SYDNEY BOYS' HIGH SCHOOL

MOORE PARK, SURRY HILLS

TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

1996

MATHEMATICS

2/3 UNIT

Time allowed - Three hours
(Plus 5 minutes reading time)

Examiners: R.Boros, D.Hespe

DIRECTIONS TO CANDIDATES

* *ALL* questions may be attempted.
* The marks allocated to each question are indicated.
* All necessary working should be shown in very question. Full marks may not be awarded for careless or badly arranged work.
* Standard integrals are printed at the back. Approved calculators may be used.
* *Each* section attempted is to be returned in a *separate* bundle, clearly marked Section A (Q1, Q2), Section B (Q3, Q4), Section C (Q5, Q6), Section D (Q7, Q8), or Section E (Q9, Q10). Each bundle must also show your name. Start each question on a new page.
* If required, additional paper maybe obtained from the Examination Supervisor upon request.

NOTE: This is a trial paper only and does not necessarily reflet the content or format of the Higher School Certificate Examination Paper for this subject.

SECTION A (Hand up separately)

Question 1 (start a new page)

(2 marks) [a] Factorise and simplify $\frac{2x^2 - 8}{x^3 - 8}$

(1 mark) [b] Solve $\frac{x-1}{2} + \frac{x+2}{3} = 5$

(3 marks) [c] Solve $|2x-1| > 5$ and graph your solution on a number line.

(2 marks) [d] If $f(p) = \begin{cases} p^2 & \text{for } p < 2 \\ 3p - 2 & \text{for } 2 \leqslant p < 3 \\ \frac{1}{3}p^3 - 2 & \text{for } p \geqslant 3 \end{cases}$

evaluate $f(-1) + f(2) + f(5)$

(2 marks) [e] Write down the exact value of $\cot 60^\circ$

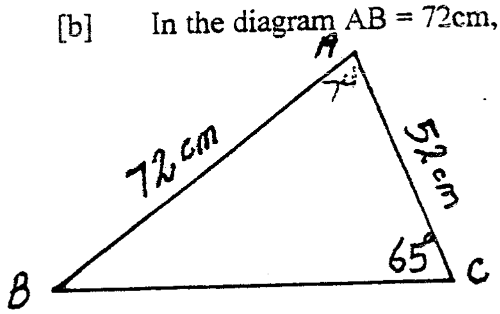
(2 marks) [f] Express $\frac{1+\sqrt{3}}{1-\sqrt{3}}$ in the form $a + b\sqrt{3}$

where a and b are rational numbers.

Question 2 [start a new page]

(2 marks) [a] Find the perpendicular distance from the origin to the line whose equation is $3x - 4y = 30$. Hence give the equation of the circle, centre the origin, to which the line $3x - 4y = 30$ is a tangent.

[b] In the diagram AB = 72cm, AC = 52cm and $A\hat{C}B = 65^\circ$



[figure not to scale]

(2 marks) (i) Find the size of $A\hat{B}C$ to the nearest degree.

(2 marks) (ii) Hence find the area of triangle ABC to 2 significant figures.

Question 2(Cont.)

[c] Given the points $A(-4, 2)$, $B(0,-4)$, $C(1, 1)$

(2 marks) (i) Find the equation of the line AB, written in the general form.

(2 marks) (ii) A line passing through the point C, perpendicular to AB, meets AB at N. Show that N has coordinates $(-2, -1)$

(2marks) [d] Solve for θ if $0^\circ \le \theta \le 360^\circ$, $3\tan^2\theta = 1$

SECTION B (Hand up separately)

Question 3 [start a new page]

[a] Differentiate with respect to x:

(1 mark) (i) $2x^2 - 4x + 9$

(1 mark) (ii) $(4-5x)^3$

(2 marks) (iii) $\dfrac{e^{2x}}{\sin x}$

(2 marks) (iv) $2x\ln(3-x)$

[b] Find

(1 mark) (i) $\int(\sin 2x - 2\sec^2 x)\,dx$

(1 mark) (ii) $\int_1^2 (3x+1)^2\,dx$

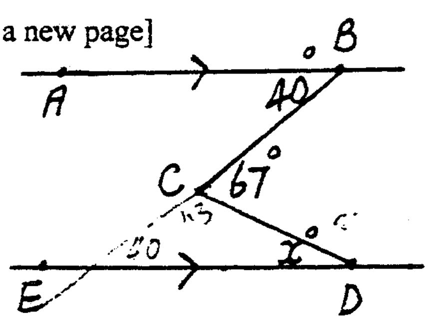
(2 marks) (iii) $\int_1^3 \left(\dfrac{1}{x+1} - \dfrac{1}{x^2}\right) dx$ (leave your answer in exact form)

(2 marks) [c] The gradient function of a curve is given by $\dfrac{dy}{dx} = 3-4x$.

Find the equation of the curve if it passes through the point $(6,-5)$.

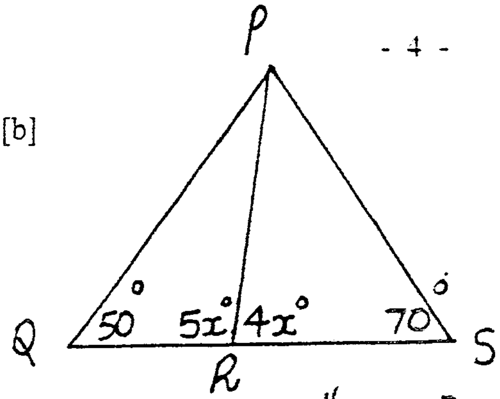
Question 4 [start a new page]

(1 mark) [a]



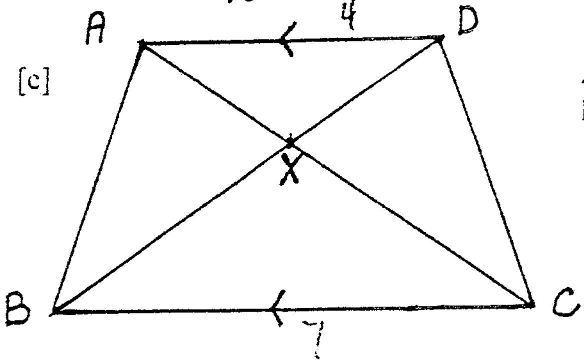
In the diagram AB∥ED.
Find x, giving reasons.

(3 marks) [b]



Given QRS is a straight line, prove that PR bisects the angle QPS.

[c]



ABCD is a trapezium. AD || BC. Intervals AC and BD intersect at X.

(2 marks) (i) Prove that $\frac{AX}{AD} = \frac{CX}{BC}$,

(1 mark) (ii) Given AD = 4cm, BC = 7cm find the ratio AX to CX.

(2 marks) [d] Prove that $\frac{1 + \cos\theta}{\sin\theta} + \frac{\sin\theta}{1 + \cos\theta} = 2\operatorname{cosec}\theta$.

(3 marks) [e] Shade on a number plane the region for which $y \leq \sqrt{16 - x^2}$ and $y \geq 2$ hold.

SECTION C (hand up separately)

Question 5 [start a new page]

(2 marks) [a] An urn contains 7 green discs and 4 blue discs. Two discs are withdrawn at random, one at a time, the first one not being replaced before the second one is drawn. Find the probability that the two discs drawn are of different colours.

[b] A particle is moving in a straight line so that its distance t seconds after the initial observation is x cm from a fixed point O. The position of the particle is given by $x = t^3 - 6t^2 + 9t$.

Question 5 [b] Cont.

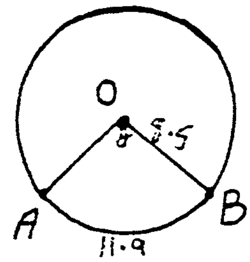
(2 marks) (i) Find the initial position and initial velocity of the particle.

(2 marks) (ii) Show that the particle is momentarily at rest on two occasions.

[c] An arc of length 11.9cm subtends an angle of θ at the centre O of a circle of radius length 8.5cm. Find:

(2 marks) (i) the value θ in both radians (to 1 decimal place) and degrees (nearest degree),

(2 marks) (ii) the area of sector AOB.



(2 marks) [d] Draw a neat sketch of the curve $y = 3\sin 2x$ for the domain $0 \leq x \leq 2\pi$.

Question 6 [start a new page]

[a] At the beginning of each year a man invests $2400 in a superannuation scheme which offers a 10.5% p.a. return on his investment. He contributes to this scheme for 24 years. The interest on his investment is compounded every year. Find to the nearest dollar:

(1 mark) (i) the value of his first $2400 investment at the end of the 24 year period.

(3 marks) (ii) the total value of the superannuation at the end of the 24 year period.

[b] Given the parabola $y = x^2 - 2x - 3$.

(2 marks) (i) express this equation in the form $(x-h)^2 = 4a(y-k)$.

(1 mark) (ii) state the coordinates of the vertex.

(1 mark) (iii) state the coordinates of the focus.

(1 mark) (iv) what is the equation of the directrix ?

Question 6 [b] Cont.

(1 mark) (v) what is the equation of the normal to this parabola at the point where this parabola crosses the y axis?

[c] If α and β are the roots of $2x^2 - 4x + 9$ what is the value of

(1 mark) (i) $\frac{1}{\alpha} + \frac{1}{\beta}$

(1 mark) (ii) $(\alpha - \beta)^2$

SECTION D (hand up separately)

Question 7 (start a new page)

(2 marks) [a] Use the trapezoidal rule with 3 function values to find an approximate value for

$$\int_0^{\frac{\pi}{2}} \sin^2 x \, dx$$

Answer correct to 3 significant figures.

(4 marks) [b] Determine the area enclosed by the curve $f(x) = 3x(2 - x)$ and the x axis from $x = 1$ to $x = 3$.

(4 marks) [c] Find the volume generated when the region bounded by the curve $y = x^2 + 1$ and the x axis between $x = o$ and $x = 5$ is rotated around the x axis. Leave your answer in exact form in terms of π.

(2 marks) [d] Find the value of x if $\log x^2 - \log 2x = \log 8$. All logs are to the same base.

Question 8 (start a new page)

(2 marks) [a] Sketch the graph of a function given that

$f(2) = 0, \; f'(2) = 0$

$f'(x) < 0$ for all $x < 2$

$f'(x) > 0$ for all $x > 2$.

(3 marks) [b] A plant is observed over a period of time. Its initial height is 40cm. It grows 6cm during the first week of observation. Each succeeding week the growth is 70% of the previous weeks growth. Assuming this pattern continues, calculate the plant's ultimate height.

[c] Consider the curve $y = x^3 + x^2 - x + 1$.

(3 marks) (i) Find any turning points and determine their nature.

(2 marks) (ii) Find any points of inflexion

(2 marks) (iii) Sketch the curve for $-2 \leq x \leq 1$

SECTION E (hand up separately)

Question 9 (start a new page)

(2 marks) [a] Solve $2^x \times 4^{x+1} = 0{\cdot}5$

(3 marks) [b] Find the value(s) of k for which $x^2 - (k - 2)x + (k + 1) = 0$ has real roots.

(3 marks) [c] Find the locus of a point $P(x,y)$ which moves so that it is equidistant from the point S $(-4, 0)$ and the y axis.

Question 9 [d]Cont.

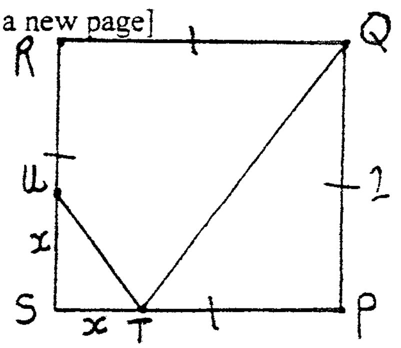
[d] A particle starts from rest at 0 and moves in a straight line with a velocity V cm/s after t seconds, given by $V = 16t - 4t^2$.

(2 marks) (i) Find the acceleration after 2 seconds.

(2 marks) (ii) Find the distance travelled in the first 2 seconds.

Question 10 [start a new page]

[a]



PQRS is a square of side length 2cm. ST = SU = xcm as shown on the diagram.

(1 mark) (i) Show that the area $A\text{cm}^2$, of the quadrilateral QRUT is given by
$A = 2 + x - \frac{1}{2}x^2$

(2 marks) (ii) Find the value of x for which A is a maximum.

(1 mark) (iii) Hence determine the maximum area for QRUT.

[b] Ron borrows $95,000 to buy a cottage. He is charged interest on the balance owing at the rate of 10.5% p.a. compounded monthly and agrees to repay the loan including interest making equal monthly instalments of $M.

(1 mark) (i) How much does Ron owe at the end of the first month just before he makes an instalment?

(1 mark) (ii) Write an expression involving M for the total amount owed by Ron just after the first instalment is paid.

Question 10 [b] Cont.

(3 marks) (iii) Calculate the value of M (to the nearest cent) which will repay the loan after 25 years.

(3 marks) (iv) In how many months (to the nearest whole month) will the loan be repaid if Ron made instalments of $1500 per month?

THIS IS THE END OF THE PAPER

STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x}\,dx = \ln x, \quad x > 0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}, \quad a \neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax, \quad a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax, \quad a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax\,dx = \frac{1}{a}\sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \ln\left(x+\sqrt{x^2-a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \ln\left(x+\sqrt{x^2+a^2}\right)$$

NOTE: $\ln x = \log_e x, \quad x > 0$

(1)(a) $\dfrac{2(x+2)}{x^2+2x+4}$

(b) $x = 4\frac{3}{5}$

(c) $\{x > 3 \text{ or } x < -2\}$

(d) $34\frac{2}{3}$

(e) $\dfrac{\sqrt{3}}{3}$

(f) $-\sqrt{3} - 2$

(2)(a)(i) $3\sqrt{10}$ (ii) $x^2 + y^2 = 90$

(b)(i) 41^0 (ii) 1800 cm^2

(c)(i) $3x + 2y + 8 = 0$ (ii) Proof

(d) $30^0, 150^0, 210^0, 330^0$

(3)(a)(i) $4(x-1)$ (ii) $-15(4-5x)^2$

(iii) $\dfrac{e^{2x}(2\sin x - \cos)}{\sin^2 x}$

(iv) $2\ln(3-x) - \dfrac{2x}{3-x}$

(b)(i) $-\frac{1}{2}\cos 2x - 2\tan x + c$ (ii) 31

(iii) $\ln 2 - \frac{2}{3}$

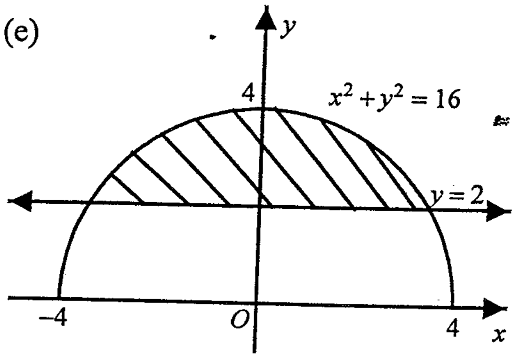
(c) $y = -2x^2 + 3x + 49$

(4)(a) $x = 27$ (b) Proof

(c)(i) Proof (ii) $\frac{4}{7}$

(d) Proof

(e)



(5)(a) $2 \times \dfrac{4}{11} \times \dfrac{7}{11} = \dfrac{56}{121}$

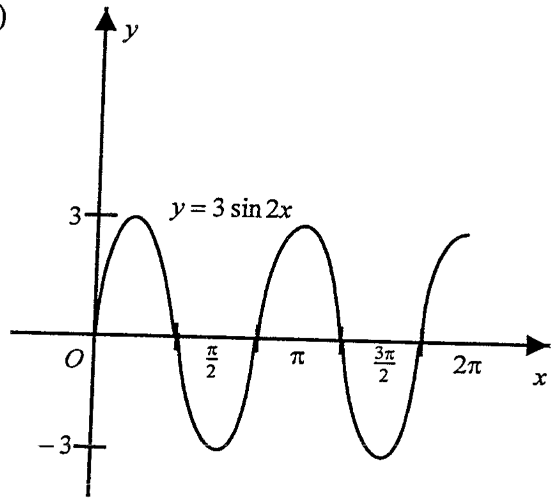
(b)(i) $x = 0, v = 9$ cm/s

(ii) Let $v = 0$

(c)(i) 1.4 rad$\approx 80^0$

(ii) 50.575 units^2 (to 3 d.p)

(d)



(6)(a)(i) \$26 358 (to the nearest dollar).

(ii) \$281 250

(b)(i) $(x-1)^2 = y + 4$

(ii) $(1, -4)$

(iii) $\left(1, -3\frac{3}{4}\right)$

(v) $x-8y-24=0$

(c)(i) $\frac{4}{9}$ (ii) -14

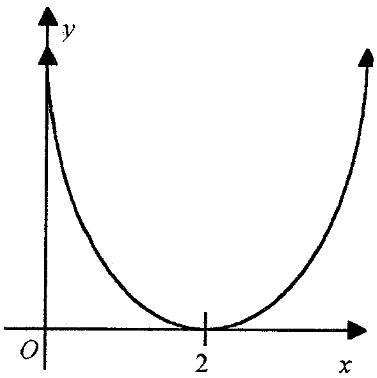
(7)(a) 0.785 (to 3 s.f.)

(b) 6 units^2

(c) 755π units^3

(d) $x=16$

(8)(a)

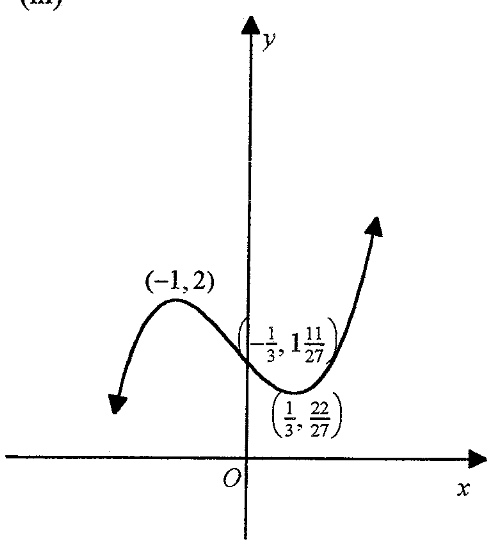


(b) 60

(c)(i) $(-1, 2)$ rel. max.; $\left(\frac{1}{3}, \frac{22}{27}\right)$ rel. min.

(ii) $\left(-\frac{1}{3}, 1\frac{11}{27}\right)$

(iii)



(9)(a) $x=-1$

(b) $x \geq 4+2\sqrt{2}$ or $x \leq 4-2\sqrt{2}$

(c) Locus is a parabola : $y^2 \doteq 8x+16$

(d)(i) $a=0$ cm/s^2 (ii) $21\frac{1}{3}$ cm

(10)(a)(i) Proof

(ii) $x=1$

(iii) Area= 2.5 cm^2

(b)(i) \$95831.25

(ii) \$(95831.25 $-M$)

(iii) \$896.97

(iv) 93 months (to the nearest month)