2009
TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics

General Instructions

- Reading Time - 5 minutes.
- Working Time - 3 hours.
- Write using a blue or black pen.
- Approved calculators may be used.
- A table of standard integrals is provided at the back of this paper.
- All necessary working should be shown for every question.
- Begin each question in a new booklet.

Total marks (**120**)

- Attempt Questions 1-10.
- All questions are of equal value.

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Question 1 (12 Marks) Use a Separate Writing Booklet **Marks**

(a) Evaluate $\dfrac{5.25}{\sqrt{9.74-3.35}}$ correct to 2 decimal places. **2**

(b) Solve $|2x-3| \geq 5$ **2**

(c) Find the exact value of $\tan\dfrac{5\pi}{6}$ **2**

(d) Solve $5x^2-2x-3=0$ **2**

(e) Express $\dfrac{5\sqrt{3}}{\sqrt{7}-2}$ with a rational denominator. **2**

(f) Paint at the local hardware store is sold at a profit of 30% on the cost price. If a drum of paint is sold for \$67·50, find the cost price to the nearest cent. **2**

Question 2 **(12 Marks)** Use a Separate Writing Booklet **Marks**

(a) Differentiate with respect to x.

(i) $2\cos(3x)$ **2**

(ii) $\dfrac{\sin x}{e^{2x}}$ **2**

(b) Find:

(i) $\int \left(6e^{6x} + \dfrac{6}{x}\right) dx$ **2**

(ii) $\int_0^{\pi} \sec^2 \dfrac{x}{4}\, dx$ **2**

(c) If α and β are the roots of the equation $3x^2 - 4x - 7 = 0$
Find:

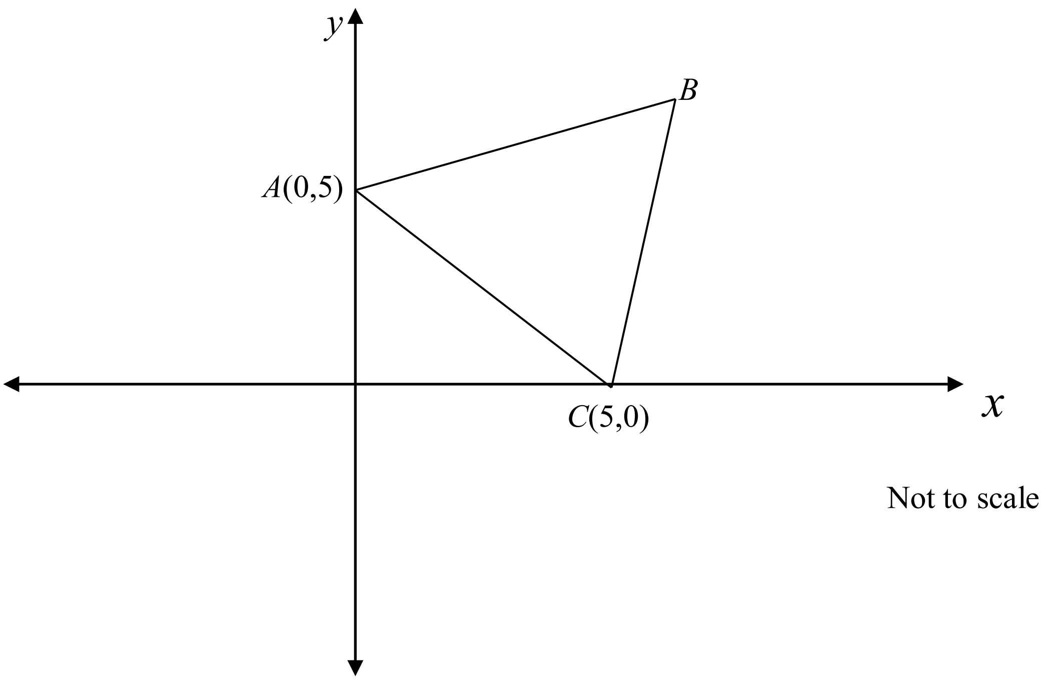
(i) $2\alpha + 2\beta$. **1**

(ii) $\dfrac{2}{\alpha} + \dfrac{2}{\beta}$. **1**

(iii) $2\alpha^2 + 2\beta^2$. **2**

Question 3 **(12 Marks)** Use a Separate Writing Booklet **Marks**

(a) The diagram below shows ΔABC.



The lines AB and CB have equations $x-6y+30=0$ and $6x-y-30=0$ respectively.

(i) Find the coordinates of the point B. **2**

(ii) Find the gradient of the line AC. **1**

(iii) Show that the line AC has equation $x+y-5=0$. **1**

(iv) Find the exact distance of AC. **2**

(v) Find the perpendicular distance from the point B to the line AC and hence find the area of the triangle ABC. **3**

(vi) State the inequalities that together define the area bounded by the triangle ABC. **1**

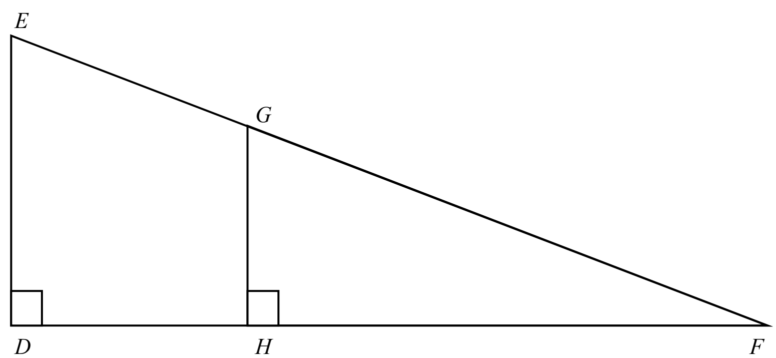
(b) Solve $\sqrt{3}\tan x=-1$ in the domain $0\le x\le 2\pi$. **2**

Question 4 (12 Marks) Use a Separate Writing Booklet **Marks**

(a) (i) Sketch $y = 4\cos 2x$ in the domain $0 \le x \le 2\pi$. **2**

(ii) Hence using the graph or otherwise find the number of solutions to the equation $4\cos 2x = -2$ in the domain $0 \le x \le 2\pi$. Justify how you derived your answer. **2**

(b) In the diagram below $\angle EDF = \angle GHF = 90^\circ$.



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(i) Show, giving reasons that $\Delta\, EDF$ is similar to $\Delta\, GHF$. **2**

(ii) Given that $ED = 9$ cm, $GH = 6$ cm and $DH = 7$ cm, find the distance HF. **2**

(c) Consider the function $f(x) = \ln(x+5)$

(i) State the domain and range of the function. **2**

(ii) Find the exact value of x when $f(x) = 5$. **2**

Question 5 **(12 Marks)** Use a Separate Writing Booklet **Marks**

(a) Consider the curve that has equation $y = 2x^3 - 9x^2 + 12x + 3$

(i) Find the coordinates of the stationary points and determine their nature. **4**

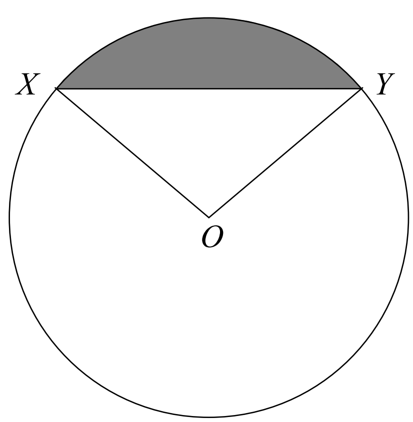
(ii) Find any points of inflexion. **2**

(iii) Graph the function showing all the main features. **2**

(b) Use Simpson's rule to evaluate $\int_1^{2.5} f(x)\,dx$, to 1 decimal place using the 7 function values in the table below. **2**

x	1·00	1·25	1·50	1·75	2·00	2·25	2·50
$f(x)$	3·43	2·17	0·38	1·87	2·65	2·31	1·97

(c) Given that in the circle below $\angle XOY = 60^\circ$ and the radius of the circle is 14 cm. Find the exact area of the minor segment shaded. **2**



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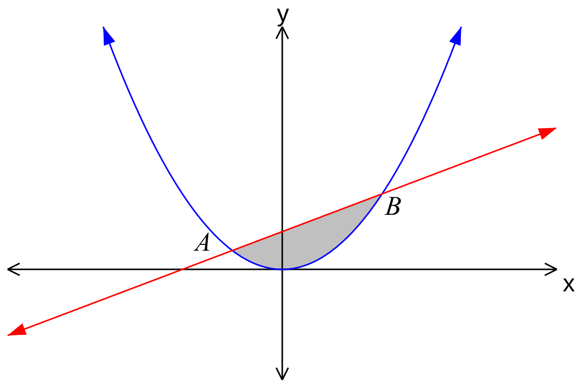
Question 6 (12 Marks) Use a Separate Writing Booklet **Marks**

(a) The parabola $y = x^2$ and the line $y = x + 2$ intersect at points A and B, as shown.



(i) Show that the points of intersection of $y = x^2$ and $y = x + 2$ are $A(-1, 1)$ and $B(2, 4)$. **2**

(ii) Write a definite integral that will give the shaded area bounded by the parabola and the line? **1**

(iii) Calculate this area. **2**

(b) The area bounded by the curve $y = 3e^{\frac{3}{2}x}$ between the lines $x = 1$ and $x = 3$ is rotated about the x-axis. Find the volume of the solid of revolution formed, in exact form. **3**

(c) Find the equation of the normal to $y = \cos 2x$ at the point where $x = \frac{\pi}{6}$. **4**

Question 7 **(12 Marks)** Use a Separate Writing Booklet **Marks**

(a) Diego borrows \$20 000 in order to buy a new car.
The interest rate is 6% per annum reducible and the loan is to be repaid in equal monthly repayments, M, over 4 years, with the interest calculated monthly.
Let \$ A_n be the amount owing after the nth repayment.

(i) Write down expressions for \$$A_1$ and \$$A_2$, the amounts owing after the first and the second repayments respectively. **2**

(ii) Find the amount of each monthly repayment. **2**

(b) A particle moves in a straight line so that its displacement (in m) from a fixed point O at time t seconds is given by $x = 2\sin 2t,\ 0 \le t \le 2\pi$.

Find:

(i) The initial velocity **1**

(ii) The acceleration after $\dfrac{\pi}{12}$ seconds. **1**

(iii) When the particle is at rest in the given domain. **2**

(iv) The displacement of the particle when it is at rest. **2**

(c) Evaluate $\displaystyle\sum_{n=2}^{15} 2^n$. **2**

Question 8 **(12 Marks)** Use a Separate Writing Booklet **Marks**

(a) The population P of a certain town grows at a rate proportional to the current population, and satisfies the equation $P = Ae^{kt}$, where A and k are constants, and t is measured in years.
If the population grows from 20 000 to 25 000 in two years :

(i) Find the values of A and k. **3**

(ii) Find the population of the town, to the nearest hundred, after a **further** 8 years. **2**

(iii) Calculate the rate of change of the population at this time. **2**

(b) If $\log_a 2 + 2\log_a x - \log_a 6 = \log_a 3$ find the value of x. **2**

(c) Solve the following equation for x, leaving your answer in exact form: **3**

$$e^{2x} - 3\cdot(e^x) - 10 = 0$$

Question 9 (12 Marks) Use a Separate Writing Booklet **Marks**

(a) In the construction of a 5 km highway a truck delivers materials from a base. After each load is deposited, the truck returns to the base to collect the next load. The first load is deposited 100 m from the base, the second 240 m from the base, the third 380 m from the base. Each subsequent load is deposited 140 m from the previous one.

(i) How far is the 20^{th} load deposited from the base? **1**

(ii) How many loads are deposited along the total length of the 5 km highway? (The last load is deposited at the end of the highway) **2**

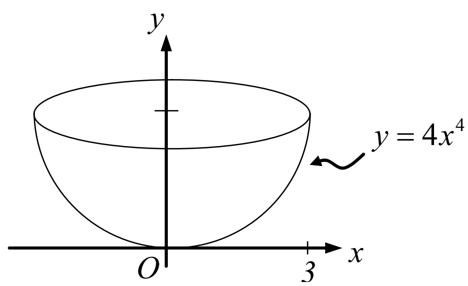
(iii) How many kilometres has the truck travelled in order to make all the deposits and then return to the base? **2**

(b) The rate at which Carbon Dioxide will be produced when conducting an experiment is given by $\dfrac{dV}{dt} = \dfrac{1}{100}\left(30t - t^2\right)$ where V cm^3 is the volume of gas produced after t minutes.

(i) At what rate is the gas being produced 15 minutes after the experiment begins. **2**

(ii) How much Carbon Dioxide has been produced during this time? **2**

(c) A bowl is formed by rotating the part of the curve $y = 4x^4$ between $x = 0$ and $x = 3$ about the y-axis. Find the volume of the bowl. **3**



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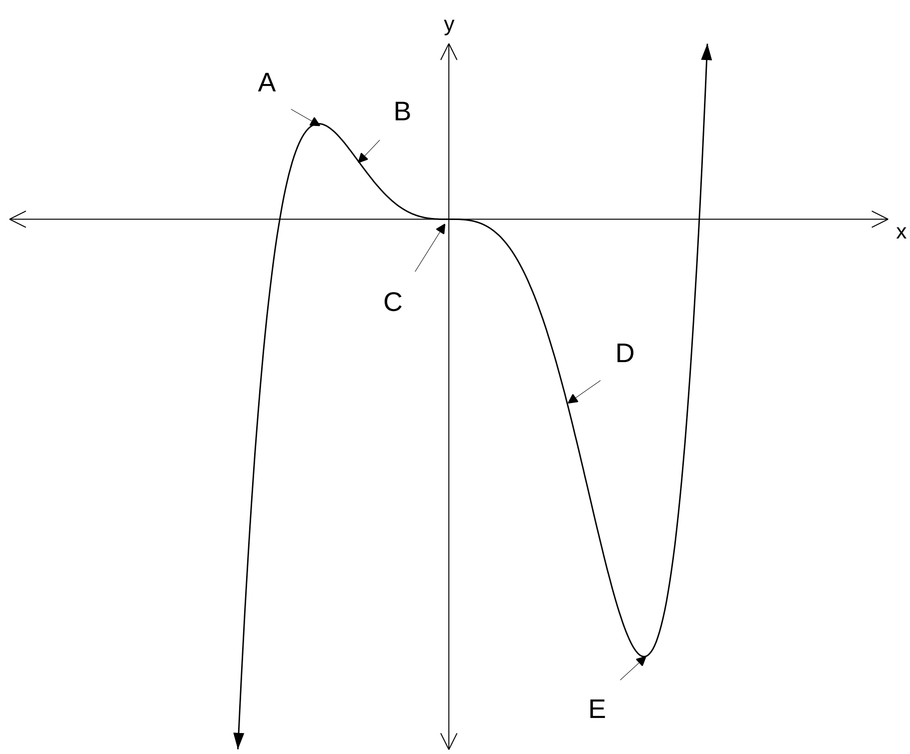
Question 10 (12 Marks) Use a Separate Writing Booklet **Marks**

(a) An open cylindrical can is made from a sheet of metal with an area of 300cm^2. Given that the surface area of an open cylinder is $SA = \pi r^2 + 2\pi rh$ and the volume is $V = \pi r^2 h$:

(i) Show that the volume of the can is given by $V = 150r - \frac{1}{2}\pi r^3$. **2**

(ii) Find the radius of the cylinder that gives the maximum volume. Justify your answer. **3**

(b) The graph of the curve *y = f(x)* is drawn below.



(i) Name the points of inflexion. **1**

(ii) When is the graph decreasing? **1**

(iii) Sketch the gradient function. **1**

(c) (i) Show that $\dfrac{(1+\tan^2\theta)\cot\theta}{\operatorname{cosec}^2\theta} = \tan\theta$ **2**

(ii) Hence, solve $\dfrac{(1+\tan^2\theta)\cot\theta}{\operatorname{cosec}^2\theta}\cdot\tan\theta = 3$ for $-\pi \le \theta \le \pi$ **2**

END OF EXAMINATION

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STANDARD INTEGRALS

$$\int x^n \, dx = \frac{1}{n+1} x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x} \, dx = \ln x, \quad x > 0$$

$$\int e^{ax} \, dx = \frac{1}{a} e^{ax}, \quad a \neq 0$$

$$\int \cos ax \, dx = \frac{1}{a} \sin ax, \quad a \neq 0$$

$$\int \sin ax \, dx = -\frac{1}{a} \cos ax, \quad a \neq 0$$

$$\int \sec^2 ax \, dx = \frac{1}{a} \tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax \, dx = \frac{1}{a} \sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2 + x^2} \, dx = \frac{1}{a} \tan^{-1} \frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} \, dx = \sin^{-1} \frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} \, dx = \ln\left(x + \sqrt{x^2 - a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} \, dx = \ln\left(x + \sqrt{x^2 + a^2}\right)$$

NOTE : $\ln x = \log_e x, \quad x > 0$