Student Number:
Teacher:

St George Girls High School

Mathematics Advanced

2020 Trial HSC Examination

General Instructions

- Reading time – 10 minutes
- Working Time – 3 hours
- Write using black pen
- Calculators approved by NESA may be used.
- A reference sheet is provided.
- For questions in **Section I,** use the multiple-choice answer sheet provided.
- For questions in **Section II:**
  - Answer the questions in the space provided.
  - Show relevant mathematical reasoning and/or calculations.
  - Extra writing space is provided at the back of this booklet. If you use this space, clearly indicate which question you are answering.
  - Marks may not be awarded for incomplete or poorly presented solutions

Total marks: 100

Section I – 10 marks (pages 3 – 8**)**

- Attempt Questions 1 - 10
- Allow about 15 minutes for this section

Section II – 90 marks (pages 9 – 30**)**

- Attempt Questions 11 – 33
- Allow about 2 hour and 45 minutes for this section

Q1 – Q10	/10
Q11 – Q15	/15
Q16 – Q19	/15
Q20	/2
Q21 – Q25	/15
Q26 – Q29	/15
Q30	/8
Q31	/9
Q32	/5
Q33	/6
Total	**/100**
	%

Section I

10 marks
Attempt Questions 1 - 10
Allow about 15 minutes for this section.

Use the multiple-choice answer sheet for Questions 1-10.

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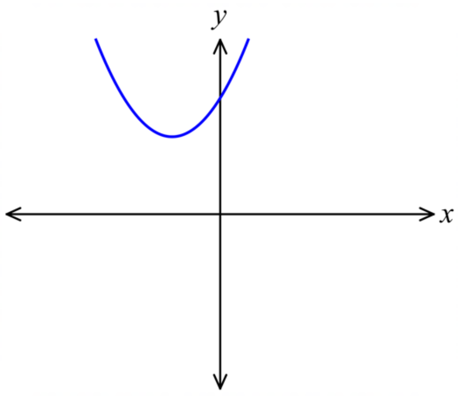
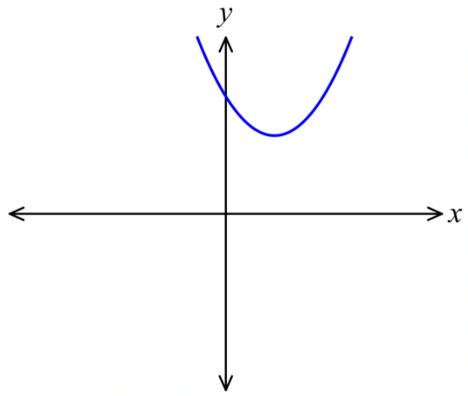
1. Which diagram best shows the graph of the parabola $y = 2 - (x + 1)^2$?

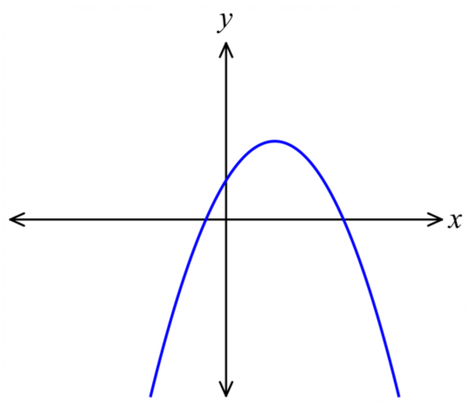
(A)



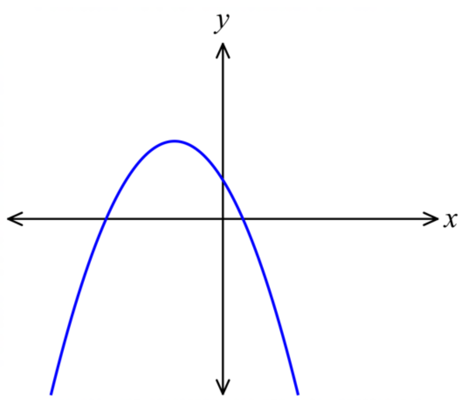
(B)



(C)

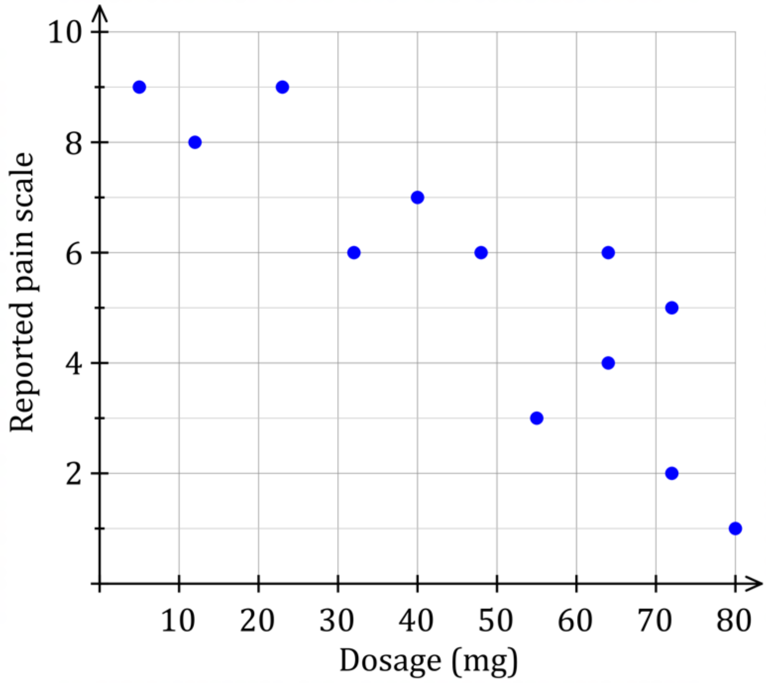


(D)



Section I (cont'd)

2. A scatterplot of pain (as reported by patients) compared to the dosage (in mg) of a drug is shown below.



How could you describe the correlation between the pain and the dosage?

(A) A moderate negative correlation

(B) A moderate positive correlation

(C) A weak positive correlation.

(D) No correlation.

3. Robina threw an ordinary die numbered 1 to 6 twice. What is the probability that the second number shown on the die is more than the first?

(A) $\frac{5}{6}$

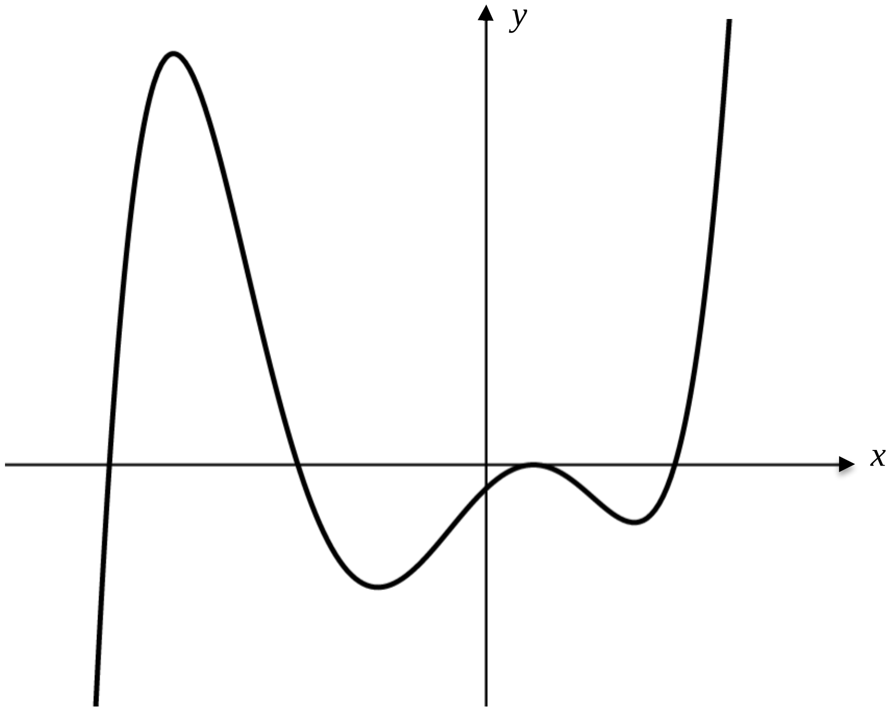
(B) $\frac{1}{6}$

(C) $\frac{5}{12}$

(D) $\frac{1}{2}$

Section I (cont'd)

4. The graph of $y = f(x)$ is as shown below.



What is the number of stationary points and points of inflection in the graph of $y = f(x)$?

(A) 3 stationary points and 3 points of inflection

(B) 3 stationary points and 4 points of inflection

(C) 4 stationary points and 3 points of inflection

(D) 4 stationary points and 4 points of inflection

5. What is the amplitude and period for the curve $y = -1 + 3\sin\ 2x$?

(A) Amplitude $= 3$, Period $= 2\pi$

(B) Amplitude $= 2$, Period $= 2\pi$

(C) Amplitude $= 3$, Period $= \pi$

(D) Amplitude $= 2$, Period $= \pi$

Section I (cont'd)

6. $y = \log_a x^{-3}$ is equivalent to:

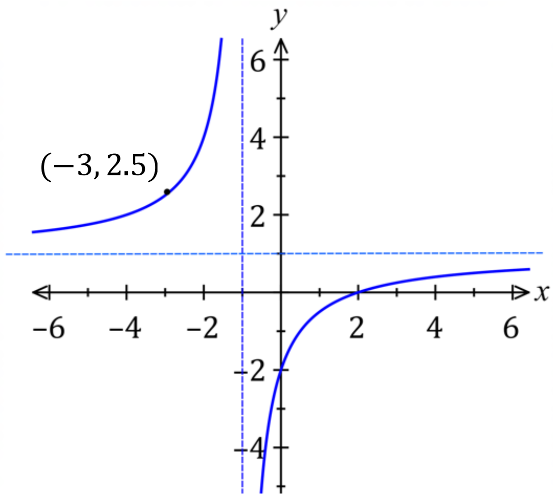
(A) $x = a^{\frac{y}{3}}$

(B) $x = a^{-\frac{y}{3}}$

(C) $x = a^{3y}$

(D) $x = a^{-3y}$

7. The diagram below shows the graph of $y = f(x)$.



Which of the following statements is false?

(A) The horizontal asymptote is $y = 1$

(B) The curve is continuous

(C) The curve is concave up for $x < -1$

(D) The equation of the function is $y = \frac{x-2}{x+1}$

Section I (cont'd)

8. Which of the following is the derivative of $y = \dfrac{3x^2 - 4}{4x^2 - 3}$?

(A) $\dfrac{14\,x}{(4\,x^2 - 3)^2}$

(B) $\dfrac{14\,x}{16\,x^4 + 9}$

(C) $\dfrac{48\,x^3 - 50\,x}{(4x^2 - 3)^2}$

(D) $\dfrac{48\,x^3 - 50\,x}{16\,x^4 + 9}$

9. The discrete random variable X has the following probability distribution.

X	0	1	2
$P(X = x)$	a	b	0.3

Given that $E(X) = 0.8$, then

(A) $a = 0.5$ and $b = 0.2$

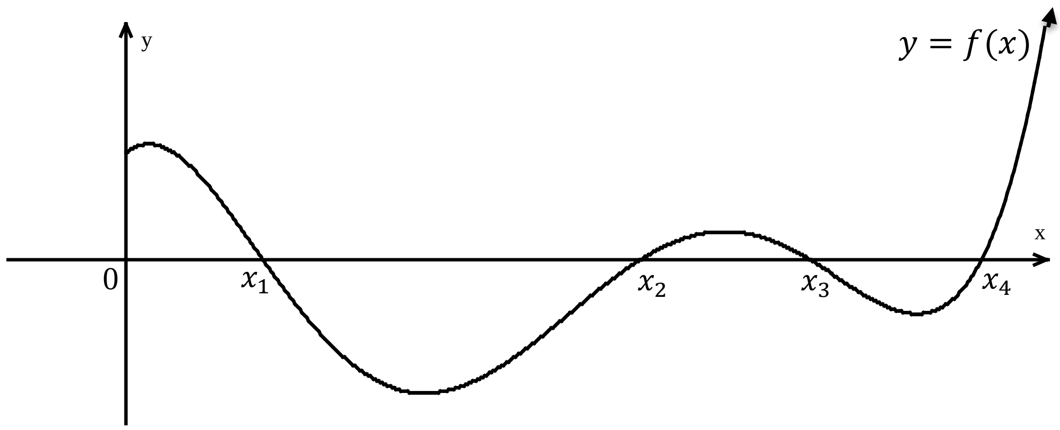
(B) $a = 0.2$ and $b = 0.5$

(C) $a = 0.3$ and $b = 0.4$

(D) $a = 0.3$ and $b = 0.2$

Section I (cont'd)

10. The graph of the function $y = f(x)$ is shown below.



Given that $\int_0^{x_2} f(x)\,dx = -3$, $\int_0^{x_3} f(x)\,dx = -1$ and $\int_0^{x_4} f(x)\,dx = -4$,

what is the value of $\int_{x_2}^{x_4} f(x)\,dx$?

(A) -1

(B) -2

(C) – 3

(D) – 4

End of Section I

Section II

90 marks
Attempt Questions 11 – 33
Allow about 2 hours and 45 minutes for this section.

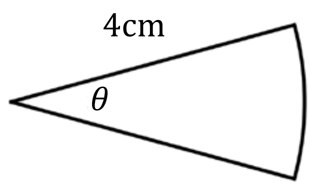
Your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (2 marks) **Marks**

Find $\int 6x^2 + 2 + \frac{1}{\sqrt{x}}\,dx$, giving each term in its simplest form. **2**

Question 12 (2 marks)

A sector has a radius of 4 cm and an area of $\frac{8\pi}{3}$ cm^2. **2**
Find the angle θ.



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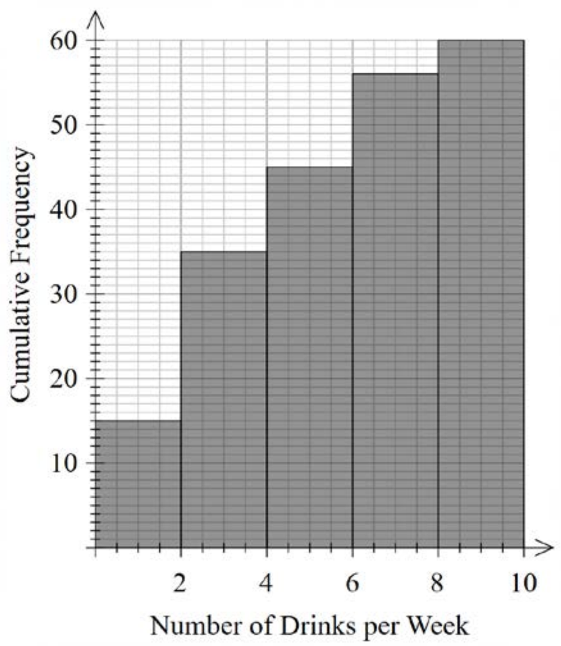
Question 13 (6 marks) **Marks**

Lizzie is a university student collecting data about her classmates' alcohol consumption. A question in her survey asks the respondent to indicate how many alcoholic drinks they typically consume each week. The data collected is displayed in the cumulative frequency histogram.



(a) How many drinks per week do each of the students in the top decile of respondents consume? **2**

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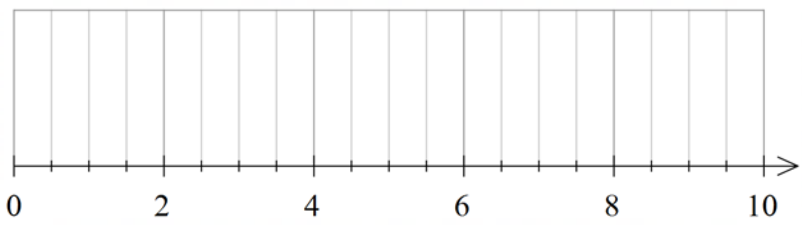
(b) Construct a box-plot to represent the data. **3**



(c) Comment on the distribution of the data. **1**

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Question 14 (4 marks) **Marks**

The random variable X has this probability distribution.

X	0	1	2	3	4
$P\,(X = x)$	0.1	0.2	0.4	0.2	0.1

(a) Find $P(1 < X \leq 3)$. **1**

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(b) Find the expected value of X, showing all working. **1**

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(c) Find the variance of X, showing all working. **2**

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Question 15 (1 mark)

Find the derivative of $16 \sin 5x$. **1**

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Question 16 (4 marks) **Marks**

A curve with the equation $y = f(x)$, has $\frac{dy}{dx} = x^3 + 2x - 7$.

(a) Find $\frac{d^2y}{dx^2}$. **1**

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(b) Show that $\frac{d^2y}{dx^2} \geq 2$ for all values of x. **1**

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(c) The point $P(2,4)$ lies on the curve. Find an equation in general form, for the normal to the curve at point P. **2**

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Question 17 (3 marks) **Marks**

Find the exact solutions to the equation $1 + 2\sin 3x = 2$ in the domain $0 \leq x \leq \pi$. **3**

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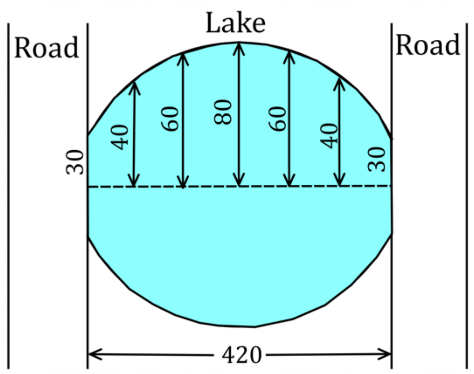
Question 18 (3 marks)

A symmetrical lake has two roads, 420 metres apart, forming two of its sides. Equally spaced measurements of the lake, in metres, are shown on the diagram.



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Use the trapezoidal rule to estimate the area of the lake. **3**

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Question 19 (5 marks) **Marks**

(a) Show that the derivative of $\ln\left(\dfrac{3+x}{3-x}\right)$ is $\dfrac{6}{9-x^2}$. **3**

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(b) Hence show that $\displaystyle\int_1^2 \frac{1}{9-x^2}\,dx = \frac{1}{6}\ln\frac{5}{2}$. **2**

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Question 20 (2 marks) **Marks**

Simplify $\dfrac{\tan\theta\sec\theta}{1+\tan^2\theta}$. **2**

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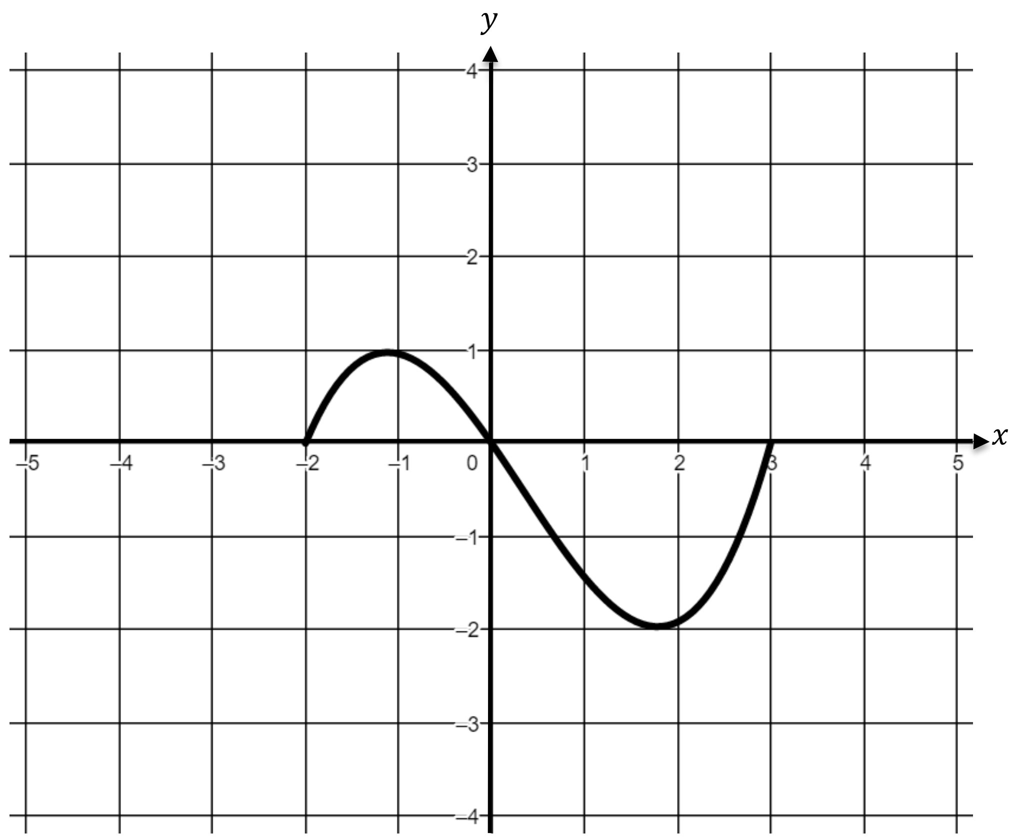
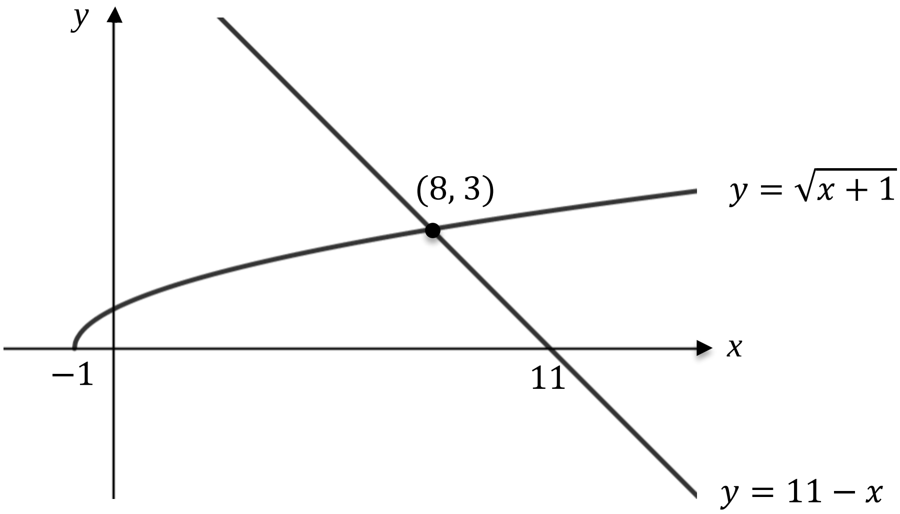
Question 21 (2 marks)

The graph below shows $y = f(x)$. **2**

On the same graph sketch $y = -2\,f(x-1)$.



Question 22 (3 marks) **Marks**



Calculate the area bounded by the curves $y = \sqrt{x+1}$ and $y = 11 - x$ and the x-axis. **3**

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Question 23 (2 marks) **Marks**

Evaluate $\int_{1}^{e^3} \frac{5}{x}\, dx$. **2**

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Question 24 (3 marks)

Consider the functions $f(x) = \ln(x)$ and $g(x) = e^{2x+1}$. **2**

(a) Show that the composite function, $g\big(f(x)\big)$, is a parabola.

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(b) Find, in interval notation, the natural domain of the composite function. **1**

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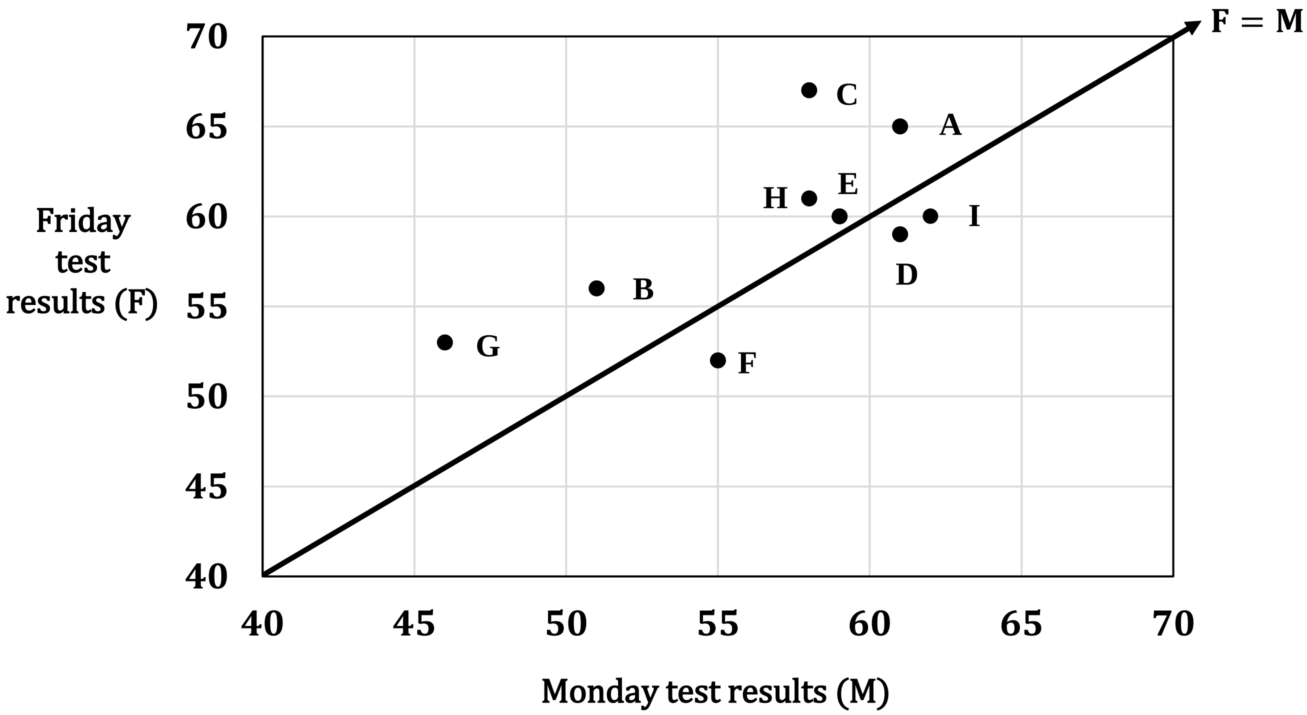
Question 25 (5 marks) **Marks**

After a class of ten students sits a test on Monday, the teacher spends time doing revision and tests the students again on Friday. Only nine of the students are present for the second test on Friday. The results, in percentages, for those nine students, are shown in the following table and scatterplot.

Student	A	B	C	D	E	F	G	H	I	J
Monday (M)	61	51	58	61	59	55	46	58	62	63
Friday (F)	65	56	67	59	60	52	53	61	60	Absent



(a) Using your calculator, find the correlation coefficient for the 9 students and explain the type and strength of correlation this data represents. **2**

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Question 25 continues on the next page

Question 25 continued **Marks**

(b) Determine the equation of the least squares-regression line for this data. **1**

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(c) Using your answer to part (b), predict the result for the student who was absent for Friday's test. Give your answer to the nearest whole mark. **1**

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(d) The scatterplot above also shows the line $F = M$. Explain the significance of a student being represented below the line $F = M$. **1**

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Question 26 (5 marks) **Marks**

A swimming pool is to be emptied for maintenance. The quantity of water, Q litres, remaining in the pool at a time, t minutes after it starts to drain, is given by:
$Q(t) = 2000(25 - t)^2, \;\; t \geq 0.$

(a) At what rate (in litres/min) is the water being removed at any time (t)? **1**

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(b) If the pool is completely full before being emptied, how long will it take to remove half of the water from the pool to the nearest minute? **3**

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(c) At what time does the rate of flow of water from the pool reach 20 kL/minute? **1**

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Question 27 (5 marks) **Marks**

(a) Show that the two functions $y = 4x - x^3$ and $y = x$ intersect when $x = 0$ and $x = \pm\sqrt{3}$. **2**

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(b) Hence find the exact area between the two functions in the first quadrant. **3**

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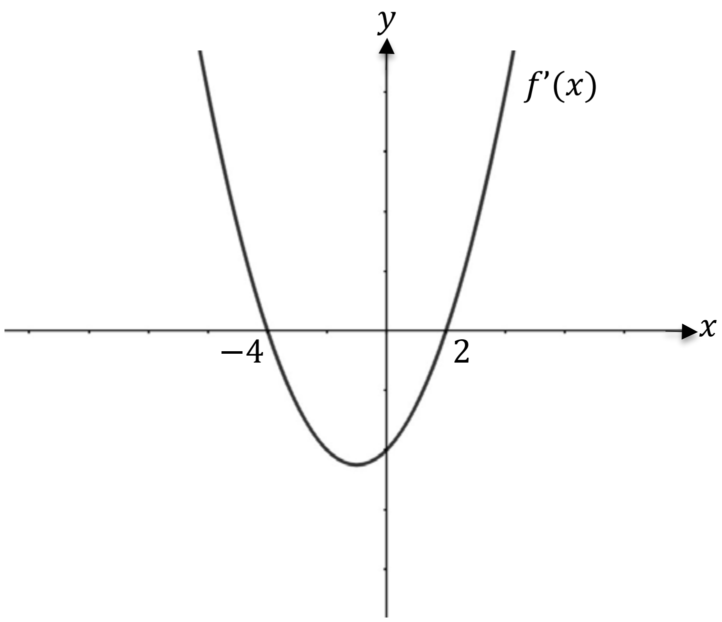
Question 28 (2 marks) **Marks**

The diagram below shows the graph of $y = f'(x)$. **2**



Sketch a possible graph of $y = f(x)$ below, including axes and any stationary points.

Question 29 (3 marks) **Marks**

A drone is used during the filming of a television show. The drone leaves its base station, at point O, and flies $50m$ on a bearing of $045°T$ to point A. It then changes direction to $130°T$ and flies a further $90m$ to point B. To the nearest metre, calculate how far the drone is from base when it reaches point B. **3**

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Question 30 (8 marks) **Marks**

The displacement of a particle is described by the equation $x = 4te^{-t} + 3$, where x is the displacement from the origin in cm and t is the time in seconds.

(a) Find the particle's initial displacement. **1**

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(b) Find an equation for the particle's velocity. **1**

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(c) Find when the particle is at maximum distance from the origin and what its displacement is at that time. **3**

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Question 30 continues on the next page

Question 30 continued **Marks**

(d) Describe what happens to the particle eventually, given that **1**

as $t \to \infty$, $\frac{t}{e^t} \to 0$.

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(e) Sketch the curve of the displacement $x = 4te^{-t} + 3$ below, **2**

showing the maximum, any asymptotes and any intercepts.

Question 31 (9 marks) **Marks**

It is known for a large population that at the beginning of winter, 15% of people will be infected with a particular virus.

(a) Four people are selected at random. **2**
Find the probability that at least one of them has the virus.

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(b) What is the smallest number of people a drug company would need to test to have a greater than 95% chance that at least one of the tested people has the virus? **3**

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Question 31 continues on the next page

Question 31 continued **Marks**

(c) As winter progresses the virus spreads further so the health authorities decide to trial a new medication to try and stop the spread of the virus.
The two-way table shows the number of people in a trial.
(**Note:** The trial consists of those taking the medication and those in a control group).

	Taking Medication	Control Group
Virus	204	205
No Virus	212	209

(i) What percentage of people in the trial had the virus? **1**

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(ii) What percentage of people in the control group had the virus? **1**

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(iii) Giving a reason, determine if it is worth the health authorities using this new medication. **2**

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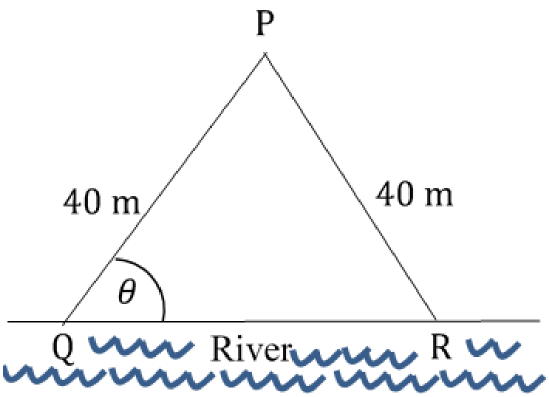
Question 32 (5 marks) **Marks**



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A triangular enclosure has been created by the fences PQ and PR, each of length 40 metres. A river forms the third boundary of the enclosure, as shown in the diagram.

Let $\angle PQR = \theta$.

(a) Show that the area of $\Delta\ PQR$ is $A = 800\sin 2\theta$. **2**

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Question 32 continues on the next page

Question 32 continued **Marks**

(b) Find the maximum possible area of this triangular enclosure, as θ changes. **3**

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Question 33 (6 marks) **Marks**

The height h(t) metres of the tide above the mean sea level on 1st April is given by the following rule: $h(t) = 4\sin\left(\frac{\pi}{8}t\right)$ where t is the number of hours after midnight.

(a) Draw a graph of $y = h(t)$ for $0 \leq t \leq 24$. **2**

(b) At which time(s) does the high tide occur? **1**

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(c) What was the height of the high tide? **1**

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(d) What was the height of the tide at 10 a.m.? **2**
(Answer correct to 1 decimal place)

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END OF EXAMINATION

Section I

10 marks
Attempt Questions 1 - 10
Allow about 15 minutes for this section.

Use the multiple-choice answer sheet for Questions 1-10.

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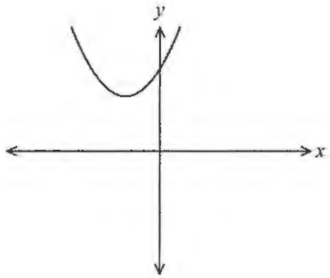
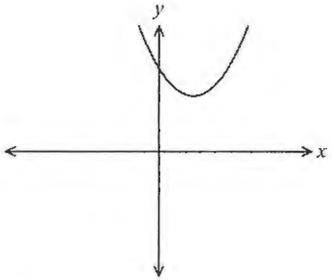
1. Which diagram best shows the graph of the parabola $y = 2 - (x+1)^2$?

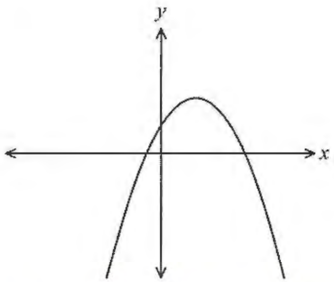
(A)



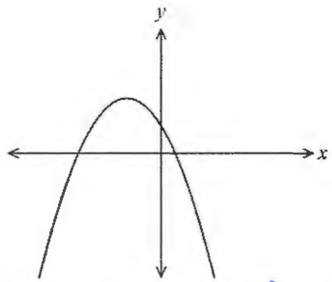
(B)



(C)



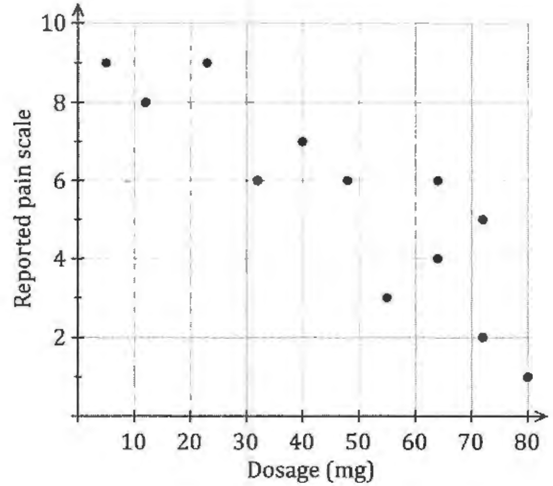
(D)



$y = x^2$ is translated 1 unit left, reflected in the x-axis and then shifted up by 2.

Section I (cont'd)

2. A scatterplot of pain (as reported by patients) compared to the dosage (in mg) of a drug is shown below.



How could you describe the correlation between the pain and the dosage?

(A) A moderate negative correlation
(B) A moderate positive correlation
(C) A weak positive correlation.
(D) No correlation.

3. Robina threw an ordinary die numbered 1 to 6 twice. What is the probability that the second number shown on the die is more than the first?

(A) $\frac{5}{6}$

(B) $\frac{1}{6}$

(C) $\frac{5}{12}$

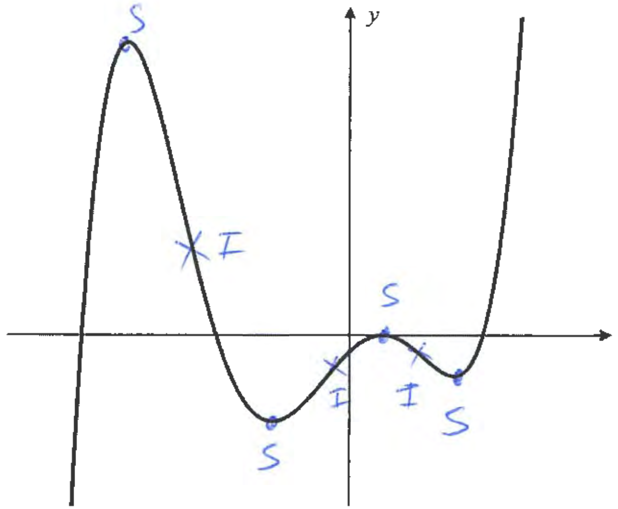
(D) $\frac{1}{2}$

$\frac{15}{36}$

Ways this can happen:
1,2 2,3 3,4 4,5 5,6
1,3 2,4 3,5 4,6
1,4 2,5 3,6
1,5 2,6
1,6

Section I (cont'd)

4. The graph of $y = f(x)$ is as shown below.



What is the number of stationary points and points of inflection in the graph of $y = f(x)$?

(A) 3 stationary points and 3 points of inflection

(B) 3 stationary points and 4 points of inflection

(C) 4 stationary points and 3 points of inflection

(D) 4 stationary points and 4 points of inflection

5. What is the amplitude and period for the curve $y = -1 + 3\sin 2x$?

(A) Amplitude = 3, Period = 2π

(B) Amplitude = 2, Period = 2π

(C) Amplitude = 3, Period = π

(D) Amplitude = 2, Period = π

amplitude

Period $= \frac{2\pi}{a} = \frac{2\pi}{2} = \pi$

Section I (cont'd)

6. $y = \log_a x^{-3}$ is equivalent to:

(A) $x = a^{\frac{y}{3}}$

(B) $x = a^{-\frac{y}{3}}$

(C) $x = a^{3y}$

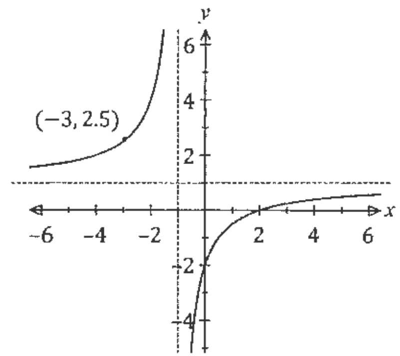
(D) $x = a^{-3y}$

$a^y = x^{-3}$

$a^{-y} = x^3$

$x = a^{-\frac{y}{3}}$

7. The diagram below shows the graph of $y = f(x)$.



Which of the following statements is false?

(A) The horizontal asymptote is $y = 1$

(B) The curve is continuous

(C) The curve is concave up for $x < -1$

(D) The equation of the function is $y = \frac{x-2}{x+1}$

Section I (cont'd)

8. Which of the following is the derivative of $y = \dfrac{3x^2 - 4}{4x^2 - 3}$?

(A) $\dfrac{14x}{(4x^2-3)^2}$

(B) $\dfrac{14x}{16x^4+9}$

(C) $\dfrac{48x^3 - 50x}{(4x^2-3)^2}$

(D) $\dfrac{48x^3 - 50x}{16x^4+9}$

Using quotient rule:

$y = \dfrac{vu' - uv'}{v^2}$

$y = \dfrac{(4x^2-3)(6x) - (3x^2-4)(8x)}{(4x^2-3)^2}$

$y = \dfrac{24x^3 - 18x - 24x^3 + 32x}{(4x^2-3)^2}$

$= \dfrac{14x}{(4x^2-3)^2}$

9. The discrete random variable X has the following probability distribution.

X	0	1	2
$P(X = x)$	a	b	0.3

Given that $E(X) = 0.8$, then

(A) $a = 0.5$ and $b = 0.2$

(B) $a = 0.2$ and $b = 0.5$

(C) $a = 0.3$ and $b = 0.4$

(D) $a = 0.3$ and $b = 0.2$

$E(X) = 0.8$

$0.8 = 0 \times a + 1 \times b + 2 \times 0.3$

$0.8 = b + 0.6$

$\therefore b = 0.2$

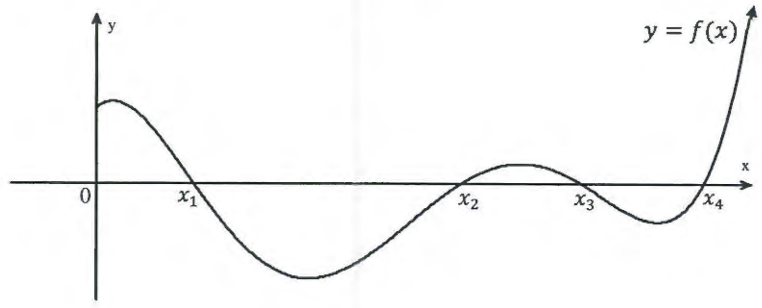
$a = 1 - b - 0.3$

$a = 1 - 0.2 - 0.5$

$a = 0.5$

Section I (cont'd)

10. The graph of the function $y = f(x)$ is shown below.



Given that $\int_0^{x_2} f(x)\,dx = -3$, $\int_0^{x_3} f(x)\,dx = -1$ and $\int_0^{x_4} f(x)\,dx = -4$,

what is the value of $\int_{x_2}^{x_4} f(x)\,dx$?

(A) −1

(B) −2

(C) −3

(D) −4

$\int_{x_2}^{x_3} f(x)\,dx = -1 - (-3) = 2$

$\int_{x_3}^{x_4} f(x)\,dx = -4 - (-1) = -3$

$\therefore \int_{x_2}^{x_4} f(x)\,dx = 2 + -3 = -1$

End of Section I

Section II

90 marks
Attempt Questions 11 – 33
Allow about 2 hours and 45 minutes for this section.

Your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (2 marks) **Marks**

Find $\int 6x^2 + 2 + \frac{1}{\sqrt{x}}\,dx$, giving each term in its simplest form. 2

$\int 6x^2 + 2 + \frac{1}{\sqrt{x}}\,dx = \int 6x^2 + 2 + x^{-1/2}\,dx$

$= \frac{6x^3}{3} + 2x + \frac{x^{1/2}}{1/2} + c$ — 1 mk

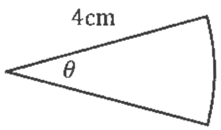
$= 2x^3 + 2x + 2\sqrt{x} + c$

– ½ mk for the answer to be written in its simplest form

– ½ mk for writing down the constant of integration.

Question 12 (2 marks)

A sector has a radius of 4 cm and an area of $\frac{8\pi}{3}$ cm². Find the angle θ. 2



Not to scale

Area of sector $= \frac{1}{2} r^2 \theta$

$\frac{8\pi}{3} = \frac{1}{2} 4^2 \times \theta$ — 1 mk for appropriate substitution into the correct formula

$\frac{8\pi}{3} = 8\theta$

$\therefore \theta = \pi/3$

1 mk for the correct answer.

EXAMINER'S COMMENTS

Q11

Generally well done.

Some students made mistakes with the calculations especially, $\frac{1}{1/2} = 2$. Others forgot to write down the constant of integration.

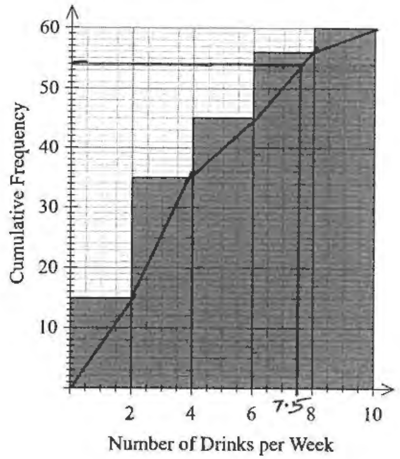
Q 12.

Some students need to re-visit the area of sector formula.

At this stage, it is advisable for students to leave their answer in terms of π rather than in degrees.

Question 13 (6 marks) **Marks**

Lizzie is a university student collecting data about her classmates' alcohol consumption. A question in her survey asks the respondent to indicate how many alcoholic drinks they typically consume each week. The data collected is displayed in the cumulative frequency histogram.



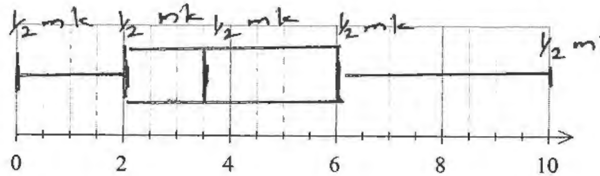
$10\% \times 60 = 6$

(a) How many drinks per week do each of the students in the top decile of respondents consume? **2**

The top 10% drink 7.5 to 10 drinks per week.

Any integer between 7.5 to 10 — 1 mk
A range (7.5 – 10 or 8 – 10) — 1 mk.

(b) Construct a box-plot to represent the data. **3**



½ mk for correct shape.

(c) Comment on the distribution of the data. **1**

The distribution is positively skewed — 1 mk

EXAMINER'S COMMENTS

13a) The top decile of respondents refers to the top 10% of the respondents. A number of students failed to realise that there will be a range of scores which are going to lie in the top decile.
If students had shown some understanding of the 'top decile' either through relevant diagrams or written down any integer between 7.5 to 10, they were awarded with 1 mark. Those students who had written down either 7.5 to 10 or 8 – 10 were awarded with 2 marks.

13 b) Generally, well done.

13c) A range of answers were accepted which included a description of the range, median, upper-quartile, lower-quartile, inter-quartile range or the skewness of the data set.

Question 14 (4 marks) **Marks**

The random variable X has this probability distribution.

X	0	1	2	3	4
$P(X = x)$	0.1	0.2	0.4	0.2	0.1

(a) Find $P(1 < X \leq 3)$. 1

$P(1 < X \leq 3) = 0.4 + 0.2 = 0.6$

½ mk ½ mk

(b) Find the expected value of X, showing all working. 1

$\mu = E(X) = \sum x\,p(x) = 0\times 0.1 + 1\times 0.2 + 2\times 0.4 + 3\times 0.2 + 4\times 0.1$ — ½ mk

$= 2$ — ½ mk

(c) Find the variance of X, showing all working. 2

$Var(X) = E(X^2) - \mu^2$

$= 0^2\times 0.1 + 1^2\times 0.2 + 2^2\times 0.4 + 3^2\times 0.2 + 4^2\times 0.1 - 2^2$ — 1 mk for appropriate usage of the formula

$= 1.2$ — 1 mk for the correct answer

Question 15 (1 mark)

Find the derivative of $16\sin 5x$. 1

$\frac{d}{dx}(16\sin 5x) = 16\times 5\cos 5x$ — ½ mk

$= 80\cos 5x$ — ½ mk

EXAMINER'S COMMENTS

14a) Generally, well done.

14 b) Some students need to re-visit the formula for finding the expected value. All working had to be displayed clearly as the question could have been done on the calculator.

14 c) $Var(X) = E(X^2) - \mu^2$

$= \sum x^2 p(x) - \sum x\,p(x)$

Some students need to re-visit the fact that $E(X^2) = \sum x^2 p(x)$.

Students who did not show any working lost 2 marks as they could have done the question on the calculator.

Q15 Generally done well.

Question 16 (4 marks) **Marks**

A curve with the equation $y = f(x)$, has $\frac{dy}{dx} = x^3 + 2x - 7$.

(a) Find $\frac{d^2y}{dx^2}$. 1

$\frac{d^2y}{dx^2} = 3x^2 + 2$

(b) Show that $\frac{d^2y}{dx^2} \geq 2$ for all values of x. 1

$x^2 \geq 0$ for all Real x

$\therefore 3x^2 \geq 0$

$\therefore 3x^2 + 2 \geq 2$

$\therefore \frac{d^2y}{dx^2} \geq 2$

(c) The point $P(2,4)$ lies on the curve. Find an equation in general form, for the normal to the curve at point P. 2

Gradient of tangent at $P(2,4)$:

$m = \frac{dy}{dx} = (2)^3 + 2(2) - 7 = 5$ — ½ gradient of tangent

$\therefore$ Gradient of normal $= \frac{-1}{m} = -\frac{1}{5}$ — ½ gradient of normal

Equation of normal:

$y - 4 = -\frac{1}{5}(x - 2)$ — ½ for using the formula

$5y - 20 = -x + 2$

$x + 5y - 22 = 0$ — ½ for answer

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EXAMINER'S COMMENTS

Question 16

a) Most students had little difficulty with this question

b) Most students approached this question from the wrong end but were awarded 1 mark if they stated $x^2 \geq 0$ for all values of x.
Some students then errorionsly wrote $x \geq 0$ lost ½
Graphs were only accepted if it was shown to have a minimum at (0, 2) using calculus.

c) Some students did not know where to get the gradient from and lost ½
Not in general form lost ½ $ax + by + c = 0$
where a, b and c are integers and $a \geq 0$

Question 17 (3 marks) **Marks**

Find the exact solutions to the equation $1 + 2\sin 3x = 2$ in the domain $0 \leq x \leq \pi$. **3**

$1 + 2\sin 3x = 2$

$2\sin 3x = 1$

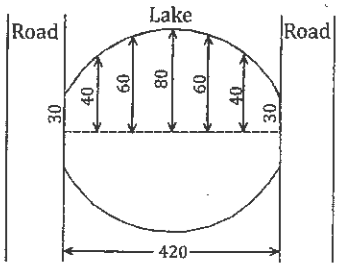
$\sin 3x = \frac{1}{2}$ $\quad 0 \leq 3x \leq 3\pi$ — (1)

$\therefore 3x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{13\pi}{6}, \frac{17\pi}{6}$ — (1)

$x = \frac{\pi}{18}, \frac{5\pi}{18}, \frac{13\pi}{18}, \frac{17\pi}{18}$ — (1)

Question 18 (3 marks)

A symmetrical lake has two roads, 420 metres apart, forming two of its sides. Equally spaced measurements of the lake, in metres, are shown on the diagram.



Not to scale

Use the trapezoidal rule to estimate the area of the lake. **3**

Width of each strip $= 420 \div 6 = 70$ m — $\frac{1}{2}$

Area of top half of lake $\approx$

$\frac{70}{2}(30 + 30 + 2(40 + 60 + 80 + 60 + 40))$ 1

$\approx 21700 \text{ m}^2$ $\frac{1}{2}$

$\therefore$ Area of whole lake $\approx 43\,400 \text{ m}^2$ 1

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EXAMINER'S COMMENTS

Question 17

Most students formed an equation with sin 3x as the subject but some did not show the domain for 3x

Question 18

Needed to find the width of each strip. This part was not well done by many students

The majority of students only found half the area.

Question 19 (5 marks) **Marks**

(a) Show that the derivative of $\ln\left(\frac{3+x}{3-x}\right)$ is $\frac{6}{9-x^2}$. 3

$$\frac{d}{dx}\left[\ln\left(\frac{3+x}{3-x}\right)\right] = \frac{d}{dx}\ln(3+x) - \frac{d}{dx}\ln(3-x)$$

(i)

$$= \frac{1}{3+x} - \frac{-1}{3-x}$$ (1)

$$= \frac{1}{3+x} + \frac{1}{3-x}$$ (i)

$$= \frac{3-x+3+x}{(3+x)(3-x)} = \frac{6}{9-x^2}$$

(b) Hence show that $\int_1^2 \frac{1}{9-x^2}\,dx = \frac{1}{6}\ln\frac{5}{2}$. 2

$$\text{LHS} = \int_1^2 \frac{1}{9-x^2}\,dx = \frac{1}{6}\int_1^2 \frac{6}{9-x^2}\,dx$$ $\frac{1}{2}$

$$= \frac{1}{6}\left[\ln\left(\frac{3+x}{3-x}\right)\right]_1^2$$ 1

$$= \frac{1}{6}\left[\ln\left(\frac{3+2}{3-2}\right) - \ln\left(\frac{3+1}{3-1}\right)\right]$$ $\frac{1}{2}$

$$= \frac{1}{6}\left[\ln 5 - \ln 2\right]$$

$$= \frac{1}{6}\ln\frac{5}{2} = \text{RHS}$$ 1

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EXAMINER'S COMMENTS

Question 19

a) Some students did not show what they were differentiating

Another method not shown

let $u = \frac{3+x}{3-x}$

$$\frac{du}{dx} = \frac{(3-x)+3+x}{(3-x)^2} = \frac{6}{(3-x)^2}$$

let $y = \ln u$

$$\frac{dy}{du} = \frac{1}{u} = 1 \div \left(\frac{3+x}{3-x}\right)$$

$$\frac{dy}{dx} = \frac{dy}{du}\times\frac{du}{dx} = \frac{3-x}{3+x}\times\frac{6}{(3-x)^2} = \frac{6}{(3+x)(3-x)} = \frac{6}{9-x^2}$$

b) Those students that recognised the connection between part a) and $\int_1^2 \frac{1}{9-x^2}\,dx$ had little difficulty with this part.

Question 20 (2 marks) **Marks**

Simplify $\dfrac{\tan\theta \sec\theta}{1+\tan^2\theta}$. **2**

Method 1:

$\dfrac{\tan\theta \sec\theta}{1+\tan^2\theta} = \dfrac{\tan\theta \sec\theta}{\sec^2\theta}$

$= \dfrac{\tan\theta}{\sec\theta} = \dfrac{\sin\theta}{\cos\theta} \div \sec\theta$

$= \dfrac{\sin\theta}{\cos\theta} \times \cos\theta = \sin\theta$

Method 2:

$\dfrac{\tan\theta \sec\theta}{1+\tan^2\theta} = \dfrac{\frac{\sin\theta}{\cos\theta} \times \frac{1}{\cos\theta}}{1 + \frac{\sin^2\theta}{\cos^2\theta}}$

$= \dfrac{\sin\theta}{\cos^2\theta} \div \dfrac{\cos^2\theta + \sin^2\theta}{\cos^2\theta}$

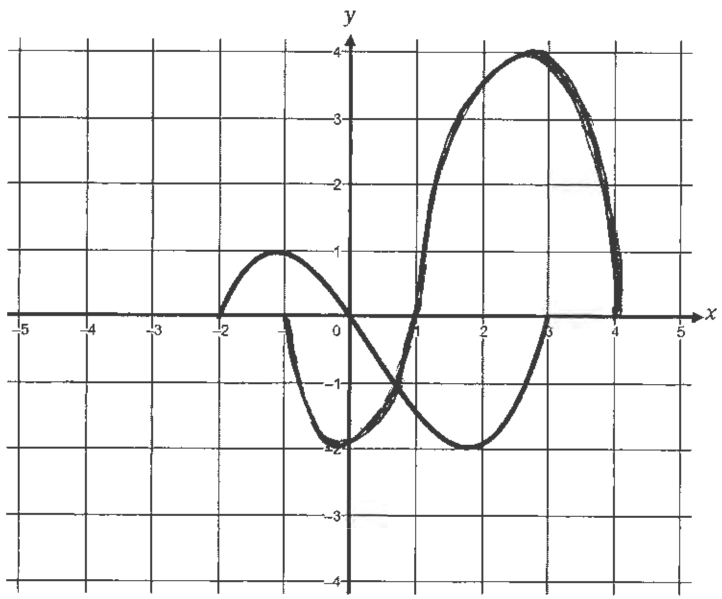
$= \dfrac{\sin\theta}{\cos^2\theta} \times \dfrac{\cos^2\theta}{1}$

$= \sin\theta$

Question 21 (2 marks)

The graph below shows $y = f(x)$. **2**

On the same graph sketch $y = -2f(x-1)$.



EXAMINER'S COMMENTS

Method 1:

A) $\frac{1}{2}$ mark for changing $\tan\theta$ to $\frac{\sin\theta}{\cos\theta}$

B) $\frac{1}{2}$ mark for changing $\sec\theta$ to $\frac{1}{\cos\theta}$

C) $\frac{1}{2}$ mark for changing $1+\tan^2\theta$ to $\sec^2\theta$

D) Lose $\frac{1}{2}$ mark if made a major error after c).

E) $\frac{1}{2}$ mark for correct final answer.

Method 2: As above except for part c) below

c) $\frac{1}{2}$ mark for forming a common denominator on $1 + \frac{\sin^2\theta}{\cos^2\theta}$.

There are three transformations:

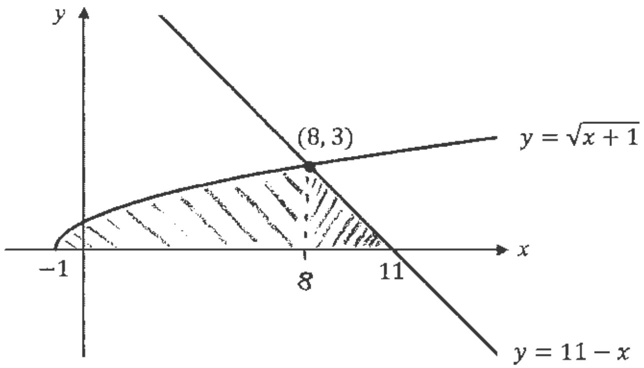
- reflection in the x-axis
- vertical dilation by a factor of 2
- shift right by 1

2 marks were awarded for showing all three transformations with no errors

$1\frac{1}{2}$ marks for all three transformations, but some small error

1 mark for evidence of any one correct transformation

Question 22 (3 marks) **Marks**



Calculate the area bounded by the curves $y = \sqrt{x+1}$ and $y = 11 - x$ and the x-axis. 3

$A = \int_{-1}^{8} (x+1)^{\frac{1}{2}} dx + \frac{1}{2} \times 3 \times 3$

$= \left[\frac{2}{3}(x+1)^{\frac{3}{2}} \right]_{-1}^{8} + 4.5$

$= \frac{2}{3} \left[9^{\frac{3}{2}} - 0 \right] + 4.5$

$= \frac{2}{3} \times 27 + 4.5$

$= 22.5$ units^2

EXAMINER'S COMMENTS

1 mark for correctly integrating $(x+1)^{\frac{1}{2}}$

1 mark for finding the area of the triangle

Final mark for complete solution.

Many students need to revise the concept of compound areas.

Question 23 (2 marks) **Marks**

Evaluate $\int_1^{e^3} \frac{5}{x}\, dx$. 2

$\int_1^{e^3} \frac{5}{x}dx = 5\int_1^{e^3} \frac{1}{x}dx$

$= 5\left[\ln x\right]_1^{e^3}$

$= 5\left[\ln e^3 - \ln 1\right]$

$= 5\left[3 - 0\right]$

$= 15$

Question 24 (3 marks)

Consider the functions $f(x) = \ln(x)$ and $g(x) = e^{2x+1}$. 2

(a) Show that the composite function, $g(f(x))$, is a parabola.

$g(f(x)) = e^{2(\ln x)+1}$

$= e^{\ln x^2 + 1}$

$= e^{\ln x^2} \times e^1$

$= x^2 \times e$

$= ex^2$

which is a parabola.

(b) Find, in interval notation, the natural domain of the composite function. 1

For $f(x) = \ln x$, domain: $(0, \infty)$

$g(x) = e^{2x+1}$, domain: $(-\infty, \infty)$

$\therefore$ for $g(f(x))$, domain: $(0, \infty)$

EXAMINER'S COMMENTS

Question 23:

1 mark for correct integration

1 mark for correctly simplifying $\ln e^3$ and achieving an answer of 15

Question 24 (a):

1 mark for correct expression for $g(f(x))$

1 mark for correctly simplifying to ex^2

Small mistakes in simplifying (e.g. $y = x^2 + 1$) were awarded $\frac{1}{2}$ mark.

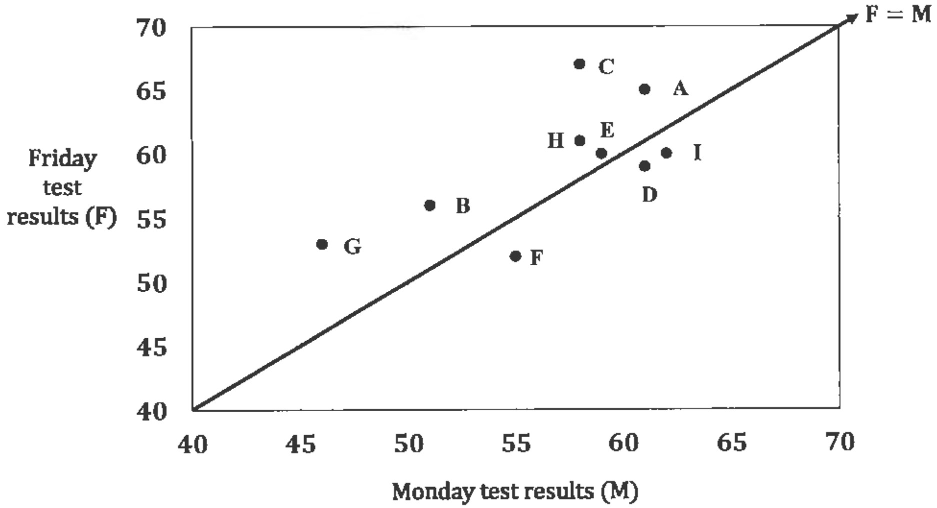
Question 24 (b):

$[0, \infty)$ was awarded $\frac{1}{2}$ mark

Question 25 (5 marks) **Marks**

After a class of ten students sits a test on Monday, the teacher spends time doing revision and tests the students again on Friday. Only nine of the students are present for the second test on Friday. The results, in percentages, for those nine students, are shown in the following table and scatterplot.

Student	A	B	C	D	E	F	G	H	I	J
Monday (M)	61	51	58	61	59	55	46	58	62	63
Friday (F)	65	56	67	59	60	52	53	61	60	Absent



(a) Using your calculator, find the correlation coefficient for the 9 students and explain the type and strength of correlation this data represents. 2

$r = 0.66020159\ldots$

$\doteqdot 0.66$ (2 d.p.)

This is a moderate positive correlation

Question 25 continues on the next page

EXAMINER'S COMMENTS

Learn to use the regression features of stats mode in your calculator!

You also need a working knowledge of what the various types of correlation look like. Regardless of what your calculator says, this cannot be a negative correlation. (such answers attracted no marks)

Use the agreed terminology; this is no place for statements like "fairly strong" or "pretty weak". Hedging your bets with phrases like "moderately strong" or "fairly weakly strongish" earned you no marks - be clear.

1 mark for correct value of r

$\frac{1}{2}$ mark each for 'moderate' and 'positive'

Question 25 continued **Marks**

(b) Determine the equation of the least squares-regression line for this data. 1

$A = 23.80666\ldots$ $B = 0.6237574\ldots$
$\doteq 23.81$ (2dp) $\doteq 0.62$ (2dp)

$\therefore F = 23.81 + 0.62M$

(c) Using your answer to part (b), predict the result for the student who was absent for Friday's test. Give your answer to the nearest whole mark. 1

$F = 23.81 + 0.62(63)$
$= 62.87$
$\doteq 63$

(d) The scatterplot above also shows the line F = M. Explain the significance of a student being represented below the line F = M. 1

Students below the line did better on Monday than Friday.

EXAMINER'S COMMENTS

1 mark for the correct answer
$F = 0.62 + 23.81M$ was awarded $\frac{1}{2}$ mark
NB: please use the stats mode in your calculator. Every other approach was unsuccessful.

1 mark for using your answer in part (b) to get 63
If you made a mistake in part (b), you could still earn this mark, provided you showed evidence of substitution into a linear function $A + BX$, where $A, B \neq 0$. Estimations from the graph, or unsupported by evidence of using the prior result, were not awarded any marks.

1 mark for the correct explanation without contradictory or incorrect information. You cannot just write down every statistic-y word you've ever heard and claim that the answer is in there somewhere.

Also, we weren't asking for your speculations on the causes of these results; stick to the facts.

Marks

Question 26 (5 marks)

A swimming pool is to be emptied for maintenance. The quantity of water, Q litres, remaining in the pool at a time, t minutes after it starts to drain, is given by:
$Q(t) = 2000(25 - t)^2, \; t \geq 0.$

$Q(t) = 0$ when $t = 25$ $\therefore 0 \leq t \leq 25$ the full will be empty when $t = 25$

(a) At what rate (in litres/min) is the water being removed at any time (t)? **1**

$Q'(t) = 2 \times 2000 \times (25-t)^1 \times -1$

$= -4000(25-t)$ or $4000t - 10\,000$

$\therefore$ It is removed at the rate of $4000(25-t)$ litres per minute.

(b) If the pool is completely full before being emptied, how long will it take to remove half of the water from the pool to the nearest minute? **3**

NB $0 \leq t \leq 25$ for this model.

When full, at $t = 0$ $Q(t) = 2000(25-0)^2$

$= 1\,250\,000$ L

$\therefore$ when half full $V = 625\,000$ L $t = ?$

Method 1

$Q(t) = 625\,000$

$625\,000 = 2000(25-t)^2$

$312.5 = (25-t)^2$

$\pm\sqrt{312.5} = 25 - t$

$t = 25 \mp \sqrt{312.5}$

$t = 42.677$ or 7.322

as $0 \leq t \leq 25$. The time taken to remove half the water is 7 minutes (to the nearest minute).

Method 2 "formula"

$Q(t) = 625\,000$

$625\,000 = 2000(25-t)^2$

$(25-t)^2 = \frac{625}{2}$

$2(t-25)^2 = 625$

$2t^2 - 100t + 625 = 0$

$t = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$ etc

(c) At what time does the rate of flow of water from the pool reach 20 kL/minute? **1**

Method 1) Rate of water removal $= 20$ kL/min $= 20\,000$ L/min

$\therefore 4000(25-t) = 20000$

$25 - t = 5$

$25 - 5 = t$

$t = 20$ minutes

Method 2

$-4000(25-t) = -20000$

$25 - t = 5$

$t = 20$ minutes.

$\therefore$ 20 minutes after it starts to empty the rate of flow of water

EXAMINER'S COMMENTS

Question 26

$Q(t) = 2000(25-t)^2$ $t \geq 0$

this graphs to be a parabola with minimum at $(25, 0)$

Students needed to note that the pool will be empty at $t = 25$ min. and $0 \leq t \leq 25$

(a) generally well done.
The form $4000(25-t)$ worked best in part (c)
(1) mark no half marks.

(b) (1) mark given if students found that at half full $Q(t) = 625\,000$ L
(1) mark given if students substituted $Q(t) = 625\,000$ into $Q(t) = 2000(25-t)^2$ to create a quadratic equation
(1) mark given if found the solution $t = 7$ minutes
$(-\frac{1}{2}$ if also included $t = 43$ minutes$)$

(c) most students did not get a mark at all
Many substituted $Q'(t) = 20\,000$ instead of $-20\,000$ into $Q'(t) = -4000(25-t)$
Students may prefer to use method 1 in future.

Question 27 (5 marks) Marks

(a) Show that the two functions $y = 4x - x^3$ and $y = x$ intersect when $x = 0$ and $x = \pm\sqrt{3}$. **2**

Method 1: solve simultaneously

$y = 4x - x^3$ — ①

$y = x$ — ②

sub ① in ② $4x - x^3 = x$

$3x - x^3 = 0$

$x(3 - x^2) = 0$

$x = 0$ or $x^2 = 3$

$x = 0$ or $x = \pm\sqrt{3}$

∴ they intersect when $x = 0$ and $x = \pm\sqrt{3}$

Method 2: substitute $x = 0$ & $x = \sqrt{3}$ & $x = -\sqrt{3}$ into both equations ① & ②

if $x = 0$: $y = 4x - x^3$, $y = 4(0)^3 - 0 = 0$ $(0,0)$

if $x = 0$: $y = x$, $y = 0$ $(0,0)$

etc...

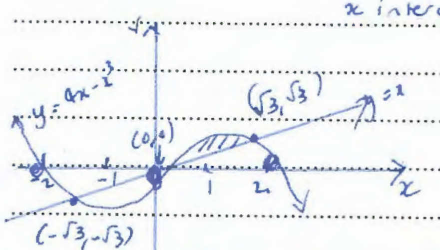
(b) Hence find the exact area between the two functions in the first quadrant. **3**

Graph: $y = x$ & $y = 4x - x^3$

$y = x$ $y = x(4 - x^2)$

$y = x(2 - x)(2 + x)$

x intercepts at $x = 0, \mp 2$.



Area $= \int_0^{\sqrt{3}} 4x - x^3\,dx - \int_0^{\sqrt{3}} x\,dx$

$= \int_0^{\sqrt{3}} 4x - x^3 - x\,dx$

$= \int_0^{\sqrt{3}} 3x - x^3\,dx$ (continued)

$= \left[\frac{3x^2}{2} - \frac{x^4}{4}\right]_0^{\sqrt{3}}$

$= \left[\left(\frac{3(\sqrt{3})^2}{2} - \frac{(\sqrt{3})^4}{4}\right) - \left(\frac{0}{2} - \frac{0}{4}\right)\right]$

$= \frac{9}{2} - \frac{9}{4}$

$= \frac{9}{4}$ square units

EXAMINER'S COMMENTS

Question 27

(a) ① mark given if working towards answer with correct method

① answer

(b) Better students drew a diagram, showing points of intersection

① mark diagram and correct integral

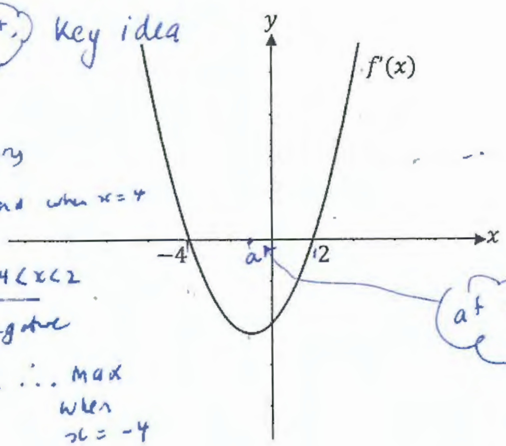
① mark integration

① mark answer

Question 28 (2 marks) **Marks**

The diagram below shows the graph of $y = f'(x)$. 2

For a stationary point, $f'(x) = 0$ — Key idea



$f'(-4) = 0$, $f'(2) = 0$ ∴ stationary points when $x = 2$ and when $x = 4$

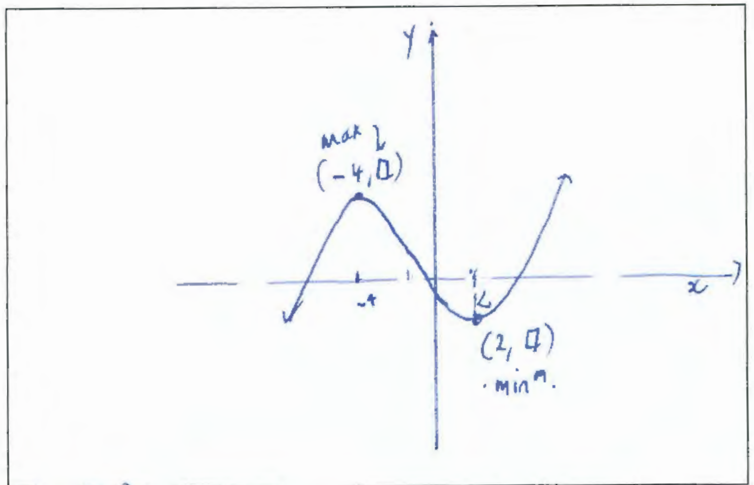
nature:

x	$x < -4$	-4	$-4 < x < 2$
$f'(x)$	positive	0	negative

∴ max when $x = -4$

at $x = a$ P.O.I. in $y = f(x)$ — it will not be a horizontal point of inflection

Sketch a possible graph of $y = f(x)$ below, including axes and any stationary points.



min (2, □)

x	$-4 < x < 2$	2	$x > 2$
$f'(x)$	negative	0	positive

Question 28

EXAMINER'S COMMENTS

The key idea in this question is to recognise that when $f'(x) = 0$ at $x = 2$ and $x = -4$ these stationary points were turning points. Most students achieved 2 marks.

- Students received 1 mark if they had the correct shape
- Students who reversed the max & the min received ① mark.

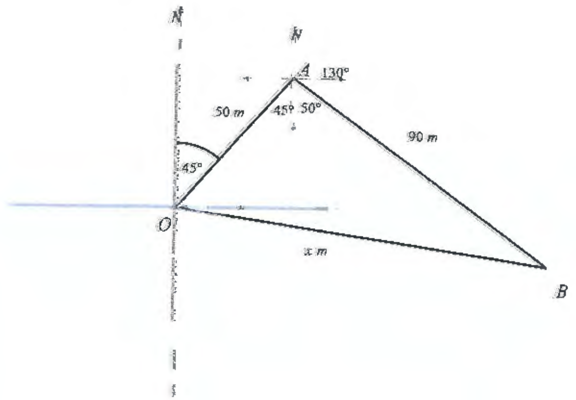
Many students inserted a horizontal point of inflection. this is not true as the point of inflection is NOT a stationary point.

- a significant number of students did not realise that $f(x)$ would be a polynomial. This is worth looking at again before the final HSC examination.

Question 29 (3 marks) **Marks**

A drone is used during the filming of a television show. The drone leaves its base station, at point O, and flies $50m$ on a bearing of $045°T$ to point A. It then changes direction to $130°T$ and flies a further $90m$ to point B. **3**

To the nearest metre, calculate how far the drone is from base when it reaches point B.



By the Cosine Rule, $x^2 = 50^2 + 90^2 - 2\times 50\times 90\cos(45^\circ + 50^\circ)$

$= 11384.40168...$

$\therefore x = 106.6977$, i.e. $x = 107\ m$, correct to nearest metre

EXAMINER'S COMMENTS

Question 29

* Many students presented excellent responses to this question, achieving full marks.
* Weaker students could not draw a diagram and had difficulty preceeding. These students should revise bearings thoroughly before their HSC examination.

Marks allocated...

(1) mark given for correct diagram

(1) correct substitution into the cosine rule

(1) calculation of the value of x.

Question 30 (8 marks) **Marks**

The displacement of a particle is described by the equation $x = 4te^{-t} + 3$, where x is the displacement from the origin in cm and t is the time in seconds.

a) Find the particle's initial displacement **1**

at $t=0$, $x = 4 \times 0 \times e^{-0} + 3 = 3$ cm.

b) Find an equation for the particle's velocity. **1**

$v = \frac{dx}{dt} = \frac{d}{dx}(4te^{-t}) + \frac{d}{dx}(3)$

$= 4 \times e^{-t} + 4t \times -e^{-t} + 0$

$= 4e^{-t}(1-t)$

Equivalent answers: $-4e^{-t}(t-1)$ or $4e^{-t} 4te^{-t}$

c) Find when the particle is at maximum distance from the origin and what its displacement is at that time. **3**

Find stationary points at $v = 0$

$4e^{-t}(1-t) = 0$

e^{-t} cannot equal zero

$\therefore 1 - t = 0 \Rightarrow t = 1$ second.

$\frac{dv}{dt} = 4e^{-t} \times -1 - 4e^{-t}(1-t)$

$= 4e^{-t}(-1-(1-t))$

$= 4e^{-t}(t-2)$

at $t=1$, $\frac{dv}{dt} = 4e^{-1}(1-2) = -\frac{4}{e} < 0$

could do this by looking at v either side of t=1

t	0	1	2
v	1.47	0	-1.4
	/	—	\

$\therefore$ the particle is at a max distance from origin at $t=1$.

at $t=1$, $x = 4 \times 1 \times e^{-1} + 3 = \frac{4}{e} + 3$ cm.

Question 30 continues on the next page

EXAMINER'S COMMENTS

Q30 a) 1 mark for 3cm.

Most students got this correct, but many forgot to add the units. They were not penalised for this.

Q30 b)

- Most students answered this correctly.
- Students received zero if they didn't use the product or quotient rule (product rule is recommended).
- Students received $\frac{1}{2}$ mark if they made a small error or they had an issue with a negative.

Q30 c)

- 1 mark for finding stationary point of $t = 1$ second.
- $\frac{1}{2}$ mark for explaining $e^{-t} = 0$ has no solution
- 1 mark for proving that at $t=1$ second there is a maximum. (lose $\frac{1}{2}$ mark if look at v either side of t ~~with no number~~ shown)
- $\frac{1}{2}$ mark for finding $x = \frac{4}{e} + 3$ cm at $t = 1$ sec.
- lose $\frac{1}{2}$ mark for set out of proving maximum if unclear the process you are following.

Question 30 continued **Marks**

(d) Describe what happens to the particle eventually, given that **1**

as $t \to \infty$, $\frac{t}{e^t} \to 0$.

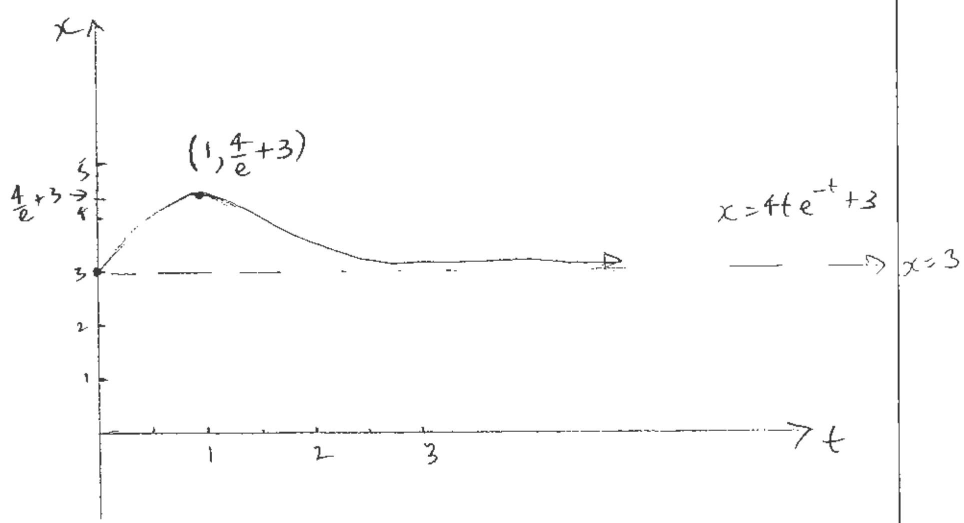
$x = \frac{4t}{e^t} + 3$ as $t \to \infty$ $\frac{1}{e^t} \to 0$

$\therefore \frac{4t}{e^t} \to 0$

$\therefore \frac{4t}{e^t} + 3 \to 3$.

$\therefore$ The particle eventually approaches $x = 3$

(e) Sketch the curve of the displacement $x = 4te^{-t} + 3$ below, showing the maximum, any asymptotes and any intercepts. **2**



EXAMINER'S COMMENTS

Q 30 d)

1 mark for stating that as $t \to \infty$, $x \to 3$.

- It was acceptable to use words to describe x as moving towards or approaching 3.
- It was not acceptable to state that $x = 3$ or x goes to or equals 3.

Q 30 e)

- $\frac{1}{2}$ mark for y-intercept at 3.
- $\frac{1}{2}$ mark for showing maximum at $(1, \frac{4}{e}+3)$
- $\frac{1}{2}$ mark for asymptote at $x=3$.
- $\frac{1}{2}$ mark for showing curve approaching 3 as $t \to \infty$.

- $\frac{1}{2}$ mark lost if $t<0$ was shown as part of function.
- $\frac{1}{2}$ mark lost for poor shape.

Question 31 (9 marks) **Marks**

It is known for a large population that at the beginning of winter, 15% of people will be infected with a particular virus.

$V = 15\% = 0.15$

$\bar{V} = 1 - 0.15 = 0.85$

(a) Four people are selected at random. **2**

Find the probability that at least one of them has the virus.

P(at least one has virus) $= 1 - P(\text{none have virus})$

$= 1 - P(\bar{V}\bar{V}\bar{V}\bar{V})$

$= 1 - (0.85)^4$ — ①

$= 0.47799375$ — ①

(b) What is the smallest number of people a drug company would need to test to have a greater than 95% chance that at least one of the tested people has the virus? **3**

let n be the least number of people

P(at least one of n has the virus) > 0.95

$1 - (0.85)^n > 0.95$ — ①

$-(0.85)^n > -0.05$

$(0.85)^n < 0.05$

$\ln (0.85)^n < \ln 0.05$

$n \ln (0.85) < \ln 0.05$

$n > \dfrac{\ln 0.05}{\ln 0.85}$ — ①

$n > 18.43312827$

∴ 19 people would need to be tested. — ①

Question 31 continues on the next page

EXAMINER'S COMMENTS

Q31 (a) & (b) was poorly done!

- Show all working
- Show calculator display to get "some" marks
- answer the question

② marks provides correct solution with correct working

① mark for some worthwhile progress

(b) You have been taught "logs" girls. There is no need for "guess & refine" method, it is time consuming!!!

① for setting up the inequality

① for reversing the inequality when you divide by ln 0.85 (since it is negative)

① for correct rounding to 19

Question 31 continued **Marks**

(c) As winter progresses the virus spreads further so the health authorities decide to trial a new medication to try and stop the spread of the virus. The two-way table shows the number of people in a trial.

	Taking Medication	Control Group	
Virus	204	205	409
No Virus	212	209	421
	416	414	830

(i) What percentage of people in the trial had the virus? **1**

$\frac{204+205}{830} = \frac{409}{830} = 0.492771084$

$= 49.27710843\ \%$

(ii) What percentage of people in the control group had the virus? **1**

$\frac{205}{205+209} = \frac{205}{414} = 0.495169082$

$= 49.51690821\ \%$

(iii) Giving a reason, determine if it is worth the ~~heath~~ health authorities using this new medication. **2**

Not worth using the new medication since the "taking medication" group ($\frac{204}{416} = 49.03846154\ \%$) and the "control" group ($\frac{205}{414} = 49.51690821\ \%$) had a similar incidence (occurrence) of the virus.

EXAMINER'S COMMENTS

Q31(c) Show all working, show calculator display as the answers were mostly 49% to the nearest percentage, and so when you were mentioning these in your answer, especially in part (iii) I had no idea which "group" you were referring to.

(i) ① mark correct answer

(ii) ① mark correct answer

(iii) • needed to compare the group taking the medication with the control group, that is intervention with no intervention. Not the over all trial group with the virus ($\frac{416}{830}$) with the control group.

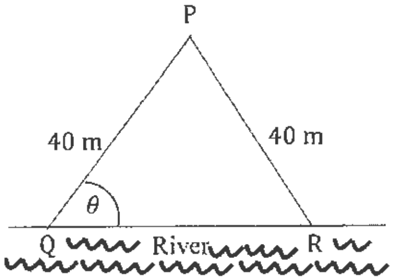
• look at the % difference of the people in the control group with virus $\frac{205}{414} = 49.51690821$ vs the people in taking medication group with virus $\frac{204}{416} = 49.03846154$, therefore a difference of $0.47844667\% = 0.5\%$. ∴ not worth it.

② marks provides correct answer with reason

① mark for "not worth it" but comparing the two wrong groups.

• No need for essays, get straight to the point using correct percentages as your back up!

Question 32 (5 marks) **Marks**



A triangular enclosure has been created by the fences PQ and PR, each of length 40 metres. A river forms the third boundary of the enclosure, as shown in the diagram.

Let $\angle PQR = \theta$.

a) Show that the area of $\Delta\, PQR$ is $A = 800 \sin 2\theta$. **2**

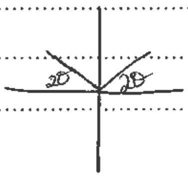
$\angle QRP = \theta$ ($\triangle QPR$ is isosceles, $PQ = PR$)

$\therefore \angle QPR = 180 - 2\theta$ (angle sum of a triangle)

$A = \frac{1}{2} \times 40 \times 40 \times \sin(180 - 2\theta)$

$= 800 \sin(180 - 2\theta)$

$= 800 \sin 2\theta$



sin (180 − 2θ) is in the second quadrant as equals sin 2θ in the first quadrant.

Question 32 continues on the next page

EXAMINER'S COMMENTS

Q32a)

Most students recognised that they needed to use the area formula including 'sine', but many could not prove the angle.

- $-\frac{1}{2}$ mark for $\frac{1}{2} \times 40 \times 40$ part of formula
- $-\frac{1}{2}$ mark for indicating $\angle P = 180 - 2\theta$
- $-\frac{1}{2}$ mark for simplifying $\frac{1}{2} \times 40 \times 40 = 800$
- $-\frac{1}{2}$ mark for $\sin(180 - 2\theta) = \sin 2\theta$.

Question 32 continued **Marks**

b) Find the maximum possible area of this triangular enclosure, as θ changes **3**

$\frac{dA}{d\theta} = 1600 \cos 2\theta$

Find stationary points at $\frac{dA}{d\theta} = 0$

$1600 \cos 2\theta = 0$

$\cos 2\theta = 0$

$2\theta = \frac{\pi}{2}, \frac{3\pi}{2}, \ldots$

$\therefore \theta = \frac{\pi}{4}, \frac{3\pi}{4}, \ldots$

θ can't be $\frac{3\pi}{4}$ as $2 \times \frac{3\pi}{4} = \frac{6\pi}{4} > \pi$

$\therefore$ angle too large

$\therefore \theta = \frac{\pi}{4}$

This step could be completed as

θ	$\frac{\pi}{8}$	$\frac{\pi}{4}$	$\frac{\pi}{2}$
A'	1131	0	-1600
	/	—	\

$\therefore$ max.

$\frac{d^2A}{d\theta^2} = -3200 \sin 2\theta$

at $\frac{\pi}{4}$, $\frac{d^2A}{d\theta^2} = -3200 \times \sin \frac{\pi}{2} = -3200 \times 1$

$= -3200 < 0$

$\therefore$ at $\theta = \frac{\pi}{4}$, the area is a maximum

$\therefore$ At $\theta = \frac{\pi}{4}$, $A = 800 \times \sin 2\left(\frac{\pi}{4}\right)$

$= 800 \times \sin \frac{\pi}{2}$

$= 800 \text{ m}^2$.

EXAMINER'S COMMENTS

Q32 b)

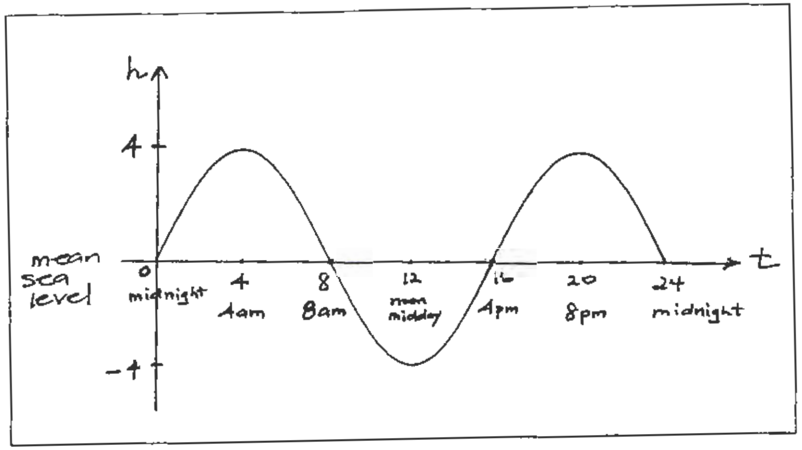
Most students recognised that this was an optimisation questions, but many struggled to complete all 3 tasks.

- $\frac{1}{2}$ mark for differentiating A
- $\frac{1}{2}$ mark for finding stationary point. Not many students explained why $\theta = \frac{3\pi}{4}$ was not a solution – fortunately they were not penalised.
- $\frac{1}{2}$ mark for differentiating A'
- $\frac{1}{2}$ mark for proving that at $\theta = \frac{\pi}{4}$, the area is a maximum.
  → (students could gain this mark by investigating sign of A' either side of $\theta = \frac{\pi}{4}$).
- $\frac{1}{2}$ mark for $A = 800 \times \sin 2\left(\frac{\pi}{4}\right)$
- $\frac{1}{2}$ mark for $A = 800 \text{ m}^2$.

Question 33 (6 marks) **Marks**

The height h(t) metres of the tide above the mean sea level on 1st April is given by the following rule: $h(t) = 4\sin\left(\frac{\pi}{8}t\right)$ where t is the number of hours after midnight.

(a) Draw a graph of $y = h(t)$ for $0 \le t \le 24$. 2



(b) At which time(s) does the high tide occur? 1

High tide occurs when t=4 or t=20 hours
∴ at 4am and 8pm

(c) What was the height of the high tide? 1

4m above sea level

(d) What was the height of the tide at 10 a.m.? 2
(Answer correct to 1 decimal place)

$h(10) = 4\sin\left(\frac{\pi}{8}\times 10\right)$
$= -2.828427125$
$= -2.8$
∴ 2.8 m below sea level

END OF EXAMINATION

EXAMINER'S COMMENTS

Q33 (a) Girls that worked out the period were more successful in achieving the correct diagram

$\text{Period} = \frac{2\pi}{n} = \frac{2\pi}{\frac{\pi}{8}} = 2\pi \times \frac{8}{\pi} = 16$

(2) marks provides correct diagram with correct labels

$\frac{1}{2}$ – for period of 16

$\frac{1}{2}$ – for correct shape

$\frac{1}{2}$ – for correct x-intercepts 0, 8, 16, 24

$\frac{1}{2}$ – for amplitude of 4 and peaks at 4, 12, 20.

(b) $\frac{1}{2}$ – for 4am

$\frac{1}{2}$ – for 8pm

(c) 4m above sea level – (1) mark

(d) (2) marks correct answer with working of 2.8 m below sea level. The negative answer is because the height is below sea level. You should not cross it out!

(1) mark – for some worthwhile progress

• markers suggestion, show working and your calculator display so you can get "some" marks!