r^2\theta - \frac{1}{2}r^2\sin\theta\right]$		
$= 0.81\pi - \left[\frac{1}{2}\times 0.9^2 \times \frac{\pi}{3} - \frac{1}{2}\times 0.9^2 \times \sin\frac{\pi}{3}\right]$	1	$-\frac{1}{2}$ mark of any minor error.
↳① ↳①	1	
$= 0.81\pi - \left[0.135\pi - 0.405 \times \frac{\sqrt{3}}{2}\right]$		
$\doteqdot 2.4713$		
$\doteqdot 2.47\ m^2$	1	
(b) $\alpha+\beta = -\frac{b}{a}$ $\quad \alpha\beta = \frac{c}{A}$		-1 for $\alpha+\beta = -k+1$ or $1-k$ many identified $b=(b-1)$
$= -\frac{-(k-1)}{1}$ $\quad = \frac{2k}{1}$		
$\therefore (k-1) = 2k$	1	
$k = -1$	1	

MATHEMATICS - QUESTION NO: 13 continued

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
Q13 (c) $\frac{1}{\sec A-1} + \frac{1}{\sec A+1} = 2\cot A \operatorname{cosec} A$		
$LHS = \frac{\sec A+1 + \sec A-1}{\sec^2 A - 1}$	1	$-\frac{1}{2}$ mark for any minor error.
$= \frac{2\sec A}{\tan^2 A}$	1	
$= 2 \times \frac{1}{\cos A} \times \frac{\cos^2 A}{\sec^2 A}$		
$= 2 \times \frac{\cos A}{\sin A} \times \frac{1}{\sin A}$	1	
$= 2\cot A \operatorname{cosec} A$		
$= RHS.$		
(d) $T_2 = a+d = 37$ —①		$-\frac{1}{2}$ mark for minor error.
$T_6 = a+5d = 17$ —②		
②−① $4d = -20$		
$d = -5$	1	
Substituting $d=-5$ into ①		
$a-5 = 37$		
$a = 42$	1	
$S_{10} = \frac{10}{2}[2(42) + 9\times(-5)]$		
$= 195$	1	

MATHEMATICS - QUESTION NO: 13 continued

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
Q13 (e)  $y = \log_e x$		
$x = e^y$	1	-½ mark for any minor error.
$V = \pi \int_a^b x^2 \, dy.$		
$= \pi \int_1^3 (e^y)^2 \, dy.$		
$= \frac{\pi}{2} \left[e^{2y} \right]_1^3$	1	
$= \frac{\pi}{2} \left[e^6 - e^2 \right]$	1	
(f)(i) $y = (e^x - 3)^4$		
$\frac{dy}{dx} = 4(e^x - 3)^3 \times e^x$	1	
$= 4e^x (e^x - 3)^3$		
(ii) $y = x \tan x$		
$\frac{dy}{dx} = x \cdot \sec^2 x + \tan x \cdot 1$	1	
$= x \sec^2 x + \tan x$		
(iii) $y = \log_e (\cos x)$		
$\frac{dy}{dx} = \frac{1}{\cos x} \times -\sin x$		
$= -\frac{\sin x}{\cos x}$	1	
$= -\tan x$		

MATHEMATICS - QUESTION NO: 14

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
a) $y = x \log_e x$ $\quad u = x \quad v = \log_e x$		Evaluate the gradient of the tangent before finding the negative reciprocal.
$u' = 1 \quad v' = \frac{1}{x}$		
$y' = x \times \frac{1}{x} + 1 \times \log_e x$		
$= 1 + \log_e x$	1	
when $x = 1$, $y = 0$, $y' = 1$		
$\therefore$ gradient of normal $= -1$	1	
$y - 0 = -1(x - 1)$		
$y = -x + 1$	1	
b) i $4.8 = 20e^{-5k}$		
$e^{-5k} = 0.24$		
$-5k = \ln 0.24$		
$k = \frac{-\ln 0.24}{5}$ $\left(\doteq 0.28542\right)$	1	
ii $1 = 20e^{-kt}$	1	
$e^{-kt} = 0.05$		
$-kt = \ln 0.05$		
$t = \frac{\ln 0.05}{-k}$		
$= 10.49575 \quad \doteq 10.5$ hours	1	

MATHEMATICS - QUESTION NO: 14 continued

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
c) i $\sin x = \cos x$		
$\tan x = 1$		
$x = \frac{\pi}{4}, \pi + \frac{\pi}{4}$	1	(for finding the related angle $\frac{\pi}{4}$)
$\therefore A\left(\frac{\pi}{4}, \frac{1}{\sqrt{2}}\right)$ and $B\left(\frac{5\pi}{4}, \frac{-1}{\sqrt{2}}\right)$	1	
ii $A = \int_{\frac{\pi}{4}}^{\frac{5\pi}{4}} (\sin x - \cos x)\, dx$		
$= \left[-\cos x - \sin x\right]_{\frac{\pi}{4}}^{\frac{5\pi}{4}}$	1	
$= \left(-\frac{-1}{\sqrt{2}} - \frac{-1}{\sqrt{2}}\right) - \left(-\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}\right)$		
$= 4/\sqrt{2}$		
$= 2\sqrt{2}\ u^2$	1	
d) i $A = 20000\ (1.07)^{10}$		
$= 39\ 343.02715$		
$\doteq \$39\ 343.03$	1	

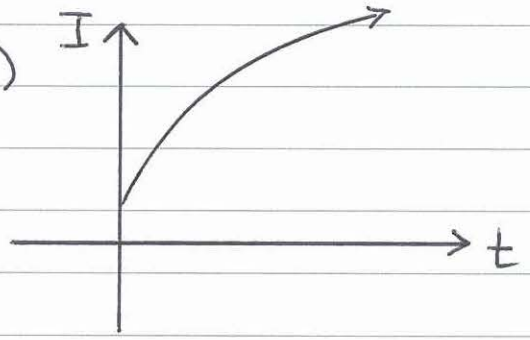
MATHEMATICS - QUESTION NO: 14 continued

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
d) ii Let A_1 be the value of the account on 01/07/2007		
$A_1 = 20000 \times 1.07 + 2000$		
$A_2 = A_1 \times 1.07 + 2000$		
$= 20000 \times 1.07^2 + 2000 \times 1.07 + 2000$		
$A_3 = A_2 \times 1.07 + 2000$		
$= 20000 \times 1.07^3 + 2000 \times 1.07^2 + 2000 \times 1.07 + 2000$		
$\vdots$		
$A_{10} = 20000 \times 1.07^{10} + 2000 \times 1.07^9 + 2000 \times 1.07^8 + \ldots$		
$\ldots 2000 \times 1.07 + 2000$	1	for correctly identifying the general pattern
$= 20000 \times 1.07^{10} + 2000(1 + 1.07 + 1.07^2 + \ldots 1.07^9)$		
GP with $a=1$, $r=1.07$ $n=10$		
$S_{10} = \frac{1(1.07^{10}-1)}{1.07-1}$	1	correct use of the formula for S_n of a G.P.
$= 13.8164\ldots$		
$\therefore A_{10} = 39343.02715 + 27632.89592$		
$= 66975.92307\ldots$		
$\div \$66\,975.92$	1	correct evaluation including both parts of the investment

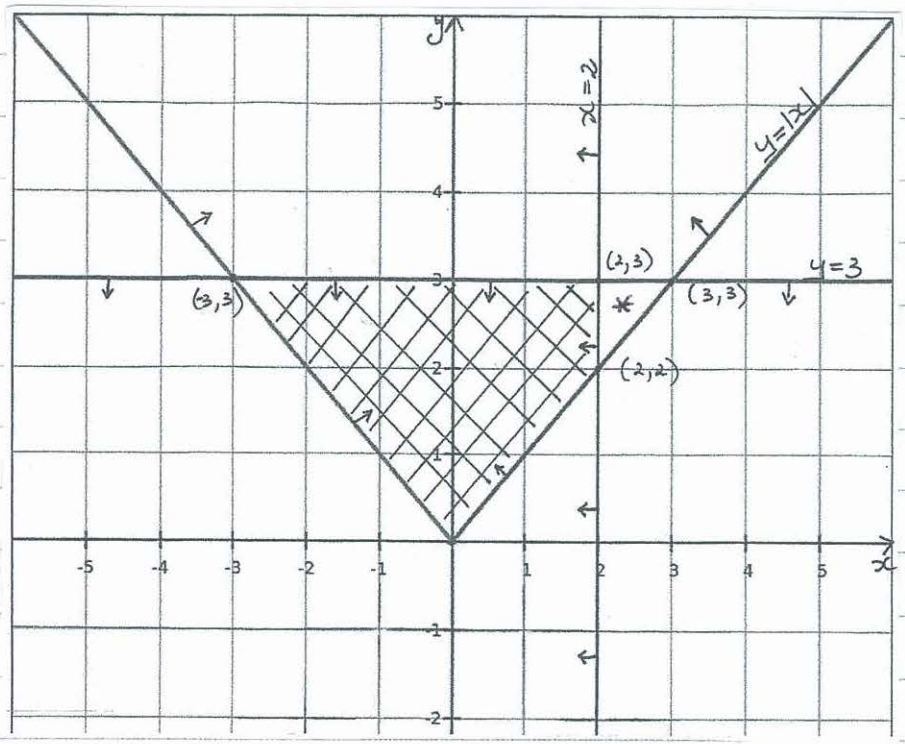
MATHEMATICS - QUESTION NO: 14 continued

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
d) iii $49565 = 20000(1+r)^{10}$	1	
$(1+r)^{10} = \frac{4000}{9913}$		
$r+1 = \sqrt[10]{\frac{4000}{9913}}$		
$= 1.09500$		
$\therefore r = 0.09500$		
$\therefore$ the interest rate is 9.5% p.a.	1	
e) let $y = \log_2 x^2$		
$= \frac{\ln x^2}{\ln 2}$	1	correct use of change of base - note that changing to base 10 is of no benefit, and no marks are awarded for this.
$= \frac{1}{\ln 2} \times \ln x^2$		
$\therefore y' = \frac{1}{\ln 2} \times \frac{2x}{x^2}$		
$= \frac{2}{x \ln 2}$		
$\therefore \frac{d}{dx} \log_2 x^2 = \frac{2}{x \ln 2}$	1	

MATHEMATICS - QUESTION NO: 15 (18 marks)

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
(a)(i) immigration was increasing $\therefore \frac{dI}{dt} > 0$	$\frac{1}{2}$	[CFPA means correct for previous answer]
(*tangent is positive) OR $\frac{dI}{dt}$ is positive		
at a decreasing rate $\therefore \frac{d^2I}{dt^2} < 0$	$\frac{1}{2}$	
(*curve is concave down) OR $\frac{d^2I}{dt^2}$ is negative		
(ii) 	1	• $(-\frac{1}{2})$ if curve was below t-axis • If (i) wrong (no marks awarded), (ii) correct graph for (i) then (1) mark was awarded.

MATHEMATICS - QUESTION NO: 15 continued

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
(b) (i) 	2	correct region shaded
	* 1½	lost ½ mark if didn't have small triangle unshaded
	½	region to the left of $x=2$
	½	region below $y=3$
	½	region inside $y=\|x\|$
	½	correct intersection of all 3.
(ii) Area of Intersection = Big Δ − Small Δ = ½ x 6 x 3 − ½ x 1 x 1 = 9 − ½ = 8½ square units = 17/2 square units OR equivalent.	1	correct answer
OR A = Δ + trapezium = ½ x 3 x 3 + ½ x 2 (3+1) = 9/2 + 4 = 8½ square units = 17/2 square units	½	CFPA in (i) as area was made easier to find

MATHEMATICS - QUESTION NO: 15 continued

SUGGESTED SOLUTIONS		MARKS	MARKER'S COMMENTS
(c)			Majority of students didn't read the question carefully to notice that time and speed were not both in the same unit of time
• changing minutes to hours ÷60	0 $\frac{1}{60}$ $\frac{1}{30}$ $\frac{1}{20}$ $\frac{1}{15}$ (hr)		
(c) t (min)	0 1 2 3 4		
V km/h	0 24 35 28 50		
÷60	0 $\frac{2}{5}$ $\frac{7}{12}$ $\frac{7}{15}$ $\frac{5}{6}$		
• changing speed to km/min			
$x = \int_0^{\frac{1}{15}} V\,dt \doteq \frac{\frac{1}{30}-0}{6}\left[0+4(24)+35\right]$			
$+\ \frac{\frac{1}{15}-\frac{1}{30}}{6}\left[35+4(28)+50\right]$			
$= \frac{1}{180}\left[0+4(25)+35+35+4(28)+50\right]$			
$= \frac{1}{180} \times 328$			
$= \frac{328}{180}$ km		3	correct working and answer
$= \frac{82}{45}$ km			
$= 1.82$ km			
OR $x = \int_0^4 V\,dt \doteq \frac{1}{3}\left[0+\frac{5}{6}+4\left(\frac{2}{5}+\frac{7}{15}\right)+2\left(\frac{7}{12}\right)\right]$	• $h = \frac{b-a}{n}$	1	for changing minutes to hours OR changing speed to km/min
$= \frac{1}{3} \times \frac{82}{15}$	$\therefore h = \frac{4-0}{4} = 1$	1	for substituting into either formula correctly
$= \frac{82}{45}$	(n subintervals OR strips)		
$= 1.82$ km			
0 marks for wrong application of formula, ie +(?) between 0 & 24, there were no "inbetween" values given.		1	for correct answer
		0	for wrong formula

MATHEMATICS - QUESTION NO: 15 continued

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
(c)		READ QUESTION!!!
		LEARN FORMULA!!!
	2	fr correct working and answer of $\frac{328}{3}$ km
		OR $109.\dot{3}$ km
	1	fr correct working inside [ ] but wrong "fraction" outside.

(c).

OR

t min	0	1	2	3	4
V km/h	0 (y_0)	24 (y_1)	35 (y_2)	28 (y_3)	50 (y_4)

NOTE

* not changing t & V:

$$x = \int_0^4 v \, dt$$

$$\div \frac{1}{3}\left[0 + 4(24+28) + 2(35) + 50\right]$$

$$= \frac{1}{3}\left[4(52) + 2(35) + 50\right]$$

$$= \frac{1}{3}\left[208 + 70 + 50\right]$$

$$= \frac{328}{3} \text{ km} = 109.\dot{3} \text{ km}$$ {2 marks out of 3}

OR $x = \int_0^4 v \, dt \div \frac{2-0}{6}\left[0 + 4(24) + 35\right]$ (131)

$$+ \frac{4-2}{6}\left[35 + 4(28) + 50\right]$$ (197)

$$= \frac{1}{3}\left[0 + 4(24+28) + 2(35) + 50\right]$$

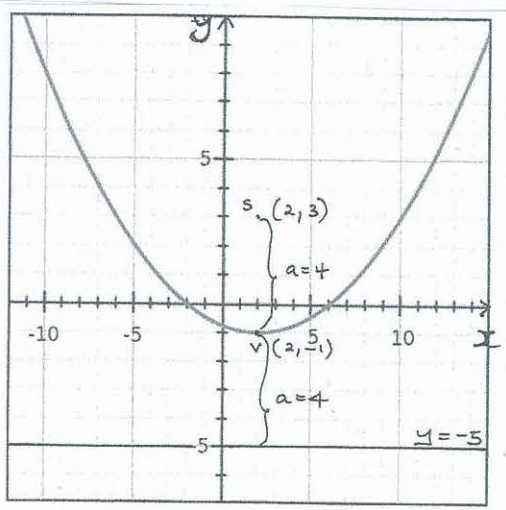
$$= \frac{328}{3}$$ {2 marks out of 3}

$$= 109.\dot{3} \text{ km}$$

MATHEMATICS - QUESTION NO: 15 continued

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
(d) $16y = x^2 - 4x - 12$ (concave up parabola)		• Learn axis of
(i) $y = \frac{x^2}{16} - \frac{4x}{16} - \frac{12}{16}$		symmetry formula
		$x = \frac{-b}{2a}$
$y = \frac{x^2}{16} - \frac{x}{4} - \frac{3}{4}$		
		• Learn how to
$a = \frac{1}{16}$ $\quad x = \frac{-b}{2a}$		complete the
$b = -\frac{1}{4}$		square
$= \frac{-\left(-\frac{1}{4}\right)}{2\left(\frac{1}{16}\right)}$		• Learn the
		importance of
$\therefore x = 2$		the focal length.
$y = \frac{2^2}{16} - \frac{2}{4} - \frac{3}{4}$	1	for finding
		axis of symmetry
$= -1 \quad \therefore$ Vertex $(2, -1)$		correctly $x=2$
	1	for $y=-1$
OR $x^2 - 4x + \left(-\frac{4}{2}\right)^2 = 16y + 12 + \left(-\frac{4}{2}\right)^2$		
$(x-2)^2 = 16y + 12 + 4$	1	for completing the
$= 16y + 16$		square to get to
		correct frm.
$(x-2)^2 = 16(y+1)$	1	for correct
$\therefore$ vertex $(2, -1)$		vertex (or CFPA)

MATHEMATICS - QUESTION NO: 15 continued

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
(ii) needed $(x-2)^2 = 16(y+1)$ frm to find focal length ie $4a = 16$ $a = 4$	$\frac{1}{2}$	if stated "a", then I was able to check focus & directrix (CFPA).
$\therefore$ Focus $(2, -1+4)$ $(2, 3)$	1	correct answer
(iii) Directrix : $y = -1 - 4$ $\therefore y = -5$	1	correct answer
A sketch will help to visualise the parabola and see where the focus & directrix are with respect to the focal length and vertex.		• Note, the directrix is a line not a point (no marks awarded if answer was given as a point)
		• students that did a quick sketch didn't make errors.

MATHEMATICS - QUESTION NO: 15 continued

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
(e) Learn basic formulas !		
For cylinder : $V = \pi r^2 h$		
$SA = 2\pi r^2 + 2\pi rh$		
(i) $V = 200$		
$\pi r^2 h = 200$		
$h = \frac{200}{\pi r^2}$	1	correct answer
(ii) $S = 2\pi r^2 + 2\pi rh$		
$= 2\pi r^2 + 2\pi r \times \frac{200}{\pi r^2}$		
$= 2\pi r^2 + \frac{400}{r}$	1	correct answer with working ie showing substitution of $h = \frac{200}{\pi r^2}$ and cancelling.

MATHEMATICS - QUESTION NO: 15 continued

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
(iii) $S = 2\pi r^2 + 400r^{-1}$		
$\frac{dS}{dr} = 4\pi r - 400r^{-2}$	1	finds the first derivative of S w.r.t. r. ie $\frac{dS}{dr}$ or S'
$= 4\pi r - \frac{400}{r^2}$		
Stationary values for S occur when $\frac{dS}{dr} = 0$		
ie $4\pi r - \frac{400}{r^2} = 0 \quad (r > 0)$		
$4\pi r = \frac{400}{r^2}$		
$4\pi r^3 = 400$		
$r^3 = \frac{400}{4\pi}$		
$r^3 = \frac{100}{\pi}$		finds the zero of the first derivative, ie $r = \sqrt[3]{\frac{100}{\pi}}$
$r = \sqrt[3]{\frac{100}{\pi}}$	1	
$= 3.169202884$		

MATHEMATICS - QUESTION NO: 15 continued

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
(e)(iii) continued	1	Verifies this
$\frac{d^2S}{dr^2} = 4\pi + 800r^{-3}$		zero corresponds
$\frac{d^2S}{dr^2} = 4\pi + \frac{800}{r^3}$		to a minimum S
		value, ie uses
when $r^3 = \frac{100}{\pi}$, $\frac{d^2S}{dr^2} = 4\pi + \frac{800}{\frac{100}{\pi}}$		$\frac{d^2S}{dr^2}$ or
$= 4\pi + 8\pi$		equivalent, with
$= 12\pi > 0$ U		numerical answers.
$= 37.69911184 > 0$		
$\therefore$ minimum area when $r = \sqrt[3]{\frac{100}{\pi}}$		
OR		

r	3	$\sqrt[3]{\frac{100}{\pi}}$	4
$\frac{dS}{dr}$	-6.74533	0	25.26548

$\therefore$ minimum area will occur when $r = \sqrt[3]{\frac{100}{\pi}}$

$\therefore S = 2\pi\left(\sqrt[3]{\frac{100}{\pi}}\right)^2 + \frac{400}{\sqrt[3]{\frac{100}{\pi}}}$ | 1 | finds the value of S correctly to the nearest cm^2 ie $S = 189\ cm^2$

$= 63.10735139 + 126.2147028$

$= 189.3220542$

$= 189\ cm^2$ (to the nearest cm^2)