SYDNEY BOYS HIGH SCHOOL

2019

YEAR 12 TRIAL HSC ASSESSMENT TASK

Mathematics

General Instructions

- Reading time – 5 minutes
- Working time – 3 hours
- Write using **black** pen
- NESA approved calculators may be used
- A reference sheet is provided with this paper
- Marks may **NOT** be awarded for messy or badly arranged work
- In Questions 11–16, show ALL relevant mathematical reasoning and/or calculations

Total Marks: 100

Section I – 10 marks (pages 2–5)

- Attempt Questions 1–10
- Allow about 15 minutes for this section

Section II – 90 marks (pages 6–15)

- Attempt Questions 11–16
- Allow about 2 hours and 45 minutes for this section

Examiner: **B.K**

Section I

10 marks
Attempt Questions 1-10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1-10.

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1 What is the value of $\int_{-2}^{2} x^3 \, dx$?

(A) -2

(B) 0

(C) 0.5

(D) 4

2 What is the derivative of $3\cos 2x$?

(A) $6\sin 2x$

(B) $-6\sin 2x$

(C) $6\cos 2x$

(D) $-6\cos 2x$

3 A particle is moving on a straight line according to the function $x = 2\sin \pi t$.
What is the period of oscillation?

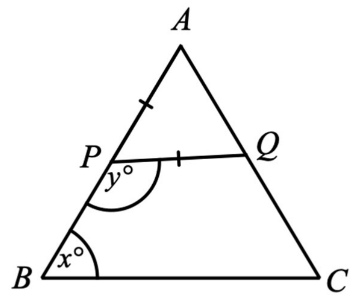
(A) π

(B) 2

(C) $\frac{\pi}{2}$

(D) $\frac{2}{\pi}$

4 In the figure ΔABC , $AB = AC$. P and Q are on AB and BC respectively such that $AP = PQ$.



What is y in terms of x ?

(A) $y = 360 - 4x$

(B) $y = 180 - 2x$

(C) $y = 180 + 2x$

(D) $y = 360 + 4x$

5 A parabola has its vertex at (2, 0) and its focus at (4, 0).
What is the equation of this parabola?

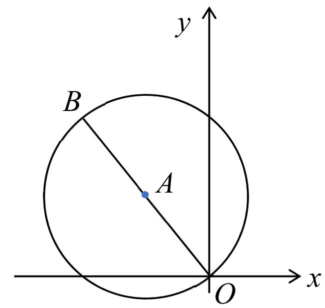
(A) $(y-2)^2 = 8x$

(B) $y^2 = 8(x-2)$

(C) $y^2 = 2(x-2)$

(D) $x^2 = 8(y-2)$

6 The diagram shows the circle with centre A and with diameter OB.
The equation of this circle is $(x+3)^2+(y-4)^2=25$.



What are the coordinates of the point B?

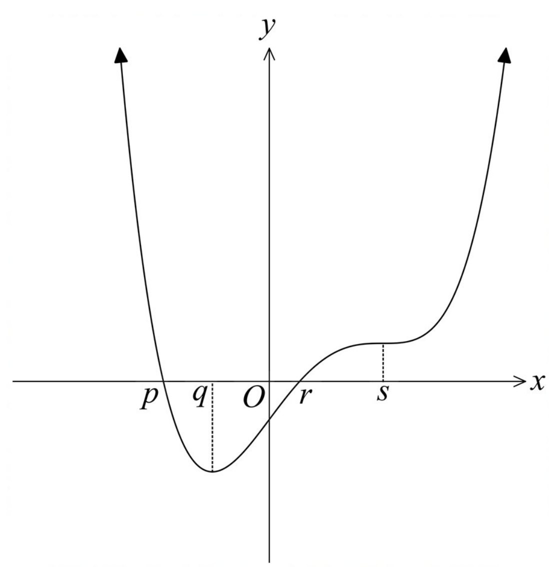
(A) $(-8,9)$

(B) $(-3,4)$

(C) $(-6,8)$

(D) $(-6,7)$

7 The diagram shows the graph of $y=f''(x)$ for the function $f(x)$.



For what value of x does the function $f'(x)$ have a maximum turning point?

(A) $x=p$

(B) $x=q$

(C) $x=r$

(D) $x=s$

8 A bag contains 4 red balls, 5 white balls and 1 blue ball.
Two balls are drawn without replacement.

What is the probability that the balls will be of different colours?

(A) $\frac{1}{3}$

(B) $\frac{16}{45}$

(C) $\frac{29}{45}$

(D) $\frac{31}{45}$

9 The three terms x, y and z are consecutive terms in in a geometric series, such that $x+y+z=26$ and $xyz=64$. What is the value of $x+z$?

(A) 18

(B) 22

(C) 28

(D) 30

10 Given that $81^{2x+3}=243^{5-x}$, what is the value of x?

(A) 0

(B) $\frac{13}{3}$

(C) 1

(D) -1

Section II

90 marks
Attempt Questions 11-16
Allow about 2 hours and 45 minutes for this section
Answer each question in a separate writing booklet.
In Questions 11-16, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) Use a SEPARATE Writing Booklet.

(a) Express $\frac{3\pi^2}{4}$ correct to 3 significant figures. **2**

(b) Rationalise the denominator : $\frac{2+\sqrt{3}}{8-2\sqrt{3}}$ **2**

(c) Factorise $8x^3 - 125y^3$. **2**

(d) Differentiate the following with respect to x.

(i) $\frac{1}{3x}$ **1**

(ii) $e^x \cos x$ **2**

(iii) $\ln(\sin x)$ **2**

(iv) $(3x^2 - 5x)^6$ **2**

(e) Find the limiting sum of $1 - \frac{1}{3} + \frac{1}{9} - \frac{1}{27} + \ldots$ **2**

End of Question 11

Question 12 (15 marks) Use a SEPARATE Writing Booklet.

(a) Find the function whose derivative is $\dfrac{6x}{1+3x^2}$ and which passes through the origin. **1**

(b) Evaluate $\displaystyle\lim_{x\to 2}\frac{4x-x^3}{2-x}$ **1**

(c) Evaluate $\displaystyle\int_0^{\frac{\pi}{4}}\cos 3x\,dx$ leaving the answer in exact form. **2**

(d) In an arithmetic series the first term is 4 and the sum of the first 20 terms is 650.

(i) Find the common difference. **2**

(ii) Find the 20$^{\text{th}}$ term. **1**

(e) A babushka doll set consists of 7 wooden dolls of decreasing size placed one inside another.

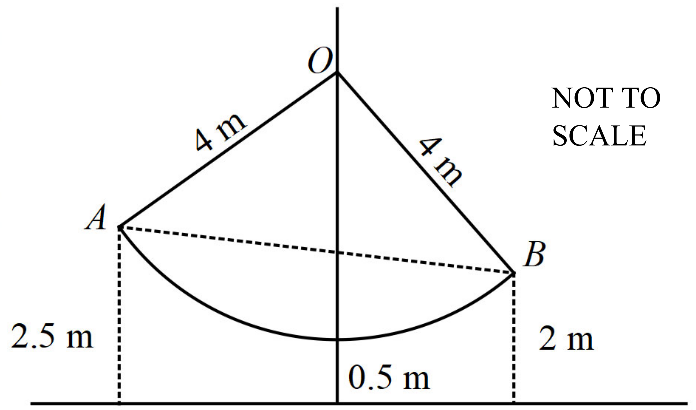
The smallest doll measures 0.5 cm in height and the tallest doll measures 32 cm.

(i) If the height of the dolls from smallest to tallest forms a geometric sequence, find the common ratio. **2**

(ii) The dolls are made from a piece of wood whose length must be at least the total height of all 7 dolls.
Calculate the minimum length of wood required. **2**

Question 12 continues on page 9

(f) The ropes of a swing are 4 m long. When the swing is at rest, the seat is 0.5 m above ground level. When a child uses the swing, the highest point, A, reached by the seat on one side is 2.5 m above ground-level while on the other side the highest point, B, is 2 m above ground-level.



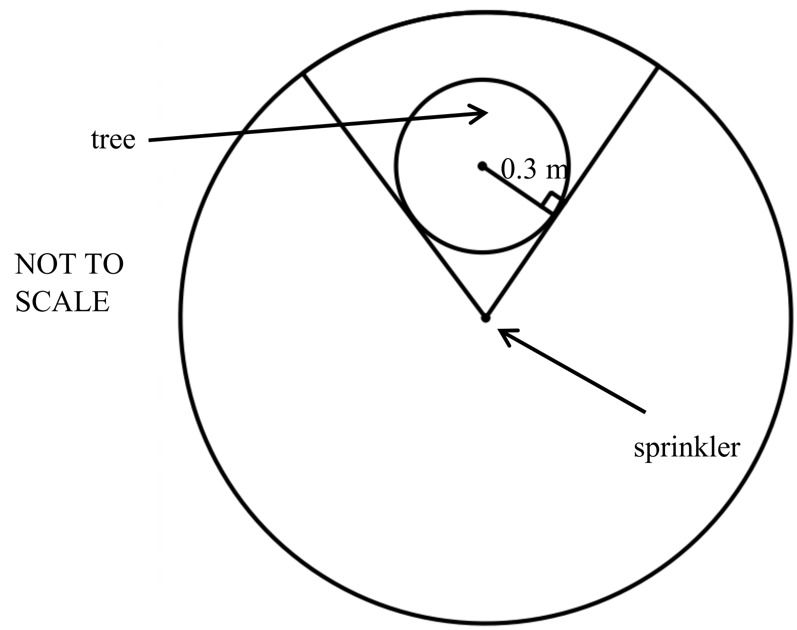
(i) Find $\angle AOB$, through which the seat swings. Give your answer to the nearest minute. **2**

(ii) Find the straight-line distance AB, correct to one decimal place, between the two highest points reached. **2**

End of Question 12

Question 13 (15 marks)   Use a SEPARATE Writing Booklet.

(a) A garden sprinkler sprays in a full circle with radius 3 m. **3**
It is placed on a lawn as shown below.
A small tree with a cylindrical trunk impedes the path of some of the spray.



The tree trunk's diameter is 60 cm.
At its closest point the tree is 1 m from the sprinkler.
If there is no breeze blowing, the area behind the tree is unwatered.

What is the area of the watered section to the nearest square metre?

(b) A bus company has established that the cost of a 1000 km journey is **3**

$$J = 0.04v^2 + \frac{17500}{v} + 275 \text{ dollars,}$$

where v is the average speed in kilometres per hour.
If the bus company wishes to reduce costs should it instruct its drivers to speed up or slow down from the present average of 80 km/h?

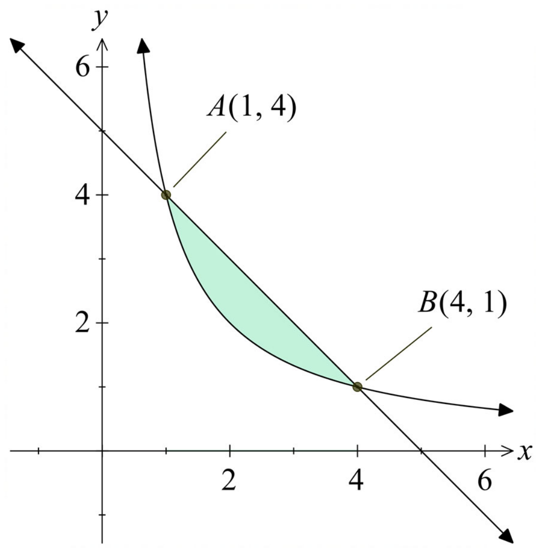
Justify your answer.

Question 13 continues on page 11

Question 13 (continued)

(c) The line $y = 5 - x$ intersects the curve $y = \dfrac{4}{x}$ at the points A (1, 4) and B (4, 1).

The region bounded by the curve and the line between the points A and B is shaded as shown in the diagram.



(i) Find the exact area of the shaded region. **2**

(ii) Use one application of Simpson's rule to find an estimate for the area of the shaded region. **2**

The point C lies on the curve in the first quadrant.
The tangent at C is parallel to the line $y = 5 - x$.

(iii) Show that C has coordinates (2, 2). **2**

(iv) Find the area of triangle ABC. **3**

End of Question 13

Question 14 (15 marks) Use a SEPARATE Writing Booklet.

(a) (i) Find $\frac{d}{dx}(\log_e(\cos x))$ **2**

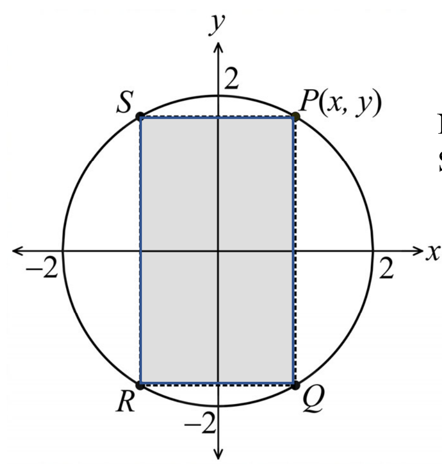
(ii) Show that $\frac{d}{dx}(\tan x - x) = \tan^2 x$. **2**

(iii) Hence find the area bounded by the curve $y = \tan x$, the x-axis and the lines $x = 0$ and $x = \frac{\pi}{4}$. **2**

(iv) Find the volume generated when this area is rotated about the x-axis. **2**

(b) (i) Verify that $\frac{d}{dx}\left(x\sqrt{4-x^2}\right) = \frac{4-2x^2}{\sqrt{4-x^2}}$ **2**

(ii) Hence or otherwise find the maximum area of a rectangle inscribed in a circle of radius 2 m. **3**
You may use the diagram below to assist you.



NOT TO SCALE

(c) The population of native noisy miner birds increases from 20 000 to 35 000 in 10 years. If the number of birds is proportional to the rate of change of the population, how many more years till the population reaches 50 000 birds? Give your answer correct to 1 decimal place **2**

End of Question 14

Question 15 (15 marks) Use a SEPARATE Writing Booklet.

(a) Inside two bags, A and B, there are 8 cards each.
In bag A, there are 2 cards labelled WIN, while in bag B there are 3 cards labelled WIN.
All the remaining cards in both bags are labelled NO WIN.

(i) Joshua is to select a card from bag A then a card from bag B. **1**
What is the probability he will select at least one WIN card?

(ii) Before selecting a card Joshua is to roll a die. **2**
If he gets 5 or 6 he is to draw 2 cards from bag B and none from A.
If he gets any other number, he will select the 2 cards from bag A and none from B.

What is the probability he will not select a WIN card?

(b) The displacement of a particle moving along the x-axis is given by

$$x = t + \ln(3t + 1)$$

where t is the time in seconds and x is measured in centimetres.

(i) Show that the particle never comes to rest. **2**

(ii) Find the distance travelled by the particle during the 3rd second. **1**
Give the answer correct to 1 decimal place.

(iii) Write down an expression for the acceleration of the particle. **1**

(iv) Is the particle slowing down or speeding up for $t > 0$? **1**
Give reasons to support your answer.

(c) A function is defined by $f(x) = x^3 - 3x^2 - 9x + 22$.

(i) Find the stationary points and their nature. **3**

(ii) Find any points of inflection. **2**

(iii) Sketch the curve showing these features and the y-intercept. **2**

End of Question 15

Question 16 (15 marks) Use a SEPARATE Writing Booklet.

(a) An amateur photographer can take a good photo 60% of the time. How many photos must he take to be 99% sure of taking at least one good photo? **2**

(b) A model rocket is launched from rest from the ground and travels vertically upwards. The rocket's acceleration for the first five seconds is given by

$$\frac{dv}{dt} = \frac{76}{5} - 5t \text{ m/s}^2.$$

(i) Find the velocity, v m/s, of the rocket after five seconds. **1**

(ii) Find the height of the rocket after five seconds. Give your answer in metres, correct to two decimal places. **2**

(iii) After five seconds, the rocket's acceleration is given by **2**

$$\frac{dv}{dt} = -9.8 \text{ m/s}^2.$$

By using calculus, find the maximum height reached by the rocket. Give your answer in metres, correct to two decimal places.

(iv) Having reached its maximum height, the rocket falls directly to the ground, and as a result **3**

$$\frac{dv}{dt} = -9.8 \text{ m/s}^2.$$

By using calculus, find the time for which the rocket was in flight. Give your answer in seconds, correct to one decimal place.

Question 16 continues on page 15

Question 16 (continued)

(c) Rob and Janet borrow \$650 000 to buy an apartment.
The loan is over 25 years with interest being charged at 6 % p.a. compounding monthly.

(i) Determine the value of the monthly repayments. **2**

(ii) After 5 years the interest rate is increased to 7.2 % p.a. compounding monthly. How much longer (in years and months) will it take them to repay the loan if they keep the same repayments as before? **3**

End of paper

SYDNEY
BOYS
HIGH
SCHOOL

2019

YEAR 12 TRIAL HSC
ASSESSMENT TASK

Mathematics

SUGGESTED SOLUTIONS

MC QUICK ANSWERS

1. B
2. B
3. B
4. A
5. B
6. C
7. A
8. C
9. B
10. C

2U Y12 THSC 2019 Multiple choice solutions

Mean (out of 10): 8.62

1. $\int_{-2}^{2} x^3\,dx$

$= \left[\frac{x^4}{4}\right]_{-2}^{2}$

$= [4] - [4]$

$= 0$ B

– OR –

odd function, limits equidistant from 0 $\Rightarrow$ 0

A	2
B	177
C	0
D	4

2. $\frac{d}{dx}(3\cos 2x)$

$= 3.(-\sin 2x).2$

$= -6\sin 2x$ B

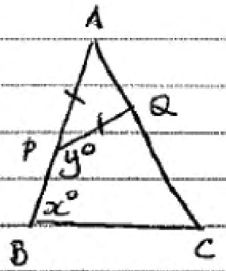
A	0
B	181
C	0
D	1

3. Period $= \frac{2\pi}{\pi}$

$= 2$ B

A	18
B	160
C	4
D	1

4.



$\angle ACB = x°$ (base angles of isosceles ΔABC)

$\angle PAQ = \angle PQA$ (base angles of isosceles ΔPAQ)

$\angle QPB = \angle PAQ + \angle PQA$ (exterior angle of ΔPAQ)

$\therefore y° = 2\angle PAQ$

$\therefore \angle PAQ = \frac{y}{2}°$

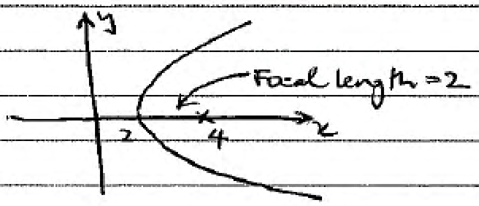
$\therefore x + x + \frac{y}{2} = 180$ (angle sum of ΔABC)

$\therefore 4x + y = 360$

$y = 360 - 4x$ A

A	161
B	15
C	3
D	4

5.



Equation is

$(y-0)^2 = 4 \times 2(x-2)$

$y^2 = 8(x-2)$ B

A	11
B	143
C	16
D	13

6. Centre of circle (A) is $(-3, 4)$

$O_x \to A_x : 0 \to -3$
$\therefore A_x \to B_x : -3 \to -6$

$O_y \to A_y : 0 \to 4$
$\therefore A_y \to B_y : 4 \to 8$

$\therefore$ B is $(-6, 8)$ C

A	3
B	6
C	173
D	1

7. Graph is $y = f''(x)$
For $f'(x)$ to have a maximum turning point, $f''(x) = 0$ and $f''(x)$ changes from positive to negative. $\therefore$ Required point is $x = p$. A

A	119
B	27
C	23
D	14

8. P (different colours)
$= 1 - P(WW) - P(RR)$
$= 1 - \frac{5}{10} \times \frac{4}{9} - \frac{4}{10} \times \frac{3}{9}$
$= 1 - \frac{32}{90}$
$= \frac{29}{45}$ C

A	0
B	9
C	169
D	5

9. $\frac{y}{x} = \frac{z}{y}$
$\therefore y^2 = xz$

$xyz = 64$
$\therefore y \cdot xz = 64$

$\therefore y^3 = 64$
$y = 4$

$x + y + z = 26$
$\therefore x + z = 26 - 4$
$= 22$ B

A	30
B	122
C	16
D	15

10. $81^{2x+3} = 243^{5-x}$

$\therefore 3^{4(2x+3)} = 3^{5(5-x)}$

$\therefore 8x + 12 = 25 - 5x$

$\therefore 13x = 13$
$x = 1$ C

A	1
B	9
C	173
D	0

2019 Mathematics Trial HSC Question 11 (15 marks)

(a) Express $\dfrac{3\pi^2}{4}$ correct to 3 sig. fig.

= 7.40

Criteria	Marks
7.40	**2**
7.402203301 or equivalent including 3 decimal places	**1**

(b) Rationalise the denominator $\dfrac{2+\sqrt{3}}{8-2\sqrt{3}}$

$$\frac{(2+\sqrt{3})}{(8-2\sqrt{3})}\times\frac{(8+2\sqrt{3})}{(8+2\sqrt{3})}$$

$$=\frac{16+4\sqrt{3}+8\sqrt{3}+6}{64-12}$$

$$=\frac{22+12\sqrt{3}}{52}$$

$$=\frac{11+6\sqrt{3}}{26}$$

Criteria	Marks
Provides the correct answer	**2**
Provides the answer but not fully simplified	**1.5**
Correct expansion with the multiplication of the conjugate	**1**

(c) Factorise $8x^3-125y^3$

$$=(2x)^3-(5y)^3$$

$$=(2x-5y)(4x^2+10xy+25y^2)$$

Criteria	Marks
Provides the factorisation	**2**
Attempt part of the cubic factorisation	**1**

(d) Differentiate

i) $\dfrac{1}{3x}$

$$\frac{d}{dx}(3x)^{-1}=-(3x)^{-2}(3)$$

$$=-\frac{3}{9x^2}$$

$$=-\frac{1}{3x^2}$$

Surprisingly not very well done. Candidates differentiate $3x^{-1}$ instead of $(3x)^{-1}$. Some differentiate $3x^{-1}$ and got to $-3x^{-2}$ which is $\dfrac{-3}{x^2}$ NOT equal to $\dfrac{-1}{3x^2}$.

Criteria	Marks
Provides the correct derivative with the correct working.	**1**

ii) $e^x \cos x$

$$\frac{dy}{dx} = e^x(-\sin x) + \cos x(e^x)$$
$$= e^x(\cos x - \sin x)$$

No penalties for not factorise the final solution.

Criteria	**Marks**
Provides correct derivative	**2**
Attempt to use the product rule, or equivalent merit.	**1**
Not applying product rule and only provides $-e^x \sin x$	**0.5**

iii) $\ln(\sin x)$

$$\frac{dy}{dx} = \frac{\cos x}{\sin x}$$
$$= \cot x$$

Criteria	**Marks**
Provides correct derivative	**2**
Recognise the log derivative form of $\frac{f'(x)}{f(x)}$ without simplifying the expression to $\cot x$	**1.5**
Recognise the log derivative form of $\frac{f'(x)}{f(x)}$ but incorrect numerator	**1**

iv) $\left(3x^2 - 5x\right)^6$

$$\frac{dy}{dx} = 6(3x^2 - 5x)^5(6x - 5)$$
$$= 6x(6x - 5)(3x - 5)^5$$

Deducted marks for not inserting brackets correctly.

Criteria	**Marks**
Provides correct derivative	**2**
Attempt to use the power rule/ chain rule, or equivalent merit.	**1**

(e) Find the limiting sum of $1 - \frac{1}{3} + \frac{1}{9} - \frac{1}{27} + \ldots$

$a = 1, \qquad r = -\frac{1}{3}$

$$S_\infty = \frac{a}{1-r}$$
$$= \frac{1}{1 - -\frac{1}{3}}$$
$$= \frac{3}{4}$$

Criteria	**Marks**
Provides correct solution	**2**
Correct value of the first term and the common ratio and correct substitution into the limiting sum formula	**1.5**
Correct substitution to the limiting sum formula.	**1**

Question 12

a) $f(x) = y = \ln(3x^2+1) + C$

$f(0) = \ln(0+1) + C = 0$

$\ln(1) = C = 0$

$f(x) = \ln(3x^2+1)$ ① Answer

Generally well done, some did not evaluate C.

b) $\lim\limits_{x \to 2} \dfrac{4x - x^2}{2-x}$

$= \lim\limits_{x \to 2} \dfrac{x\cancel{(2-x)}(2+x)}{\cancel{2-x}}$ ← $\frac{1}{2}$

$= 2(2+2)$

$= 8$ ← ①

Generally well done some attempted direct substitution. Some overcomplicated. some made arithmetic errors here!

c) $\int_0^{\pi/4} \cos 3x \, dx$

$= \left[\frac{1}{3}\sin 3x\right]_0^{\pi/4}$ ← ①

$= \frac{1}{3}\left(\sin \frac{3\pi}{4} - \sin 0\right)$

$= \frac{1}{3}\left(\frac{1}{\sqrt{2}}\right)$

$= \frac{1}{3\sqrt{2}}$ ← ① e.c.f. [error carried forward]

Reasonably well done occasional errors finding integral.
- Method was well understood.

d) $a = 4 \quad S_{20} = 650$

i) $S_n = \frac{n}{2}(2a + (n-1)d)$

$S_{20} = 20/2\,(2(4) + 19d) = 650$ ← (1)

$80 + 19d = 650$

$d = 3$

Very well done. A few substitution errors.

$T_n = a + (n-1)d$

$T_{20} = 4 + 19(3)$

$= 61$ ← (1)

Very well done. error carried forward(i)

e) $a = 0.5 \quad T_7 = 32$

i) $T_n = ar^{n-1}$

$T_7 = 0.5(r)^6 = 32$ ← (1)

$r^6 = 64$

$r = 2$ ← (1)

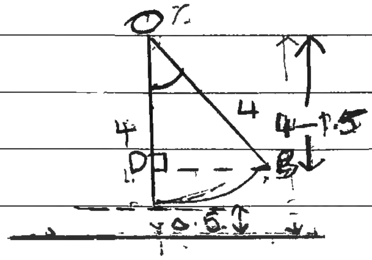
V.Well done. Some found $r = 1/2$ - question explicitly states "from smallest to tallest." But method well understood.

$S_n = \frac{a(r^n - 1)}{r-1}$

$S_7 = \frac{0.5(2^7 - 1)}{2-1}$ ← (1)

$= 63.5$ ← (1)

Well done. 2^6 was occasionally used.

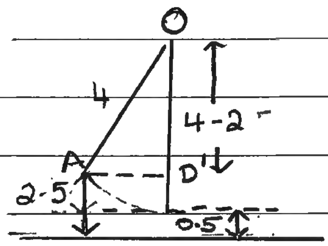


Reasonably poorly done. Many did not attempt. Method of solution sometimes not clearly understood or conveyed well. Some tried to use length of arc.

i) $OD = 4 - 1.5 = 2.5$

$\angle BOD = \cos^{-1}\left(\frac{2.5}{4}\right)$

$= 1^\circ \; ' \; 4.13''$ ①



Some attempted to rotate the swing so AB was horizontal

$OD' = 4 - 2 = 2$

$\angle AOD' = \cos^{-1}\left(\frac{2}{4}\right)$

$= 60^\circ$ ①

ii) $\angle AOB \doteqdot 60 + 51^\circ 19'$ (nearest minute)

$\approx 111^\circ 1'$

iii) $AB^2 = AO^2 + \cancel{OB^2} - 2(AO)(OB)\cos(AOB)$ ①

$= 4^2 + 4^2 - 2(4)(4)\cos 111^\circ 19'$

$= 43.633$ (43.6327)

$AB = 6.6056$ (6.6056)

≈ 6.6 (1 dp) ①

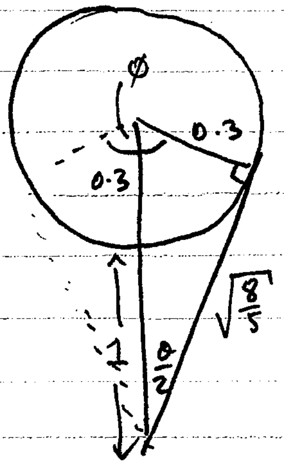
Error carried forward (e.c.f) from part (ii).

no sq rt $\frac{1}{2}$

$AB = \dfrac{4\sin(111^\circ 19')}{\sin\left(\frac{180 - 111^\circ 19'}{2}\right)}$

Formula only ①.

Question 13 (a)



Considering this right-angled triangle,

$$\sin\frac{\theta}{2} = \frac{0.3}{1.3}$$

$$\frac{\theta}{2} = \sin^{-1}\left(\frac{3}{13}\right)$$

$$= 0.2328681783 \text{ radians}$$

$$= 13^\circ 21'$$

$$\theta = 0.4657363565 \text{ radians}$$

$$\underline{= 26^\circ 41'}$$

$$\phi = 180^\circ - \theta = \pi - \theta$$

$$= 153^\circ 19' \quad = 2.675856297 \text{ rad}$$

$$\text{Area of kite} = 2 \times \text{Area of kite}$$

$$= 2 \times \frac{1}{2} \times 0.3 \times \sqrt{\frac{8}{5}}$$

$$\approx 0.3794733192$$

$$\text{Area of sector} = \frac{1}{2} r^2 \phi$$

$$= \frac{1}{2} \times 3^2 \times 2.6759$$

$$= 0.1204135334$$

$$\text{Area wet in front of tree} = \text{kite} - \text{sector}$$

$$= 0.2590597858$$

$$\text{Area wet away from tree} = \frac{1}{2} \times 3^2 \times (2\pi - \theta)$$

$$= 26.17852028$$

$$\text{Total area wet} = 26.179 + 0.259$$

$$\underline{= 26.43758006}$$

$$\approx 26 m^2$$

b) $J(v) = 0.04v^2 + 17500v^{-1} + 275$

$J'(v) = 0.08v - 17500v^{-2}$

$J'(80) = 0.08 \times 80 - 17500 \div 80^2$

$= 3.666$

$\therefore$ Since the rate of change is positive at $v=80$, an increase in v will increase J, and equivalently a decrease in v will decrease J.

$\therefore$ Driver should reduce speed.

Alternate Method.

Solving for $J'(v) = 0$

$0.08v = 17500v^{-2}$

$v^3 = \frac{17500}{0.08}$

$v = \sqrt[3]{\frac{17500}{0.08}} = 60.25$

$\therefore$ $v = 60.25$ km/hr represents a stationary pt.

Test by concavity

$J''(v) = 0.08 + 35000v^{-3}$

$J''(60.25) = 0.08 + 35000 \div 60.25^3$

$= \frac{6}{25} > 0$

$\therefore$ $v = 60.25$ represents a minimum as its concave up

$\therefore$ Driver should reduce speed to get closer to minimum of J.

Test by derivative

v	50	60.25	80
$J'(v)$	-3	0	3.666

$\therefore$ $v = 60.25$ is a minimum ~~as~~

$\therefore$ Driver should reduce speed to get closer to minimum of J.

c) i) Since the y-value of the line is greater than or equal to the y-value of the curve for all $1 \le x \le 4$, the area is found as

$$A = \int_1^4 (5-x) - \left(\frac{4}{x}\right) dx$$

$$= \left[5x - \frac{x^2}{2} - 4\ln(x) \right]_1^4$$

$$= \left(5\times4 - \frac{4^2}{2} - 4\ln(4)\right) - \left(5\times1 - \frac{1^2}{2} - 4\ln(1)\right)$$

$$= \frac{15}{2} - 4\ln(4) = \frac{15}{2} - 8\ln(2)$$

ii) $A \approx \frac{b-a}{6}\left[f(a) + 4f\left(\frac{a+b}{2}\right) + f(b)\right]$, where $f(x) = 5 - x - \frac{4}{x}$

$$= \frac{4-1}{6}\left[0 + 4\left(5 - 2.5 - \frac{4}{2.5}\right) + 0\right]$$

$$= \frac{9}{5}$$

iii) For $y = 5 - x$, gradient $= -1$.

For $y = \frac{4}{x}$,

$$\frac{dy}{dx} = \frac{-4}{x^2}$$

Solving $\frac{-4}{x^2} = -1$

$x^2 = 4$, $x = 2$ (since $x > 0$ in the 1st quadrant)

$y = \frac{4}{2} = 2$

$\therefore C = (2,2)$

iv) The distance from $(2,2)$ to the line $x + y - 5 = 0$ is

$$d = \frac{|2+2-5|}{\sqrt{1^2+1^2}} = \frac{1}{\sqrt{2}}$$

$$AB = \sqrt{(1-4)^2 + (4-1)^2} = 3\sqrt{2}$$

$$\text{Area}_{ABC} = \frac{1}{2} \times \frac{1}{\sqrt{2}} \times 3\sqrt{2}$$

$$= \frac{3}{2} \text{ units}^2$$

Question 13 - Marking Feedback

(a)

Essentially the mark breakdown is

① For $\frac{\theta}{2}$ or other relevant angle

① For area of large sector

① For area in front of tree.

- Students found this question very difficult. Many left it blank, or attempted to solve it using quadrants.
- Most successful attempts had a clear diagram copied onto their booklet, with points labelled.
- Many students took the second short side of the triangle to be 1 m, or the hypotenuse as 1 m, showing hasty reading of the question.
- Only a small proportion of students thought to include the area in front of the tree.
- The best solutions worked exclusively in radians.

(b) Overall this was done well.

- However a surprisingly large number of students used a trial-and error approach (eg found J(79), J(80) & J(81)) to determine whether to reduce or increase speed. This was awarded only ①, as it did not demonstrate advanced skills, nor account for the possibility of a turning point at, say 79.5 km/hr or 80.5 km/hr.

c) i) Generally done well.

- Some students don't know how to integrate $\frac{4}{x}$ though, with common result of $\frac{-4}{x^2}$ or $\frac{1}{4}\ln(x)$.

ii) Done poorly, considering the simplicity of the question.

- Many students attempted to evaluate as $\frac{1}{3}[f(1)+4f(2)+2f(3)+f(4)]$ or similar. This method resulted in zero marks, as it missed the point completely.
- Many transcription errors in this question, such as suddenly evaluating from $x=1$ to $x=5$.

iii) Overall was done well.

- Students who tried to reverse-engineer their solution and show that the line with $m=-1$ through $(2,2)$ is $y=-x+4$, did not get any marks unless they also showed it is tangent to $y=\frac{4}{x}$.
- Students who failed to show that $y=2$ when $x=2$ lost $\frac{1}{2}$. Although trivial, make sure to <u>show</u> everything you are asked to <u>show</u>.

iv) Done well.

- Some students incorrectly evaluated perpendicular distance as $\frac{|+2+2-5|}{\sqrt{2^2+2^2}}$
- Students used a variety of other methods successfully. Remember to use the quickest, simplest method in order to minimise risk of mistakes.

Question 14.

Comments:
some Ss
forgot − sign
or sin/cos = tan

a. i) $\frac{d}{dx}\left[\log_e(\cos x)\right] = \frac{-\sin x}{\cos x}$

$= -\tan x$

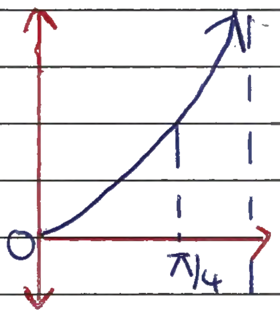
ii) $\frac{d}{dx}\left[\tan x - x\right] = \sec^2 x - 1$

$= \tan^2 x + 1 - 1$

$= \tan^2 x$

Generally well done.

iii)



$A = \int_0^{\pi/4} \tan x \, dx$

$= \left[-\log_e(\cos x)\right]_0^{\pi/4}$ from part i)

$= \left[-\log_e \cos\frac{\pi}{4} + \log_e \cos 0\right]$

$\doteqdot 0.3465735\ldots$

$= 0.35$ (2dp)

$= -\log_e \frac{1}{\sqrt{2}}$

Generally well done

iv) $V = \pi \int_0^{\pi/4} (\tan x)^2 dx$

$= \pi \int_0^{\pi/4} \tan^2 x \, dx$

$= \pi \left[\tan x - x\right]_0^{\pi/4}$

$$= \pi \left[\left(\tan \frac{\pi}{4} - \frac{\pi}{4} \right) - \left(\tan 0 - 0 \right) \right]$$

$$= \pi \left[1 - \frac{\pi}{4} \right]$$

$$= \frac{4\pi - \pi^2}{4}$$

$\boxed{\text{Many Ss thought } \pi\left(1-\frac{\pi}{4}\right) = \frac{3\pi^2}{4}}$

b.i) $\frac{d}{dx}\left(\underset{u}{x}\underset{v}{\sqrt{4-x^2}} \right)$

$u = x \quad v = (4-x^2)^{1/2}$

$u' = 1 \quad v' = \frac{1}{2}(4-x^2)^{-1/2} \cdot -2x$

$= -x(4-x^2)^{-1/2}$

$$= uv' + vu'$$

$$= -x^2(4-x^2)^{-1/2} + (4-x^2)^{1/2}$$

$$= \frac{-x^2}{\sqrt{4-x^2}} + \sqrt{4-x^2}$$

$$= \frac{-x^2 + \sqrt{4-x^2}\cdot\sqrt{4-x^2}}{\sqrt{4-x^2}}$$

$\boxed{\text{Generally well done}}$

$$= \frac{-x^2 + 4 - x^2}{\sqrt{4-x^2}}$$

$$= \frac{4-2x^2}{\sqrt{4-x^2}}$$

ii $x^2 + y^2 = 4$

$\therefore y^2 = 4 - x^2$

$y = \sqrt{4-x^2}$

$A = 2x \cdot 2y$

$= 4xy$

$\therefore A = 4x\sqrt{4-x^2}$

$$\frac{dA}{dx} = 4 \cdot \left(\frac{4-2x^2}{\sqrt{4-x^2}}\right)$$

let $\frac{dA}{dx} = 0$

$$0 = 4 \cdot \left(\frac{4-2x^2}{\sqrt{4-x^2}}\right)$$

$$0 = \frac{4-2x^2}{\sqrt{4-x^2}}$$

$$0 = 4 - 2x^2$$

$$2x^2 = 4$$

$$x^2 = 2$$

$$x = \pm\sqrt{2}$$

x	1	$\sqrt{2}$	1.5
dA/dx	1.15	0	-0.3
	/	—	\

$\therefore$ max

$$y = \sqrt{4-(\sqrt{2})^2}$$

$$= \sqrt{2} \qquad \therefore \quad (x,y) = (\sqrt{2}, \sqrt{2})$$

$$\therefore \text{ max area } = 2\sqrt{2} \cdot 2\sqrt{2}$$

$$= 8m^2$$

Many Ss did not prove x was a max st pt.

when using table to prove max point you must use values. stating +/- is not enough.

common error: not realising Area is $2x \cdot 2y$

c) $t=0$ $P=20000$
$t=10$ $P=35000$

$\underline{P = Ae^{kt}}$

$20000 = Ae^{0}$
$\therefore A = 20\,000$

$35000 = 20000e^{10k}$

$7/4 = e^{10k}$

$\ln 7/4 = 10k$

$k = \dfrac{\ln 7/4}{10}$

$\underline{50000} = \underline{20000e^{kt}}$

$5/2 = e^{kt}$

$\ln 5/2 = kt$

$t = \dfrac{\ln 5/2}{k}$

$= 16.37356829$

$\doteqdot 16.4$ years.

$\therefore 16.4 - 10$ years later

$\therefore 6.4$ years.

Generally well done

Some Ss didn't account for the 10 year difference.

Question 15

Solution	Marking Criteria	Marker's Comments
a) i) P(At least 1 WIN) = 1 – P(no "WIN") $=1-\frac{3}{4}\times\frac{5}{8}$ $=\frac{17}{32}$	**1 mark** for the correct answer.	Majority of the candidates did well in this question. Candidates whom made error in the question was due to considering 1 case (i.e. either 1 WIN or 2 WIN cards but not both.)
a) ii) P(not select a WIN card) = P(bag *B* and 2 no WIN cards in Bag *B*) + P(bag *A* and 2 no WIN cards in bag *A*) $=\frac{2}{6}\times\frac{5}{8}\times\frac{4}{7}+\frac{4}{6}\times\frac{6}{8}\times\frac{5}{7}$ $=\frac{10}{21}$	**1 mark** for the correct working out. **1 mark** for the correct answer.	Significant number of candidates did not do well in this question. Many candidates who lost marks in this question were due to not considering taking two cards from each bag, not considering Joshua rolling a die or simply not understanding what to do.
b) i) $x=t+\ln(3t+1)$ $\frac{dx}{dt}=1+\frac{3}{3t+1}$ $=\frac{3t+4}{3t+1}$ Let $\frac{dx}{dt}=0$ to find stationary point i.e. when the particle comes to rest. $\frac{3t+4}{3t-1}=0$ $\therefore 3t+4=0$ $t=-\frac{4}{3}$, but $t\geq 0$ as it a physical quantity $\therefore v=\frac{dx}{dt}\neq 0$ ∴ The particle never comes to rest.	**1 mark** for finding the correct $\frac{dx}{dt}$. **1 mark** for correct explanation of why particle never comes to rest.	Acceptable solution also included finding the correct $\frac{dx}{dt}$ and stating that it cant equal to 0 when $t\geq 0$. Significant number of candidates forgot to differentiate *t*, only differentiating $\ln(3t+1)$.

b) ii) Distance travelled in 3rd second = $x_{t=3} - x_{t=2}$ $= 3 + \ln(3\times 3+1) - 2 - \ln(3 \times 2 + 1)$ $= 1 + \ln(10) - \ln(7)$ $\approx 1.4cm$ (1 d.p.)	**1 mark** for the correct answer	This was done poorly by many candidates. The common errors made by candidates included: simply substituting $t = 3$ which gave the displacement at $t = 3$; looking at $t = 3$ to $t = 4$ which is the 4th second or making careless mistakes in simplifying the expression. Significant number of candidates did not read the question carefully, to leave their answer to 1 decimal place.
b) iii) $a = \dfrac{d(1+\frac{3}{3t+1})}{dt}$ $= \dfrac{-3\times 3}{(3t+1)^2}$ $= -\dfrac{9}{(3t+1)^2}$	**1 mark** for the correct answer	Done well by majority of the candidates.
b) iv) $t \ge 0$, $v > 0$ and $a < 0$ (As $(3t+1)^2 > 0$ and $-9 < 0$) $\therefore v \times a < 0$ $\therefore$ The particle is slowing down.	**1 mark** for correct answer WITH correct explanation.	This was done poorly by many candidates. Candidates got penalised for simply stating that the acceleration was negative, then the particle is slowing down. This is only true if the velocity is positive, if velocity is negative then the particle would be speeding up.

		Candidates must be explicit on their paper with this to gain full marks. Candidates who mentioned the particle was slowing down due to the velocity was getting smaller as $t \to \infty$ and showing the values were rewarded with full marks.
c) i) $f(x) = x^3 - 3x^2 - 9x + 22$ $f'(x) = 3x^2 - 6x - 9$ Let $f'(x) = 0$ to find stationary points $3(x^2 - 2x - 3) = 0$ $x^2 - 2x - 3 = 0$ $(x-3)(x+1) = 0$ $\therefore x = 3$ and $x = -1$ When $x = 3$, $y = 3^3 - 3(3)^2 - 9\times 3 + 22$ $= -5$ When $x = -1$, $y = (-1)^3 - 3(-1)^2 - 9\times(-1) + 22$ $= 27$ $\therefore$ Stationary points: (3, -5) and (-1, 27) Using 2nd derivative (or table) to find nature of the stationary points. $f''(x) = 6x - 6$ At $(3, -5)$ $f''(3) = 6(3) - 6$ $= 12$ > 0 (concave up) $\therefore$ (Local) Minimum turning point at (3, -5) At (-1, 27) $f''(-1) = 6(-1) - 6$ $= -12$ < 0 (concave down) $\therefore$ (Local) Maximum turning point at (-1, 27)	**1 mark** for finding the 2 x-values of the stationary points **1 mark each** for finding the nature of the stationary points with correct working out.	Done well by many candidates. Candidates got penalised if they show NO WORKING to justify the nature of the turning points. Candidates were not penalised but should be mindful of stating Minimum **turning** point and Maximum **turning** point, not just “min” and “max”.

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c) ii) From (i) ,

$f''(x) = 6x - 6$

Let $f''(x) = 0$ to find POSSIBLE point(s) of inflection.

$6x - 6 = 0$

$x = 1$

When $x = 1$, $y = 1^3 - 3(1)^2 - 9(1) + 22$

$= 11$

Test if $(1, 11)$ is a point of inflection.

<table>
<tr><td>x</td><td>0</td><td>1</td><td>2</td></tr>
<tr><td>$f''(x)$</td><td>-6</td><td>0</td><td>6</td></tr>
<tr><td></td><td>$\cap$</td><td></td><td>$\cup$</td></tr>
</table>

Since change of concavity of either side of (1, 11)

$\therefore$ Point of inflection = (1, 11)

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1 mark to find correct point of inflection.

1 mark testing if the possible point of inflection is the actual inflection point.

</td>
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Many candidates didn't test the possible point of inflection and lost 1 mark. Using the second derivative only gives possible point(s) of inflection and in some cases, they are not points of inflection.

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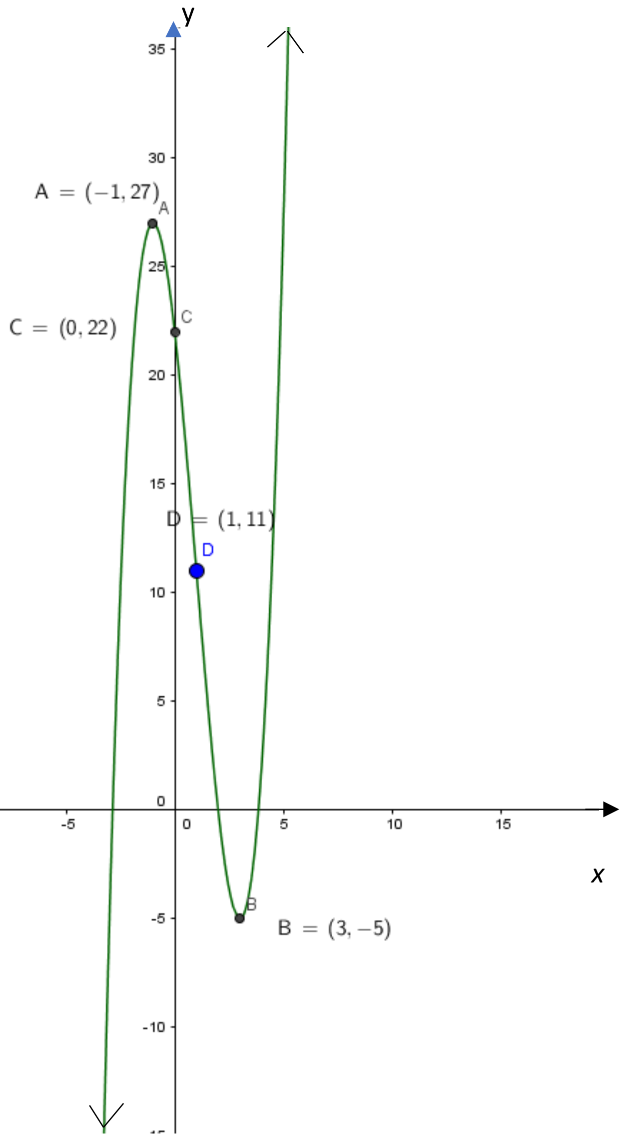
c) iii)



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1.5 mark for the correct graph including the y - intercept.

0.5 mark for labelling the two turning points, y – intercept and inflection point.

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Done well by majority of candidates.

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Question 16 SOLUTIONS

(a)	An amateur photographer can take a good photo 60% of the time. How many photos must he take to be 99% sure of taking at least one good photo?	**2**

Let n be the number of photos taken.

P(At least 1 good photo) = 1 – P(no good photos)

In n goes the probability of getting no good photos is 0.4^n.

To be 99% sure:

$$1-0.4^n = 0.99$$
$$\therefore 0.4^n = 0.01$$
$$\therefore n = \frac{\ln 0.01}{\ln 0.4}$$
$$\doteqdot 5.025883189...$$

So the photographer needs to take 6 photos.

Comment

Many students didn't see or understand the clue in the question i.e. "at least one …"

Some students were successful in getting to a value of n, but then rounded down instead of up.

Note that it was '99% sure' and not 'at least 99% sure' : some students caused errors unnecessarily by treating the problem as an inequality.

Question 16 SOLUTIONS (continued)

(b) A model rocket is launched from rest from the ground and travels vertically upwards. The rocket's acceleration for the first five seconds is given by

$$\frac{dv}{dt} = \frac{76}{5} - 5t \text{ m/s}^2.$$

(i) Find the velocity, v m/s, of the rocket after five seconds. **1**

Let the ground be $y = 0$.
$\therefore t = 0, y = 0, v = 0$

$$\frac{dv}{dt} = 15.2 - 5t \Rightarrow v = 15.2t - 2.5t^2 + C$$

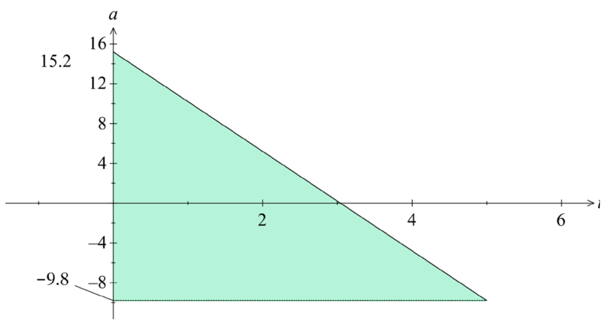
$$t = 0, v = 0 \Rightarrow v = 15.2t - 2.5t^2$$

$$t = 5 \Rightarrow v = 13.5 \text{ m/s}$$

The velocity is 13.5 m/s

Alternative

The area of the triangle gives the change in velocity i.e. $\frac{15.2 + 9.8}{2} \times 5 = 13.2$ m/s



As $t = 0$, $v = 0$, then $v = 13.2$ at $t = 5$
This is still "using calculus".

(ii) Find the height of the rocket after five seconds. **2**
Give your answer in metres, correct to two decimal places.

$$v = \frac{76}{5}t - \frac{5}{2}t^2 \Rightarrow y = \frac{38}{5}t^2 - \frac{5}{6}t^3 + K$$

$$t = 0, y = 0 \Rightarrow y = \frac{38}{5}t^2 - \frac{5}{6}t^3$$

$$t = 5 \Rightarrow y = \frac{515}{6} \text{ m}$$

The rocket reached a height of $85\frac{5}{6}$ m after 5 seconds.

Comment

Parts (i) and (ii) were generally well done by all students.
Constants of integration had to be shown or explained.

Many students are using $\int_0^5 a\,dt$ and $\int_0^5 v\,dt$ to get the answers to parts (i) and (ii).
These expressions represent the change in velocity and change in displacement over 5 seconds.
Fortunately $t = 0, y = 0, v = 0$.

(b)	(iii)	After five seconds, the rocket's acceleration is given by $$\frac{dv}{dt} = -9.8 \text{ m/s}^2.$$ By using calculus, find the maximum height reached by the rocket. Give your answer in metres, correct to two decimal places.	**2**

'Re-starting' time at this point i.e. $t = 0,\ y = \dfrac{515}{6},\ v = 13.5$ from parts (i) and (ii).

$$\frac{dv}{dt} = -9.8 \Rightarrow v = -9.8t + C_1$$
$$t = 0,\ v = 13.5 \Rightarrow v = -9.8t + 13.5$$

Now $v = -9.8t + 13.5 \Rightarrow x = -4.9t^2 + 13.5t + C_2$

$$t = 0,\ y = \frac{515}{6} \Rightarrow y = -4.9t^2 + 13.5t + \frac{515}{6}$$

Maximum height will occur when $v = 0$ i.e. $-9.8t + 13.5 = 0 \Rightarrow t = \dfrac{135}{98}$

$$\begin{aligned} y_{\max} &= -4.9 \times \left(\frac{135}{98}\right)^2 + 13.5 \times \frac{135}{98} + \frac{515}{6} \\ &\doteqdot 95.13180272... \\ &\doteqdot 95.13 \end{aligned}$$

The maximum height reached by the rocket was 95.13 m

Alternative approach on next page.

(b) (iii) **Alternative:** Not 're-starting' time.

$$\frac{dv}{dt} = -9.8 \Rightarrow v = -9.8t + C_1$$
$$t = 5,\ v = 13.5 \Rightarrow 13.5 = -9.8 \times 5 + C_1$$
$$\therefore C_1 = 62.5$$

$$\therefore v = -9.8t + 62.5$$
$$\therefore y = -4.9t^2 + 62.5t + C_2$$
$$t = 5,\ y = \frac{515}{6} \Rightarrow \frac{515}{6} = -4.9 \times 5^2 + 62.5 \times 5 + C_2$$
$$\therefore C_2 = -104\tfrac{1}{6}$$
$$\therefore y = -4.9t^2 + 62.5t - 104\tfrac{1}{6}$$

Maximum height will occur when $v = 0$ i.e. $-9.8t + 62.5 = 0 \Rightarrow t = \dfrac{625}{98}$ $\left(6\tfrac{37}{98}\right)$

$$y_{\max} = -4.9 \times \left(\frac{625}{98}\right)^2 + 62.5 \times \frac{625}{98} - 104\tfrac{1}{6}$$
$$\doteqdot 95.13180272...$$
$$\doteqdot 95.13$$

Comment

Not done well by most students or set out well. Failure to show constants was also penalised.

The biggest mistake made by students was not realising that the governing equation of motion had changed i.e. $a = -9.8$

Students who didn't use a calculus approach in a section could not gain any marks for the parts that section.

(b)	(iv)	Having reached its maximum height, the rocket falls directly to the ground, and as a result $$\frac{dv}{dt} = -9.8 \text{ m/s}^2.$$ By using calculus, find the time for which the rocket was in flight. Give your answer in seconds, correct to one decimal place.	**3**

The particle will fall i.e. starting from rest.
Let the point of maximum height be $y = 0$ and so the ground will be $y = -y_{\text{max}}$.

$\therefore t = 0, y = 0$ and $v = 0$

Let T be the time for the particle to drop from its maximum height
i.e. the time for $y = -y_{\text{max}}$.

Using $t = 0, y = 0$ and $v = 0$ and $\frac{dv}{dt} = -9.8$ then $v = -9.8t$ and $y = -4.9t^2$

How long until it reaches the ground?
Solve $-y_{\text{max}} = -4.9T^2$

$$\therefore T = \sqrt{\frac{y_{\text{max}}}{4.9}}$$
$$\doteqdot 4.406206261...$$
$$\doteqdot 4.4$$

Alternative

Students could just solve the equation from (b) (iii) for t (> 0)
i.e. $y = -4.9t^2 + 13.5t + 85\tfrac{5}{6} = 0$
and then add on 5 seconds.

So the rocket was in flight for $5 + \frac{135}{98} + T \doteqdot 10.8$
The particle has been in the air for approx. 10.8 s

Alternative approach on next page.

(b) (iv) **Alternative:** Not 're-starting' time.

From Alternative (b) (iii): $y = -4.9t^2 + 62.5t - 104\tfrac{1}{6}$

Solve $y = 0$

$$y = 0 \Rightarrow -4.9t^2 + 62.5t - 104\tfrac{1}{6} = 0$$
$$\therefore 4.9t^2 - 62.5t + 104\tfrac{1}{6} = 0$$

$$\therefore t = \frac{62.5 \pm \sqrt{62.5^2 - 4\times 4.9\times 104\frac{1}{6}}}{9.8}$$
$$\doteqdot 10.8 \qquad (t > 0)$$

The particle has been in the air for approx. 10.8 s

Comment

Not done well by most students or set out well. Failure to show constants was also penalised.

Students wasted time and ink by justifying the "maximum" nature of the problem. The particle goes up and stops and then falls down. This is a maximum distance.

As in the previous part, biggest mistake made by students was not realising that the governing equation of motion had changed i.e. $a = -9.8$

Students who didn't use a calculus approach in a section could not gain any marks for the parts that section.

(c) Rob and Janet borrow \$650 000 to buy an apartment.
The loan is over 25 years with interest being charged at 6 % p.a. compounding monthly.

(i) Determine the value of the monthly repayments. **2**

6% p.a. = 0.5 % per month

25 years = 300 months

Let $\$A_n$ = the amount owing after n months
Let $\$M$ = amount of the monthly repayment

$$A_1 = 650\,000 \times 1.005 - M$$

$$\begin{aligned} A_2 &= A_1 \times 1.005 - M \\ &= (650\,000 \times 1.005 - M) \times 1.005 - M \\ &= 650\,000 \times 1.005^2 - M(1 + 1.005) \end{aligned}$$

$$\begin{aligned} A_3 &= A_2 \times 1.005 - M \\ &= \left[650\,000 \times 1.005^2 - M(1 + 1.005)\right] \times 1.005 - M \\ &= 650\,000 \times 1.005^3 - M(1 + 1.005) \times 1.005 - M \\ &= 650\,000 \times 1.005^3 - M(1 + 1.005 + 1.005^2) \end{aligned}$$

$$\begin{aligned} \therefore A_n &= 650\,000 \times 1.005^n - M(\underbrace{1 + 1.005 + 1.005^2 + \ldots + 1.005^{n-1}}_{\text{GP: } a=1,\, r=1.005,\, n \text{ terms}}) \\ &= 650\,000 \times 1.005^n - M \times \frac{1 \times (1.005^{n-1} - 1)}{1.005 - 1} \\ &= 650\,000 \times 1.005^n - 200M(1.005^{300} - 1) \end{aligned}$$

Now $A_{300} = 0$ (the loan is paid off after 300 months)

$$\therefore 650\,000 \times 1.005^{300} - 200M(1.005^{300} - 1) = 0$$

$$\begin{aligned} \therefore M &= \frac{650\,000 \times 1.005^{300}}{200(1.005^{300} - 1)} \\ &\doteqdot 4187.95911 \\ &\doteqdot 4187.96 \end{aligned}$$

To the nearest dollar the monthly repayment is \$4188

Comment

Some students still don't 'remember' that they have to show derivation of the formula.
Just stating that $A_2 = 650\,000 \times 1.005^2 - M(1 + 1.005)$ or equivalent was not good enough.

Many mistakes were due to incorrect usage of formulae – these are in the Reference sheet!

(c)	(ii)	After 5 years the interest rate is increased to 7.2 % p.a. compounding monthly. How much longer (in years and months) will it take them to repay the loan if they keep the same repayments as before?	**3**

5 years = 60 months

How much do they owe after 5 years?

$$A_{60} = 650\,000 \times 1.005^{60} - 200 \times M \times (1.005^{60} - 1)$$
$$= 584\,558.5643...$$

7.2% p.a = 0.6% per month

Let $\$B_n$ = the amount owing after n months and let M be the same value as in (i)

Using the results from part (i):

$$B_n = A_{60} \times 1.006^n - M(\underbrace{1 + 1.006 + 1.006^2 + ... + 1.006^{n-1}}_{\text{GP: } a=1,\, r=1.005,\, n \text{ terms}})$$

$$= A_{60} \times 1.006^n - M \times \frac{1 \times (1.006^n - 1)}{1.006 - 1}$$

$$= A_{60} \times 1.006^n - \frac{500}{3} M(1.006^n - 1)$$

(c) (ii) (continued)

So when is $B_n = 0$?

$$A_{60} \times 1.006^n - \frac{500}{3}M(1.006^n - 1) = 0$$

$$\therefore 1.006^n\left(\frac{500}{3}M - A_{60}\right) = \frac{500}{3}M$$

$$\begin{aligned}\therefore 1.006^n &= \frac{\frac{500}{3}M}{\frac{500}{3}M - A_{60}} \\ &= \frac{500M}{500M - 3A_{60}} \\ &\doteqdot 6.153264153...\end{aligned}$$

$$\begin{aligned}\therefore n &= \frac{\ln(6.153264153...)}{\ln 1.006} \\ &\doteqdot 303.7380352\end{aligned}$$

$\therefore$ n = 304 months
$\therefore$ total time = 364 months
$\therefore$ extra time = 64 months = 5 years and 4 months.

Comment

Some students would have benefitted by using $u = 1.006^n$ and then solving for u. After all, this is about you!

Many students forgot to factor in the original 5 years or 60 months.

End of solutions