SYDNEY BOYS HIGH SCHOOL
MOORE PARK, SURRY HILLS

2015

HIGHER SCHOOL CERTIFICATE
TRIAL PAPER

Mathematics

General Instructions

- Reading time – 5 minutes
- Working time – 3 hours
- Write using black or blue pen
  Black pen is preferred
- Board-approved calculators may be used
- All necessary working should be shown in every question if full marks are to be awarded.
- Marks may **NOT** be awarded for messy or badly arranged work.
- Leave your answers in the simplest exact form, unless otherwise stated.
- In Questions 11-16, show relevant mathematical reasoning and/or calculations
- Start each **NEW** question in a separate answer booklet.

Total Marks – 100

Section I Pages 2–6

10 Marks

- Attempt Questions 1–10
- Allow about 15 minutes for this section.

Section II Pages 7–15

90 marks

- Attempt Questions 11–16
- Allow about 2 hour and 45 minutes for this section

Examiner: *P Bigelow*

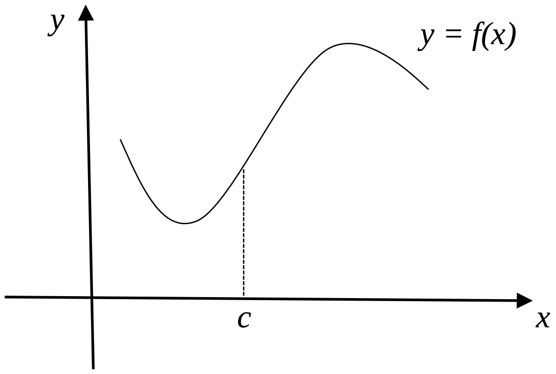
This is an assessment task only and does not necessarily reflect the content or format of the Higher School Certificate.

Section I

10 marks
Attempt Questions 1-10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1-10.

1



Which of the following statements is true?

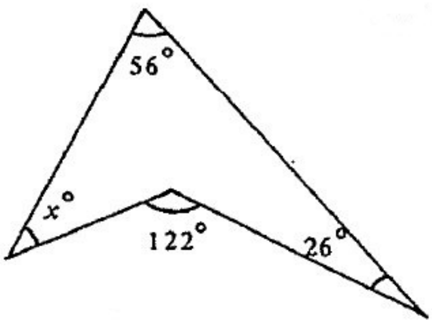
(A) $f'(c) > 0$ and $f''(c) < 0$

(B) $f'(c) > 0$ and $f''(c) > 0$

(C) $f'(c) < 0$ and $f''(c) < 0$

(D) $f'(c) < 0$ and $f''(c) > 0$

2



The value of *x* is

(A) 31

(B) 34

(C) 48

(D) 40

3 What is the focus of $(x-5)^2 = 12y$?

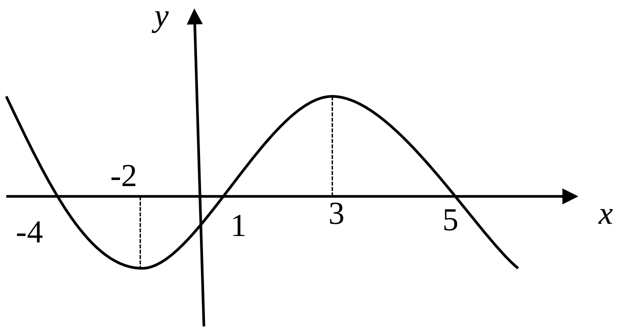
(A) (5, 0)

(B) (5, 3)

(C) (3, 5)

(D) (0, 5)

4



The graph of the function $y = f'(x)$ is given.
At which point is there a minimum turning point of $f(x)$?

(A) -4

(B) -2

(C) 1

(D) 3

5 If $a \neq \pm 1$ then $1 + a^2 + a^4 + a^6 + \cdots + a^{2n}$
is equal to

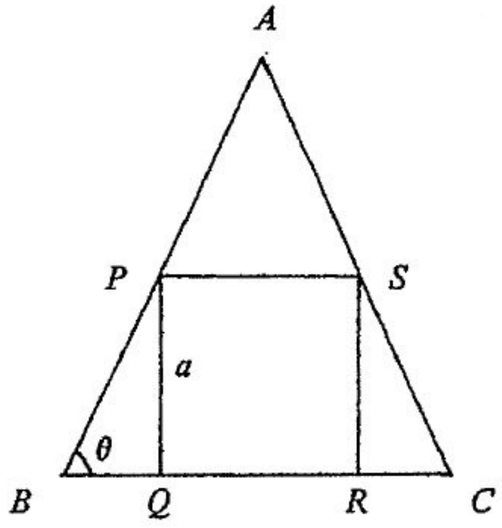
(A) $\frac{1-a^{2n}}{1-a^2}$

(B) $\frac{1-a^{2n+1}}{1-a^2}$

(C) $\frac{1-a^{2n+2}}{1-a^2}$

(D) $\frac{1-a^{2n}}{1-a}$

6 *PQRS* is a square inscribed in ΔABC, $AB = AC$ and $PQ = a$.



$\therefore$ *AB* is equal to

(A) $a\left(\frac{1}{\sin\theta}+\frac{1}{2\cos\theta}\right)$

(B) $a\left(\frac{1}{\cos\theta}+\frac{1}{2\sin\theta}\right)$

(C) $a\left(\sin\theta+\frac{1}{2}\cos\theta\right)$

(D) $\frac{2a}{\sin\theta}$

7 A particle is moving along the *x*-axis. The displacement of the particle at time *t* seconds is *x* metres. At a certain time $\dot{x} = 4ms^{-1}$ and $\ddot{x} = -3ms^{-2}$.
Which statement explains the motion at that time?

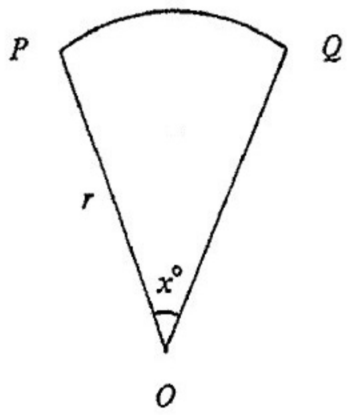
(A) The particle is moving to the right with increasing speed

(B) The particle is moving to the left with increasing speed

(C) The particle is moving to the right with decreasing speed

(D) The particle is moving to the left with decreasing speed

8



In the sector, the radius is r and $\angle POQ = x°$. If the area of the sector is A then the value of x is

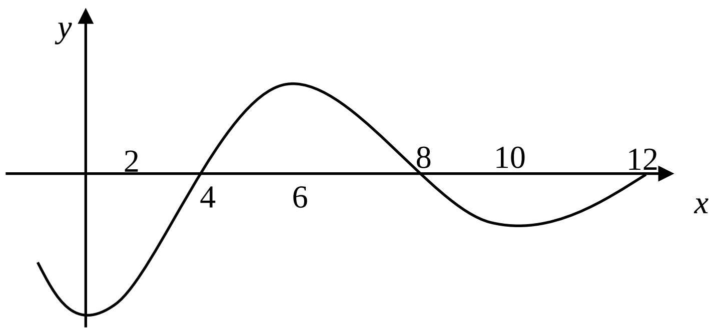
(A) $\frac{2A}{r^2}$

(B) $\frac{180A}{\pi r^2}$

(C) $\frac{360A}{r^2}$

(D) $\frac{360A}{\pi r^2}$

9



The graph represents $y = f(x)$.
Which of the following has the least value?

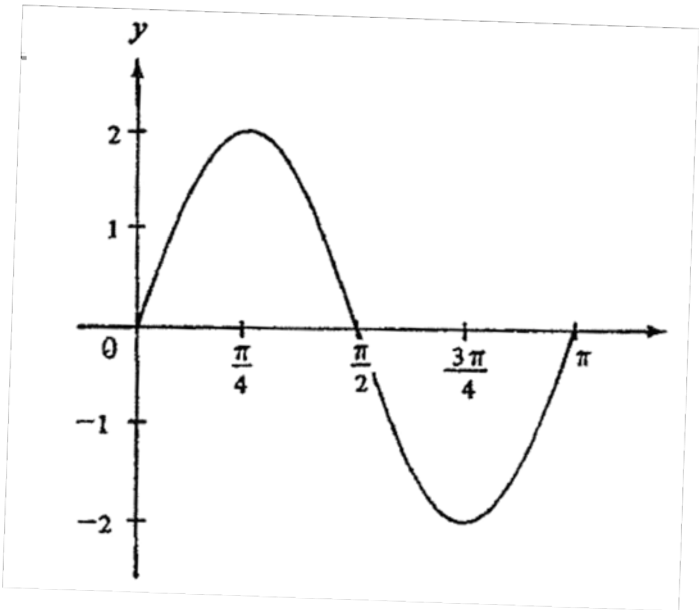
(A) $\int_0^2 f(x)dx$

(B) $\int_0^4 f(x)dx$

(C) $\int_0^{10} f(x)dx$

(D) $\int_0^{12} f(x)dx$

10



If the figure shows the graph of $y = a \sin k\theta$ then

(A) $a = 1$ and $k = \frac{1}{2}$

(B) $a = 1$ and $k = 2$

(C) $a = 2$ and $k = 2$

(D) $a = 2$ and $k = \frac{1}{2}$

Section II

90 marks
Attempt Questions 11-16
Allow about 2 hours and 45 minutes for this section

Answer each question in a separate writing booklet. Extra writing booklets are available.

In Questions 11-16, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) USE A SEPARATE WRITING BOOKLET

(a) Evaluate $\frac{\pi}{2+e}$ correct to three decimal places. **1**

(b) Write down the exact value of $\sin \frac{7\pi}{4}$. **1**

(c) Graph $|3x - 1| \geq 8$ on a number line. **2**

(d) Differentiate the following with respect to x:

(i) $\dfrac{x}{x-2}$ **1**

(ii) $x^2 \ln x$ **1**

(iii) $\sin(1 + x^2)$ **1**

(e) What is the value of $\log_2 \sqrt{32}$ **1**

(f) Find the equation of the normal to $y = \ln(3x + 4)$ at the point where x = -1. **2**

(g) Express 1.27 radians as an angle in degrees, correct to the nearest minute. **2**

(h) Given the ΔPQR where PR = 8 cm, RQ = 9 cm and $\angle PRQ$ = 60°, find

(i) the length of PQ **2**

(ii) the exact area of ΔPQR. **1**

Question 12 (15 marks) USE A SEPARATE WRITING BOOKLET

(a) Factorise

(i) $6x^2 + 23x - 4$ **1**

(ii) $8a^3 - b^3$ **1**

(b) Write down a primitive of the following:

(i) $1 + \sqrt{x}$ **1**

(ii) $\cos\dfrac{4x}{9}$ **1**

(iii) $\dfrac{x}{1 + 3x^2}$ **1**

(iv) $e^{-\frac{x}{3}}$ **1**

(c) Evaluate **2**

$$\sum_{r=1}^{4} 3^{1-r}$$

(d) Solve $\sin\left(x + \frac{\pi}{3}\right) = 0$ for $0 \le x \le 2\pi$ **2**

(e) Express $\frac{\sqrt{3}+1}{\sqrt{3}-1}$ in the form $a + b\sqrt{3}$. **2**

(f) Given the arithmetic series 32 + 25 + 18 + ….

(i) Find S_{15} . **1**

(ii) Hence, find the sum of the next 20 terms. **2**

Question 13 (15 marks) USE A SEPARATE WRITING BOOKLET

(a) Given $4x^2 + 5x + 6 \equiv A(x+1)^2 + B(x+1) + C$, find the values of *A*, *B* and *C*. **3**

(b) Change the subject of $y = 3\ln 2x + 1$ to x. **2**

(c) The curve $f(x) = ax^2 + bx$ has a tangent with gradient $\frac{1}{2}$ at the origin and the curve passes though (1, 2).
Find the values of *a* and *b*. **2**

(d) Given that the equation $y = 2x^2 - 5x + 5$ has roots α and β, write down the values of

(i) $\alpha + \beta$ **1**

(ii) $\alpha\beta$ **1**

(iii) $\alpha^2 + \beta^2$ **1**

(iv) $\alpha^3 + \beta^3$ **2**

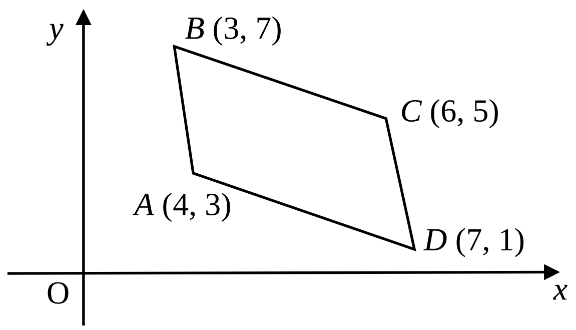
(e) (i) Show that $\frac{d}{dx}(\tan x - x) = \tan^2 x$ **1**

(ii) Hence evaluate **2**

$$\int_0^{\frac{\pi}{4}} \tan^2 x \, dx$$

Question 14 (15 marks) USE A SEPARATE WRITING BOOKLET

(a)



ABCD is a parallelogram.

(i) Show that the equation of the line *AD* is $2x + 3y - 17 = 0$ **1**

(ii) Find the length of *BC*. **1**

(iii) Find the perpendicular distance between *C* (6, 5) and the line *AD*. **2**

(iv) Hence, or otherwise, find the area of *ABCD*. **2**

(b) The volume V cm^3 of a balloon is increasing such that the volume at any time t seconds is given by **2**

$$V = \frac{\pi t^3}{3} - \frac{\pi t^2}{6} + \frac{1}{2}$$

Find the rate at which the volume is increasing when $t = 2$.

(c) A particle moves in a straight line such that its distance, x metres from a fixed point after t seconds is given by $x = 6t - 2t^3 + 5$, for $t \geq 0$.

(i) Find the equation of the velocity after t seconds. **1**

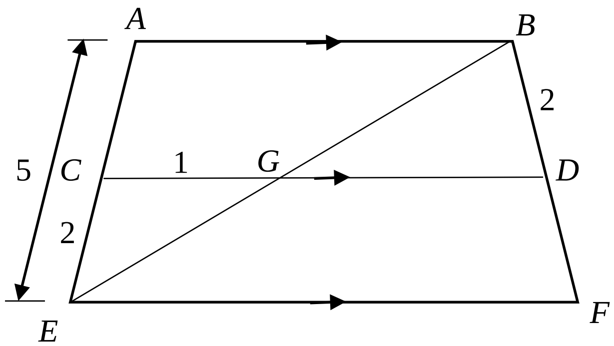
(ii) At what time does the particle stop? **1**

(iii) Find the total distance travelled in the first 2 seconds. **2**

Question 14 continues on page 11

Question 14 (continued)

(d)



In the diagram *AB*, *CD* and *EF* are parallel and *BE* cuts *CD* at *G*.
AE is 5 cm, *CE* is 2 cm, *CG* is 1 cm and *BD* is 2 cm.

(i) Write down the length of *AB*. **1**

(ii) Find the length of *DF*. **2**

End of Question 14

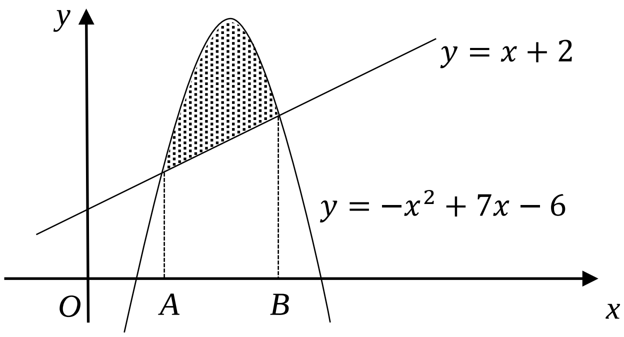
Question 15 (15 marks) USE A SEPARATE WRITING BOOKLET

(a) Two dice are biased so that the probability of throwing a six, face up, is $\frac{3}{8}$ and the probability the probability of each of the other five numbers is $\frac{1}{8}$. Find the probability of:

(i) rolling a double six, face up; **1**

(ii) only one six appearing face up when the two dice are rolled. **2**

(b)



The diagram shows the graphs of $y = x + 2$ and $y = -x^2 + 7x - 6$.

(i) Show that the values of *A* and *B* are 2 and 4 respectively. **1**

(ii) Calculate the area of the shaded region. **2**

(c) Given the function $f(x) = 2x(x-3)^2$:

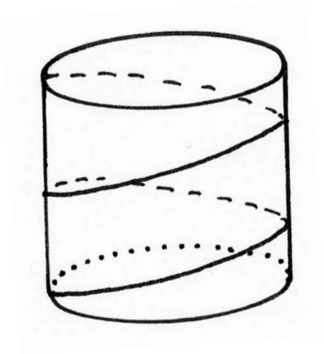
(i) Find the co-ordinates of the points where the curve $y = f(x)$ cuts the *x*-axis; **1**

(ii) Find the co-ordinates of any turning points on the curve $y = f(x)$ and determine their nature **2**

(iii) Sketch the curve $y = f(x)$ in the domain $-1 \leq x \leq 4$. **2**

(iv) Hence solve $2x^3 - 12x^2 + 18x - 8 = 0$ **2**

Question 15 continues on page 13

(d) 

A piece of string is wound tightly around a cylinder of radius r and height h, as shown. One end of the string is attached to the bottom of the cylinder and the other end is attached to the top of the cylinder, directly above the other end. The midpoint of the string is also directly above the end attached to the bottom of the cylinder. Find the length of the piece of string (in terms of r and h). **2**

End of Question 15

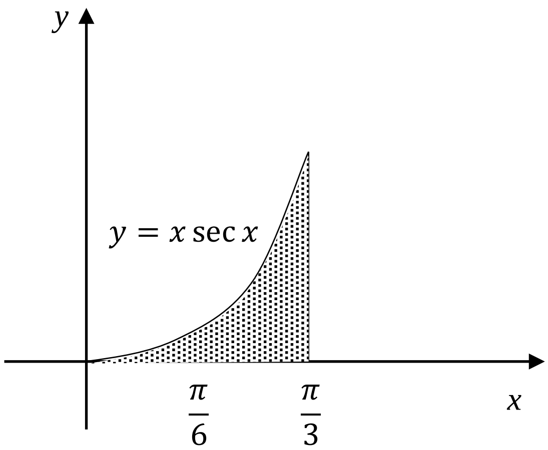
Question 16 (15 marks) USE A SEPARATE WRITING BOOKLET

(a) On their daughter Catherine's 11^{th} birthday, Mr and Mrs Lee deposited \$1000 in an account paying 6% annual compound interest. They continue to deposit \$1000 on successive birthdays up to her 20^{th} birthday, giving her the accumulated amount on her 21^{st} birthday.

(i) What will be the amount of Catherine's 21^{st} birthday present? **2**

(ii) What single deposit on her 11^{th} birthday would have, under the same conditions (ie 6% annual compound interest), provided her the same birthday present (to the nearest \$10)? **2**

(b) **3**



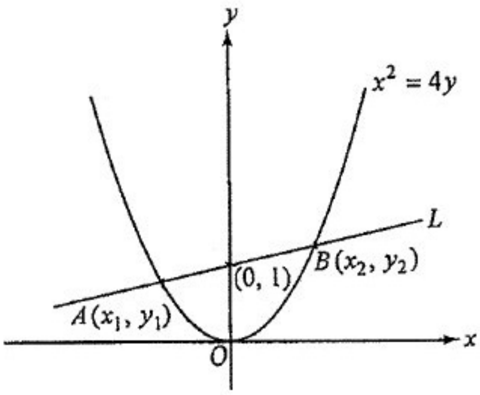
The area bounded by $y = x \sec x$ and the x-axis from $x = 0$ to $x = \frac{\pi}{3}$ is rotated about the x-axis.

Use Simpson's Rule with three function values to find the volume generated (correct to 3 significant figures).

Question 16 continues on page 15

Question 16 (continued)

(c)



L is a straight line of slope m which passes through (0, 1) and cuts the parabola $x^2 = 4y$ at the points $A(x_1, y_1)$ and $B(x_2, y_2)$.

(i) Show that x_1 and x_2 are the roots of the equation $x^2 - 4mx - 4 = 0$. **1**

(ii) Find $(x_1 - x_2)^2$ in terms of m. **1**

(iii) Hence, show that $AB = 4(1 + m^2)$ **2**

(iv) C is a circle with AB as diameter. Find in terms of m, the co-ordinates of the centre of C and its radius. **2**

(v) Find, in terms of m, the distance from the centre of C to the line $y + 1 = 0$. **1**

(vi) State the geometrical relationship between C and the line $y + 1 = 0$. **1**

End of paper

STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x}\,dx = \ln x, \quad x > 0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}, \quad a \neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax, \quad a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax, \quad a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax\,dx = \frac{1}{a}\sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \ln\left(x+\sqrt{x^2-a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \ln\left(x+\sqrt{x^2+a^2}\right)$$

NOTE : $\ln x = \log_e x, \quad x > 0$

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TRIAL PAPER

Mathematics (2 Unit)

Sample Solutions

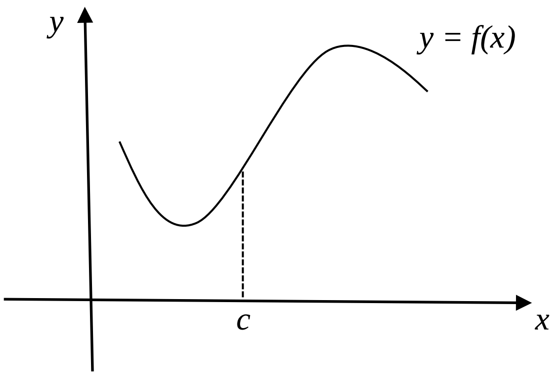
Question	Teacher
Q11	**JM**
Q12	**AMG**
Q13	**AF**
Q14	**PP**
Q15	**EC**
Q16	**AYW**

MC Answers

Q1	B
Q2	D
Q3	B
Q4	B and C
Q5	C
Q6	A
Q7	C
Q8	D
Q9	B
Q10	C

Section 1

1



Which of the following statements is true?

(A) $f'(c) > 0$ and $f''(c) < 0$

(B) $f'(c) > 0$ and $f''(c) > 0$

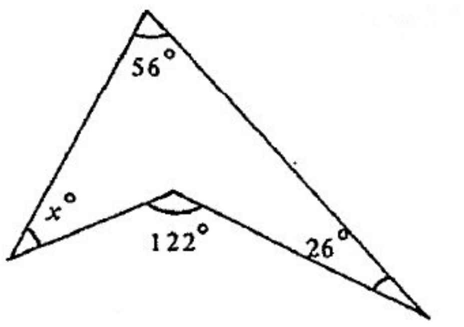
(C) $f'(c) < 0$ and $f''(c) < 0$

(D) $f'(c) < 0$ and $f''(c) > 0$

ANSWER: B

Gradient is positive at c $f'(c) > 0$ al so concavity at c is concave up hence $f''(c) > 0$

2



The value of x is

(A) 31

(B) 34

(C) 48

(D) 40

ANSWER:D

Quadrilateral has angle sum is 360^{o}:

$360 = x + 56 + 26 + (360 - 122)$

$x = 360 - 26 - 56 - 238$

$x = 40$

3 What is the focus of $(x-5)^2 = 12y$?

(A) (5, 0)

(B) (5, 3)

(C) (3, 5)

(D) (0, 5)

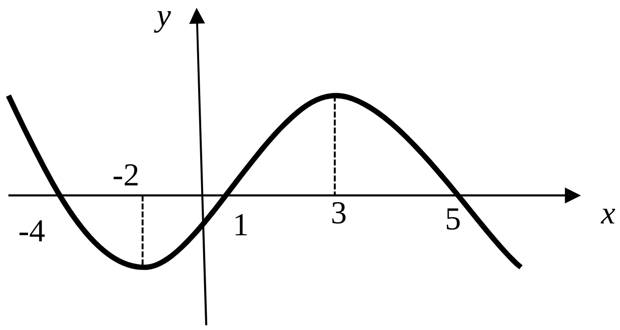
ANSWER: B

$(x-h)^2 = 4a(y-k)$

The focus is: $(x, k+a)$ $a=3$ is the focus length

$\therefore$ Focus is $(5,3)$

4



The graph of the function $y = f'(x)$ is given.
At which point is there a minimum turning point of $f(x)$?

(A) -4

(B) -2

(C) 1

(D) 3

ANSWER: C or **B**

For $x = 1^-$, $y' < 0$ and for $x = 1^+$, $y' > 0$.

$\therefore$ $x = 1$ is a minimum turning point of $y = f(x)$

5 If $a \neq \pm 1$ then $1 + a^2 + a^4 + a^6 + \cdots + a^{2n}$ is equal to

(A) $\frac{1-a^{2n}}{1-a^2}$

(B) $\frac{1-a^{2n+1}}{1-a^2}$

(C) $\frac{1-a^{2n+2}}{1-a^2}$

(D) $\frac{1-a^{2n}}{1-a}$

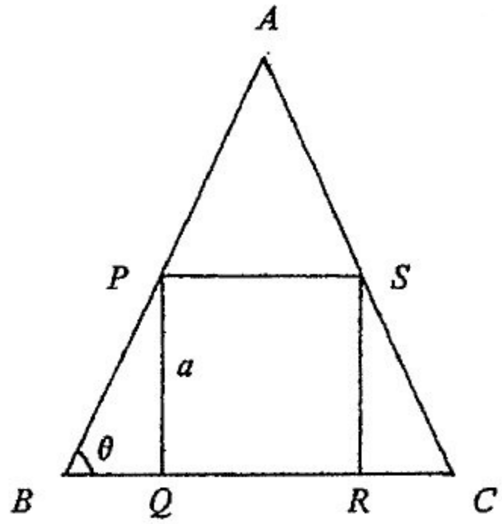
ANSWER: C

Common ratio is a^2, number of terms is $1+2n$

$$\frac{a\left(1-r^n\right)}{1-r} = \frac{1\left(1-a^{2(2n+1)}\right)}{1-a^2}$$

$$= \frac{\left(1-a^{2n+2}\right)}{1-a^2}$$

6 *PQRS* is a square inscribed in ΔABC, $AB = AC$ and $PQ = a$.



$\therefore$ *AB* is equal to

(A) $a\left(\frac{1}{\sin\theta} + \frac{1}{2\cos\theta}\right)$

(B) $a\left(\frac{1}{\cos\theta} + \frac{1}{2\sin\theta}\right)$

(C) $a\left(\sin\theta + \frac{1}{2}\cos\theta\right)$

(D) $\frac{2a}{\sin\theta}$

ANSWER: A

In ΔPBQ let $PB = x$

$$\sin\theta = \frac{a}{x}$$

$$x = \frac{a}{\sin\theta}$$

In ΔAPS let $AP = y$

$$\cos\theta = \frac{0.5a}{y}$$

$$y = \frac{0.5a}{\cos\theta}$$

$$AB = x + y$$

$$= a\left(\left(\frac{1}{\sin\theta}\right) + \left(\frac{1}{2\cos\theta}\right)\right)$$

7 A particle is moving along the x-axis. The displacement of the particle at time t seconds is x metres. At a certain time
$\dot{x} = 4ms^{-1}$ and $\ddot{x} = -3ms^{-2}$.
Which statement explains the motion at that time?

(A) The particle is moving to the right with increasing speed

(B) The particle is moving to the left with increasing speed

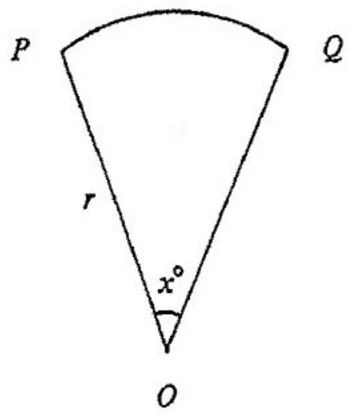
(C) The particle is moving to the right with decreasing speed

(D) The particle is moving to the left with decreasing speed

ANSWER: C

The acceleration is opposite sign to velocity so it is decreasing speed.

8



In the sector, the radius is r and $<POQ = x°$. If the area of the sector is A then the value of x is

(A) $\frac{2A}{r^2}$

(B) $\frac{180A}{\pi r^2}$

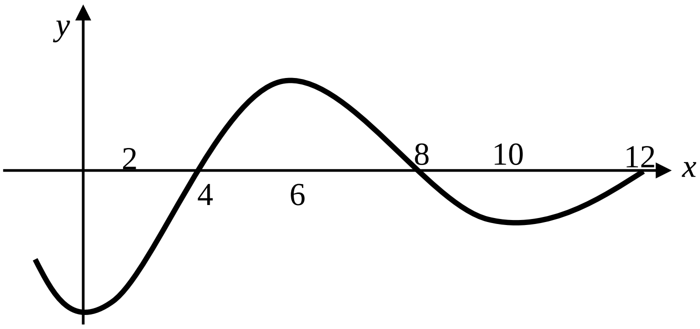
(C) $\frac{360A}{r^2}$

(D) $\frac{360A}{\pi r^2}$

ANSWER: D

$$A = \frac{1}{2}r^2\theta$$
$$= \frac{1}{2}r^2\left(\frac{\pi x}{180}\right)$$
$$x = \frac{A(180)(2)}{r^2\pi}$$
$$= \frac{360A}{\pi r^2}$$

9



The graph represents $y = f(x)$. Which of the following has the least value?

(A) $\int_0^2 f(x)dx$

(B) $\int_0^4 f(x)dx$

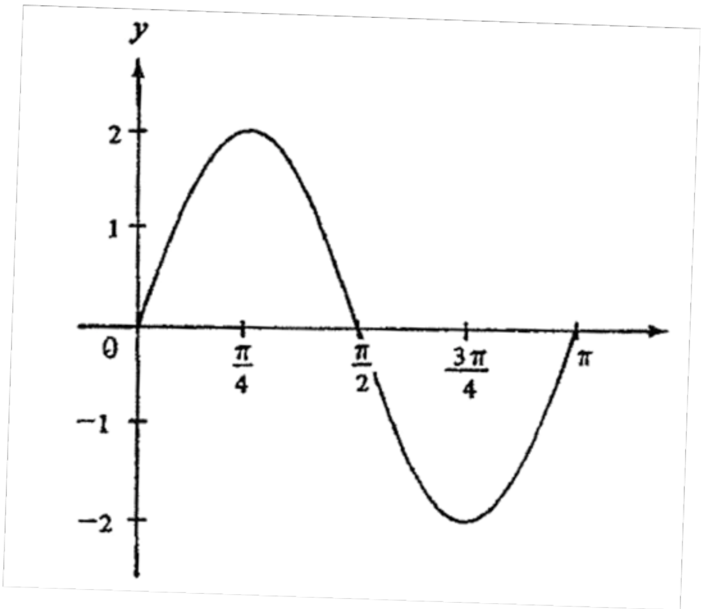
(C) $\int_0^{10} f(x)dx$

(D) $\int_0^{12} f(x)dx$

ANSWER: B

$\int_0^4 f(x)\,dx$ is completely below the *x*-axis hence producing the largest negative value the other integrals are larger due to being partly above the *x*-axis and the sum of the areas is larger.

10



If the figure shows the graph of $y = a\sin k\theta$ then

(A) $a = 1$ and $k = \frac{1}{2}$

(B) $a = 1$ and $k = 2$

(C) $a = 2$ and $k = 2$

(D) $a = 2$ and $k = \frac{1}{2}$

ANSWER: C

a is amplitude which is 2 as is the height of the curve
k is related to the period, T i.e. $T = \frac{2\pi}{k}$.
$\therefore T = \frac{2\pi}{k} = \pi \Rightarrow k = 2$

Section II

Question 11

(a) $\dfrac{\pi}{2+e} = 0.6658340404$

$\approx 0.666 \quad (3.\text{d.p})$ [1]

(b) $\sin\left(\dfrac{7\pi}{4}\right) = \sin\left(2\pi - \dfrac{\pi}{4}\right)$

$= -\sin\left(\dfrac{\pi}{4}\right)$

$= -\dfrac{1}{\sqrt{2}} \quad \left(-\dfrac{\sqrt{2}}{2}\right)$ [1]

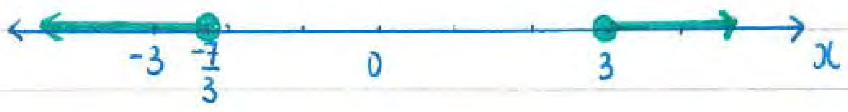
Comments:
Parts (a) and (b) where generally done well by everyone.

(c) $|3x - 1| \geq 8$

$3x - 1 \geq 8$ or $3x - 1 \leq -8$

$3x \geq 9 \qquad 3x \leq -7$

$x \geq 3 \qquad x \leq -\dfrac{7}{3}$



[2]

Comments:
Students generally forgot to consider both the positive and negative cases of the absolute value. Also, with the negative case some students didn't change the inequality.

(d) i) $\frac{d}{dx}\left(\frac{x}{x-2}\right)$ Let $u = x$, $v = x - 2$; $u' = 1$, $v' = 1$

$= \frac{vu' - uv'}{v^2}$

$= \frac{(x-2).1 - x.1}{(x-2)^2}$

$= \frac{x - 2 - x}{(x-2)^2}$

$= \frac{-2}{(x-2)^2}$ [1]

ii) $\frac{d}{dx}\left(x^2 \ln x\right)$ Let $u = x^2$, $v = \ln x$; $u' = 2x$, $v' = \frac{1}{x}$

$= vu' + uv'$

$= \ln x . 2x + x^2 . \frac{1}{x}$

$= 2x \ln x + x$

$= x(2 \ln x + 1)$ [1]

iii) $\frac{d}{dx}\left(\sin(1 + x^2)\right)$

$= \cos(1 + x^2) . 2x$

$= 2x \cos(1 + x^2)$ [1]

Comments:

All parts in (d) where generally done well.

Students who made a mistake, didn't use the appropriate differentiation rules related to the question.

(e) $\log_2 \sqrt{32} = \log_2 (2^5)^{1/2}$

$= \log_2 2^{5/2}$

$= \frac{5}{2} \log_2 2$

$= \frac{5}{2}$ [1]

Comments:
This part was generally done well by everyone.

(f) $y = \ln(3x+4)$, equation of the normal at $x = -1$.

when $x = -1$, $y = \ln(3(-1)+4)$
$= \ln(-3+4)$
$= \ln(1)$
$= 0$ $\therefore (-1, 0)$

$\frac{dy}{dx} = \frac{3}{3x+4}$

when $x = -1$; $\frac{dy}{dx} = \frac{3}{3(-1)+4}$
$= 3$

$\therefore m_t = 3$
$m_n = \frac{-1}{3}$

$$y - 0 = -\frac{1}{3}(x - (-1))$$

$$-3y = x + 1$$

$$x + 3y + 1 = 0$$

or

$$y = -\frac{1}{3}x - \frac{1}{3}$$

[2]

Comments:

Students generally made mistakes in:

- finding the correct *y* – value.
- finding the correct gradient of the normal at the point $x = -1$.
- leaving the equation of a straight line in either the form, $y = mx + b$ or $y = ax^2 + bx + c$.

(g) $$1.27 \times \frac{180}{\pi} = 72^\circ 45' 56.3''$$

$$= 72^\circ 46'$$

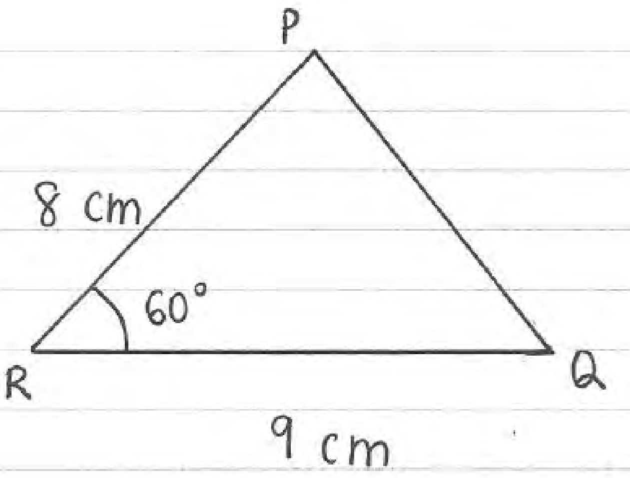
[2]

Comments:

Common mistakes where that students:

- didn't multiply correctly, and
- rounded to the nearest minute incorrectly.

(h)



(N.T.S)

i) $PQ^2 = QR^2 + PR^2 - 2 \times QR \times PR \times \cos \angle PRQ$

$= 9^2 + 8^2 - 2 \times 9 \times 8 \times \cos 60$

$= 81 + 64 - 144 \times \frac{1}{2}$

$= 73$

$\therefore PQ = \sqrt{73}$ cm [2]

≈ 8.54 (2.d.p) cm

ii) $A = \frac{1}{2} \times 8 \times 9 \times \sin 60$

$= 36 \times \frac{\sqrt{3}}{2}$

$= 18\sqrt{3}$ cm^2 [1]

Comments:

Common mistakes where that students:

- did not know the correct trigonometry formulas, and
- did not leaving the answer for part (ii) in exact form.

SBHS THSC Maths (2U) 2015

Question 12

(a) (i) $6x^2 + 23x - 4 = (6x-1)(x+4)$

(ii) $8a^3 - b^3 = (2a-b)(4a^2 + 2ab + b^2)$

Comment: Very well answered. Some were careless.

(b) (i) $\int \left(1 + x^{\frac{1}{2}}\right) dx = x + \dfrac{2x^{\frac{3}{2}}}{3} + C$

$= x + \frac{2}{3}x\sqrt{x} + C$

(ii) $\int \cos \frac{4}{9} x \, dx = \frac{9}{4} \sin \frac{4}{9} x + C$

(iii) $\int \dfrac{x}{1+3x^2} dx = \dfrac{1}{6} \int \dfrac{6x dx}{1+3x^2}$

$= \frac{1}{6} \ln\left(1 + 3x^2\right) + C$

(iv) $\int e^{-x/3} dx = -3e^{-x/3} + C$

Comment: These parts were all well answered. Some made the error of differentiating in part (iv).

(c) $\sum_{r=1}^{4} 3^{1-r} = 3^0 + 3^{-1} + 3^{-2} + 3^{-3}$

$= 1 + \dfrac{1}{3} + \dfrac{1}{9} + \dfrac{1}{27}$

$= \dfrac{40}{27}$

Comment: Very few stumbled on this notation-based question.

(d) $\sin\left(x + \frac{\pi}{3}\right) = 0$

$x + \dfrac{\pi}{3} = ...0, \pi, 2\pi, ...$

$x = ... -\dfrac{\pi}{3}, \ \dfrac{2\pi}{3}, \ \dfrac{5\pi}{3}, ...$

In the domain $0 \le x \le 2\pi$:

$x = \dfrac{2\pi}{3}, \ \dfrac{5\pi}{3}$

Comment: Many included the negative root shown above. There seems to be a lack of understanding of domains.

(e) $\frac{\sqrt{3}+1}{\sqrt{3}-1}\times\frac{\sqrt{3}+1}{\sqrt{3}+1}=\frac{3+2\sqrt{3}+1}{3-1}$

$$=\frac{4+2\sqrt{3}}{2}$$

$$=2+\sqrt{3},\quad a=2,\ b=1$$

Comment: Very well answered. A few made careless errors.

(f) Arithmetic Series: 32 + 25 + 18 + …

$a=32,\ d=-7$

(i) $S_{15}=\frac{15}{2}\left(64+14\times-7\right)$

$=-255$

(ii) Sum of the next 20 terms:

$S_{35}-S_{15}=-3045-\left(-255\right)$

$=-2790$

Comment: Whilst the majority had no difficulty, a fair number misunderstood what was required. They were given marks only if they explained what they were doing.

13) a) $4x^2+5x+6 \equiv A(x+1)^2+B(x+1)+C$

equate coefficient of x^2

$A=4$

let $x=-1$

$4(-1)^2+5(-1)+6=C$

$C=5$

let $x=0$

$6=A+B+C$

$6=4+B+5$

$B=-3$

$\therefore A=4,\ B=-3,\ C=5$

b) $y=3\ln 2x+1$

$y-1=3\ln 2x$

$\ln 2x=\frac{y-1}{3}$

$2x=e^{\frac{y-1}{3}}$

$x=\frac{1}{2}e^{\frac{y-1}{3}}$

c) $f(x)=ax^2+bx$

$f'(x)=2ax+b$

$f'(0)=2a(0)+b=\frac{1}{2}$

$b=\frac{1}{2}$.

$f(x)=ax^2+\frac{x}{2}$

$f(1)=a(1)^2+\frac{(1)}{2}=2$

$a+\frac{1}{2}=2$

$a=\frac{3}{2}$.

d) $2x^2 - 5x + 5 = 0$ has roots α, β

i) $\alpha + \beta = -\frac{b}{a}$

$\alpha + \beta = \frac{-(-5)}{2}$

$= \frac{5}{2}$

ii) $\alpha\beta = \frac{c}{a}$

$= \frac{5}{2}$

iii) $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$

$= \left(\frac{5}{2}\right)^2 - 2\left(\frac{5}{2}\right)$

$= \frac{5}{4}$

iv) $\alpha^3 + \beta^3 = (\alpha + \beta)(\alpha^2 - \alpha\beta + \beta^2)$

$= \left(\frac{5}{2}\right)\left(\frac{5}{4} - \frac{5}{2}\right)$

$= -\frac{25}{8}$

e) i) $\frac{d}{dx}(\tan x - x) = \sec^2 x - 1$

$= (1 + \tan^2 x) - 1$

$= \tan^2 x$

ii) $\int_0^{\frac{\pi}{4}} \tan^2 x \, dx = \left[\tan x - x\right]_0^{\frac{\pi}{4}}$

$= \tan\frac{\pi}{4} - \frac{\pi}{4} - (0)$

$= 1 - \frac{\pi}{4}$. or $\frac{4 - \pi}{4}$

COMMENTS:

a) This question was generally done well

b) The answer could be written in many equivalent forms.

$x = \frac{1}{2}e^{\frac{y-1}{3}}$, $x = \frac{\sqrt[3]{e^{y-1}}}{2}$, $x = e^{\frac{y-1}{3} - \ln 2}$, ...

Spot the mistake

$y = 3\ln 2x + 1$

$y - 1 = 3\ln 2x$

$y - 1 = \ln 2x^3$ — some students did this

It should be

$y - 1 = \ln(2x)^3$

$(2x)^3 = e^{y-1}$

$2x = \sqrt[3]{e^{y-1}}$

$x = \frac{\sqrt[3]{e^{y-1}}}{2}$

c) Solve using simultaneous equations formed from the given conditions ie $f'(0) = \frac{1}{2}$, $f(1) = 2$.

d) $2x^2 - 5x + 5 = 0$ actually has no real roots.

$\Delta = b^2 - 4ac$

$= (-5)^2 - 4(2)(5)$

$= -15$

< 0

This doesn't stop us from finding the values of $\alpha+\beta$, $\alpha\beta$, $\alpha^2+\beta^2$, $\alpha^3+\beta^3$.

iv) $\alpha^3+\beta^3$ could be found using

$$\alpha^3+\beta^3 = (\alpha+\beta)(\alpha^2-\alpha\beta+\beta^2)$$

<u>Note</u>: $\alpha^2+\beta^2$ was found in (iii)

OR $\alpha^3+\beta^3 = (\alpha+\beta)^3 - 3\alpha\beta(\alpha+\beta)$

Many students used

$$\alpha^3+\beta^3 = (\alpha+\beta)(\alpha^2+\alpha\beta+\beta^2)$$

OR

$$\alpha^3+\beta^3 = (\alpha+\beta)(\alpha^2-2\alpha\beta+\beta^2)$$

which are incorrect.

e) i) Generally done well. However, many students did a lot of working.

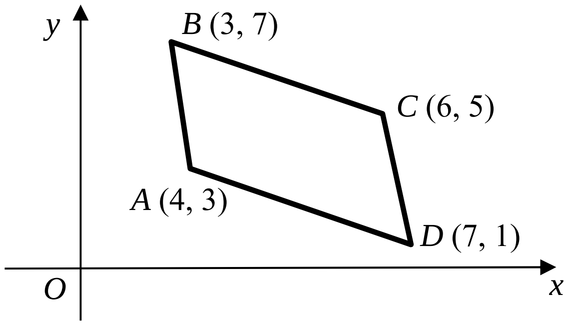
eg. $$\frac{d(\tan x - x)}{dx} = \sec^2 x - 1$$
$$= \frac{1}{\cos^2 x} - 1$$
$$= \frac{1-\cos^2 x}{\cos^2 x}$$
$$= \frac{\sin^2 x}{\cos^2 x}$$
$$= \tan^2 x.$$

ii) Answer should be in exact form.

Many students simplified $1-\frac{\pi}{4}$ to $\frac{3\pi}{4}$.

Question 14 (15 marks)

(a)



ABCD is a parallelogram.

(i) Show that the equation of the line *AD* is $2x+3y-17=0$ **1**

Method 1:

Test *A*: $\text{LHS} = 2\times 4 + 3\times 3 - 17 = 0 = \text{RHS}$

Test *D*: $\text{LHS} = 2\times 7 + 3\times 1 - 17 = 0 = \text{RHS}$

$\therefore$ the equation of *AD* is $2x+3y-17=0$

Method 2:

$$m_{AD} = \frac{3-1}{4-7} = -\frac{2}{3}$$

$$\therefore y-3 = -\tfrac{2}{3}(x-4)$$

$$\therefore 3y-9 = -2x+8$$

$$\therefore 2x+3y-17=0$$

Comment

This part was generally done well by everyone.

(ii) Find the length of *BC*. **1**

$$BC = \sqrt{(3-6)^2+(7-5)^2}$$

$$= \sqrt{9+4}$$

$$= \sqrt{13}$$

Comment

This part was generally done well by everyone.

(iii) Find the perpendicular distance between *C* (6, 5) and the line *AD*. **2**

$$\Big(\underbrace{6}_{x_1},\ \underbrace{5}_{y_1}\Big)$$

$$d = \frac{|\,2\times x_1 + 3\times y_1 - 17\,|}{\sqrt{2^2+3^2}}$$

$$d = \frac{|\,2\times 6 + 3\times 5 - 17\,|}{\sqrt{2^2+3^2}}$$

$$= \frac{|\,10\,|}{\sqrt{13}} = \frac{10}{\sqrt{13}}$$

Comment

This part was generally done well by everyone who knew the formula.

Success was limited for those who either don't know the formula or wrong variations of it.

(a) (iv) Hence, or otherwise, find the area of *ABCD*. **2**

$$\text{Area} = BC \times d$$
$$= \sqrt{13} \times \frac{10}{\sqrt{13}} = 10 \text{ u}^2$$

Comment

It was surprising the number of people who don't know the area of a parallelogram formula or other properties of parallelograms.

(b) The volume V cm^3 of a balloon is increasing such that the volume at any **2**

time t seconds is given by

$$V = \frac{\pi t^3}{3} - \frac{\pi t^2}{6} + \frac{1}{2}$$

Find the rate at which the volume is increasing when $t = 2$.

$$\frac{dV}{dt} = \pi t^2 - \frac{\pi t}{3}$$
$$t = 2 \Rightarrow \frac{dV}{dt} = \pi \times 2^2 - \frac{\pi \times 2}{3}$$
$$= \frac{10\pi}{3} \text{ cm}^3/\text{s}$$

Comment

It was surprising to see that many students left their answer as $4\pi - \frac{2\pi}{3}$ i.e. they didn't simplify far enough. They lost a $\frac{1}{2}$ mark.

(c) A particle moves in a straight line such that its distance, x metres from a fixed point after t seconds is given by $x = 6t - 2t^3 + 5$ for $t \geq 0$.

(i) Find the equation of the velocity after t seconds. **1**

$$v = \dot{x}$$
$$= 6 - 6t^2$$

Comment

There were too many careless results with the differentiation.

Use the correct notation i.e. $v = \frac{dx}{dt} = \dot{x}$.

(ii) At what time does the particle stop? **1**

A particle stops when $v = 0$

$$\therefore 6 - 6t^2 = 0$$
$$\therefore 6(1 - t^2) = 0$$
$$\therefore t = \pm 1$$
$$\therefore t = 1$$

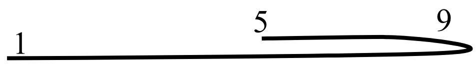
Comment

Stopping or at rest is when $v = 0$, it has nothing to do with acceleration.

(c) (iii) Find the total distance travelled in the first 2 seconds. **2**

Method 1:

$t = 0$: $x = 5$
$t = 1$: $x = 9$
$t = 2$: $x = 1$



$\therefore$ Distance travelled = 4 + 8 = 12

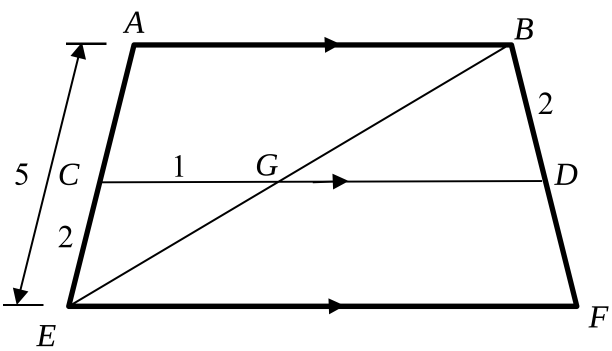
Method 2:

$$\begin{aligned}\text{Distance travelled} &= \int_0^1 v\,dt + \left| \int_1^2 v\,dt \right| \\ &= \left[x\right]_0^1 + \left| \left[x\right]_1^2 \right| \\ &= \left[6t - 2t^3 + 5\right]_0^1 + \left| \left[6t - 2t^3 + 5\right]_1^2 \right| \\ &= 9 - 5 + \left| 1 - 9 \right| \\ &= 12\end{aligned}$$

Comment

There are many students who don't know the difference between position/displacement; change in displacement and change in distance.
Substituting $t = 2$ only gets the displacement/position of the particle at $t = 2$.
Those students who calculated $x(2) - x(0)$, only found the change in displacement.

(d)



In the diagram *AB*, *CD* and *EF* are parallel and *BE* cuts *CD* at *G*.
AE is 5 cm, *CE* is 2 cm, *CG* is 1 cm and *BD* is 2 cm.

(i) Write down the length of *AB*. **1**

$\Delta AEB \;|||\; \Delta CEG$ (equiangular)

$\therefore \dfrac{AE}{CE} = \dfrac{AB}{CG} = \dfrac{EB}{EG}$ (matching sides of sim. Δs)

$\therefore \dfrac{5}{2} = \dfrac{AB}{1} \Rightarrow AB = 2.5$

Comment

There was a lot written down for 1 mark.

Question 14 (continued)

(d) (ii) Find the length of *DF*. **2**

$CA = 5 - 2 = 3$

$CE : CA = DF : DB$ (parallel lines preserve ratio)

$\therefore 2:3 = DF:2$

$\therefore \dfrac{2}{3} = \dfrac{DF}{2}$

$\therefore DF = \dfrac{4}{3}$

Comment

Some students who didn't know this theorem about the intercepts on parallel lines were still able to follow on from (i) and use similarity. This approach was long.

When proving non triangular shapes similar, there are many students who don't realise they have to show more than just equiangular. This is not sufficient.

Question (15) (15)

(a) (i) $\left(\frac{3}{8}\right)^2 = \frac{9}{64}$. [1]

(ii) $\left(\frac{3}{8}\right)\left(\frac{5}{8}\right) \times 2$

$= \frac{30}{64}$

$= 15/32$. [2]

(b) (i) $\begin{cases} y = x+2 \\ y = -x^2+7x-6 \end{cases}$

$x+2 = -x^2+7x-6$

$\therefore -x^2+6x-8=0$

$x^2-6x+8=0$

$(x-4)(x-2)=0$

$\therefore x = 2, 4$

i.e. $A=2$, $B=4$. [1]

(ii) $\int_2^4 (-x^2+6x-8)\,dx$

$= \left[-\frac{x^3}{3} + 3x^2 - 8x\right]_2^4$

$= \left(\frac{-64}{3} + 16\right) - \left(\frac{-8}{3} - 4\right)$

$= 4/3$ [2]

(c) $f(x) = 2x(x-3)^2$

(i) When $x=0$, $y=0$ $(0,0)$ [1]

$x=3$, $y=0$ $(3,0)$.

(ii) $f'(x) = 2(x-3)^2 + 4x(x-3)$

$= 6(x-1)(x-3)$.

$f''(x) = 12x - 24$

$= 12(x-2)$.

$f'(x) = 0$ When $x=1$, $y=8$

$(1,8)$

When $x=3$, $y=0$

$\therefore (1,8)$ $(3,0)$ are stationary pts.

$f''(1) < 0$

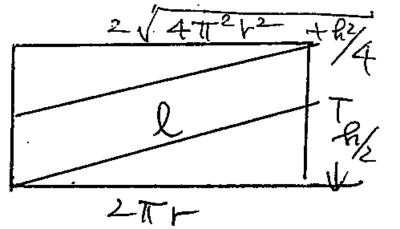
$f''(3) > 0$

$\therefore (1,8)$ is max

$(3,0)$ is min [2]

See the next page for (c) (iii) & (iv)

(d)



$\therefore l = \sqrt{(2\pi r)^2 + \frac{h^2}{4}}$ [2]

$= \frac{1}{2}\sqrt{16\pi^2 r^2 + h^2}$

$\therefore$ length $= 2l = (16\pi^2 r^2 + h^2)^{1/2}$

15 (c) (iii) & (iv)

$$f(x) = 2x(x-3)^2$$
$$= 2x(x^2-6x+9)$$
$$= 2x^3-12x^2+9x.$$

Now

$$2x^3-12x^2+18x-8=0$$
$$\Rightarrow \quad 2x^3-12x^2+18x=8.$$

i.e.

$$2x(x-3)^2=8.$$

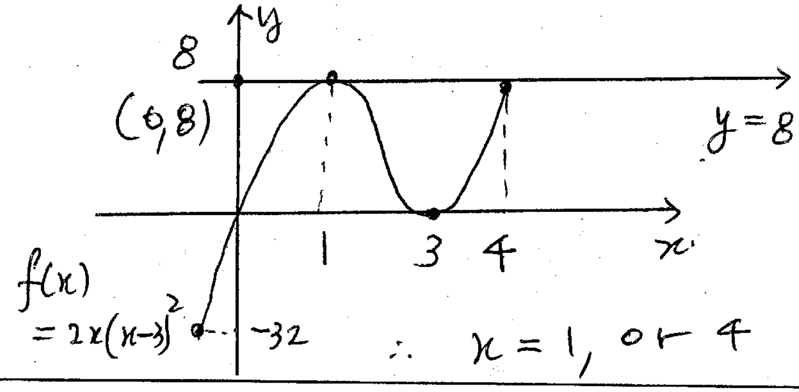
$\therefore$ To solve

$$2x^3-12x^2+18x-8=0$$

<u>either</u> sketch

$$\begin{cases} y = 8 \\ \text{and } y = 2x(x-3)^2 \end{cases}$$

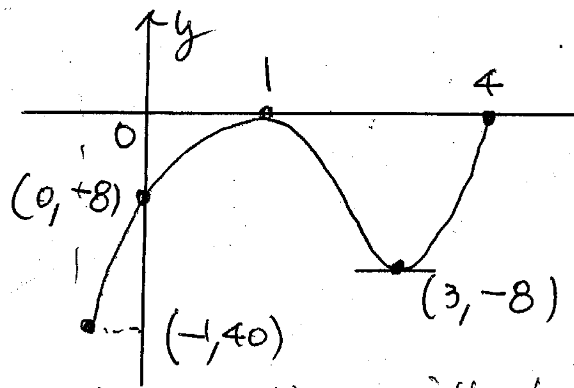
on the same number plane



$\therefore \quad x = 1, \text{ or } 4$

<u>or alternatively</u>, translate (shift) the graph of $y = 2x(x-3)^2$ 8 units down vertically to derive the solutions $x=1$ and $x=4$

i.e. solve
$$\begin{cases} y=0 \\ y=2x(x-3)^2-8 \end{cases}$$
simultaneously



Use factor thm: will also get the full mark even though its not in syllabus.

Question 16: (No half marks were given)

a) i) A_n is the amount of money Catherine has on her birthday.

$A_0 = \$1000$ (11 years old)

$A_1 = \$1000 \times 1.06 + 1000$ (12 years old)

$A_2 = (\$1000 \times 1.06 + 1000) \times 1.06 + \$1000$ (13 years old)

$= \$1000 \times 1.06^2 + 1000 \times 1.06 + \$1000$

$\vdots$

$A_{10} = \$1000 \times 1.06^{10} + 1000 \times 1.06^9 + 1000 \times 1.06^8 + \dots + 1000 \times 1.06$

(Note: No deposit on her 21st birthday)

$= \$1000(1.06 + 1.06^2 + 1.06^3 + \dots + 1.06^{10})$

GP series with $a = 1.06$ $r = 1.06$ $n = 10$

$\therefore A_{10} = \$1000\left[\frac{1.06(1.06^{10}-1)}{1.06-1}\right]$ (1 mark for working)

$= \$13971.64$ (1 mark for answer.)

Marker's Comments

- This question was not well done by many candidates.
- Some of the major mistakes that candidates made in this question included:
  - Having the first term of the GP series being 1 rather 1.06.
  - The time amount (n) not being 10.
  - Compounded monthly rather than yearly.
  - Careless errors such as 6% being 0.006.
- Candidates who made **only** 1 mistake listed above would have been given 1 mark out of 2. Two or more mistakes would have no marks being rewarded.

ii) Using compound interest formula

$A = P(1+r)^n$ (1 mark for working)

$\$13971.64 = P(1+0.06)^{10}$ (1 mark for answer.)

$P \doteqdot \$7800$

Marker's Comments

- Majority of the candidates wrote the right working for this question.
- Candidates who made a mistake in a) i) got 1 mark out of 2 for CFPA (Correct from Previous Answer).
- Candidates lost a mark if they did not follow the direction of the question, which was to round the answer to nearest $10.

b) $V = \pi \int_0^{\frac{\pi}{3}} y^2 \, dx$

x	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$
$x^2\sec^2 x$	0	$\frac{\pi^2}{27}$	$\frac{4\pi^2}{9}$

Since $y = x\sec x$

$y^2 = x^2\sec^2 x$ (1) mark.

By Simpson's Rule ($\approx \frac{b-a}{6}(f(0) + 4f(\frac{\pi}{6}) + f(\frac{\pi}{3}))$)

$V \doteqdot \pi \times \left[\frac{\frac{\pi}{6}}{3} \left(0 + 4 \times \frac{\pi^2}{27} + \frac{4\pi^2}{9} \right) \right]$ (1) mark

$\doteqdot 3.21 \text{ units}^3$ (1) mark.

Marker's Comments

- This question was not done well by many candidates.
- Common mistakes by candidates included:
  - Just finding the approximation of the area, and not the volume.
  - Forgetting to multiply the extra π (that arises from the formula Volume = $\boldsymbol{\pi} \times \int y^2 \, dx$)
  - Not substituting the values into y^2.
  - Not multiplying by $\frac{\pi}{18}$ (which arises from $\frac{b-a}{6}$) but rather other values or no values.
  - Wrong substitution of the function values or using wrong function values i.e. using $\frac{\pi}{4}$.
- Candidates who made a minor mistake i.e. 1 wrong substitution or not multiplying the required π, lost 1 mark for CFPE (Correct from Previous Error). Otherwise candidates who made more mistakes or only found area were given 1 or 0 mark out of 3.

c) i) $y = mx + 1$ (1) $(y - 1 = m(x - 0))$

$x^2 = 4y$ (2)

Sub (1) into (2)

$\therefore x^2 = 4(mx+1)$

$x^2 = 4mx + 4$

$\therefore x^2 - 4mx - 4 = 0$

(1) mark.

Since A and B are points of intersection of the 2 ~~lines~~ functions above,

$\therefore x_1$ and x_2 are roots of equation $x^2 - 4mx - 4 = 0$.

Marker's comments

- Only 1 mark, so the mark was ONLY given if candidates derived the equation **AND** wrote A and B are the intercepts (intersection) of the two functions.

(PTO)

- No mark was given if candidates only derived the equation as they did not state why x_1 and x_2 are the roots of the equation.

(ii) $x^2 - 4mx - 4 = 0$

Sum and products of the factors

$x_1 + x_2 = 4m \qquad (\alpha + \beta = \frac{-b}{a})$

$x_1 x_2 = -4 \qquad (\alpha\beta = \frac{c}{a})$

$\therefore (x_1 - x_2)^2 = (x_1 + x_2)^2 - 4x_1x_2$

$= (4m)^2 - 4 \times (-4)$

$= 16m^2 + 16$ (1) mark.

Marker's Comment

- This question was done correctly by many candidates; however most candidates took an inefficient method. The method above saved time and all candidates who used this approach answered the question correctly.

(iii) At point A : $(x_1)^2 = 4y_1 \qquad \therefore y_1 = \frac{x_1^2}{4}$

At point B : $(x_2)^2 = 4y_2 \qquad y_2 = \frac{x_2^2}{4}$

$\therefore y_1 - y_2 = \frac{x_1^2}{4} - \frac{x_2^2}{4}$

$= \frac{1}{4}(x_1^2 - x_2^2)$

$= \frac{1}{4}(x_1 - x_2)(x_1 + x_2)$

$\therefore (y_1 - y_2)^2 = \left(\frac{1}{4}\right)^2 (x_1 - x_2)^2 (x_1 + x_2)^2$ (1) mark.

$= \frac{1}{16}(16m^2 + 16)(4m)^2$

$= (m^2 + 1) \times 16m^2$

$= 16m^4 + 16m^2$

$\therefore AB = \sqrt{(x_1 - x_2)^2 + (y_2 - y_1)^2}$ (distance formula)

$= \sqrt{16m^2 + 16 + 16m^4 + 16m^2}$

$= \sqrt{16m^4 + 32m^2 + 16}$ (1) mark.

$= 4\sqrt{m^4 + 2m^2 + 1}$

$= 4(\sqrt{(m^2+1)^2}$

$= 4(1 + m^2)$

Marker's Comments

- Many candidates did not do this question well.
- A significant number of candidates tried to make LHS = RHS by making incorrect assumptions or tried to write a solution that started with the RHS. The solution of those candidates was incorrect and was given 0 out of 2 marks.
- Candidates were only given 1 mark out 2 if they started LHS by using the distance formula and made significant working towards the correct solution without getting the correct solution.

(iv) Centre $= \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

$y_1+y_2 = \frac{x_1^2+x_2^2}{4}$

$\therefore$ Centre $= \left(\frac{x_1+x_2}{2}, \frac{x_1^2+x_2^2}{8}\right)$

Since $x_1+x_2 = 4m$ and $x_1x_2 = -4$ (from (ii))

$(x_1^2+x_2^2) = (x_1+x_2)^2 - 2x_1x_2$

$= (4m)^2 - 2\times(-4)$

$= 16m^2+8$

$\therefore$ Centre $= \left(\frac{4m}{2}, \frac{16m^2+8}{8}\right)$

$= (2m, 2m^2+1)$ (1 mark)

Since $d_{AB} = 4(1+m^2)$ (from question (iii))

which is the diameter (given in the question)

$\therefore$ Radius $= 2(1+m^2) = 2+2m^2$

(1 mark)

Marker's comments

- Majority of the candidates were able to find the radius as it stated in the question that AB is the diameter. Candidates just needed to half the result given in question iii).
- Many candidates failed to follow the direction of the question that the centre must be in terms of m. Most candidates whom just gave an answer of (0, 1) from the diagram, had v) and vi) incorrect. **No marks were given for (0, 1) as it was not in terms of m.**
- The correct answer for the centre in this question would allow students to answer v) and especially vi) correctly.

(v) $y + 1 = 0$ $\quad C = (2m, 2m^2 + 1)$

$y = -1$

$\therefore$ Distance $= 2m^2 + 1 - (-1)$

$= 2m^2 + 2$; 1 mark

Marker's comments

- This was done incorrectly by a significant amount of candidates due to careless mistake.
- This was done incorrectly if students weren't able to get iv) correctly.
- Only 1 mark for the correct answer.

(vi) distance from the line $y + 1 = 0$ to the centre

= length of the radius

$\therefore$ $y + 1 = 0$ is a tangent to the circle.

1 mark

Marker's Comments

- Candidates were successful in answering this question (and get 1 mark) if they found iii) and iv) correctly.