STELLA MARIS

MANLY

IN OMNIBUS GLORIFICETUR DEUS

2008
TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics

General Instructions
- Reading Time – 5 minutes
- Working Time – 3 hours
- Write using black or blue pen
- Board approved calculators may be used
- A table of standard integrals is provided at the back of this paper
- All necessary working should be shown in every question

Total Marks – 120
- Attempt Questions 1 – 10
- All questions are of equal value

BLANK PAGE

Total Marks – 120
Attempt Questions 1 – 10
All questions are of equal value

Begin each question in a SEPARATE writing booklet. Extra booklets are available.

Question 1 (12 marks) Use a SEPARATE writing booklet. **Marks**

a) Evaluate $\left(\frac{1}{e^{2\cdot5}}-1\right)^2$ correct to 3 significant figures. **2**

b) Solve $|2x-1|\leq 3$ **2**

c) If $\frac{4}{2-\sqrt{3}}=a+b\sqrt{3}$ find the values of a and b. **2**

d) Find the tenth term of the arithmetic series **2**
$4\frac{1}{2} + 3 + 1\frac{1}{2} + \ldots$

e) Factorise $2x^2+7x-4$ **2**

f) Find the perpendicular distance from the point (1, 3) to the line **2**
$6x-8y+5=0$

End of Question 1

Question 2 (12 marks) Use a SEPARATE writing booklet. **Marks**

a) Differentiate with respect to x

(i) $x\sin x$ **2**

(ii) $\left(1+e^{x}\right)^{5}$ **2**

b) (i) Find $\int \sec^{2} 3x \; dx$ **2**

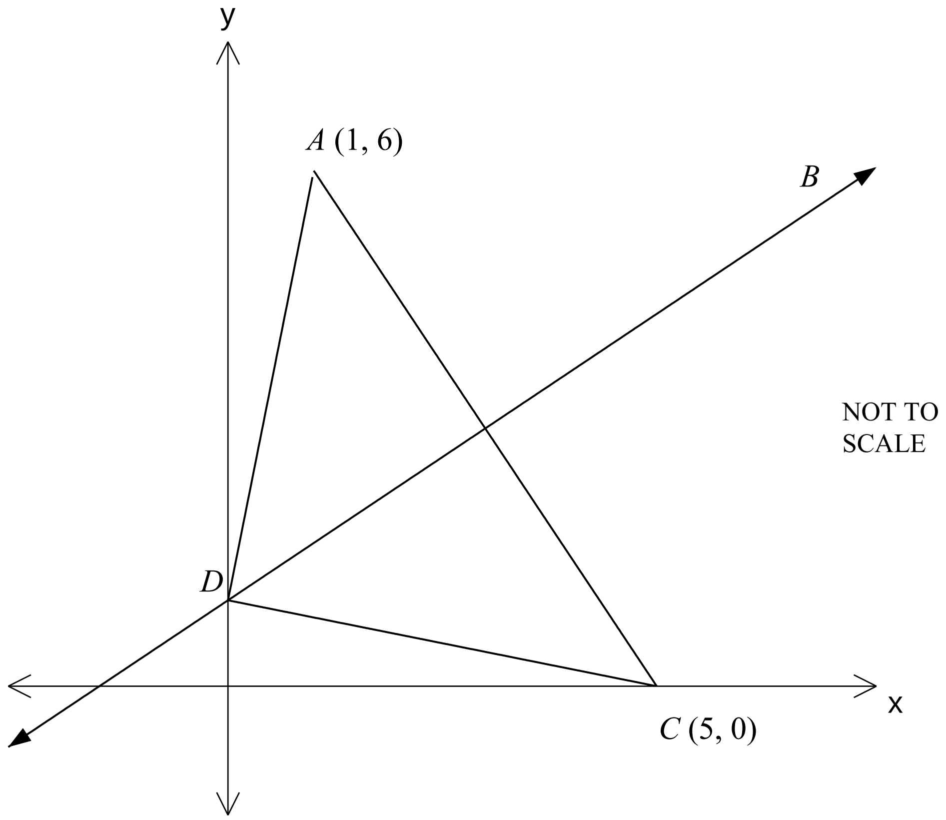
(ii) Evaluate $\int_{0}^{3} \frac{6x}{1+x^{2}} \, dx$ **3**

c) Find the equation of the tangent to the curve $y=\cos 3x$ at the point whose x-coordinate is $\frac{\pi}{6}$. **3**

End of Question 2

Question 3 (12 marks) Use a SEPARATE writing booklet. **Marks**

a)



The points A and C have coordinates (1, 6) and (5, 0) respectively.
The line BD has an equation of $2x-3y+3=0$ and meets the y axis in D.

i) The point M is the midpoint of AC. Show that M has coordinates (3, 3). **1**

ii) Show that M lies on BD. **1**

iii) Find the gradient of the line AC. **1**

iv) Show that BD is perpendicular to AC. **2**

v) Find the distance AC. **1**

vi) Explain why the quadrilateral $ABCD$ is a kite regardless of the position of B. **1**

Question 3 continues on next page.

Question 3 continued **Marks**

b) Michael is training for a local marathon. He has trained by completing practice runs over the marathon course. So far he has completed three practice runs with times shown below.

Week 1	Week 2	Week 3
3 hours	2 hours 51 minutes	2 hours 42 minutes 27 seconds

i) Show that these times form a geometric series with a common ratio $r = 0.95$. **1**

ii) If this series continues, what would be his expected time in Week 5, to the nearest second? **1**

iii) How many hours, minutes and seconds (to the nearest second) will he have run, in total, in his practice runs in these 5 weeks? **1**

iv) If the previous winning time for the marathon was 2 hours and 6 minutes, how many weeks must he keep practicing to be able to run the marathon in less that the previous winning time? **2**

End of Question 3

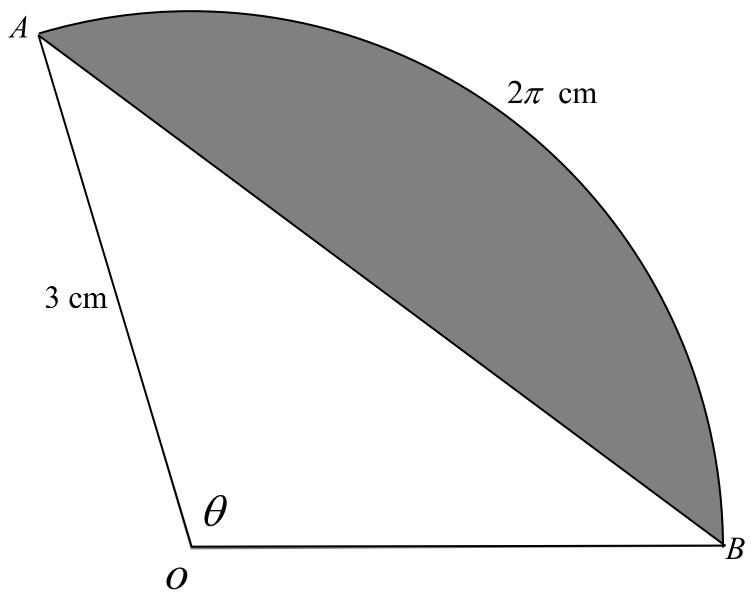
Question 4 (12 marks) Use a SEPARATE writing booklet. **Marks**

a) Show that: **2**

$$\sqrt{\frac{\operatorname{cosec}^2 x - \cot^2 x - \cos^2 x}{\cos^2 x}} = \tan x$$

b) For the function $y = x^3 - 3x^2 - 9x + 1$, find the values of x for which the curve is decreasing. **3**

c)



NOT TO SCALE

AOB is a sector of a circle, centre O and radius 3 cm.
The length of arc AB is 2π cm. AB is a chord.

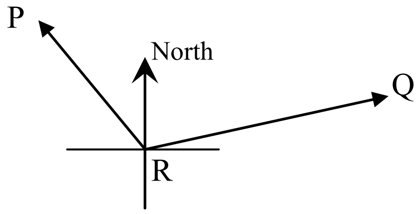
(i) Calculate the angle θ subtended at the centre of the circle by the arc AB. **1**

(ii) Calculate the exact area of the shaded segment. **2**

Question 4 continues on next page.

Question 4 continued

d) Peta and Quentin are pilots of two light planes which leave Resthaven station at the same time. Peta flies on a bearing of 330^o at a speed of 180 km/h and Quentin flies on a bearing of 080^o at a speed of 240 km/h. Copy the diagram below onto your answer page and mark the information on the diagram.



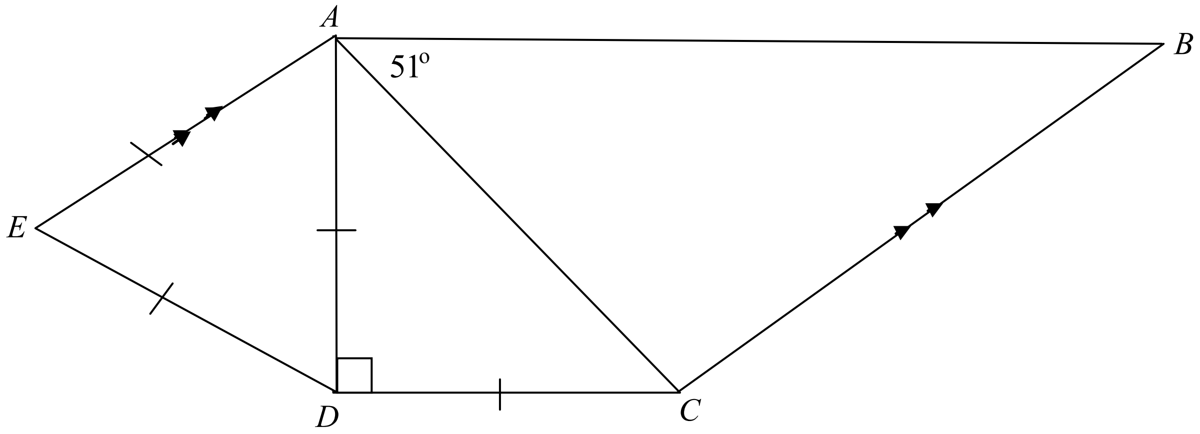
i) Show that Peta and Quentin are 692 km (to the nearest km) apart after 2 hours? **2**

ii) What is the bearing of Quentin from Peta after 2 hours. (Answer to the nearest degree.) **2**

End of Question 4

Question 5 (12 marks) Use a SEPARATE writing booklet. **Marks**

a) In the diagram below $AE = ED = AD = DC$, $\angle ADC = 90^\circ$ and $AE \parallel BC$. $\angle BAC = 51^\circ$



i) Find the size of $\angle EAB$. Give reasons for your answer. **3**

ii) Find the size of $\angle ABC$. Give reasons for your answer. **1**

b) A particle moves in a straight line so that its displacement, in metres, is given by

$x = \dfrac{4t^2 + t + 8}{4t + 1}$ where t is measured in seconds.

i) Find the initial displacement of the particle. **1**

ii) Find an expression for the velocity of the particle. **1**

iii) Show that the particle is stationary when $t = \dfrac{-1 + 4\sqrt{2}}{4}$ seconds (or $1 \cdot 2$ seconds to one decimal place). **2**

iv) Find the total distance travelled in the first two seconds. (Answer to 1 decimal place.) **2**

c) Solve the pair of simultaneous equations **2**

$$3x - y = 10$$
$$x = y + 2$$

End of Question 5

Question 6 (12 marks) Use a SEPARATE writing booklet. **Marks**

a) For the function $y = x^6 - 6x^4$

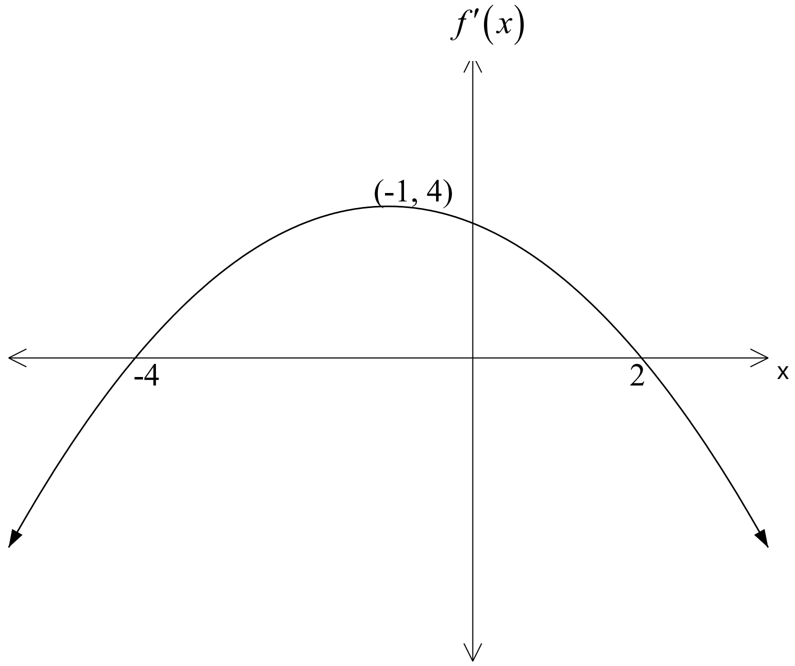
i) Find the x coordinates of the points where the curve crosses the axes. **2**

ii) Find the coordinates of the stationary points and determine their nature. **4**

iii) Find the coordinates of any points of inflexion. **2**

iv) Sketch the graph of $y = x^6 - 6x^4$ indicating clearly the intercepts, stationary points and points of inflexion. **2**

b) For a certain function $y = f(x)$, the sketch of $y = f'(x)$ is shown.



Give the x coordinates of the stationary points on $y = f(x)$ and indicate if they are maxima or minima. **2**

End of Question 6

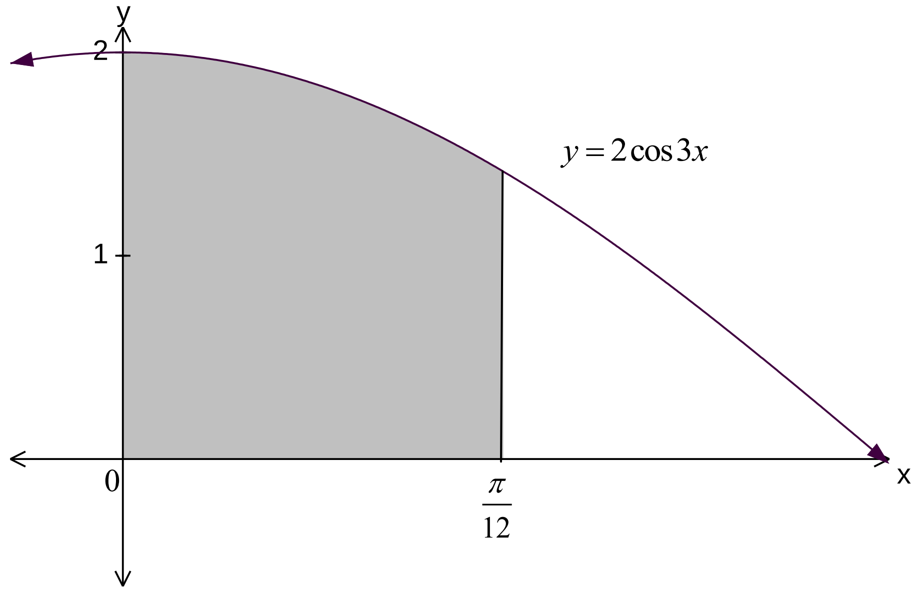
Question 7 (12 marks) Use a SEPARATE writing booklet. **Marks**

a) Let $\log_a 2 = x$ and $\log_a 5 = y$. Find an expression, in terms of *x* and *y,* for

i) $\log_a 0 \cdot 4$ **1**

ii) $\log_a 20$ **2**

b)



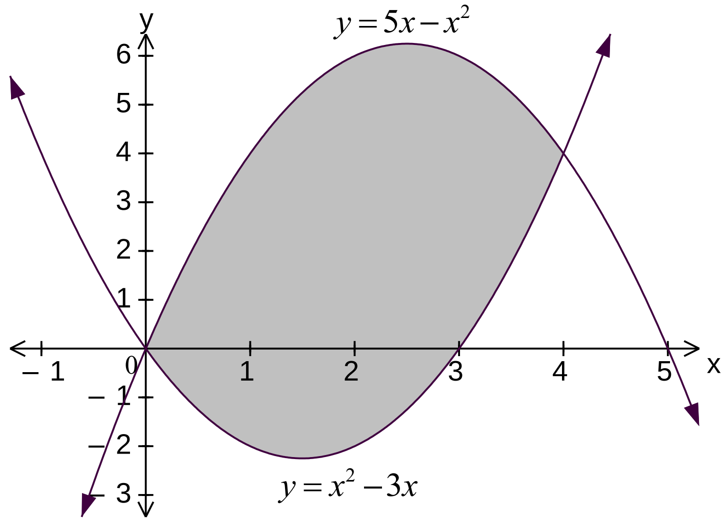
The diagram shows the graph of $y = 2\cos 3x$. **3**

Find the area enclosed by the curve $y = 2\cos 3x$, the line $x = \frac{\pi}{12}$ and the *x* and *y* axes.

Question 7 continues on next page.

Question 7 continued

c)



i) Show that the curves $y = x^2 - 3x$ and $y = 5x - x^2$ intersect at the points (0, 0) and (4, 4). **2**

ii) Find the area enclosed between the two curves. **3**

d) Find $\frac{d}{dx}\log_e(\cos x)$. **1**

End of Question 7

Question 8 (12 marks) Use a SEPARATE writing booklet. **Marks**

a) A city has a population which is growing at a rate that is proportional to the current population. The population at time t years is given by

$$P = Ae^{kt}$$

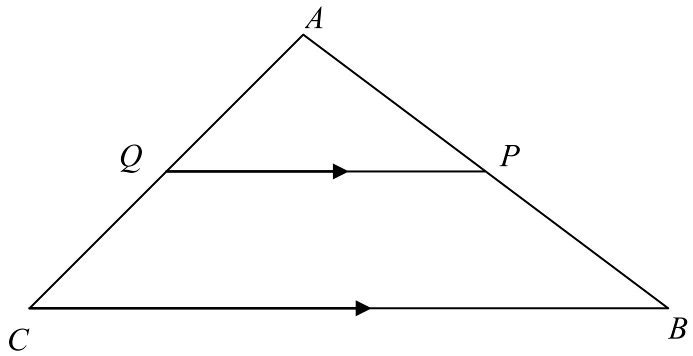
i) Show that $P = Ae^{kt}$ satisfies the equation $\frac{dP}{dt} = kP$. **1**

ii) If the population at the start of 2006 when $t = 1$ was 147 200 and at the start of 2007 when $t = 2$ was 154 800, find the values of A and k. **2**

iii) Find the population at the start of 2009. **1**

iv) Find during which year the population will first exceed 200 000. **1**

b) In the diagram below, P is the midpoint of the side AB of the ΔABC. PQ is drawn parallel to BC.



i) Prove that $\Delta ABC \parallel\!| \Delta APQ$. **2**

ii) Explain why Q is the midpoint of AC. **2**

c) Find an approximation for $\int_1^3 g(x)\,dx$ by using Simpson's Rule with the values in the table below. **2**

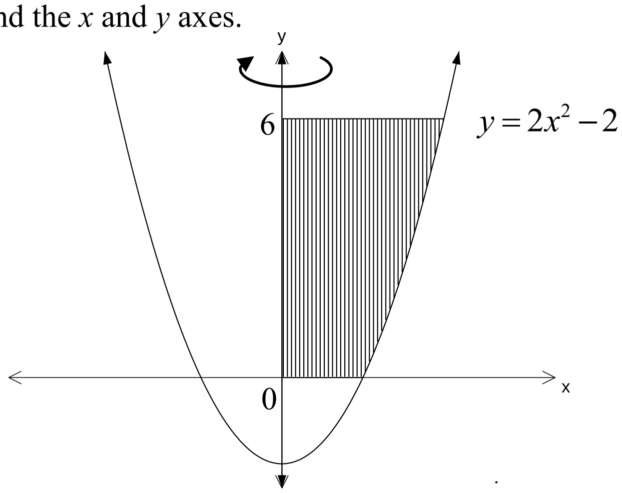
x	1	1.5	2	2.5	3
$g(x)$	12	8	0	3	5

d) Evaluate $\sum_{n=2}^{5} (n^2 - 1)$ **1**

End of Question 8

Question 9 (12 marks) Use a SEPARATE writing booklet. **Marks**

a) The diagram shows the region bounded by the curve $y = 2x^2 - 2$ the line $y = 6$ and the x and y axes. **3**



Find the volume of the solid of revolution formed when the region is rotated about the y axis.

b) Sketch the function $y = \ln(x+1)$, showing its essential features. **2**

c) Given that $\frac{d}{dx}\left(e^{x^5}\right) = 5x^4e^{x^5}$, find $\int 2x^4e^{x^5}\,dx$. **1**

d) A car dealership has a car for sale for a cash price of \$20 000. It can also be bought on terms over three years. The first six months are interest free and after that interest is charged at the rate of 1% per month on that months balance. Repayments are to be made in equal monthly instalments beginning at the end of the first month.

A customer buys the car on these terms and agrees to monthly repayments of $\$M$. Let $\$A_n$ be the amount owing at the end of the nth month.

i) Find an expression for A_6. **1**

ii) Show that $A_8 = (20\,000 - 6M)1{\cdot}01^2 - M(1 + 1{\cdot}01)$ **1**

iii) Find an expression for A_{36}. **2**

iv) Find the value of M. **2**

End of Question 9

Question 10 (12 marks) Use a SEPARATE writing booklet. **Marks**

a) A plant nursery has a watering system which repeatedly fills a storage tank then empties its contents to water different sections of the nursery.
The volume of water (in cubic metres) in the tank at a time t is given by the equation

$V = 2 - \sqrt{3}\cos t - \sin t$ where t is measured in minutes.

i) Give an equation for $\frac{dV}{dt}$, the rate of change of the volume at a time t. **1**

ii) Is the tank initially filling or emptying? **1**

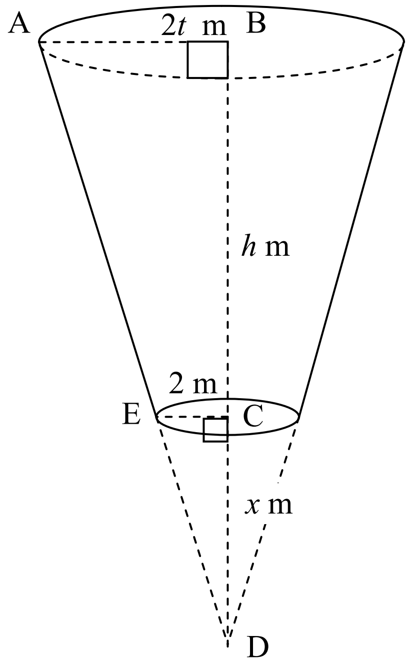
iii) At what time does the tank first become completely full and what is its capacity when full? **3**

Question 10 continues on next page.

Question 10 continued

Marks

b) A truncated cone is to be used as a part of a hopper for a grain harvester. It has a total height of h metres. The top radius is to be t times greater than the bottom radius which is 2 metres.



AB = $2t$ metres
BC = h metres
EC = 2 metres
CD = x metres

i) If x is the height of the removed section of the original cone, show using similar triangles that $x = \dfrac{h}{t-1}$ **2**

ii) Show that the volume of the truncated cone is given by **2**
$V = \left(\dfrac{4\pi h}{3}\right)\left(t^2 + t + 1\right)$

iii) If the upper radius plus the lower radius plus the height of the truncated cone must total 12 metres, calculate the maximum volume of the hopper. **3**

End of Examination

STANDARD INTEGRALS

$$\int x^n \, dx = \frac{1}{n+1} x^{n+1}, \ n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x} \, dx = \ln x, \quad x > 0$$

$$\int e^{ax} \, dx = \frac{1}{a} e^{ax}, \quad a \neq 0$$

$$\int \cos ax \, dx = \frac{1}{a} \sin ax, \ a \neq 0$$

$$\int \sin ax \, dx = -\frac{1}{a} \cos ax, \ a \neq 0$$

$$\int \sec^2 ax \, dx = \frac{1}{a} \tan ax, \ a \neq 0$$

$$\int \sec ax \tan ax \, dx = \frac{1}{a} \sec ax, \ a \neq 0$$

$$\int \frac{1}{a^2 + x^2} \, dx = \frac{1}{a} \tan^{-1} \frac{x}{a}, \ a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} \, dx = \sin^{-1} \frac{x}{a}, \ a > 0, \ -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} \, dx = \ln\left(x + \sqrt{x^2 - a^2}\right), \ x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} \, dx = \ln\left(x + \sqrt{x^2 + a^2}\right)$$

NOTE: $\ln x = \log_e x, \quad x > 0$

2008
TRIAL HSC
EXAMINATION

Mathematics

SOLUTIONS

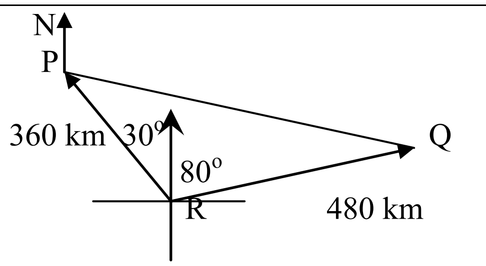
Question 1	Trial HSC Examination- Mathematics	
Part	Solution	Marks
(a)	$\left(\frac{1}{e^{2.5}}-1\right)^2=0.84256=0.843$ (3 sig fig)	2
(b)	$\lvert 2x-1\rvert\le 3$ $-3\le 2x-1\le 3$ $-2\le 2x\le 4$ $-1\le x\le 2$	2
(c)	$\frac{4}{2-\sqrt{3}}=a+b\sqrt{3}$ $\frac{4}{2-\sqrt{3}}=\frac{4}{2-\sqrt{3}}\times\frac{2+\sqrt{3}}{2+\sqrt{3}}$ $=\frac{8+4\sqrt{3}}{4-3}$ $a+b\sqrt{3}=8+4\sqrt{3}$ $a=8$ and $b=4$	2
(d)	4½ + 3 + 1½ + … Series is arithmetic with $a=4\frac{1}{2}$ and $d=-1\frac{1}{2}$ $T_n=a+(n-1)d$ $T_{10}=4\cdot5+9\times-1\cdot5$ $T_{10}=-9$	2
(e)	$(x+4)(2x-1)$	2
(f)	$d=\left\lvert\frac{6(1)-8(3)+5}{\sqrt{6^2+(-8)^2}}\right\rvert$ $=\left\lvert\frac{-13}{\sqrt{100}}\right\rvert$ $=1.3\,\text{units}$	2

Question 2	Trial HSC Examination- Mathematics	
Part	Solution	Marks
(a) i)	$x\cos x+\sin x$	2
ii)	$5\left(1+e^x\right)^4\times e^x$ $=5e^x\left(1+e^x\right)^4$	2
(b) i)	$\frac{1}{3}\tan 3x+c$	2

Question 2	Trial HSC Examination- Mathematics	
Part	Solution	Marks
ii)	$\left[3\ln\left(1+x^2\right)\right]_0^3$ $=3\left[\ln 10-\ln 1\right]$ $=3\ln 10$	3
(c)	$y'=-3\sin 3x$ when $x=\frac{\pi}{6}$, $y'=-3\sin\frac{\pi}{2}$ $=-3$ $\therefore$ gradient of tangent is -3 when $x=\frac{\pi}{6}$, $y=0$. Equation of tangent is $3x+y-\frac{\pi}{2}=0$	3

Question 3	Trial HSC Examination- Mathematics	
Part	Solution	Marks
(a)i)	Midpoint of (1, 6) and (5, 0). $MP=\left(\frac{1+5}{2},\frac{6+0}{2}\right)=\left(\frac{6}{2},\frac{6}{2}\right)=\left(3,3\right)$	1
ii)	Show that (3,3) lies on $2x-3y+3=0$ $LHS=2\left(3\right)-3\left(3\right)+3$ $=6-9+3$ $=0=RHS$ So M lies on BD.	1
iii)	Gradient AC $=m_1=\frac{6-0}{1-5}=\frac{6}{-4}=-\frac{3}{2}$	1
iv)	Find gradient m_2 of BD $2x-3y+3=0$ $2x-3y+3=0$ $3y=2x+3$ $y=\frac{2}{3}x+1$ $\therefore m_2=\frac{2}{3}$ $m_1.m_2=-\frac{3}{2}.\frac{2}{3}=-1$ $\therefore$ BD is perpendicular to AC	2

Question 3	Trial HSC Examination- Mathematics	
Part	Solution	Marks
v)	$AC=\sqrt{(5-1)^2+(0-6)^2}$ $=\sqrt{16+36}$ $=\sqrt{52}$ $=2\sqrt{13}$ units	1
(vi	The lines AC and BD would form the diagonals of the quadrilateral ABCD. BD is the perpendicular bisector of AC from ii and iv above. The diagonals of a kite meet at right angles and one diagonal bisects the other, so ABCD meets the criteria for a kite.	1
(b)i)	$\frac{\text{2 hours 51 minutes}}{\text{3 hours}}=\frac{2.85}{3}=0.95$ $\frac{\text{2 hours 42 minutes 27 seconds}}{\text{2 hours 51 minutes}}=\frac{2.7075}{2.85}=0.95$ $\therefore$ forms a geometric series with $r=0.95$	1
ii)	$a=3, n=5$ and $r=0.95$ $u_5=ar^{n-1}$ $=3\times 0.95^4$ $=2.4435...$ $=2$ hours 26 min 37 s	1
iii)	$s_n=\frac{a(1-r^n)}{(1-r)}$ $=\frac{3(1-0.95^5)}{(1-0.95)}$ $=13.573...$ $=13$ hours 34 min 23 s (to nearest s))	1
iv)	$u_n=ar^{n-1}$ 2 hours 6 min $=3\times 0.95^{n-1}$ $3\times 0.95^{n-1}=2.1$ $0.95^{n-1}=2.1\div 3=0.7$ $\ln(0.95^{n-1})=\ln(0.7)$ $(n-1)\ln(0.95)=\ln(0.7)$ $n-1=\frac{\ln(0.7)}{\ln(0.95)}$ $n-1=6.953$ $n=7.953...$ Would need to continue for 8 weeks to better the time.	2

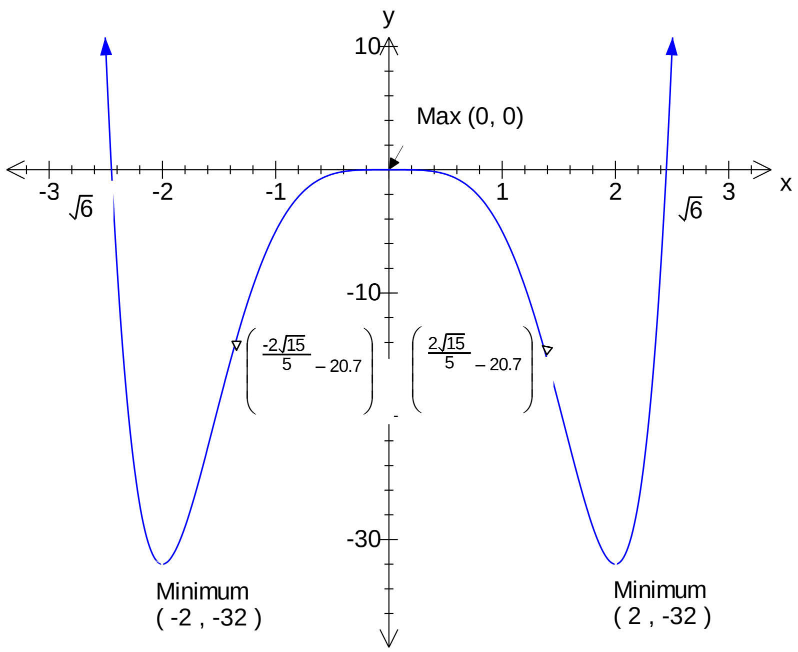
Question 4	Trial HSC Examination- Mathematics	
Part	Solution	Marks
(a)	$\sqrt{\dfrac{\text{cosec}^2 x-\cot^2 x-\cos^2 x}{\cos^2 x}}=\sqrt{\dfrac{1-\cos^2 x}{\cos^2 x}}$ $=\sqrt{\dfrac{\sin^2 x}{\cos^2 x}}$ $=\dfrac{\sin x}{\cos x}$ $=\tan x$	2
(b)	$y'=3x^2-6x-9$ $x^2-2x-3<0$ $(x+1)(x-3)<0$ $\therefore$ function is decreasing for $-1<x<3$	3
(c)i)	$l=r\theta$ $2\pi=3\theta$ $\theta=\dfrac{2\pi}{3}$	1
(c)ii)	$A=\dfrac{1}{2}r^2(\theta-\sin\theta)$ $A=\dfrac{1}{2}\times 3^2\left(\dfrac{2\pi}{3}-\sin\dfrac{2\pi}{3}\right)$ $A=\dfrac{9}{2}\left(\dfrac{2\pi}{3}-\dfrac{\sqrt{3}}{2}\right)$ $A=\dfrac{9}{2}\left(\dfrac{4\pi-3\sqrt{3}}{6}\right)=\dfrac{3}{4}\left(4\pi-3\sqrt{3}\right)\text{units}^2$	2
(d)i)	 $PQ^2=360^2+480^2-2\times 360\times 480\cos 110°$ $PQ^2=478202$ $PQ=692$ km (nearest km)	2

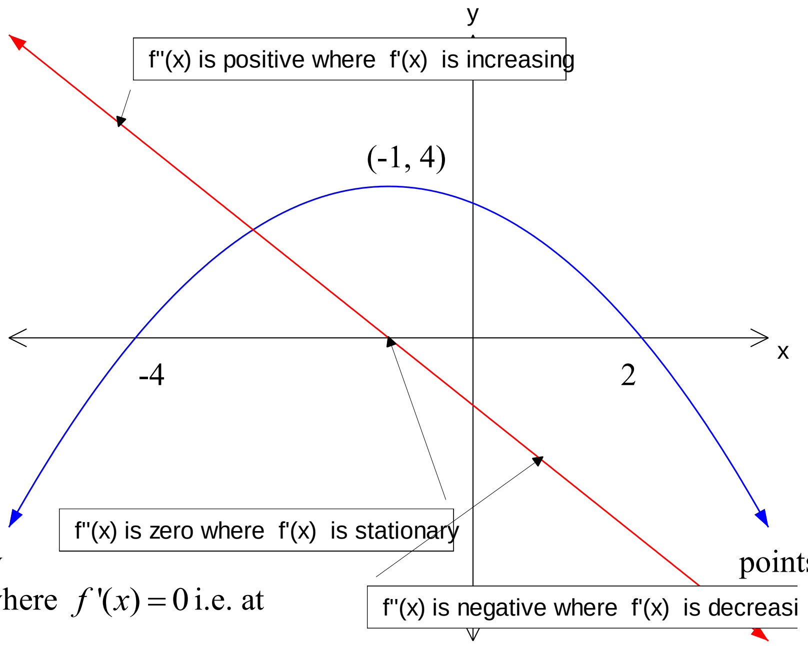
Question 4	Trial HSC Examination- Mathematics	
Part	Solution	Marks
(d)ii)	First find $\angle QPR$ $\frac{\sin\angle QPR}{480}=\frac{\sin110°}{692}$ $\sin\angle QPR=\frac{480\times\sin110°}{692}$ $\sin\angle QPR=0.652$ $\angle QPR=41°$ $\angle NPR=150°$ $Bearing(\angle NPQ)=150°-41°$ $=109°$ Can also be found using cos rule using the 3 sides..	2

Question 5	Trial HSC Examination- Mathematics	
Part	Solution	Marks
(a)i)	$\angle EAD=60°$ (equilateral ΔEAD) $\angle DAC=45°$ (isosceles right-angled ΔDAC) $\therefore\angle EAB=\angle EAD+\angle DAC+\angle CAB$ $=60°+45°+51°$ $=156°$	3
ii)	$\angle ABC=180°-156°$ (cointerior $\angle$ on || lines, AE and BC) $=24°$	1
(b)i)	$x=\frac{4t^2+t+8}{4t+1}$ $x=\frac{4(0)^2+(0)+8}{4(0)+1}$ when t = 0 $=8$ $\therefore$initial dispalcement is 8 m	2
(b)ii)	$x=\frac{4t^2+t+8}{4t+1}$ $\therefore\ \frac{(4t+1)(8t+1)-(4t^2+t+8)(4)}{(4t+1)^2}$ $=\frac{32t^2+12t+1-16t^2-4t-32}{(4t+1)^2}$ $=\frac{16t^2+8t-31}{(4t+1)^2}$	1

Question 5	Trial HSC Examination- Mathematics	
Part	Solution	Marks
(b)iii)	$\therefore$ $\dfrac{16t^2+8t-31}{(4t+1)^2}=0$ $16t^2+8t-31=0$ $t=\dfrac{-8\pm\sqrt{8^2-4(16)(-31)}}{2(16)}$ $=\dfrac{-8\pm\sqrt{2048}}{32}$ $=\dfrac{-8\pm 32\sqrt{2}}{32}$ $=\dfrac{-1\pm 4\sqrt{2}}{4}$ So it is stationary when t $=\dfrac{-1\pm 4\sqrt{2}}{4}$ Only us positive value so t $=\dfrac{-1+4\sqrt{2}}{4}\approx 1\cdot 2\text{s}$ (to one dec pl)	2
(b)iv)	$t=\dfrac{-1+4\sqrt{2}}{4}\approx 1.2\text{s}$ $v=0, x\approx 2\cdot 58$ (to 2 dec pl) When $t=0$ $x=8$ When $t=2$ $x=2.89$ (to 2 dec pl) Particle starts 8 units to the right of the origin, moving toward the origin, it stops after 1.2 sec at 2.58 m to right of origin, then begins to move away from the origin, being 2.89 units to the right of the origin after 2 sec. $\therefore$total distance travelled is $(8-2\cdot 58)+(2\cdot 89-2\cdot 58)$ $=5\cdot 73\text{m}$ or $5\cdot 7$ m (to 1 dec place)	2
(c)	$3x-y=10$ (1) $x=y+2$ (2) $3(y+2)-y=10$ (3) sub (2) in (1) $3y+6-y=10$ $2y=4$ $y=2$ $x=(2)+2$ sub y in (2) $x=4$ solution (4, 2)	2

Question 6	Trial HSC Examination- Mathematics	
Part	Solution	Marks
(a)(i)	$y = x^6 - 6x^4$ Crosses axis where $x^6 - 6x^4 = 0$ $x^4(x^2 - 6) = 0$ $x^4(x - \sqrt{6})(x + \sqrt{6}) = 0$ Crosses axis where $x = 0$ and $x = \pm\sqrt{6}$	2
(a)(ii)	$y = x^6 - 6x^4$ $y' = 6x^5 - 24x^3$ $= 6x^3(x^2 - 4)$ $= 6x^3(x-2)(x+2)$ $y'' = 30x^4 - 72x^2$ Stationary points where $x = 0, y = 0, y'' = 0$ $x = 2, y = -32, y'' = 192$ $x = -2, y = -32, y'' = 192$ Stationary points $(-2,-32),(0,0)(2,-32)$ $y'' = 30x^4 - 72x^2$ At $x = 0$ $y'' = 0$ so test either side At $x = 1$ $y'' = -42$ $\therefore$ concave down. At $x = -1$ $y'' = -42$ $\therefore$ concave down $\therefore$ maximum at (0, 0) At $x = 2$ $y'' = 192$ $\therefore$ minimum at (2, -32). At $x = -2$ $y'' = 192$ $\therefore$ minimum at $(-2$, -32).	4

Question 6	Trial HSC Examination- Mathematics	
Part	Solution	Marks
(a)(iii)	$y'' = 30x^4 - 72x^2$ $= 6x^2(5x^2 - 12)$ $= 6x^2(\sqrt{5}x - 2\sqrt{3})(\sqrt{5}x + 2\sqrt{3})$ $x = 0 \quad y = 0$ $x = \frac{2\sqrt{3}}{\sqrt{5}} = \frac{2\sqrt{15}}{5} \quad y = -20.736$ $x = -\frac{2\sqrt{3}}{\sqrt{5}} = -\frac{2\sqrt{15}}{5} \quad y = -20.736$ Check for changes of concavity From above, no change at (0, 0) but there is a change at $\left(\pm\frac{2\sqrt{15}}{5}, -20.736\right)$ Inflexions at $\left(\pm\frac{2\sqrt{15}}{5}, -20.736\right)$	2
		2

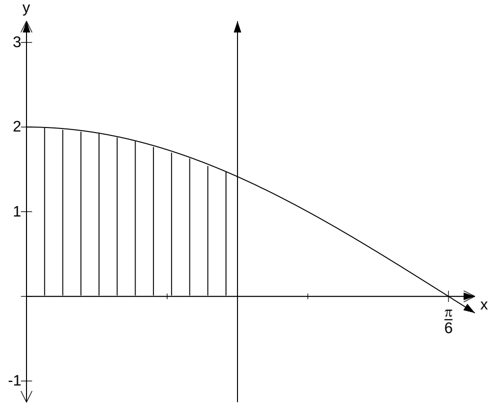
Question 6	Trial HSC Examination- Mathematics	
Part	**Solution**	**Marks**
(b)	 Stationary points on y occur where $f'(x)=0$ i.e. at $x=-4$ here $f''(x)$ is positive $\therefore$ min turning point at $x=-4$ and at $x=2$ here $f''(x)$ is negative $\therefore$ max turning point at $x=2$ or using first derivative test (explanation or table)	2

Question 7	Trial HSC Examination- Mathematics	
Part	**Solution**	**Marks**
(a)i)	$\log_a 0\cdot 4 = \log_a \frac{2}{5}$ $= \log_a 2 - \log_a 5$ $= x - y$	1
ii)	$\log_a 20 = \log_a \left(2^2 \times 5\right)$ $= 2\log_a 2 + \log_a 5$ $= 2x + y$	2

Question 7	Trial HSC Examination- Mathematics	
Part	**Solution**	**Marks**

b) (3 marks)

$y = 2\cos 3x$



$$Area = \int_0^{\frac{\pi}{12}} 2\cos 3x \; dx$$

$$= \left[\frac{2\sin 3x}{3}\right]_0^{\frac{\pi}{12}}$$

$$= \frac{2\sin\frac{\pi}{4}}{3} - \frac{2\sin 0}{3}$$

$$= \frac{2}{3\sqrt{2}} - 0$$

$$= \frac{\sqrt{2}}{3} \text{ square units.}$$

(c)i) (2 marks)

Substitute $y = x^2 - 3x$ into $y = 5x - x^2$

$5x - x^2 = x^2 - 3x$

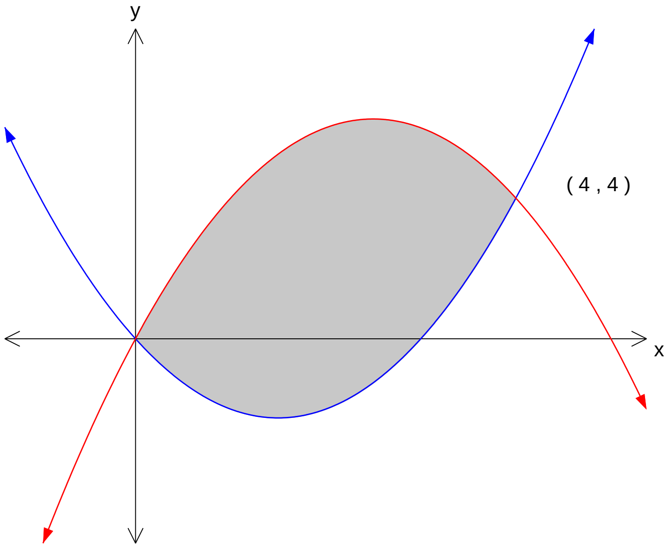
$2x^2 - 8x = 0$

$2x(x-4) = 0$

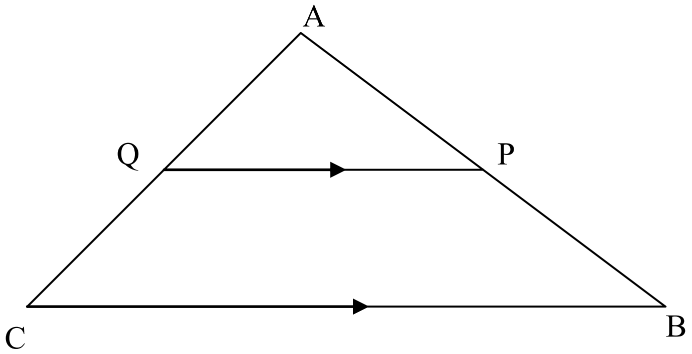
$x = 0, \quad y = 0$

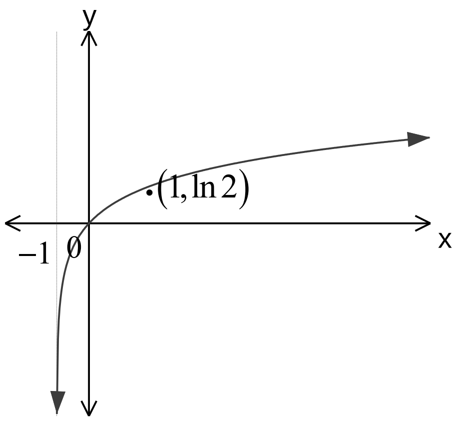
$x = 4, \quad y = 4$

Intersect at (0, 0) and (4, 4).

ii)	 $Area = \int_0^4 5x - x^2 dx - \int_0^4 x^2 - 3x dx$ $= \int_0^4 8x - 2x^2 \ dx$ $= \left[4x^2 - \frac{2x^3}{3} \right]_0^4$ $= \left(64 - \frac{128}{3} \right) - 0$ $= \frac{64}{3} = 21\frac{1}{3} u^2$	3
d)	$\frac{d}{dx} \log_e (\cos x)$ $= \frac{-\sin x}{\cos x}$ $= -\tan x$	

Question 8	Trial HSC Examination- Mathematics	
Part	Solution	Marks
(a)i)	$P = Ae^{kt}$ $\frac{dP}{dt} = Ae^{kt}.k$ $= kAe^{kt}$ $= kP$	1
ii)	$t = 1$ was 147 200 $P = Ae^{kt}$ $147200 = Ae^{k}$ (i) $t = 2$ was 154 800 $154800 = Ae^{2k}$ (ii) $\frac{154800}{147200} = \frac{Ae^{2k}}{Ae^{k}}$ (ii) $\div$(i) $1.0516 = e^{k}$ $k = \ln(1.0516)$ $k \approx 0.05$ $147200 = Ae^{0.05(1)}$ $A = \frac{147200}{e^{0.05}} = 139973$	2
iii)	When $t = 4$ $P = Ae^{kt}$ $P = 139973e^{0.05(4)}$ $= 171197$	1
iv)	$P = Ae^{kt}$ $200000 = 139973e^{0.05t}$ $\frac{200000}{139973} = e^{0.05t}$ $1.429 = e^{0.05t}$ $\ln(1.429) = \ln\left(e^{0.05t}\right)$ $0.05t = \ln(1.429)$ $t = \frac{\ln(1.429)}{0.05}$ $= 7.1$ $t = 7$ is start of 2012 Population will reach 200 000 in 2012	1

Question 8	Trial HSC Examination- Mathematics	
Part	Solution	Marks
(b)i)	 In ΔAPQ and ΔABC ∠A is common ∠AQP=∠ACB (Corresponding ∠s on || lines) ∠APQ= ∠ABC (Corresponding ∠ on || lines) ∴ ΔAPQ ||| ΔABC (Corresponding angles equal)	2
ii)	$\frac{AP}{AB}=\frac{1}{2}$ (P is midpoint of AB) $\frac{AP}{AB}=\frac{AQ}{AC}$ (sides of similar triangle in same ratio) $\frac{AQ}{AC}=\frac{1}{2}$ (from above) ∴ Q is midpoint of AC.	2
(c)	$\int_1^3 g(x)\,dx \approx \frac{1}{6}\{12+4(8)+2(0)+4(3)+5\}$ $\approx \frac{61}{6}$ $\approx 10\frac{1}{6}$	2
(d)	$\sum_{n=2}^{5}(n^2-1)=(2^2-1)+(3^2-1)+(4^2-1)+(5^2-1)$ $=3+8+15+24$ $=50$	1

Question 9	Trial HSC Examination- Mathematics	
Part	Solution	Marks
(a)	$y=2x^2-2$ $V=\pi\int_0^6 x^2dy$ $=\pi\int_0^6 \frac{y+2}{2}dy$ $=\pi\left[\frac{y^2}{4}+y\right]_0^6$ $=\pi\left[\left(\frac{36}{4}+6\right)-(0)\right]$ $=15\pi\ \ u^3$	3
(b)		2
(c)	$\frac{2}{5}\int 5x^4e^{x^5}dx=\frac{2}{5}e^{x^5}+c$	1
(c)i)	$A_6=20000-6M$	1
ii)	$A_7=(20000-6M)1.01-M$ $A_8=\left[(20000-6M)1.01-M\right]1.01-M$ $=(20000-6M)1.01^2-1.01M-M$ $=(20000-6M)1.01^2-M(1+1.01)$	1
iii)	$A_9=(20000-6M)1.01^3-M\left(1+1.01+1.01^2\right)$ $A_n=(20000-6M)1.01^{n-6}-M\left(1+1.01+1.01^2+.......1.01^{n-7}\right)$ $A_{36}=(20000-6M)1.01^{30}-M\left(1+1.01+1.01^2+.......1.01^{29}\right)$	2

Question 9	Trial HSC Examination- Mathematics	
Part	Solution	Marks
iv)	Since repaid after 36 months $A_{36} = 0$ $(20000-6M)1.01^{30} - M(1+1.01+1.01^2+.......1.01^{29}) = 0$ $M(1+1.01+1.01^2+.......1.01^{29}) = (20000-6M)1.01^{30}$ Need to evaluate $1+1.01+1.01^2+.......1.01^{29}$ Geometric series with $a=1$, $r=1.01$, $n=30$ $S_{30} = \frac{a(r^n-1)}{r-1}$ $= \frac{1(1.01^{30}-1)}{1.01-1}$ $=34.785$ $34.785M = (20000-6M)1.01^{30}$ $\frac{34.785M}{1.01^{30}} = 20000-6M$ $6M + \frac{34.785M}{1.01^{30}} = 20000$ $M\left(6+\frac{34.785}{1.01^{30}}\right) = 20000$ $31.8M = 20000$ $M = \frac{20000}{31.8}$ $= \$629$ (nearest dollar)	2

Question 10	Trial HSC Examination- Mathematics	
Part	Solution	Marks
a) i)	$V=2-\sqrt{3}\cos t-\sin t$ $\frac{dV}{dt}=\sqrt{3}\sin t-\cos t$	1
ii)	When $t=0$ $\frac{dV}{dt}=\sqrt{3}\sin 0-\cos 0$ $=-1$ $\therefore$ the tank is emptying at this time.	1
iii)	Full (or empty) when $\frac{dV}{dt}=0$ $\frac{dV}{dt}=0$ $\sqrt{3}\sin t-\cos t=0$ $\sqrt{3}\sin t=\cos t$ $\frac{\sin t}{\cos t}=\frac{1}{\sqrt{3}}$ $\tan t=\frac{1}{\sqrt{3}}$ $=\frac{\pi}{6},\frac{7\pi}{6},\frac{13\pi}{6}...$ As tank is initially emptying, second value corresponds to when full. Tank is first full when $t=\frac{7\pi}{6}$ $V=2-\sqrt{3}\cos t-\sin t$ $=2-\sqrt{3}\cos\frac{7\pi}{6}-\sin\frac{7\pi}{6}$ $=4$	3

(b)i)	$\angle$D is common $\angle C = \angle B = 90^o$ $\angle E = \angle A$ (corresponding angles on|| lines) $\Delta ABD \parallel\!\mid \Delta ECD$ (equiangular) $\frac{x+h}{x} = \frac{2t}{2} = t$ $x + h = tx$ $tx - x = h$ $x(t-1) = h$ $x = \frac{h}{t-1}$	2
(b)ii)	$V = \frac{1}{3}\pi\left(2t\right)^2.\left(h+x\right) - \frac{1}{3}\pi\left(2\right)^2.x$ $= \frac{1}{3}\pi\left(2t\right)^2.\left(h + \frac{h}{t-1}\right) - \frac{1}{3}\pi\left(2\right)^2.\left(\frac{h}{t-1}\right)$ $= \frac{1}{3}\pi\left(2t\right)^2.\left(\frac{ht}{t-1}\right) - \frac{1}{3}\pi\left(2\right)^2.\left(\frac{h}{t-1}\right)$ $= \frac{1}{3}\pi\left(2\right)^2.\left(\frac{h}{t-1}\right)\left(t^3 - 1\right)$ $= \frac{4}{3}\pi.\left(\frac{h}{t-1}\right)\left(t-1\right)\left(t^2+t+1\right)$ $= \left(\frac{4\pi h}{3}\right)\left(t^2+t+1\right)$	2

iii)	Sum of radii and height =12 $2+h+2t=12$ $h=10-2t$ $V=\left(\frac{4\pi h}{3}\right)\left(t^2+t+1\right)$ $=\left(\frac{4\pi}{3}\right)(10-2t)\left(t^2+t+1\right)$ $=\left(\frac{4\pi}{3}\right)\left(10t^2+10t+10-2t^3-2t^2-2t\right)$ $=\left(\frac{4\pi}{3}\right)\left(8t^2+8t-2t^3+10\right)$ $V=\left(\frac{4\pi}{3}\right)\left(8t^2+8t-2t^3+10\right)$ $\frac{dV}{dt}=\left(\frac{4\pi}{3}\right)\left(16t+8-6t^2\right)$ $\frac{dV}{dt}=0$ $\left(16t+8-6t^2\right)=0$ $6t^2-16t-8=0$ $3t^2-8t-4=0$ $t=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$ $=\frac{8\pm\sqrt{(-8)^2-4(3)(-4)}}{2(3)}$ $=\frac{8\pm\sqrt{112}}{6}$ $=-0.43$ or 3.10 (to 2 dec pl) $t>0$ so $t=3\cdot 10$ m $\frac{d^2V}{dt^2}=\left(\frac{4\pi}{3}\right)(16-12t)$ $\frac{d^2V}{dt^2}=\left(\frac{4\pi}{3}\right)(16-12(3.10))=\left(\frac{4\pi}{3}\right)(-21.2)$ $=-88.7$ When t = 3.10 $\therefore \frac{d^2V}{dt^2}<0$ $\therefore$ V is a maximum $V=\left(\frac{4\pi}{3}\right)\left(8(3.10)^2+8(3.10)-2(3.10)^3+10\right)$ $=218.2\ \text{m}^3$ (to 1 dec pl)	3