SYDNEY BOYS HIGH SCHOOL
MOORE PARK, SURRY HILLS

2002

TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics

General Instructions

- Reading time – 5 minutes.
- Working time – 3 hours
- Write using black or blue pen.
- Board approved calculators may be used.
- All necessary working should be shown in every question if full marks are to be awarded.
- Marks may not be awarded for messy or badly arranged work.

Total Marks - 120 marks

- All questions are of equal value.

Examiners: *P. Bigelow, P. Parker*

NOTE: This is a trial paper only and does not necessarily reflect the content or format of the final Higher School Certificate Examination Paper for this subject.

Question 1: (12 Marks) **Marks**

(a) Solve $\frac{3}{x} = \frac{4}{5}$ **2**

(b) If $a = 2 \cdot 673$ and $b = 1 \cdot 049$, evaluate $\frac{a^2 + b^2}{ab}$ correct to 2 decimal places. **2**

(c) Factorise $cd - c - dy + y$ **2**

(d) The line $kx - y = 29$ passes through the point $(4, -1)$, find the value of k. **2**

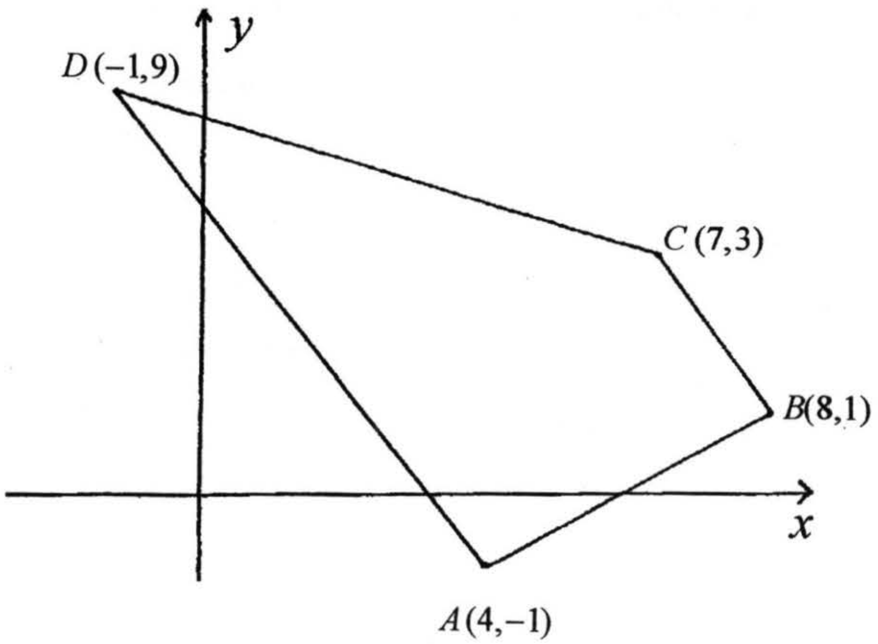
(e) Graph the solution to $|x + 3| \leq 1$ on a number line. **2**

(f) Find integers a and b such that **2**

$$\frac{1}{\sqrt{3} + 2} = a\sqrt{3} + b$$

START A NEW BOOKLET

Question 2: (12 Marks) **Marks**



A, B, C and D are the points $(4,-1)$, $(8,1)$, $(7,3)$ and $(-1,9)$ respectively.

(a) Show that the equation of AC is $4x-3y-19=0$ **2**

(b) Show $BC \parallel AD$ **2**

(c) Show $\angle ACD = 90^\circ$ **2**

(d) Show the length of AC is 5 units **2**

(e) Find the perpendicular distance of B from AC **2**

(f) Find the area of the trapezium $ABCD$. **2**

START A NEW BOOKLET

Question 3: (12 Marks) **Marks**

(a) Differentiate

(i) $(2x-1)^7$ **1**

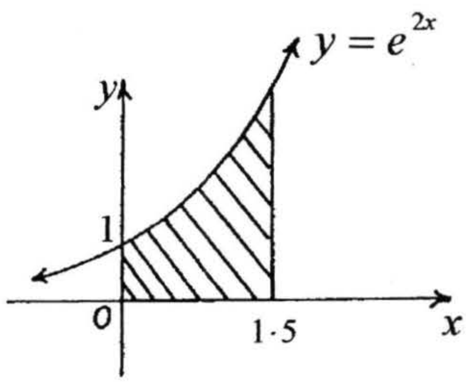
(ii) $\log_e(4+5x)$ **2**

(iii) $x^2 \sin x$ **2**

(b) Find $\int \sec^2 3x \, dx$ **1**

(c) Evaluate $\int_1^{e^2} \frac{3}{x} \, dx$ **3**

(d) **3**



The diagram above shows the region bounded by the curve $y = e^{2x}$, the line $x = 1 \cdot 5$ and the coordinate axes.
Find the EXACT area of this region.

START A NEW BOOKLET

Question 4: (12 Marks) **Marks**

(a) A parabola has equation $x^2 - 2x + 25 = 8y$.

(i) By completing the square, express this in the general form $(x-p)^2 = 4a(y-q)$ **2**

(ii) State the coordinates of the vertex. **1**

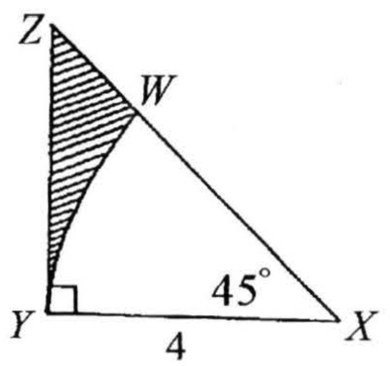
(iii) State the coordinates of the focus. **1**

(iv) State the equation of the directrix. **1**

(b) Find the equation of the line through the intersection of the lines $x - y = 0$ and $x + 2y - 6 = 0$ and which is parallel to the line $3x - 2y + 7 = 0$. **4**

Leave your answer in general form.

(c) **3**



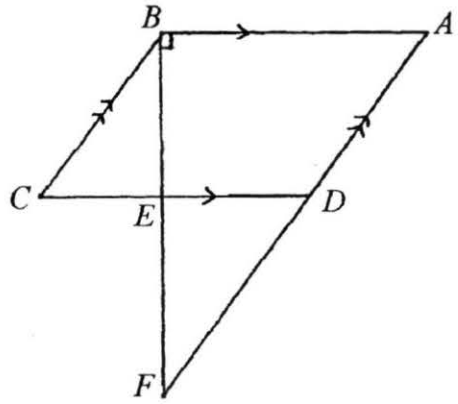
In the diagram above, XYZ is a right-angled triangle, $\angle ZXY = 45^\circ$ and $XY = 4$ units. A circular arc, centre X and radius YX cuts the side XZ at W.
Find the EXACT area of the shaded region WZY.

START A NEW BOOKLET

Question 5: (12 Marks) **Marks**

(a) Find the equation of the normal to $y = x^2 - 3x + 5$ at the point $(3,5)$. Leave your answer in general form. **2**

(b)

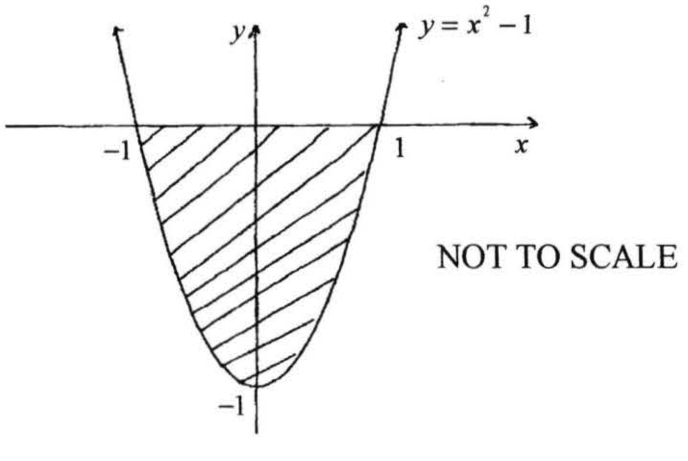


$ABCD$ is a parallelogram and $FB \perp AB$

(i) Prove $\Delta CBE \;|||\; \Delta AFB$ **3**

(ii) If $CE = 3$ cm, $BC = 7$ cm and $AF = 15$ cm, find AB. **2**

(c) **3**



The area bounded by the curve $y = x^2 - 1$ and the x-axis is rotated about the x-axis.
Find the volume of the solid of rotation. Leave your answer in EXACT form.

(d) A continuous curve $y = f(x)$ has the following properties for the closed interval $x_1 \le x \le x_2$: **2**

$$f(x) > 0, \quad f'(x) < 0, \quad f''(x) > 0$$

Sketch a curve satisfying these conditions.

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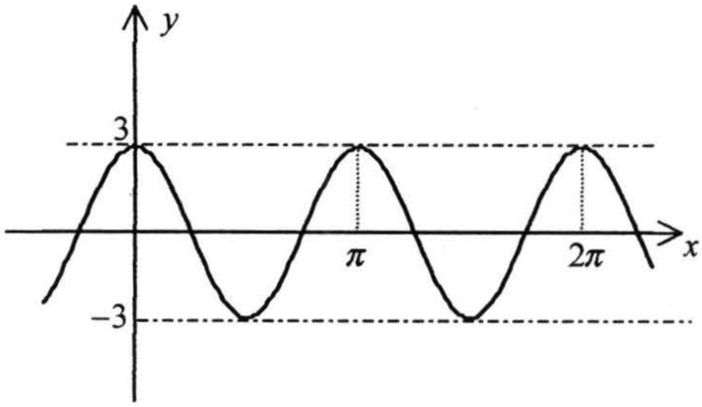
Question 6: (12 Marks) **Marks**

(a) The curve $y = x^3 + mx + n$ has a stationary point at $P(1,5)$. Find the values of the constants m and n. **3**

(b)



The graph above represents $y = a\cos mx$. **2**

Write down the values of a and m.

(c) Express as a single logarithm in simplest form **2**

$$\ln 2 + 2\ln 18 - \tfrac{3}{2}\ln 36$$

(d) Simplify $\sqrt{\dfrac{1-\cos^2\theta}{1+\tan^2\theta}}$ **2**

(e) The table shows the value of a function $f(x)$ for five values of x. **3**

x	-1	1	3	5	7
$f(x)$	5	9	2	-1	-6

Use Simpson's rule with these five values to estimate $\int_{-1}^{7} f(x)\,dx$

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Question 7: (12 Marks) **Marks**

(a) The value $\$V$ of a particular make of car can be calculated using the equation

$$V = 50000e^{-0.25t}, \text{ where } t \text{ years is the age of the car}$$

(i) State the value of the car when it is new **1**

(ii) What is the value of the car after 3 years? (to the nearest \$100). **2**

(iii) How long will it take for the car to be worth \$15 000? (Answer to the nearest tenth of a year) **2**

(b) A total of \$15 600 is to be shared among three people A, B and C. If A is to get the smallest share of \$1600, find the value of the remaining shares when

(i) the values of the three shares form an arithmetic series **2**

(ii) the values of the three shares form a geometric series **2**

(c) Two cards are chosen at random, without replacement from the seven cards below.

2	2	4	6	6	6	6

What is the probability that

(i) both cards are 2? **1**

(ii) the sum of the two numbers on the cards chosen, is greater than 8? **2**

START A NEW BOOKLET

Question 8: (12 Marks) **Marks**

(a) Two artillery guns are situated 3 kilometres apart at positions X and Y respectively. They are both aiming at a target T. **3**

The angles XYT and YXT are respectively 72° and 78°.

Find the distance between the target and the gun nearer to it

(b) Given that $x = -\frac{1}{2}$ is one root of the following quadratic equation **3**

$$mx^2 - 20x + m = 0,$$

find the exact value of the other root.

(c) Consider the function $y = x^3 + 3x^2 - 9x$

(i) Find the coordinates of the stationary points. **2**

(ii) Find the domain in which the curve is concave upwards. **1**

(iii) Sketch the curve for $-5 \le x \le 3$. **3**

START A NEW BOOKLET

Question 9: (12 Marks) **Marks**

(a) Solve $|x+1| = 2x+7$ **3**

(b) Comment on the following reasoning ie JUSTIFY whether the following statement is TRUE or FALSE **2**

"When two coins are tossed, they can either fall as two heads or two tails or as a tail and a head.
As there are three possibilities the probability of 2 heads is $\frac{1}{3}$."

(c) The acceleration a metres per second per second of a moving object is given at time t seconds $(t \geq 0)$ by

$$a = 2\pi^2 \cos \pi t$$

At time $t = 0$, the object is at the point $x = 0$, and travelling with velocity $v = \pi$ metres per second.

(i) Find the velocity v and the displacement x as a function of t, for $t \geq 0$ **3**

(ii) Find, for t in the range $0 \leq t \leq 4$, the values of t for which the object is stationary. **2**

(iii) Show that, for t in the range $2 \leq t \leq 4$, the largest value of x is **2**

$$2 + \sqrt{3} + \frac{19\pi}{6}.$$

START A NEW BOOKLET

Question 10: (12 Marks) **Marks**

(a) $C(n) = 1000(0\cdot 8)^n + 10\,000[1 + 0\cdot 8 + (0\cdot 8)^2 + \ldots + (0\cdot 8)^{n-1}]$ **3**

Find the limiting sum of C as the number of terms increases indefinitely

(b) A scientist has found that the amount, $Q(t)$, of a substance present in a mineral at time $t \geq 0$ satisfies

$$4\frac{d^2Q}{dt} + 4\frac{dQ}{dt} + Q = 0$$

(i) Verify that $Q(t) = A(1+t)e^{-0.5t}$ satisfies this equation for any constant $A > 0$ **4**

(ii) If $Q(0) = 10$ mg, find the maximum value of $Q(t)$ and the time at which this occurs. **4**

(iii) Describe what happens to $Q(t)$ as $t \to \infty$ **1**

THIS IS THE END OF THE PAPER

SBHS MATHEMATICS TRIAL 2002

1

(a) $4x = 15$

$x = \frac{15}{4}$, $3\frac{3}{4}$ — 2

(b) 2.94 — 2

(c) $c(d-1) - y(d-1)$

$(c-y)(d-1)$ — 2

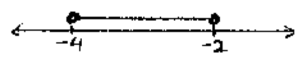
(d) $4k + 1 = 29$

$k = 7$ — 2

(e) $x + 3 \le 1$ or $-x - 3 \le 1$

$x \le -2$ $\quad -x \le 4$

$x \ge -4$



2

(f) $\frac{1}{\sqrt{3}+2} \times \frac{\sqrt{3}-2}{\sqrt{3}-2}$

$= \frac{\sqrt{3}-2}{-1}$

$-\sqrt{3} + 2 = a\sqrt{3} + b$

$a = -1 \quad b = 2$ — 2

2

(a) $(4,-1)\ (7,3)$

$m = \frac{4}{3}$

$y + 1 = \frac{4}{3}(x-4)$

$3y + 3 = 4x - 16$

$4x - 3y - 19 = 0$ as req'd — 2

(b) $BC\ (7,3)(8,1)$ $\quad AD(4,-1)(-1,9)$

$m_1 = -\frac{2}{1} = -2 \quad m_2 = \frac{10}{-5} = -2$

$m_1 = m_2 \quad BC \parallel AD$ — 2

(c) $DC\ (1,9)\ (7,3)$

$m = -\frac{6}{8} = -3/4$

grad $AC = \frac{4}{3}$ → from (a)

$m_1 m_2 = -1 \quad DC \perp AC$

$\therefore \angle ACD = 90°$ — 2

(d) $AC\ (4,-1)\ (7,3)$

$d = \sqrt{(7-4)^2 + (3--1)^2}$

$= \sqrt{25} = 5u$ — 2

(e) $d = \left| \frac{ax_1 + by_1 + c}{\sqrt{a^2+b^2}} \right|$

$4x - 3y - 19 = 0 \quad (8,1)$

$d = \left| \frac{4\times8 - 3\times1 - 19}{\sqrt{16+9}} \right|$

$= \frac{10}{5} = 2u$ — 2

(f) length $DC\ (-1,9)\ (7,3)$

$d = \sqrt{8^2+6^2} = 10u$

Area $= \triangle DCA + \triangle ACB$

$A = \frac{1}{2}\times10\times5 + \frac{1}{2}\times5\times2$

$= 30u^2$ — 2

Mathematics Trial, Assessment Task #4, 2002

$\boxed{1}$ 3. (a) i. $7 \times 2(2x-1)^6 = 14(2x-1)^6$.

$\boxed{2}$ ii. $\dfrac{5}{4+5x}$.

$\boxed{2}$ iii. $2x sin x + x^2 cos x$.

$\boxed{1}$ (b) $\int sec^2 3x dx = \frac{1}{3} tan 3x + c$.

$\boxed{3}$ (c)
$$\begin{aligned} \int_1^{e^2} \frac{3}{x} dx &= 3\left[ln x\right]_1^{e^2}, \\ &= 3\{2-0\}, \\ &= 6. \end{aligned}$$

$\boxed{3}$ (d)
$$\begin{aligned} \text{Area} &= \int_0^{\frac{3}{2}} e^{2x} dx, \\ &= \left[\frac{e^{2x}}{2}\right]_0^{\frac{3}{2}}, \\ &= \tfrac{1}{2}\left\{e^3 - 1\right\}. \end{aligned}$$

$\boxed{2}$ 4. (a) i.
$$\begin{aligned} x^2 - 2x + 1 &= 8y - 25 + 1, \\ \therefore (x-1)^2 &= 4 \times 2(y-3). \end{aligned}$$

$\boxed{1}$ ii. $(1,3)$.

$\boxed{1}$ iii. $(1,5)$.

$\boxed{1}$ iv. $y = 1$.

$\boxed{4}$ (b) First method: using λ,
$$\begin{aligned} x + 2y - 6 + \lambda(x-y) &= 0, \\ (2-\lambda)y &= -(1+\lambda)x + 6. \end{aligned}$$
The slope of line $3x - 2y + 7 = 0$ is $\frac{3}{2}$.
$$\begin{aligned} i.e.\ \frac{3}{2} &= \frac{\lambda+1}{\lambda-2}, \\ 3k - 6 &= 2\lambda + 2, \\ \lambda &= 8. \end{aligned}$$
$\therefore$ New line is $x + 2y - 6 + 8x - 8y = 0$.
$$\begin{aligned} i.e.\ 9x - 6y - 6 &= 0, \\ 3x - 2y - 2 &= 0. \end{aligned}$$

Second method: first find the intersection,
$$\begin{aligned} -y &= 2y - 6, \\ y &= 2, \\ \text{and } x &= 2. \end{aligned}$$
$\therefore$ Intersection is $(2,2)$.
The slope of line $3x - 2y + 7 = 0$ is $\frac{3}{2}$.
$$\begin{aligned} \therefore \text{New line is: } y - 2 &= \tfrac{3}{2}(x-2), \\ 2y - 4 &= 3x - 6, \\ i.e.\ 3x - 2y - 2 &= 0. \end{aligned}$$

$\boxed{3}$ (c)
$$\begin{aligned} \text{Area } \Delta XYZ &= \tfrac{1}{2} \times 4 \times 4, \\ &= 8\,\text{unit}^2. \\ \text{Area of Sector } XYW &= \tfrac{1}{2} \times 4^2 \times \frac{\pi}{4}, \\ &= 2\pi\,\text{unit}^2. \\ \therefore \text{Shaded area} &= 8 - 2\pi\,\text{unit}^2. \end{aligned}$$

TRIAL 2U HSC 2002
SOLUTIONS

QUESTION 5

a) $y = x^2 - 3x + 5$

(2 marks) $y' = 2x - 3$

at $(3, 5)$

$y' = 2 \times 3 - 3$

$= 3$

nor $m_2 = \dfrac{-1}{m_1}$

$= \dfrac{-1}{3}$

using

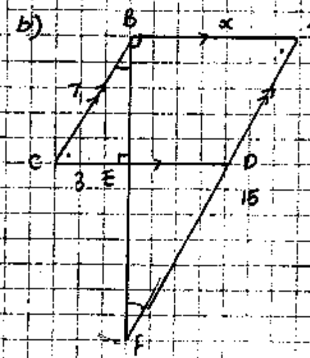
$y - y_1 = m(x - x_1)$

$y - 5 = \frac{-1}{3}(x - 3)$

$3y - 15 = -x + 3$

$\therefore \underline{x + 3y - 18 = 0}$

b)



i) In ΔCBE and ΔAFB

(3 marks) $\angle CEB = \angle ABF$ (alt. $\angle$s, AB||

$\angle BCE = \angle BAF$ (opp. $\angle$s, || gr

$\angle CBE = \angle AFB$ ($\angle$ sum of Δ)

$\therefore \Delta CBE ||| \Delta AFB$ (equiangular

ii) In ΔCBE and ΔAFB

(2 marks) $\dfrac{CB}{CE} = \dfrac{AF}{AB}$ (corr. sides of ||| Δs)

$\dfrac{7}{3} = \dfrac{15}{x}$

$7x = 45$

$x = 6\frac{3}{7}$

$\therefore \underline{AB = 6\frac{3}{7} \text{ cm}}$

c) $y = x^2 - 1$

(6 marks) $y^2 = (x^2 - 1)^2$

$= x^4 - 2x^2 + 1$

$V = \pi \int y^2 \, dx$

$= 2\pi \int_0^1 (x^4 - 2x^2 + 1) \, dx$

$= 2\pi \left[\frac{x^5}{5} - \frac{2x^3}{3} + x \right]_0^1$

$= 2\pi \left[\left(\frac{(1)^5}{5} - \frac{2(1)^3}{3} + 1 \right) - \left(\frac{(0)^5}{5} - \frac{2(0^3)}{3} - 0 \right) \right]$

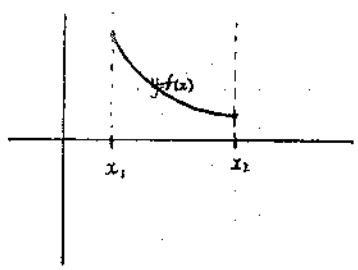
$= 2\pi \left(\frac{1}{5} - \frac{2}{3} + 1 \right)$

$= 2\pi \left(\frac{8}{15} \right)$

$V = \frac{16\pi}{15}$ units^3

d) $y = f(x)$ $\quad x_1 \le x \le x_2$

(2 marks) $f(x) > 0$, $f'(x) < 0$, $f''(x) > 0$



$f(x)$ above x-axis

$f'(x)$ negative gradient

$f''(x)$ concave up

QUESTION 6

a) $y = x^3 + mx + n$ $\quad P(1,5)$ when $y' = 0$

(3marks)

$y' = 3x^2 + m$

at $P(1,5)$

$0 = 3(1)^2 + m$

$\therefore m = -3$

$y = x^3 - 3x + n$

at $P(1,5)$

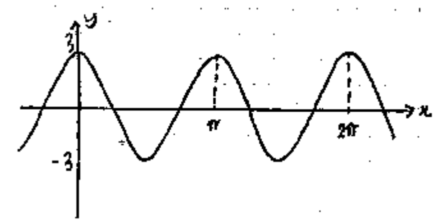
$5 = (1)^3 - 3(1) + n$

$5 = 1 - 3 + n$

$7 = n$

$\therefore n = 7$

b) (2marks)



$y = a\cos mx$ $\quad$ where amplitude $(a) = 3$

$\therefore y = 3\cos 2x$ $\quad$ period $T = \frac{2\pi}{m}$

$\pi = \frac{2\pi}{m}$

$\therefore m = 2$

c) (2marks) $\ln 2 + 2\ln 18 - \frac{3}{2}\ln 36$

$= \ln 2 + \ln 324 - \ln 216$

$= \ln \left(\frac{2 \times 324}{216} \right)$

$= \ln 3$

d)
(3 marks)

$$\sqrt{\frac{1-\cos^2\theta}{1+\tan^2\theta}}$$

Using $\cos^2\theta + \sin^2\theta = 1$

$\sin^2\theta = 1-\cos^2\theta$

$1 + \tan^2\theta = \sec^2\theta$

$= \frac{1}{\cos^2\theta}$

$$= \sqrt{\frac{\sin^2\theta}{\frac{1}{\cos^2\theta}}}$$

$$= \sqrt{\sin^2\theta\cos^2\theta}$$

$$= \sin\theta\cos\theta$$

e)
(3 marks)

$$\int_{-1}^{7} f(x)\,dx \doteqdot \frac{h}{3}\left[(y_0+y_8)+4(y_1+y_3)+2(y_2)\right]$$

where $h = \frac{b-a}{n}$

$= \frac{7--1}{4}$

$= 2$

$y_0 = 5$
$y_1 = 9$
$y_2 = 2$
$y_3 = -1$
$y_4 = -6$

$$\int_{-1}^{7} f(x)\,dx \doteqdot \frac{2}{3}\left[(5-6)+4(9-1)+2(2)\right]$$

$$\doteqdot \frac{2}{3}(-1+32+4)$$

$$\doteqdot \frac{70}{3}$$

$$\doteqdot 23\tfrac{1}{3}$$

QUESTION 7

a) i) $V = 50000e^{-0.25t}$
(1 mark)

when $t=0$, $V=?$

$V = 50000e^{-0.25\times 0}$

$= 50000$

$\therefore$ value of new car is $50000

ii) when $t=3$, $V=?$
(2 marks)

$V = 50000e^{-0.25\times 3}$

$= 23618.33$

$\therefore$ value of car after 3 years is $23 600. (nearest $100)

iii) $V = 16000$ $t=?$
(2 marks)

$15000 = 50000e^{-0.25t}$

$\frac{15000}{50000} = e^{-0.25t}$

$0.3 = e^{-0.25t}$

$\ln 0.3 = -0.25t$

$t = \frac{\ln 0.3}{-0.25}$

$= 4.815$

$\therefore$ time to taken for the car to be worth $15000 is 4.8 years (nearest tenth of a year).

b) i)
(2 marks)

$S_n = a + (a+d) + (a+2d)$

where $S_n = 15600$

$a = 1600$

$15600 = 1600 + (1600+d) + (1600+2d)$

$10800 = 3d$

$d = 3600$

$\therefore$ A = \$1600 B = \$5200 C = \$8800

ii) $S_n = \frac{a(r^n-1)}{r-1}$
(2 marks)

$S_n = a + ar^1 + ar^2$

$15600 = 1600 + 1600r + 1600r^2$

$14000 = 1600r^2 + 1600r$

$0 = 8r^2 + 8r - 70$

$r = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$= \frac{-8 \pm \sqrt{64+2240}}{16}$

$\therefore r = \pm 2\frac{1}{2}$

$r = 2.5$

$\therefore$ A = \$1600 B = \$4000 C = \$10000

c) i) $P(2,2) = \frac{2}{7} \times \frac{1}{6}$
(1 mark)

$= \frac{1}{21}$

ii) $P(4+6) = \frac{1}{7} \times \frac{4}{6}$
(2 marks)

$= \frac{4}{42}$

$P(6+4) = \frac{4}{7} \times \frac{1}{6}$

$= \frac{4}{42}$

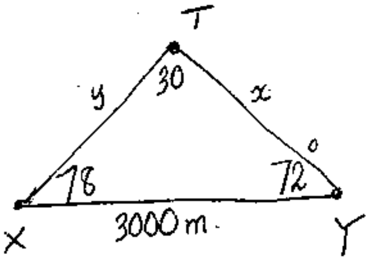
$P(6+6) = \frac{4}{7} \times \frac{3}{6}$

$= \frac{12}{42}$

$P(\geq 8) = \frac{8}{42} + \frac{12}{42}$

$= \frac{10}{21}$

(a)



$\frac{y}{\sin 72^\circ} = \frac{3000}{\sin 30}$

$y = \frac{3000 \times \sin 72^\circ}{\sin 30^\circ}$ (3)

$\doteq 5706$ m $\doteq 5.706$ km

(b) $mx^2 - 20x + m = 0$

let $x = -\frac{1}{2}$ $\quad \frac{m}{4} + 10 + m = 0$

$m + 40 + 4m = 0$

$5m = -40$

$m = -8$

So we have $-8x^2 - 20x - 8 = 0$

and $-\frac{1}{2} + \alpha = \frac{20}{-8}$

$\alpha = -\frac{20}{8} + \frac{1}{2}$

$= -2$

r $(2x+1)(x+2) = 0$
$x = -\frac{1}{2}, x = -2$.

Other root is -2. (3)

(c) $y = x^3 + 3x^2 - 9x$

$= x(x^2 + 3x - 9)$

$y' = 3x^2 + 6x - 9$

$= 3(x^2 + 2x - 3)$

$y'' = 6x + 6 = 6(x+1)$

$3(x^2 + 2x - 3) = 0$

$x^2 + 2x - 3 = 0$

$(x+3)(x-1) = 0$

$x = -3, \quad x = 1$

$y = 27 \quad y = -5$

stat points are $(-3, 27)$ and $(1, -5)$ (2)

(ii) Curve is concave upwards when $y'' > 0$

$6(x+1) > 0$

$x + 1 > 0$

$x > -1$ (1)

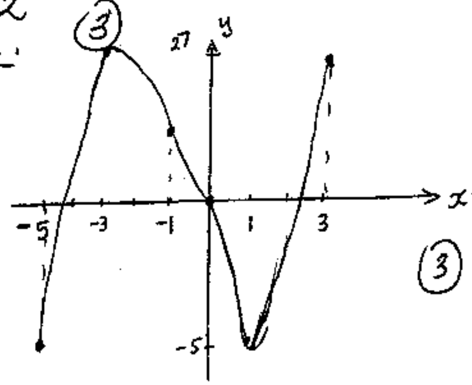
(iii) $(-3, 27)$ $y'' < 0$ MAX

$(1, -5)$ $y'' > 0$ min

$(-1, 11)$ inflection

At $x = -5, y = -5$

$x = 3, \quad y = 27$.



(3) (3)

(a) $|x+1| = 2x + 7$

$x + 1 = 2x + 7$

$-6 = x$

LHS ≠ RHS

$x + 1 = -2x - 7$

$3x = -8$

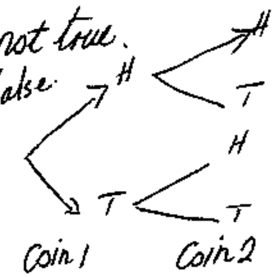
$x = -\frac{8}{3}$

LHS $= 1\frac{2}{3}$

RHS $= 1\frac{2}{3}$.

$x = -\frac{8}{3} = -2\frac{2}{3}$ $(-2.\dot{6})$ (3)

(b) not true.
ie: false



4 possibilities not 3. (2)

(c) (i) $a = 2\pi^2 \cos \pi t$

data $t = 0$, $x = 0$, $V = \pi$

$V = \int 2\pi^2 \cos \pi t \, dt$

$= \frac{2\pi^2}{\pi} \int \pi \cos \pi t \, dt$

$V = 2\pi \sin \pi t + C_1$

data $\pi = C_1$

$V = 2\pi \sin \pi t + \pi$ $(1\frac{1}{2})$

$x = \int (2\pi \sin \pi t + \pi) \, dt$

$x = -2\cos \pi t + \pi t + C_2$

C (ii) when $V = 0$

$2\pi \sin \pi t + \pi = 0$

$2\pi \sin \pi t = -\pi$

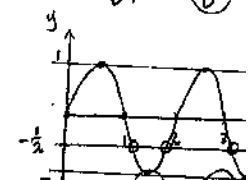
$\sin \pi t = -\frac{\pi}{2\pi}$

$\sin \pi t = -\frac{1}{2}$

$\pi t = \frac{7\pi}{6}, \frac{11\pi}{6}$

$t = \frac{7\pi}{6\pi} = \frac{7}{6}$ se

$t = \frac{11\pi}{6\pi} = \frac{11}{6}$



also $t = \frac{19}{6}, \frac{23}{6}$

(iii) $t = \frac{19}{6}$ $x = -2\cos\frac{19\pi}{6}$

$= -2 \times -\frac{\sqrt{3}}{2} + \frac{19}{6}$

$= 2 + \sqrt{3} + 19$

$t = \frac{23}{6}$

$x = -2\cos\frac{23\pi}{6} + \frac{23\pi}{6}$

$= -2 \times \frac{\sqrt{3}}{2} + \frac{23\pi}{6} +$

$= 2 + \frac{23\pi}{6} - \sqrt{3}$

At $t = \frac{19}{6}$ x is larg

So $x = -2\cos \pi t + \pi t$ $(1\frac{1}{2})$

(a) $C(n) = 1000(0.8)^n + 10{,}000[1 + .8 + .8^2 + \cdots + .8$

as $n \to \infty$ $1000(0.8)^n \to 0$

now $10000[1 + .8^1 + .8^2 + \cdots + .8^{n-1}]$

GP $a = 1$, $r = .8$, $n = n$ $\Big\}$ $S_n = \dfrac{1(.8^n - 1)}{.8 - 1} = \dfrac{.8^n - 1}{-.2} = \dfrac{1 - .}{.2}$

so we have $10{,}000 \times 5(1 - .8^n)$

$= 50{,}000(1 - .8^n)$ (3)

as $n \to \infty$ $50000(1 - .8^n) \to 50{,}000$

(b) (i) $\varphi(t) = A(1+t)e^{-0.5t}$

$\varphi(t) = Ae^{-0.5t} + Ate^{-0.5t}$

$\varphi'(t) = -0.5Ae^{-0.5t} + At \times -0.5e^{-0.5t} + Ae^{-0.5t}$

$= 0.5Ae^{-0.5t} - 0.5Ate^{-0.5t}$

$\varphi''(t) = -0.25Ae^{-0.5t} - (0.5At \times -0.5e^{-0.5t} + 0.5Ae^{-0.5t})$

$= -0.25Ae^{-0.5t} + 0.25Ate^{-0.5t} - 0.5Ae^{-0.5t}$

$= -0.75Ae^{-0.5t} + 0.25Ate^{-0.5t}$

$4(-0.75Ae^{-0.5t} + 0.25Ate^{-0.5t}) + 4(0.5Ae^{-0.5t} - 0.5Ate^{-0.5t}) + Ae^{-0.5t} + Ate^{-0.5t}$

$= 0$ (4)

(ii) $\varphi(0) = 10$

So when $t = 0$ $A(1+t)e^{-0.5t} = 10$

$A(1+0)e^{-0.5 \times 0} = 10$

$A = 10$

So $\varphi(t) = 10(1+t)e^{-0.5t}$

$\varphi'(t) = 0.5 \times 10e^{-0.5t} - 0.5 \times 10te^{-0.5t}$

$= 5e^{-0.5t} - 5te^{-0.5t}$

$= 5e^{-0.5t}(1-t)$

let $\varphi'(t) = 0 \Rightarrow t = 1$

Sub $t = 1$, $\varphi''(t) = -0.75 \times 10e^{-0.5} + 0.25 \times 10 \times e^{-0.}$

$= -7.5e^{-0.5} + 2.5e^{-0.5}$

$= -5e^{-0.5} < 0$

at $t = 1$, $\varphi(t)$ is a MAX.

When $t = 1$, $\varphi(t) = 10(2)e^{-0.5}$

$= \dfrac{20}{e^{.5}} \doteq 12.13$ (4)

(iii) $\varphi(t) = 10(1+t)e^{-0.5t}$

$= 10e^{-0.5t} + 10te^{-0.5t}$

$= \dfrac{10}{e^{0.5t}} + \dfrac{10t}{e^{0.5t}}$ As $t \to \infty$ $\varphi(t) \to 0$ (1)