Student Number: ______________________ Class Teacher: ____________________

St George Girls High School

Trial Higher School Certificate Examination

2019

Mathematics

General Instructions

- Reading time – 5 minutes
- Working time – 3 hours
- Write using black pen
- Board-approved calculators may be used
- A reference sheet is provided
- Diagrams are not necessarily to scale
- In Questions 11 – 17, show relevant mathematical reasoning and/or calculations
- Marks may not be awarded for careless or poorly presented solutions

Total Marks – 100

Section I Pages 2 – 5

10 marks

- Attempt Questions 1 – 10
- Allow about 15 minutes for this section
- Answer on the multiple choice answer sheet provided at the back of this paper

Section II Pages 6 – 13

90 marks

- Attempt Questions 11 – 17
- Allow about 2 hours and 45 minutes for this section
- Begin each question in a new writing booklet

Section I	**/10**
Section II	
Question 11	/13
Question 12	/13
Question 13	/13
Question 14	/13
Question 15	/13
Question 16	/13
Question 17	/12
Total	**/100**

Section I

10 marks
Attempt Questions 1 - 10
Allow about 15 minutes for this section.

Use the multiple-choice answer sheet for Questions 1-10.

1. What is the solution to the equation $3(2y - 1) = 27$?

   (A) $y = 4$
   (B) $y = 4\frac{1}{3}$
   (C) $y = 5$
   (D) $y = 5\frac{4}{5}$

2. What is the derivative of $\frac{x}{\sin x}$?

   (A) $\frac{\sin x + x\cos x}{\sin^2 x}$
   (B) $\frac{\sin x - x\cos x}{\sin^2 x}$
   (C) $\frac{x\cos x - \sin x}{\sin^2 x}$
   (D) $\frac{-x\cos x - \sin x}{\sin^2 x}$

3. What are the coordinates of the turning point to the curve $y = e^x - ex$?

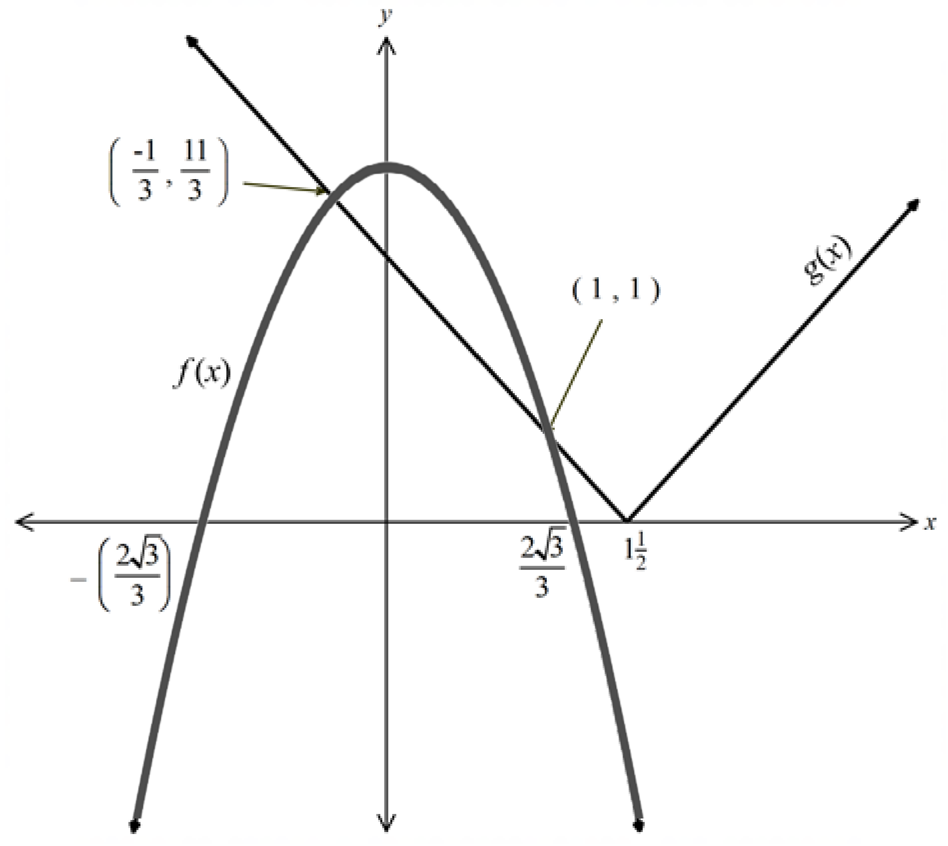
   (A) $(0,1)$
   (B) $(1,0)$
   (C) $(1, e)$
   (D) $(e, 1)$

4. The point P moves such that it remains equidistant from two fixed points.
   Which of the following equations might describe the locus of P?

   (A) $3x + 2y - 5 = 0$
   (B) $(x - 1)^2 + (y + 3)^2 = 9$
   (C) $y = \frac{3}{x}$
   (D) $x^2 = 12y$

Section I (cont'd)

5. The graph below shows the functions $f(x) = -3x^2 + 4$ and $g(x) = |2x - 3|$.



For what values of x is $|2x - 3| \geq -3x^2 + 4$?

(A) $-\frac{1}{3} \leq x \leq 1$

(B) $x \leq -\frac{1}{3}$ and $x \geq 1$

(C) $-\frac{2\sqrt{3}}{3} \leq x \leq 1$

(D) $x \leq -\frac{2\sqrt{3}}{3}$ and $x \geq 1\frac{1}{2}$

6. The solutions to $e^{6x} - 5e^{3x} + 6 = 0$ are:

(A) $x = 2, \ x = 3$

(B) $x = \log_e 2, \ x = \log_e 3$

(C) $x = e^{2x}, \ x = e^{3x}$

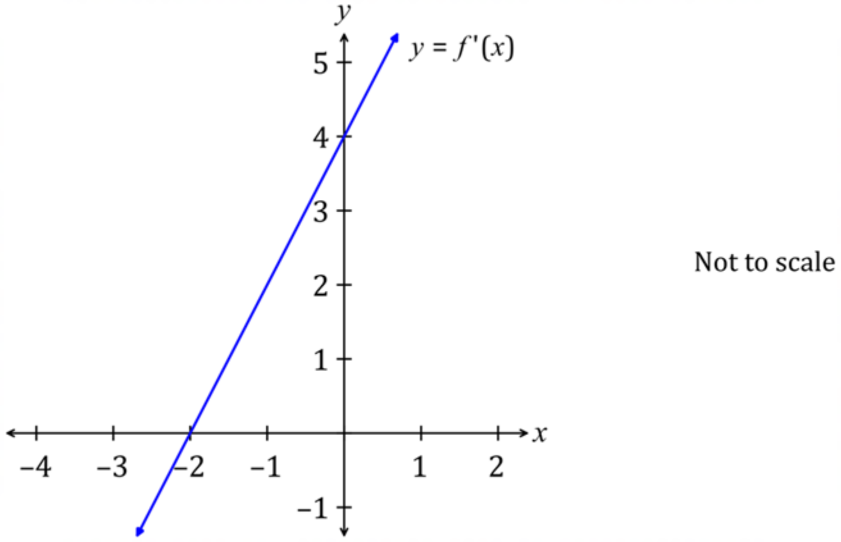
(D) $x = \frac{1}{3}\log_e 3, \ x = \frac{1}{3}\log_e 2$

Section I (cont'd)

7. If α and β are the roots of $3x^2 - 4x + 9 = 0$, then $\alpha^2 + \beta^2 =$

(A) 5
(B) $1\frac{7}{9}$
(C) $10\frac{7}{9}$
(D) $-4\frac{2}{9}$

8. The graph of $y = f'(x)$ is shown below.



The curve $y = f(x)$ has a minimum value of 6. What is the equation of the curve?

(A) $y = x^2 - 4x + 2$
(B) $y = x^2 - 4x + 10$
(C) $y = x^2 + 4x + 2$
(D) $y = x^2 + 4x + 10$

9. The value of $\displaystyle\int_0^{\pi} \sin x\, dx = 2.$

What is the value of $\displaystyle\int_0^{2n\pi} \sin x\, dx$, where n is a positive integer?

(A) $4n$
(B) 4π
(C) $2n$
(D) 0

Section I (cont'd)

10. The function $y = f(x)$ is continuous for all x.

Given that $\int_0^1 f(x)dx = 6$ and $\int_0^3 f(x)dx = 5$, what is the value of $\int_1^3 (1 + f(x))dx$?

(A) -1

(B) 0

(C) 1

(D) 2

End of Section I

Section II

90 marks
Attempt Questions 11 – 17
Allow about 2 hours and 45 minutes for this section.

Start each question in a new writing booklet.

Your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (13 marks) Start a New Writing Booklet. **Marks**

a) Evaluate $\sqrt{500} \times 2.6^2$, correct to 2 decimal places. 2

b) Solve $-1 - 3x < 11$. 1

c) Find the points of intersection of $y = 4 - x^2 + 2x$ and $x + y = 0$. 3

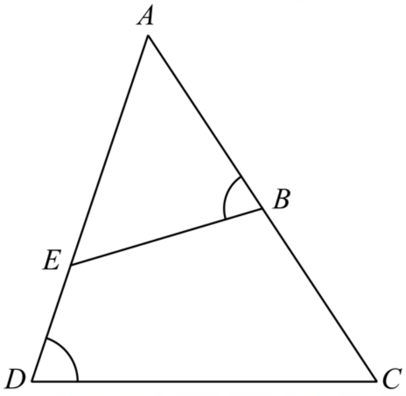
d) Find the integers a and b such that $\left(2\sqrt{3} - 1\right)^2 = a\sqrt{3} + b$. 2

e) In an arithmetic series the first term is 12 and the sum of the first 20 terms is 620.

i) Find the 20th term. 1

ii) Find the common difference. 2

f) ΔABE is similar to ΔADC. $AE = 6, AB = 4$, and $ED = 2$. Find the length of BC. 2



Not to scale

Question 12 (13 marks) Start a New Writing Booklet. **Marks**

a) Differentiate with respect to x:

i) $(4x^3 - x)^7$ 1

ii) $e^x \sin x$ 2

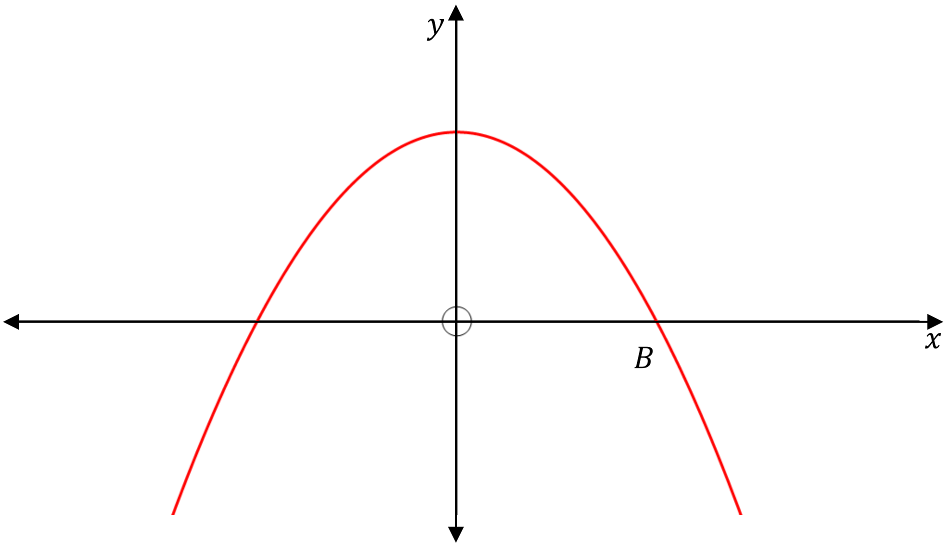
iii) $\ln(\sqrt{x})$ 2

b) Consider the parabola $4y = x^2 - 2x + 5$.

i) Find the coordinates of the vertex. 2

ii) Find the coordinates of the focus. 2

c) The parabola $y = -2x^2 + 32$ cuts the x-axis at B, as shown below.



i) Show that the coordinates of B are $(4,0)$. 1

ii) The area enclosed by the curve, the x-axis, the y-axis, and $x = 4$ is rotated about the x-axis. Find the volume of the solid formed. 3

Question 13 (13 marks) Start a New Writing Booklet. **Marks**

a) Consider the function $f(x) = 2 + 9x - \frac{x^3}{3}$

i) Find the coordinates of the turning points and determine their nature. 3

ii) Find the coordinates of the point of inflexion. 2

iii) Sketch the curve, showing the stationary points and the point of inflexion. 2
Note: You are **not** required to find the x-intercepts.

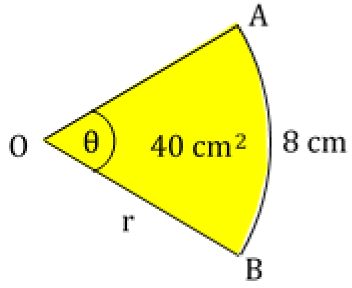
b) Consider the function $y = \sqrt{5^x + 2}$.

i) Copy and complete the table for the function, correct to 3 decimal places. 1

x	0	0.5	1	1.5	2
y			2.646		

ii) Use the trapezoidal rule with 5 function values to find an approximation for the value of $\int_0^2 \sqrt{5^x + 2}\, dx$. 2

c) The diagram shows a sector OAB with measurements as shown.



i) Find the length r of the radius. 2

ii) Find the size of angle θ. 1

Question 14 (13 marks) Start a New Writing Booklet. **Marks**

a) Consider the geometric series $(e-1)+\left(\frac{e-1}{e}\right)+\left(\frac{e-1}{e^2}\right)+\cdots$

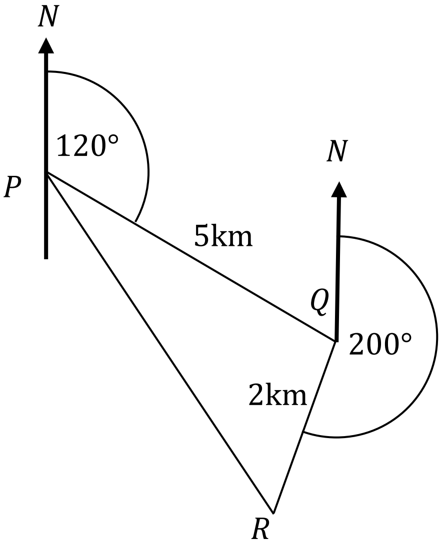
i) Explain why this geometric series has a limiting sum. 1

ii) Find the exact value of the limiting sum. 2

b) Find $\int (\sin 3x + 2)\,dx$. 2

c) Evaluate $\int_2^3 \frac{x^3}{x^4-2}\,dx$. 2

d) David starts walking from a camping place P on a bearing of 120° for 5 km to a place Q. He then walks on a bearing of 200° for 2 km to a place R.



i) What is the size of $\angle PQR$? 1

ii) What is the distance between the camping place P and the place R? 2
Answer correct to 2 decimal places.

e) Solve $2\sin^3 x - 3\sin^2 x - 2\sin x = 0$, for $0 \le x \le 2\pi$. 3

Question 15 (13 marks) Start a New Writing Booklet. **Marks**

a) i) Find $\frac{d}{dx}(x\ln x - x)$. 1

ii) Hence evaluate $\int_2^4 \ln x \, dx$. 2

b) The velocity of a particle moving in a straight line has velocity, in metres per second, given by

$$v = -\frac{7}{t+1}$$

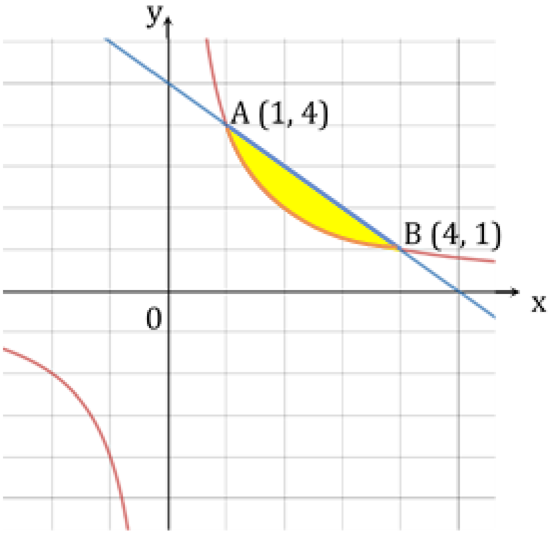
Initially the particle's displacement is 8 metres to the right of the origin.

i) Calculate the displacement of the particle at $t = 3$ seconds, to 2 decimal places. 3

ii) Show that the acceleration of the particle is always positive. 2

iii) Is the particle ever at rest? Give reasons for your answer. 1

c) The line $y = 5 - x$ intersects the curve $y = \frac{4}{x}$ at the points $A\ (1,4)$ and $B\ (4,1)$. The region bounded by the curve and the line between the points A and B is shaded in the diagram below.

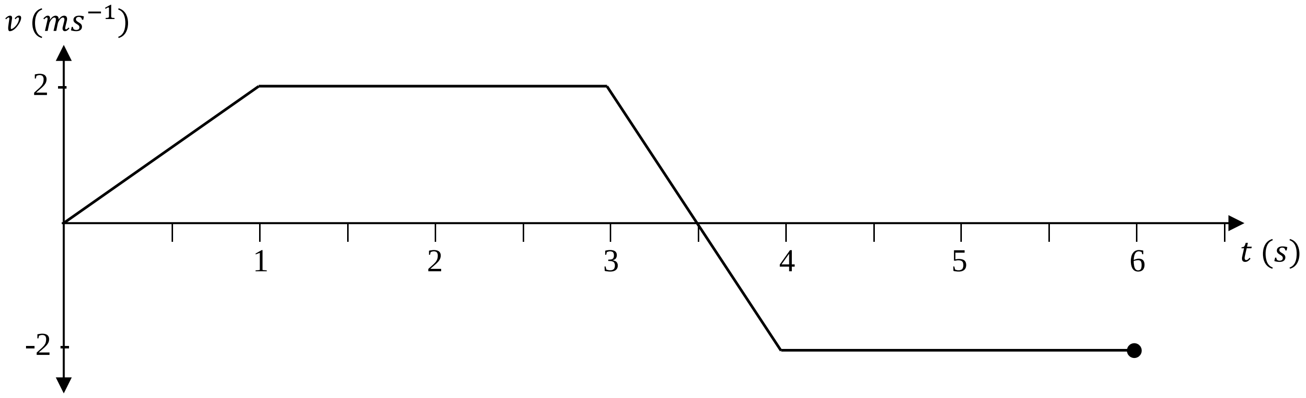


i) Use integration to find the exact area of the shaded region. 2

ii) Use one application of Simpson's Rule to estimate the shaded area. 2

Question 16 (13 marks) Start a New Writing Booklet. **Marks**

a) A particle moving along the x-axis has velocity as shown in the graph below.



The particle is initially 2m to the left of the origin, and it moves for 6 seconds.

i) In which direction does the particle initially move? 1

ii) Determine the instantaneous acceleration at $t = 3.5s$. 1

iii) When is the particle at the origin? 2

iv) When is the particle farthest from the origin, and what is its displacement then? 2

b) Kelsey borrowed \$600 000 for the purchase of a home. The interest rate on the loan is 3.6% per annum, compounded monthly, and the loan term is 30 years. Let $\$A_n$ be the amount owing at the end of n months and $\$M$ be the monthly repayment amount.

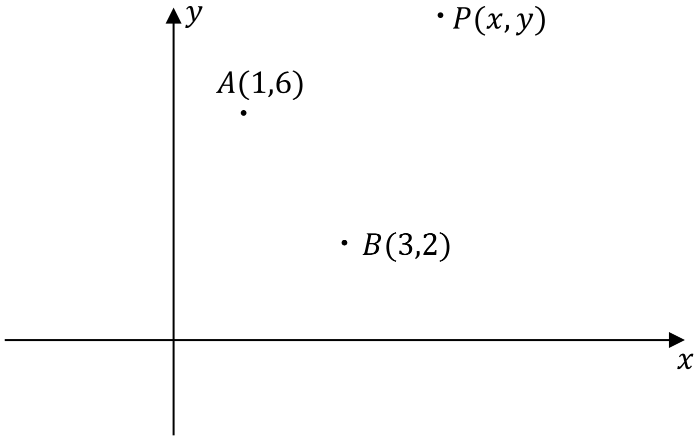
i) Show that $A_2 = 600\ 000(1.003)^2 - M(1.003 + 1)$. 1

ii) Show that $A_n = 600\ 000(1.003)^n - M\frac{1.003^n - 1}{0.003}$. 2

iii) The monthly repayments are set at \$2 728 in order for Kelsey to repay the loan by the end of 30 years. Calculate Kelsey's total saving on the home loan if she decides to pay \$2 800 per month so that the loan is paid out sooner. Assume that the interest rate remains the same. 4

Question 17 (12 marks) Start a New Writing Booklet. **Marks**

a) The points $A\ (1,6)$, $B\ (3,2)$, and $P(x,y)$ are marked on the axes below.



i) Find the gradient of PA in terms of x and y. 1

ii) The point P moves such that the gradient on PB is twice the gradient of PA. 3

Find the values of a, b, and c when the locus is expressed in the form

$$y = a + \frac{b}{x - c}$$

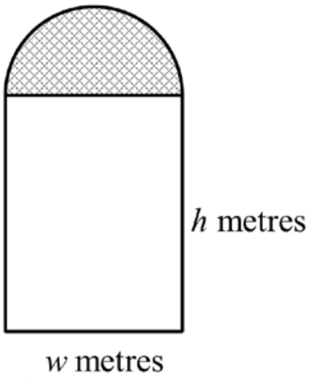
iii) Describe the locus geometrically. 1

Question 17 (Continued). **Marks**

b) A design for a new mirror is shown below. The semi-circular section at the top will be made of decorative pressed tin. The bottom section is rectangular and will be made from mirrored glass.

The frame of the window, including the horizontal piece that separates the two sections, will be made from thin metal which is 24 metres in length.

The width of the mirror is w metres. The height of the rectangular section is h metres.



The company's profit on pressed tin is \$10 per square metre. They make \$60 profit per square metre on their mirrored glass.

Let the total profit per mirror be represented by \$$P$.

i) Show that $h = 12 - w\left(1 + \frac{\pi}{4}\right)$ metres. 2

ii) Show that $P = 720w - 10w^2\left(6 + \frac{11\pi}{8}\right)$ dollars. 2

iii) Find the values of w and h that will maximise the profit made per mirror. 3

END OF EXAMINATION

MATHEMATICS – Multiple Choice

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS

① $3(2y-1)=27$

$2y-1=9$

$2y=10$

$y=5$ $\therefore$ C

② let $u=x$ $\quad v=\sin x$

$u'=1$ $\quad v'=\cos x$

$$\frac{d}{dx}\frac{x}{\sin x}=\frac{vu'-uv'}{v^2}$$

$$=\frac{\sin x-x\cos x}{\sin^2 x}$$

$\therefore$ B

③ $y=e^x-ex$

$y'=e^x-e$

Stationary points when $y'=0$

$e^x-e=0$

$e^x=e$

$\therefore x=1$

when $x=1$, $y=e^1-e(1)$

$=0$

$\therefore (1,0)$ $\therefore$ B

④ The locus of a point moving such that it is equidistant from two fixed points must be a straight line.

$\therefore$ A

MATHEMATICS – Multiple Choice

SUGGESTED SOLUTIONS | **MARKS** | **MARKER'S COMMENTS**

⑤ $|2x-3| \geqslant -3x^2+4$

Solutions where $y=|2x-3|$ is above or equal to $y=-3x^2+4$

From the graphs,

$x \leq -\frac{1}{3}$, $x \geqslant 1$ $\therefore$ B

⑥ $e^{6x}-5e^{3x}+6=0$

let $u=e^{3x}$

$\therefore u^2-5u+6=0$

$(u-3)(u-2)=0$

$u=3 \quad u=2$

$\therefore e^{3x}=3 \quad , \quad e^{3x}=2$

$3x=\ln 3 \quad 3x=\ln 2$

$x=\frac{\ln 3}{3} \quad x=\frac{\ln 2}{3}$ $\therefore$ D

⑦ $3x^2-4x+9=0$

$a=3$

$b=-4$

$c=9$

$\alpha+\beta=\frac{-b}{a}$

$=\frac{4}{3}$

$\alpha\beta=\frac{c}{a}$

$=3$

$\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta$

$=\left(\frac{4}{3}\right)^2-2(3)$

$=\frac{16}{9}-6$

$=-4\frac{2}{9}$ $\therefore$ D

MATHEMATICS – Multiple Choice

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS

⑧ $f'(x) = 2x + 4$

$\therefore f(x) = x^2 + 4x + C$

when $x = -2$, $f(x) = 6$

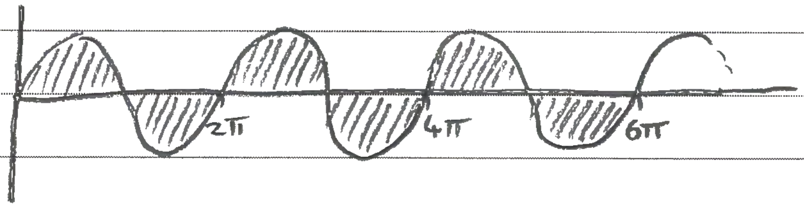
$6 = (-2)^2 + 4(-2) + C$

$= 4 - 8 + C$

$\therefore C = 10$

$\therefore f(x) = x^2 + 4x + 10$ $\therefore$ D

⑨



For any n, areas above and below the x-axis are equal.
$\therefore$ the value of the integral is zero. $\therefore$ D

⑩

$$\int_1^3 (1 + f(x))\,dx = \int_1^3 1\,dx + \int_1^3 f(x)\,dx$$

$$= \int_1^3 1\,dx + \int_0^3 f(x)\,dx - \int_0^1 f(x)\,dx$$

$$= 2 + 5 - 6$$

$$= 1$$ $\therefore$ C

MATHEMATICS TRIAL HSC 2019 – QUESTION 11 (13 marks)

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS

Marker's Comments and ADVICE!!!

Students:

- Should write their calculator display before rounding.
- Must take care to read the question carefully and need to take care when copying into their writing booklet, to avoid transcription errors. These transcriptions errors can make the question easier or harder!
- Are encouraged to show clear substitutions to avoid careless errors. Students should also state the relevant formulas and the information they use to substitute into the formulas when answering parts of questions. Remember you are given a Reference Sheet, USE IT!
- Should take note of the mark value for each part as an indication of the amount of working required. If a question is worth more than 1 mark, working is expected to be shown. The examiners expect more than just a bald answer!

(a) $\sqrt{500} \times 2.6^2 = 151.1581953$ ① for calculator display

$\doteqdot 151.16$ ① for correct answer to 2 dec. pls.

(b) $-1-3x < 11$

$-3x < 12$

$\therefore x > -4$ ① for correct answer

- many students lost the mark because they forgot the -ve sign or forgot to reverse the inequality.
- the importance of clear & logical setting out is stressed here as many solutions included confusing setting out, particularly when dividing by a negative number.

MATHEMATICS TRIAL HSC 2019 – QUESTION 11 (13 marks)

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
(c) $y = 4 - x^2 + 2x \quad \cdots$ ①		
$x + y = 0 \quad \cdots$ ② <u>OR</u>	<u>$y = -x$</u>	
subst. ① into ②		
$x + 4 - x^2 + 2x = 0$		
$4 - x^2 + 3x = 0$		
$x^2 - 3x - 4 = 0$		
$(x-4)(x+1) = 0$		
$\underline{x = 4}$ <u>&</u> $\underline{x = -1}$	②	to this stage
$4 + y = 0$ $-1 + y = 0$		
$\therefore \underline{y = -4}$ $\therefore \underline{y = 1}$	($2\frac{1}{2}$)	to this stage
∴ the points of intersection		
are $(4, -4)$ and $(-1, 1)$.	③	provides correct solution & writing points in the correct form, eg (x, y).

③ provides correct solution

② obtains $x^2 - 3x - 4 = 0$ and solves for x, or equivalent merit.

① attempts to eliminate x or y, or equivalent merit.

Common problems were: • not finding the correct quadratic or incorrectly solving $x^2 - 3x - 4 = 0$.

• not showing the substitution for x to find y and making careless errors.

MATHEMATICS TRIAL HSC 2019 – QUESTION 11 (13 marks)

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
• not writing the points of intersection in the correct form, (x, y).		
(d) $(2\sqrt{3}-1)^2 = a\sqrt{3}+b$ $(2\sqrt{3}-1)(2\sqrt{3}-1) = a\sqrt{3}+b$ $12-2\sqrt{3}-2\sqrt{3}+1 = a\sqrt{3}+b$ $-4\sqrt{3}+13 = a\sqrt{3}+b$ $\therefore a=-4$ & $b=13$	② provides correct solution.	
① for correct expansion leading to $-4\sqrt{3}+13$, or equivalent merit. ① correct a or correct b. Be careful with algebra and using $(a-b)^2 = a^2-2ab+b^2$ expansion. $(\overset{a}{\boxed{2\sqrt{3}}}-\overset{b}{\boxed{1}})^2 = (2\sqrt{3})^2-2(2\sqrt{3})(1)+(1)^2$ $= 12-4\sqrt{3}+1$ $= 13-4\sqrt{3}$ $= -4\sqrt{3}+13$ then equate coefficients.		• be careful with 12+1=13!
• many put b as −1 !		

MATHEMATICS TRIAL HSC 2019 – QUESTION 11 (13 marks)

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS

(e) $T_1 = a = 12$ • Use your Reference Sheet

$S_{20} = 620$ for correct formulas!

(i) $620 = \frac{20}{2}(12 + l)$ • wrong formula, NO marks.

$620 = 10(12+l)$ $T_n = a + (n-1)d$

$62 = 12 + l$ $S_n = \frac{n}{2}(a+l)$

$\therefore l = 50$ — ① provides correct solution.

∴ the 20th term is 50.

(ii) $T_{20} = a + 19d$

$50 = 12 + 19d$ — ① or equivalent merit

$19d = 38$

$\therefore d = 2$ — ①

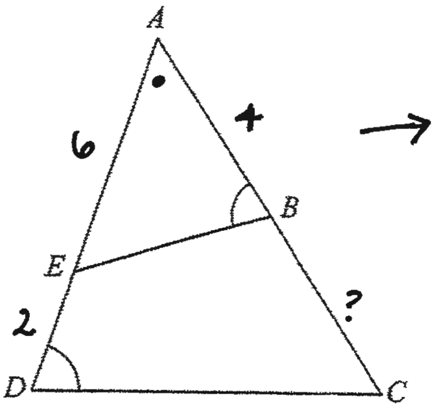
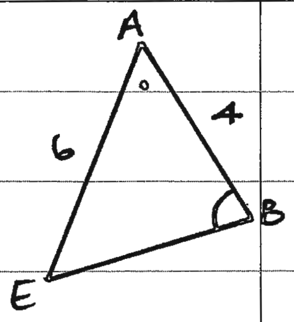
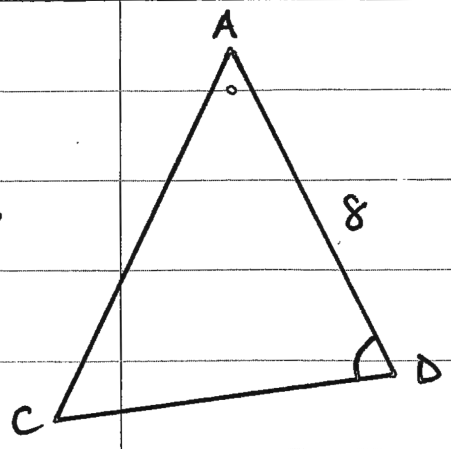
∴ the common difference is 2.

• Poor setting out in this question, eg doing (ii) before (i) and working out $d=2$ twice. Waste of valuable exam time!!!

• Using the correct formula to begin with, eg $S_n = \frac{n}{2}(a+l)$ would have avoided this.

• Careless errors made because students didn't show working, eg substitution into correct formulas.

MATHEMATICS TRIAL HSC 2019 – QUESTION 11 (13 marks)

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
(f)  → 		
$\frac{AC}{AE} = \frac{AD}{AB}$ (corresponding sides of similar triangles are in the same ratio). $\frac{AC}{6} = \frac{8}{4}$ $AC = 2 \times 6$ $\therefore AC = 12$ — ① $BC = AC - AB$ $= 12 - 4$ $\therefore BC = 8$ — ①		• ② provides correct solution. • ① attempts to use the correct ratio to find a correct equation involving AC, or equivalent merit.
OR $\frac{AC}{AE} = \frac{AD}{AB}$ $\frac{4 + BC}{6} = \frac{8}{4}$ — ① $4 + BC = 2 \times 6$ $BC = 12 - 4$ $\therefore BC = 8$ — ①		NOTE: $AC = AB + BC$ $\therefore AC = 4 + BC$ $BC = AC - AB$

MATHEMATICS TRIAL HSC 2019 – QUESTION 11 (13 marks)

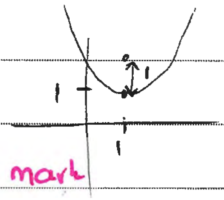
SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
OR $\dfrac{AB}{AE} = \dfrac{AD}{AC}$		
$\dfrac{AB}{AE} = \dfrac{AD}{4+BC}$		
$\dfrac{4}{6} = \dfrac{8}{4+BC}$		
$\dfrac{4+BC}{8} = \dfrac{6}{4}$		
$4+BC = \dfrac{6}{4} \times 8$		
$4+BC = 12$		
$BC = 12-4$		
$\therefore BC = 8$		

- Students that seperated their triangles and re-drew them in their answer booklets were more successful in achieving the correct answer.
- Better solutions clearly showed the ratio of corresponding sides, $\dfrac{AC}{AE} = \dfrac{AD}{AB}$ before substituting the lengths given in the question.
- If you are going to introduce a variable, say "x", you need to either state that $x = BC$ or re-draw the diagram in your answer booklet clearly labelling x !

MATHEMATICS – QUESTION 12

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
a) i) $\frac{d}{dx}(4x^3-x)^7$ $= 7(4x^3-x)^6 \times \boxed{(12x^2-1)}$ $= 7(12x^2-1)(4x^3-x)^6$ OR $(84x^2-7)(4x^3-x)^6$		Quite a few number of students forget to use the grouping symbols. Several uses the brackets inconsistently.
ii) $\frac{d}{dx}(e^x\sin x)$ $u=e^x \quad v=\sin x$ $u'=e^x \quad v'=\cos x$ $= vu'+uv'$ $= \sin x(e^x) + e^x(\cos x)$ OR $e^x(\sin x+\cos x)$ } 1 mark	1	mark
iii) $\frac{d}{dx}\ln(\sqrt{x})$ $\frac{d}{dx}\ln[f(x)] = \frac{f'(x)}{f(x)}$ $f(x) = \sqrt{x}$ $= x^{\frac{1}{2}}$ $f'(x) = \frac{1}{2}x^{-\frac{1}{2}}$ $= \frac{1}{2}\times\frac{1}{\sqrt{x}}$ $= \frac{1}{2\sqrt{x}}$ ① mark $= \frac{\frac{1}{2\sqrt{x}}}{\sqrt{x}}$ ← $\frac{1}{2}$ mark $= \frac{1}{2\sqrt{x}} \times \frac{1}{\sqrt{x}}$ $= \frac{1}{2x}$ ← $\frac{1}{2}$ mark OR $\frac{d}{dx}\ln x^{\frac{1}{2}} = \frac{d}{dx}\left(\frac{1}{2}\ln x\right)$ ← 1 mark $= \frac{1}{2}\times\frac{1}{x}$ $= \frac{1}{2x}$ ← 1 mark		Students are more successful when applying the method on the right. Students with weak indices and algebraic skills struggles to set out their work clearly and simplify.

MATHEMATICS – QUESTION 12

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
b) $4y = x^2 - 2x + 5$ i) Arrange it in the form $(x-h)^2 = 4a(y-k)$ to get vertex (h,k) $4y = (x^2 - 2x + 1) + 4$ $4y = (x-1)^2 + 4$ $4y - 4 = (x-1)^2$ } 1 mark for completing the square $(x-1)^2 = 4y - 4$ $(x-1)^2 = 4(y-1)$ } ← $\frac{1}{2}$ $\therefore$ vertex is $(1,1)$ ← $\frac{1}{2}$		i) Most knows the process but some did not complete the square correctly. Some uses the axis of symmetry to find the vertex successfully.
ii) focus $(h, k+a)$ where a is the focal length  $4a = 4$ $a = 1$ ← 1 mark $\therefore$ focus $(1,2)$ ← 1 mark		ii) Students who states the focal length, and either a quick sketch of diagram or realised that the curve is concave up, has no trouble giving the coordinates of the focus.

MATHEMATICS – QUESTION 12

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
c) i) $y = -2x^2 + 32$ B is a point on the curve that also cuts the x-axis. Substitute $(4, 0)$ into $y = -2x^2 + 32$ LHS $= 0$ RHS $= -2(4)^2 + 32$ $= -2(16) + 32$ $= -32 + 32$ $= 0$ $=$ LHS ∴ coordinates of B are $(4, 0)$	←	1 mark
OR $0 = -2x^2 + 32$ (x-intercepts) $2x^2 = 32$ $x^2 = 16$ $x = \pm 4$ ← $\frac{1}{2}$ mark on the diagram, B has positive x-values ← $\frac{1}{2}$ mark ∴ B$(4, 0)$		Must justify why $x = 4$ for x coordinate of B to get the other $\frac{1}{2}$ mark.
ii) $V = \pi\int_a^b y^2\,dx$ ① $= \pi\int_0^4 (1024 - 128x^2 + 4x^4)\,dx$ $= \pi\left[1024x - \frac{128x^3}{3} + \frac{4x^5}{5}\right]_0^4$ ① integration $= \pi\left(1024 \times 4 - \frac{128(4^3)}{3} + \frac{4}{5}(4^5) - 0\right)$ $= \pi\left(\frac{32768}{15}\right)$ $y = -2x^2 + 32$ $y^2 = (32 - 2x^2)^2$ $= 1024 - 128x^2 + 4x^4$		Many students cannot get the first line of expression correct for volume. All done well in carrying out integration then finding the volume.

$= \frac{32768\pi}{15}$ u^3 ①

MATHEMATICS – QUESTION 13

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
a) i $f(x) = 2 + 9x - \frac{x^3}{3}$		
$f'(x) = 9 - x^2$		
Stationary points when $f'(x) = 0$		
$9 - x^2 = 0$		
$x = 3, x = -3$	1	
$f''(x) = -2x$		
$\therefore f''(3) = -6$		
< 0 ∴ concave down	1	
$f''(-3) = 6$		
> 0 ∴ concave up		
$f(3) = 2 + 27 - 9$		
$= 20$		This question needs a concluding statement; don't make the examiner search through your working for the answers.
$f(-3) = 2 - 27 + 9$		
$= -16$		
∴ $(3, 20)$ is a maximum turning point.	1	
$(-3, -16)$ is a minimum turning point.		
ii) Possible points of inflexion when		To find a point of inflexion, <u>two</u> things must be satisfied:
$f''(x) = 0$		
$-2x = 0$		
$x = 0$		
When $x = 0, y = 2$ ∴ $(0, 2)$	1	a) $f''(x) = 0$, and b) concavity changes about the point.

MATHEMATICS – QUESTION 13 (continued)

SUGGESTED SOLUTIONS | **MARKS** | **MARKER'S COMMENTS**

Test concavity:

x	-1	0	1
$f''(x)$	2	0	-2
	∪	*	∩

Since concavity changes, $(0,2)$ is a point of inflexion.

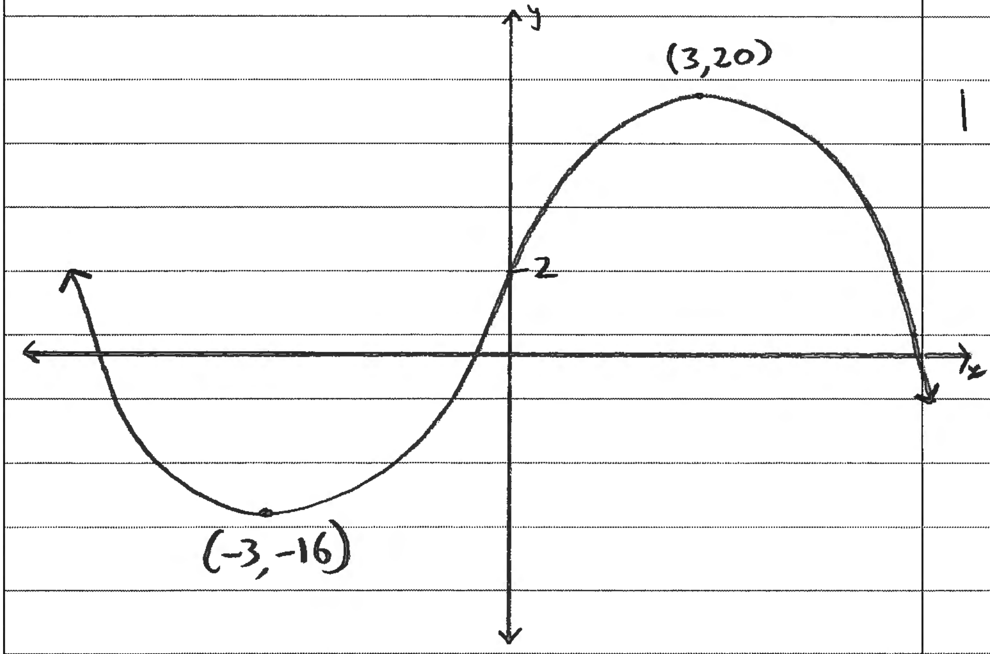
1

<u>NB</u>: A table such as below, which confuses concavity with gradient, is <u>wrong</u>.

x	-1	0	1
$f''(x)$	2	0	-2
	/	—	\

iii)

1 All points correctly drawn and labelled



1 Overall shape

Your graph should be one smooth curve, with no linear sections. Note also that points of inflexion are not always horizontal.

MATHEMATICS – QUESTION 13 (continued)

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
b) i		
$\begin{array}{c\|c\|c\|c\|c\|c} x & 0 & 0.5 & 1 & 1.5 & 2 \\ \hline y & 1.732 & 2.058 & 2.646 & 3.630 & 5.196 \end{array}$	1	
ii		
$\int_0^2 \sqrt{5^x+2}\,dx \doteqdot \frac{0.5}{2}\left[1.732+5.196+2(2.058+2.646+3.630)\right]$		1
$= 5.899$	1	
NB: you can only earn marks for this question if you use the trapezoidal rule.		
c) i $l=r\theta$ $A=\frac{1}{2}r^2\theta$		
$8=r\theta$ ① $40=\frac{1}{2}r^2\theta$	1	for either
$r^2\theta=80$ ②		equation
② ÷ ①		
$\frac{r^2\theta}{r\theta}=\frac{80}{8}$		
$r=10$ $\therefore r=10\text{cm}$	1	
ii sub $r=10$ into ①		
$8=10\theta$		
$\theta=\frac{4}{5}$	1	
Note that this measurement is in radians. Answers like $\frac{4^\circ}{5}$, or $\frac{4\pi}{5}$, attracted half marks.		

MATHEMATICS – QUESTION 14

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
a)(i) $r = \frac{T_2}{T_1}$ $= \frac{\frac{\cancel{e}-1}{\cancel{e}} \times \cancel{e}}{(e-1) \times e}$ $= \frac{1}{e} \approx 0{\cdot}368$ (to 3d·p) $\therefore \quad -1 < r < 1 \quad$ or $\quad \lvert r \rvert < 1$ $\therefore$ There is a limiting sum	①	students need to learn that $-1 < r < 1$ for a limiting sum to exist.
(ii) $S_\infty = \frac{a}{1-r}$ $= \frac{e-1 \quad \times e}{1 - \frac{1}{e} \quad \times e}$	①	subbing into $S_\infty = \frac{a}{1-r}$
$= \frac{e(e-1)}{e-1}$ $= e$	①	simplifying.
b) $\int (\sin 3x + 2) \quad dx$ $= \underbrace{-\frac{1}{3} \cos 3x}_{①} + \underset{\overset{\uparrow}{(\frac{1}{2})}}{2x} + \underset{\overset{\uparrow}{(\frac{1}{2})}}{C}$ Many students thought the question meant $\int \sin(3x+2)$ instead of $\int (\sin 3x + 2) dx$		

MATHEMATICS – QUESTION 14

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
c) $\int_2^3 \frac{x^3}{x^4-2}\,dx$		
$= \frac{1}{4}\int_2^3 \frac{4x^3}{x^4-2}\,dx$	①	
$= \frac{1}{4}\left[\ln(x^4-2)\right]_2^3$	½	
$= \frac{1}{4}\left[\ln(3^4-2) - \ln(2^4-2)\right]$		
$= \frac{1}{4}(\ln 79 - \ln 14)$ $= \frac{1}{4}\ln\frac{79}{14}$	½	
d) (i) $\theta = 360 - 60 - 200$ $= 100^\circ$		Too much time wasted with reasoning. Question did no ask for reason
(ii) $c^2 = a^2 + b^2 - 2ab\cos C$ $PR = \sqrt{5^2 + 2^2 - 2(5)(2)\cos 100}$	①	
$PR = 5.70$	①	
Many students had their calculators in radian mode instead of degrees!		

MATHEMATICS – QUESTION 14

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
e) $2\sin^3 x - 3\sin^2 x - 2\sin x = 0$ $0 \le x \le 2\pi$		
$\sin x(2\sin^2 x - 3\sin x - 2) = 0$		
$\sin x = 0$ $2\sin^2 x - 3\sin x - 2 = 0$		
let $m = \sin\theta$		
$2m^2 - 3m - 2 = 0$		
$(2m+1)(m-2) = 0$		1 mark
$2m+1 = 0$ or $m-2 = 0$		
$m = -\frac{1}{2}$ $m = 2$		
$\sin x = 0$ ∴ $\sin x = -\frac{1}{2}$ $\sin x \neq 2$		
$x = 0, \pi, 2\pi$ acute $x = \frac{\pi}{6}$ as		encourage students to justify why they reject $\sin x = 2$. (took off ½ if not stated)
$-1 \le \sin x \le 1$		
$x = \pi + \frac{\pi}{6}, 2\pi - \pi/6$		
$= \frac{7\pi}{6}, \frac{11\pi}{6}$ — ①		
① (minus $\frac{1}{2}$ if one missing, zero if two missing)		
$x = 0, \pi, 2\pi, \frac{7\pi}{6}, \frac{11\pi}{6}$		took 1 mark off if final answer in degrees

MATHEMATICS ~~EXTENSION 1~~ – QUESTION 15

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
Question 15 (contd)		
(b)(ii) $v = -7(t+1)^{-1}$		
$a = \frac{d}{dx} -7(t+1)^{-1}$		
$= -7 \times -1 \times (t+1)^{-2}$		
$= \frac{7}{(t+1)^2}$	1	
$t > 0$, $(t+1) > 0$ & $(t+1)^2 > 0$		
∴ acceleration is always positive.		
(iii) for particle to be at rest $v = 0$		
$v = \frac{-7}{(t+1)}$		
as $t > 0$, $(t+1) > 0$	½	
$v \neq 0$		
∴ Particle is never at rest.	½	
(c)(i) $A = \int_1^4 (5-x) - \frac{4}{x}\, dx$	½	
$= \left[5x - \frac{x^2}{2} - 4\ln(x)\right]_1^4$	½	
$= (20 - 8 - 4\ln 4) - (5 - \frac{1}{2} - 0)$		
$= 7\frac{1}{2} - 4\ln 4$ units^2	1	
$\doteqdot 1.95\ u^2$ (2 dp)		
(ii) $\int_1^4 5 - x - \frac{4}{x}\, dx$		

x	1	2.5	4
fx	0	$\frac{9}{10}$	0

$\Rightarrow$ ½

$\doteqdot \frac{1.5}{3}(0 + 4 \times \frac{9}{10} + 0)$	½	
$\doteqdot \frac{9}{5}\ u^2$	1	
$\doteqdot 1.8\ u^2$		

MATHEMATICS ~~EXTENSION 1~~ – QUESTION 15

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
Question 15		
(a)(i) $\frac{d}{dx} x\ln x - x$ $\quad$ $u = x$, $u' = 1$ | $v = \ln x$, $v' = \frac{1}{x}$		
$= 1\times\ln x + x\times\frac{1}{x} - 1$		
$= \ln x + 1 - 1$	①	
$= \ln x$		
(ii) $\int_2^4 \ln x\,dx$		
$= \left[x\ln x - x\right]_2^4$	①	Full marks given for
$= (4\ln 4 - 4) - (2\ln 2 - 2)$		$\ln 64 - 2$
$= 4\ln 4 - 2\ln 2 - 2$	①	$2(3\ln 2 - 1)$
$= 8\ln 2 - 2\ln 2 - 2$		$3\ln 4 - 2$
$= 6\ln 2 - 2$		
$\doteqdot 2.16$		
(b) $v = -\frac{7}{t+1}$		
$x = \int \frac{-7}{t+1}\,dt$		
$= -7\int \frac{1}{t+1}\,dt$		
$= -7\ln(t+1) + c$	①	
when $t = 0$, $x = 8$		
$8 = -7\ln(1) + c$		
$8 = c$		
$\therefore x = -7\ln(t+1) + 8$	①	
When $t = 3$		
$x = -7\ln(3+1) + 8$		
$= -1.70406\ldots$		$-\frac{1}{2}$ mark for
$\doteqdot -1.70$ (2dp)	①	rounding error

MATHEMATICS – QUESTION 16 - page1

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
a)i) Preferred answer: Right Other acceptable answers: - towards the origin - in a positive direction.	1	- Most students answered correctly. - Forwards or upwards were not acceptable.
a) ii) Gradient at $t=3.5$ is $-4\ ms^{-2}$	1	- Answered poorly. - Students did not understand to find gradient. - Many said $a=0$
a)iii) Between $t=0$ and $t=1.5s$, the area under the curve is 2. Therefore at $t=1.5s$ the particle has travelled 2m right and is at the origin. Then between $t=1.5s$ and $t=3.5s$, the particle travels 3.5m right of the origin. Between $t=3.5s$ and $t=5.5s$, the particle travels 3.5m to the left (calculated using the area under the curve) ∴ The particle is at the origin at $t=1.5s$ and $t=5.5s$.	1 mark each	- Answered poorly - many said $t=1$, incorrectly calculating the area. - Many other incorrect versions. - If a students first answer was incorrect, but their second answer was correct after assuming the first, then they received one mark.

MATHEMATICS EXTENSION I – QUESTION 16 – page 2

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
a) iv) As discussed above, the particle is farthest from the origin at $t = 3.5\text{s}$ and is 3.5m from the origin – some students understood the concept, but forgot the particle was initially 2m to the left of the origin, and so answered 5.5m.	1 mark each	– Answered poorly – Some students wrote $t=0$ and $t=3.5\text{s}$ – and so received a half mark for the time. Students need to learn that $v=0$ signals that the particle stops moving in that direction, and does not alone indicate displacement
b) i) $A_1 = 600000 \times 1.003 - m$	½	
$A_2 = A_1 \times 1.003 - m$ OR $A_2 = (600000 \times 1.003 - m)1.003 - m$ $= 600000(1.003)^2 - m(1.003 + 1)$	½	
b) ii) $A_3 = 600000 \times 1.003^3 - m(1.003^2 + 1.003 + 1)$ $\vdots$ $\therefore A_n = 600000 \times 1.003^n - m(\underbrace{1 + 1.003 + 1.003^2 + \ldots + 1.003^{n-1}}_{\text{GP with } a=1,\ r=1.003 \text{ and } n \text{ terms}})$ Need to show this long version of A_n. Could not prove from A_3.	1 mark	– No half marks given in this part. Some students had n instead of $n-1$.
$S_n = \dfrac{1(1.003^n - 1)}{1.003 - 1}$	1 mark as long as finish accurately	* Some students incorrectly had $1 - 1.003$ on denominator, and lost the mark.

MATHEMATICS EXTENSION I – QUESTION 16 - page 3

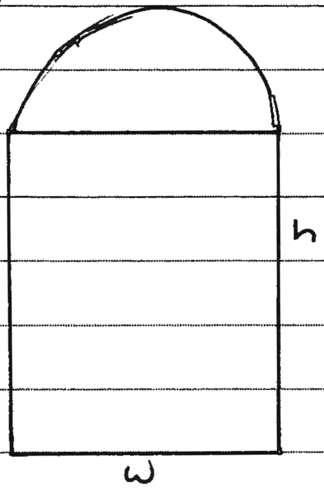
SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
$\therefore A_n = 600000 \times 1.003^n - m\left(\frac{1.003^n - 1}{0.003}\right)$		
b) iii) Original total repayments = $30 \times 12 \times 2728 = \$982\,080$	1	
If m changes to \$2800, need to find n when $A_n = 0$		
$A_n = 600000(1.003)^n - 2800\left(\frac{1.003^n - 1}{0.003}\right) = 0$		
$\therefore 2800\left(\frac{1.003^n - 1}{0.003}\right) = 600000(1.003)^n$	1	$\frac{1}{2}$ mark awarded if used trial and error instead of $A_n = 0$ $\frac{1}{2}$ if power was incorrectly n-1.
$2800(1.003^n) - 2800 = 1800 \times 1.003^n$ $1000(1.003^n) = 2800$ $1.003^n = 2.8$ $n = \log_{1.003} 2.8$		
$= \frac{\ln 2.8}{\ln 1.003} = 343.7210251$	1	
$\therefore$ 344 payments of \$2800 = \$963200		
Saving = \$982080 − 963200 = \$18880	1	−lost $\frac{1}{2}$ marks if didn't round up to 344 payments.

MATHEMATICS – QUESTION 17

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
a) i) $m_{PA} = \frac{y-6}{x-1}$ or $\frac{6-y}{1-x}$	1	Generally well attempted
ii) $m_{PB} = \frac{y-2}{x-3}$ or $\frac{2-y}{3-x}$	½	Well done, except a few had the formula as $\frac{\text{run}}{\text{rise}}$
For locus, $m_{PB} = 2m_{PA}$		Some students 'switched' the 2. A number of students multiplied both the numerator and the denominator by 2
$\frac{y-2}{x-3} = 2\times\frac{y-6}{x-1}$	½	
$(y-2)(x-1) = 2(y-6)(x-3)$		
$xy-y-2x+2 = 2xy-6y-12x+36$	½	
$5y+10x-xy-34=0$		
$5y-xy = 34-10x$		
$y(5-x) = 34-10x$		
$y = \frac{34-10x}{5-x}$		
$= \frac{10x-34}{x-5}$		
$= \frac{10x-50+50-34}{x-5}$	1	Very few students were able to use this 'split strategy'
$= \frac{10x-50}{x-5} + \frac{16}{x-5}$		
$= \frac{10\cancel{(x-5)}^{1}}{\cancel{x-5}_{1}} + \frac{16}{x-5}$		Students need to improve on their application skills.
$= 10 + \frac{16}{x-5}$		
Compare with $a+\frac{b}{x-c}$	½	

$\therefore a=10,\ b=16,\ c=5$

MATHEMATICS – QUESTION 17 continued

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
iii) locus is a hyperbola	$1/2$	Not well
with a vertical asymptote	$1/2$	attempted
of $x=5$ (or shifted right 5 units or		as some
any other relevant feature		students were
such as horizontal asymptote		not able to
of $y=10$ (or shifted 10		
units up), or stretched vertically by 16.		
b)		
		
$P = 2h + 2w + \frac{1}{2} \times 2\pi \times \frac{w}{2}$		Quite well done
$24 = 2h + 2w + \frac{w\pi}{2}$	1	
		Since it is a
$12 = h + w + \frac{w\pi}{4}$		'show' question
and		every step had
$\therefore h = 12 - w - \frac{w\pi}{4}$	1	to be displayed
		explicitly.
$h = 12 - w\left(1 + \pi/4\right)$		If students
		make a mistake
		in any of
		the steps and
		cannot arrive
		at the required
		answer, they
		should make
		an attempt to
		rectify their
		errors.

MATHEMATICS – QUESTION 17 Continued

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
ii) $P = 10 \times \frac{1}{2} \times \pi \times \left(\frac{w}{2}\right)^2 + 60 \times h \times w$	1	If the students did not display this step, they were not awarded 1 mark. No ½ marks were awarded simply because either you clearly understand the question or you don't. In 'show' questions, students should be encouraged to show <u>all</u> the steps.
$= \frac{5\pi w^2}{4} + 60w\left(12 - w\left(1 + \pi/4\right)\right)$	½	
$= \frac{5\pi w^2}{4} + 720w - 60w^2 - \frac{60\pi w^2}{4}$		
$= 720w - 60w^2 - \frac{55\pi w^2}{4}$		
$= 720w - 10w^2\left(6 + \frac{5.5\pi}{4}\right)$	½	
$= 720w - 10w^2\left(6 + \frac{11\pi}{8}\right)$		
iii) $P = 720w - 10w^2\left(6 + \frac{11\pi}{8}\right)$		Students should be encouraged to write decimal answers in these sort of questions (especially in the case of h)
$\frac{dP}{dw} = 720 - 10 \times 2 \times \left(6 + \frac{11\pi}{8}\right)w$ Max when $\frac{dP}{dw} = 0$	½	
$720 = 20w\left(6 + \frac{11\pi}{8}\right)$		
$\therefore w = \frac{720}{20\left(6 + \frac{11\pi}{8}\right)} = 3.48847\ldots \doteqdot 3.49\text{m}$ (2dp)	1	
$= \frac{36}{6 + 11\pi/8}$		
$h = 12 - \frac{36}{6 + \frac{11\pi}{8}}\left(1 + \frac{\pi}{4}\right) = 5.771679\ldots \doteqdot 5.77\text{m}$ (2dp)	1	When using the 'gradient' function test, all values need to be calculated.
$\frac{d^2P}{dw^2} = -20\left(6 + \frac{11\pi}{8}\right)$	½	

$< 0 \quad \therefore$ max at $w \doteqdot 3.49\text{m}$ and $h \doteqdot 5.77\text{m}$

Or using gradient function