SYDNEY BOYS HIGH SCHOOL

NESA Number:

Name: ______________________

Class: ______________________

2020

YEAR 12
TERM 3
TRIAL HSC

Mathematics Advanced

General Instructions

- Reading time – 10 minutes
- Working time – 3 hours
- Write using black pen
- NESA approved calculators may be used
- A reference sheet is provided with this paper
- Marks may **NOT** be awarded for messy or badly arranged work
- For questions in Section II, show ALL relevant mathematical reasoning and/or calculations

Total Marks: 100

Section I – 10 marks (pages 2 – 6)

- Attempt Questions 1 – 10
- Allow about 15 minutes for this section

Section II – 90 marks (pages 7 – 32)

- Attempt all Questions in Section II
- Allow about 2 hours and 45 minutes for this section

Examiner: BK

Section I

10 marks
Attempt Questions 1–10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1–10

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1 What is the derivative of $x\cos x$ with respect to x ?

A. $-x\sin x$

B. $-\sin x$

C. $x\sin x - \cos x$

D. $-x\sin x + \cos x$

2 Evaluate $\int_1^2 e^{2x+1}\, dx$.

A. $\frac{1}{2}e^2$

B. $\frac{1}{2}e^3(e^2 - 1)$

C. e^2

D. $e^2 + 1$

3 Which of the following is a primitive function of $4 + \sqrt{x}$?

A. $4x + \frac{2\sqrt{x^3}}{3}$

B. $4x + \frac{3\sqrt{x^2}}{2}$

C. $4x + \frac{1}{2\sqrt{x}}$

D. $\frac{1}{2\sqrt{x}}$

4 A raffle consists of 20 tickets in which there are two prizes.
David buys 5 tickets.
First prize is two movie vouchers and second prize is one movie voucher.
What is the probability that David wins at least one movie voucher?

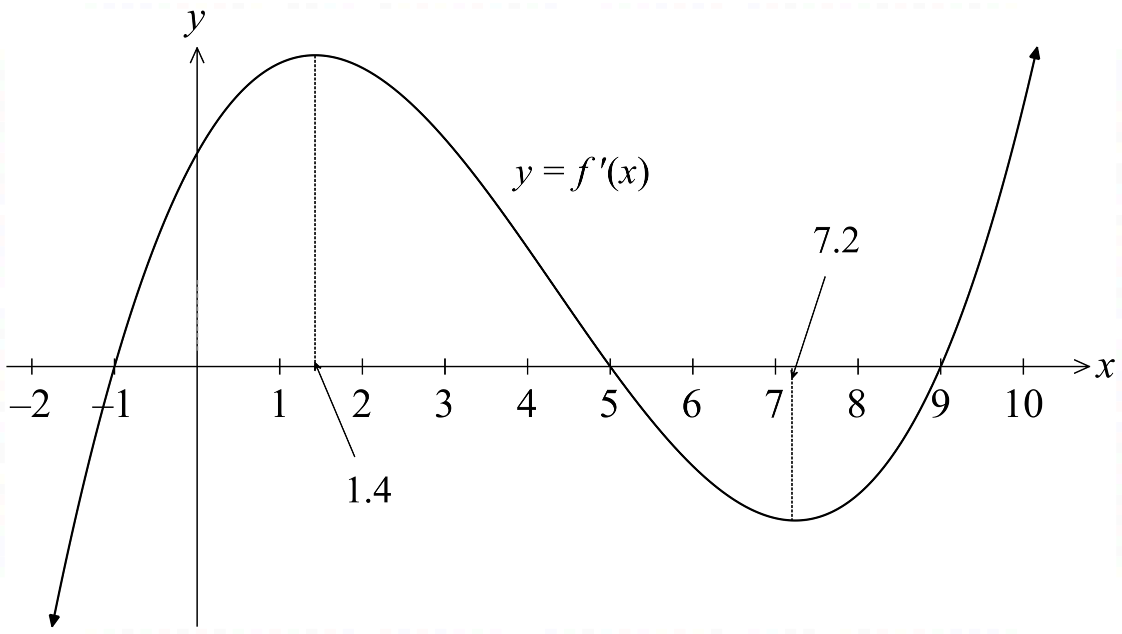
A. $\frac{5}{20}$

B. $\frac{27}{76}$

C. $\frac{7}{16}$

D. $\frac{17}{38}$

5 The graph of the derivative $y = f'(x)$ is drawn below.



At which of the following points is there a maximum turning point on $y = f(x)$?

A. $x = -1$

B. $x = 1.4$

C. $x = 5$

D. $x = 7.2$

6 What is the solution of $5^x = 4$?

A. $x = \dfrac{\log_e 4}{5}$

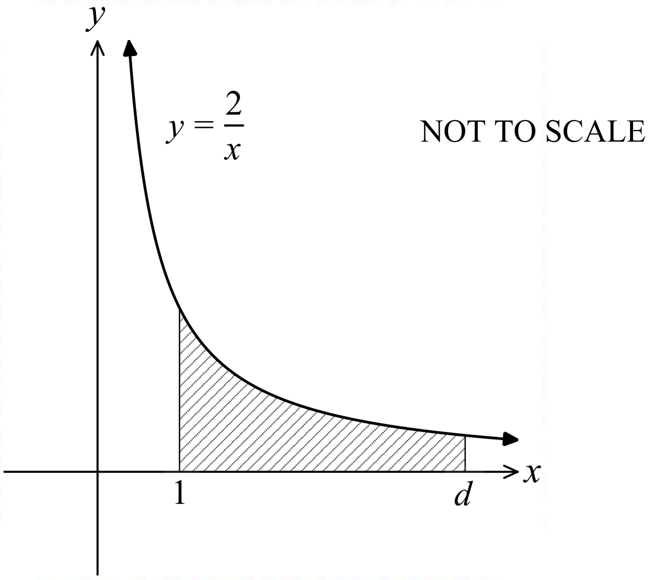
B. $x = \dfrac{4}{\log_e 5}$

C. $x = \dfrac{\log_e 4}{\log_e 5}$

D. $x = \log\left(\dfrac{4}{5}\right)$

7 The diagram shows the area under the curve $y = \frac{2}{x}$ from $x = 1$ to $x = d$.

What value of d makes the shaded area equal to 2?



A. e

B. $e+1$

C. $2e$

D. e^2

8 What is the domain of the function $f(x) = \frac{1}{\sqrt{4x^2 - 1}}$?

A. $\left(-\frac{1}{2}, \frac{1}{2}\right)$

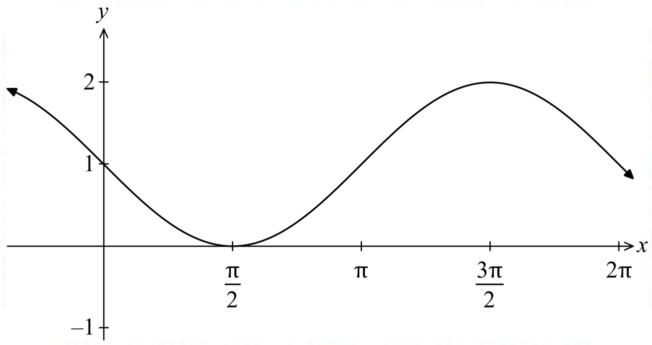
B. $\left(-\infty, -\frac{1}{2}\right) \cup \left(\frac{1}{2}, \infty\right)$

C. $\left(-\infty, -\frac{1}{2}\right] \cup \left(\frac{1}{2}, \infty\right)$

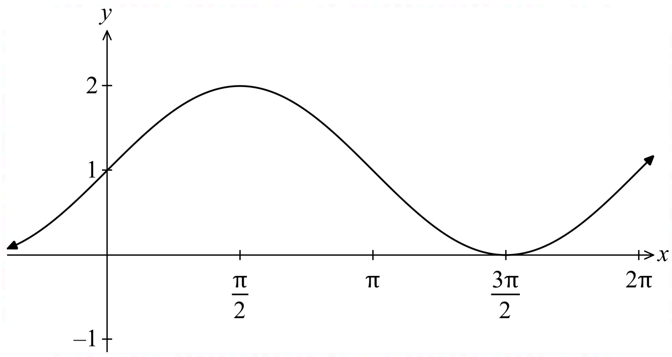
D. $\left[-\frac{1}{2}, \frac{1}{2}\right]$

9 Which one of the following best shows the graph of the function $y = 1 - \sin x$ for $[0, 2\pi]$?

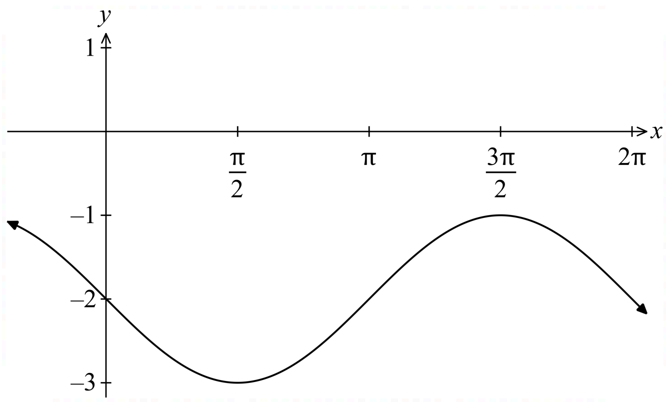
A.



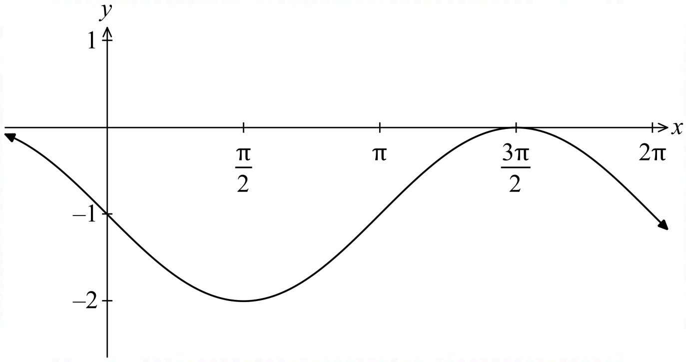
B.



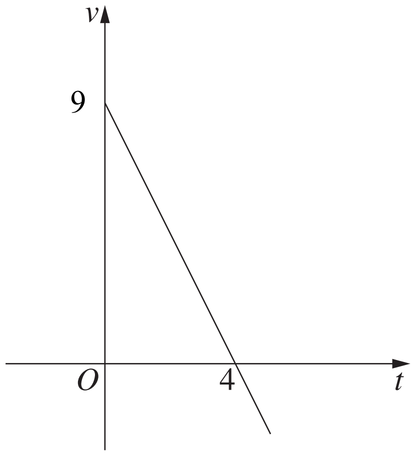
C.



D.



10 A particle is moving along the x-axis. The graph shows its velocity v metres per second at time t seconds.



When $t = 0$ the displacement x is equal to 3 metres.

What is the maximum value of the displacement x?

A. 9 m

B. 15 m

C. 18 m

D. 21 m

NESA Number										Q11-15	

YEAR 12 Mathematics Advanced

TERM 3 Cohort Task #3 (THSC)

Part A

Section II

Part A 14 marks
Attempt Questions 11–15

Answer each question in the space provided. A blank page is provided at the end of this question to allow rewriting of a part.

Your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (1 mark)

Factorise $8x^3 + 125$ **1**

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Question 12 (1 mark)

Express $260°$ as an exact radian value. **1**

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Question 13 (2 marks)

Solve $|2x-1| = 5$ **2**

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Question 14 (6 marks)

Differentiate the following with respect to x.

(a) $\dfrac{2}{\sqrt{x}}+\dfrac{1}{3x}$ **2**

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(b) $8xe^{x}$ **2**

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(c) $\log_e(4x^2+3)$ **2**

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Question 15 (4 marks)

Find the following:

(a) $\int 6x^2-7x+1\,dx$ **1**

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(b) $\int \left(4e^{2x}+\dfrac{3}{x}\right)dx$ **2**

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(c) $\int 6\cos 5x\,dx$ **1**

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Use this space to re-write any questions for Part A.

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End of Part A

NESA Number										Q16-19	

YEAR 12 Mathematics Advanced

TERM 3 Cohort Task #3 (THSC)

Part B

Section II

Part B **17 marks**
Attempt Questions 16–19

Answer each question in the space provided.

Your responses should include relevant mathematical reasoning and/or calculations.

Question 16 (1 mark)

Classify the function $y = \sin x$ as one-to-many, many-to-one, many-to-many or one-to-one. **1**

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Question 17 (5 marks)

Given that $f''(x) = 6x - 2$ and that there is a stationary point on $f(x)$ at $(1, 2)$, find

(a) $f(x)$ **3**

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(b) The co-ordinates of any point(s) of inflection. **2**

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Question 18 (7 marks)

(a) Differentiate $y = e^{\sin x}$ **1**

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(b) Show $y = e^{\sin x}$ has 2 stationary points for $0 \le x \le 2\pi$ and find the coordinates of these stationary points in simplest exact form. **2**

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(c) Find the nature of these stationary points. **2**

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(d) Sketch $y = e^{\sin x}$ for $0 \le x \le 2\pi$. **2**

Question 19 (4 marks)

Given $y = x\sqrt{x+1}$,

(a) Show $\dfrac{dy}{dx} = \dfrac{3x+2}{2\sqrt{x+1}}$ **2**

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(b) Hence, or otherwise, evaluate $\displaystyle\int_3^8 \frac{3x+2}{\sqrt{x+1}}\,dx$ **2**

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Use this space to re-write any questions for Part B.

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End of Part B

NESA Number										Q20-25	

YEAR 12 Mathematics Advanced

TERM 3 Cohort Task #3 (THSC)

Part C

Section II

Part C 16 marks
Attempt Questions 20–25

Answer each question in the space provided.

Your responses should include relevant mathematical reasoning and/or calculations.

Question 20 (1 mark)

Find $\int_{-4}^{4} x^3 \, dx$ **1**

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Question 21 (1 mark)

Given that $f(x) = 2x + 1$ and $g(x) = x^2 + 5$, find $f\big(g(-3)\big)$. **1**

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Question 22 (3 marks)

Find the value(s) of *k* for which the equation $y = (k+1)x^2 - (2+k)x + 3$ is positive definite. **3**

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Question 23 (4 marks)

Differentiate the following.

(a) $y = \log_e\left(\frac{x+4}{x-3}\right)$ **2**

(b) $y = 8\sin x \ln x$ **2**

Question 24 (3 marks)

Determine

(a) $\int 1+\tan^2\theta \, d\theta$ **1**

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(b) $\int \frac{8x+10}{2x^2+5x} \, dx$ **2**

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Question 25 (4 marks)

The velocity, v metres per second, of a body t seconds after starting from rest, is given by $v = 3t - t^2$.

(a) Find how far the body has travelled when it next comes to rest. **2**

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(b) Show that the acceleration of the body at this time is negative. **2**
Interpret your answer.

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End of Part C

NESA Number										Q26-29	

YEAR 12 Mathematics Advanced

TERM 3 Cohort Task #3 (THSC)

Part D

Section II

Part D **15 marks**
Attempt Questions 26–29

Answer each question in the space provided.

Your responses should include relevant mathematical reasoning and/or calculations.

Question 26 (5 marks)

For the curve $y = \dfrac{\sin x}{1 + \cos x}$

(a) Show $\left(\dfrac{\pi}{3}, \dfrac{\sqrt{3}}{3}\right)$ lies on the curve. **1**

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(b) Show $\dfrac{dy}{dx} = \dfrac{1}{1 + \cos x}$ **2**

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(c) Find the equation of the tangent to the curve at $x = \dfrac{\pi}{3}$ **2**

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Question 27 (3 marks)

If the sum of the first 10 terms of the series $\log_2\left(\frac{1}{x}\right)+\log_2\left(\frac{1}{x^2}\right)+\log_2\left(\frac{1}{x^3}\right)+\ldots$ is -110, **3**

find the value of x.

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Question 28 (4 marks)

The probability that a certain girl wakes up late in the morning is 0.6.
When she wakes up late, the probability that she is late to work is 0.85.
When she wakes up on time, the probability that she is late to work is 0.4.

(a) What is the probability that she is late to work on any morning? **2**

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(b) Upon seeing the girl arriving late, her boss asks her if she overslept.
What is the probability that the truthful answer is 'yes'? **2**

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Question 29 (3 marks)

The rate at which liquid is flowing into a vessel after t minutes is given by $\frac{dV}{dt} = \frac{1}{t+1}$. **3**

If the volume of liquid in the vessel is $(\log_e 2)$ cubic metres after 3 minutes, what is the volume after 8 minutes? Answer correct to 3 significant figures.

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Use this space to re-write any questions for Part D.

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End of Part D

NESA Number										Q30-33	

YEAR 12 Mathematics Advanced

TERM 3 Cohort Task #3 (THSC)

Part E

Section II

Part E **12 marks**
Attempt Questions 30–33

Answer each question in the space provided.

Your responses should include relevant mathematical reasoning and/or calculations.

Question 30 (2 marks)

Find the area bounded by the curve $y=(x+2)^3$ and the x-axis between $x=-3$ and $x=2$. **2**

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Question 31 (4 marks)

(a) Sketch the curve of $y=\log_e(x+1)$ including intercept(s) and asymptotes. **1**

(b) Using the trapezoidal rule with 4 sub-intervals, estimate $\int_1^2 \ln(x+1)\,dx$, correct to 3 decimal places. **2**

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(c) Would the trapezoidal rule used in this instance provide a value that is greater than or less than $\int_1^2 \ln(x+1)\,dx$? Explain your answer. **1**

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Question 32 (5 marks)

The population of a town can be estimated by using the formula $P = P_0 e^{kt}$, where t is the time in years after the year 2020 and P_0 and k are constants.

The population at the beginning of the year 2020 was 300, and at the beginning of 2024 it was 450.

(a) Find the annual growth rate, k, of the population correct to 3 significant figures. **2**

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(b) In what year will the town's population be double that of the year 2020? **3**

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Question 33 (3 marks)

The water height in an estuary fluctuates according to the tide. Flooding will occur when the water height reaches 1.5 m. One day the high tide of 1.6 m occurred at 3:00 am and the following low tide at 9:30 am. Assume that the following high tide was also 1.6 m and the height of the tide is modelled by the equation **3**

$$h = 1 + 0.6\sin\left(\frac{2\pi}{13}(t + 0.25)\right),$$

where h is the height in metres and t is the time in hours after midnight.
Find the time when the next flooding would be expected to occur, to the nearest minute.

Use this space to re-write any questions for Part E.

End of Part E

NESA Number										Q34-36	

YEAR 12 Mathematics Advanced

TERM 3 Cohort Task #3 (THSC)

Part F

Section II

Part F **16 marks**
Attempt Questions 34–36

Answer each question in the space provided. Blank pages are provided at the end of this question to allow rewriting of a part.

Your responses should include relevant mathematical reasoning and/or calculations.

Question 34 (6 marks)

Michael borrows \$650 000 to buy a unit at 6% per annum with interest compounded monthly.
The loan is to be repaid with equal monthly repayments of \$4200.
Let $\$A_n$ be the amount owing after n months.

(a) Show that $A_n = 840\,000 - 190\,000 \times 1.005^n$ **3**

Question 34 continues on page 28

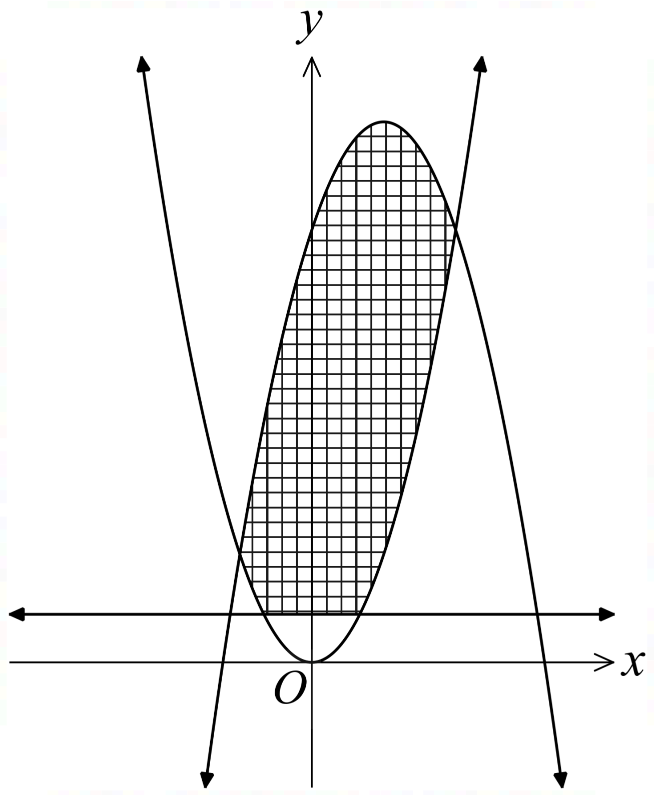
Question 34 (continued)

(b) After 15 years, the amount owing is $373 722 to the nearest dollar. **3**

At this time Michael borrows a further $200 000 to build an extension.
He adds this onto his previous mortgage. If the monthly repayments remain the same, what is the minimum remaining number of months it will take him to pay off the balance of the loan?

Question 35 (4 marks)

Find the shaded area enclosed by the curves $y = x^2$ and $y = 9 + 3x - x^2$ and the line $y = 1$. **4**



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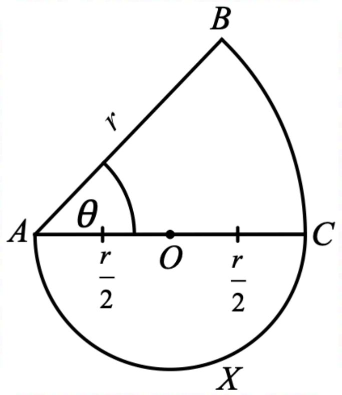
Question 36 (6 marks)

A rotating camera is formed with a cross-section as shown in the diagram.



The cross-section consists of a semi-circle AXC with centre O and radius $\frac{r}{2}$,

and a sector ABC of radius r, centre A and angle θ.

(a) Show that the perimeter, P, of the cross-section $ABCX$ is given by $P = r\left(\frac{\pi}{2} + \theta + 1\right)$. **1**

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(b) If the area of cross-section $ABCX$ is 1 square unit, show that $P = \frac{2}{r} + r\left(1 + \frac{\pi}{4}\right)$. **2**

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Question 36 continues on page 32

(c) Show that the least perimeter occurs when $r^2 = \dfrac{8}{\pi + 4}$ and calculate θ. **3**

Write the size of this angle to the nearest degree.

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End of paper

Use this space to re-write any questions for Part F

Use this space to re-write any questions for Part F.

End of Part F

SYDNEY BOYS HIGH SCHOOL

2020	YEAR 12 TERM 3 TRIAL HSC - ASSESSMENT TASK 3

Mathematics Advanced

Sample Solutions

Quick MC Answers

1	D
2	B
3	A
4	D
5	C
6	C
7	A
8	B
9	A
10	D

NOTE: Before putting in an appeal re marking, first consider that the mark is not linked to the amount of ink you have used.

Just because you have shown 'working' does not justify that your solution is worth any marks.

2020 Y12 Adv THSC Multiple choice solutions

Mean (out of 10): 8.86

1. $\frac{d}{dx}(x\cos x)$

$= \cos x . 1 + x.(-\sin x)$

$= \cos x - x\sin x$ (D)

A	4
B	0
C	1
D	175

2. $\int_1^2 e^{2x+1}\,dx$

$= \frac{1}{2}\left[e^{2x+1}\right]_1^2$

$= \frac{1}{2}\{e^5 - e^3\}$

$= \frac{1}{2}e^3(e^2-1)$ (B)

A	6
B	173
C	1
D	0

3. $\int (4+\sqrt{x})\,dx$

$= \int (4 + x^{\frac{1}{2}})\,dx$

$= 4x + \frac{2}{3}x^{3/2} + c$

$= 4x + \frac{2\sqrt{x^3}}{3} + c$ (A)

A	170
B	3
C	6
D	1

4. P(at least 1 voucher)

$= 1 - P(\text{no vouchers})$

$= 1 - \frac{15}{20} \times \frac{14}{19}$

$= 1 - \frac{21}{38}$

$= \frac{17}{38}$ D

A	9
B	17
C	9
D	143

5. Max turning pt $\Rightarrow f'(x) = 0$

$f'(x^-) > 0$

$f'(x^+) < 0$

$\Rightarrow x = 5$ (C)

A	9
B	23
C	142
D	5

6. $5^x = 4$

$\therefore \ln(5^x) = \ln 4$

$\therefore x\ln 5 = \ln 4$

$\therefore x = \frac{\ln 4}{\ln 5}$ (C)

A	2
B	2
C	175
D	1

7. $\int_1^d \frac{2}{x}\,dx$

$= 2\left[\ln x\right]_1^d$

$= 2[\ln d - \ln 1]$

$= 2\ln d$

$= 2$

$\therefore \ln d = 1$

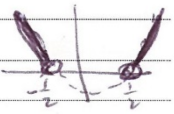
$\therefore d = e$

(A)

A	165
B	4
C	2
D	8

8. $4x^2 - 1 > 0$

$\therefore (2x-1)(2x+1) > 0$

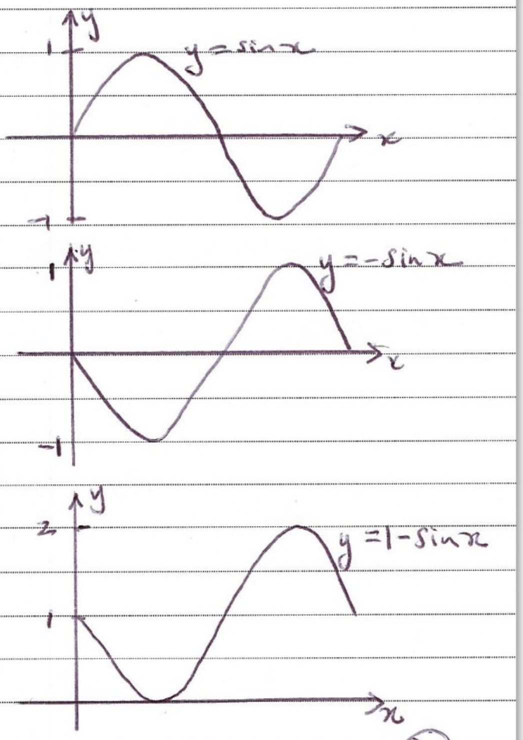


$\therefore x < -\frac{1}{2}$ OR $x > \frac{1}{2}$

(B)

A	10
B	158
C	9
D	2

9.



(A)

A	170
B	3
C	1
D	6

10. $v = -\frac{9}{4}t + 9$

$x = -\frac{9}{8}t^2 + 9t + c$

When $t = 0$

$3 = 0 + 0 + c$

$\therefore c = 3$

$\therefore x = -\frac{9}{8}t^2 + 9t + 3$

Max displacement when $\frac{dx}{dt} = 0$

$\therefore \frac{9}{4}t = 9$

$t = 4$ OR Read off graph

$\frac{d^2x}{dt^2} = -\frac{9}{4} < 0$

$\therefore$ Max displacement

$= -\frac{9}{8} \times 4^2 + 9 \times 4 + 3$

$= -18 + 36 + 3$

$= 21$ (D)

NOTE: There is no maximum distance from the origin.

As $t \to \infty$, $x \to -\infty$

A	2
B	12
C	41
D	124

Section II

Part A 14 marks
Attempt Questions 11–15

Answer each question in the space provided. A blank page is provided at the end of this question to allow rewriting of a part.

Your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (1 mark)

Factorise $8x^3 + 125$ **1**

$(2x)^3 + 5^3$

$= (2x+5)(4x^2 - 10x + 25)$ ✓ (1)

Most students knew the formula but some thought it was -2ab for the quadratic term.

Question 12 (1 mark)

Express $260°$ as an exact radian value. **1**

$260 \times \frac{\pi}{180} = \frac{13\pi}{9}$ ✓ (1)

Done well.

Question 13 (2 marks)

Solve $|2x-1| = 5$ **2**

$2x - 1 = 5$ or $2x - 1 = -5$

$2x = 6$ ✓ $2x = -4$

$x = 3$ or $x = -2$ ✓ (2)

Done well.

Question 14 (6 marks)

Differentiate the following with respect to x.

(a) $\frac{2}{\sqrt{x}}+\frac{1}{3x}$ $= 2x^{-1/2}+\frac{1}{3}x^{-1} \Rightarrow y' = -1x^{-3/2} - \frac{1}{3}x^{-2}$ **2**

or $y' = \frac{-1}{x^{3/2}} - \frac{1}{3x^2}$ (2)

Lots of errors here. Some thought x^3/2 was the cube root of x squared so they need to learn their index laws. The other very common error was to write the second term as (3x^-2)^-1 but then forget to use the chain rule.

(b) $8xe^x$ $= u.v$ **2**

$y' = uv' + vu'$

$= 8x.e^x + e^x.8$ (2)

$= 8e^x(x+1)$

Done very well.

(c) $\log_e(4x^2+3)$ **2**

$y' = \frac{8x}{4x^2+3}$ (2)

Done very well.

Question 15 (4 marks)

Find the following:

(a) $\int 6x^2-7x+1\,dx$ **1**

$+C = \frac{1}{2}m$

I/2 mark for the constant of integration in this question. It was not penalised in the following two parts.

$= 2x^3 - \frac{7x^2}{2} + x + C$ (1)

(b) $\int \left(4e^{2x}+\frac{3}{x}\right) dx$ **2**

$= \frac{4e^{2x}}{2} + 3\ln|x| + C$ (2)

$= 2e^{2x} + 3\ln|x| + C$

(c) $\int 6\cos 5x\,dx$ **1**

$= \frac{6\sin 5x}{5} + C$ (1)

Some students forgot to divide by the derivative of 5x.

Question 16 1 mark

Classify the function $y = \sin x$ as one to many, many to one, many to many or one to one.

Solution	Comments
Many to one.	No half marks. Students generally performed poorly in this question.

Question 17 5 marks

Given that $f''(x) = 6x - 2$ and that there is a stationary point on $f(x)$ at $(1, 2)$, find:

A. $f(x)$ 3

Solution	Comments
Integrate $f''(x)$ with respect to x to obtain $f'(x)$: $f'(x) = \int 6x - 2\, dx = \frac{6x^2}{2} - 2x + c = 3x^2 - 2x + c$ A stationary point at $(1, 2)$ implies $f'(1) = 0$. $\therefore 0 = 3 - 2 + c$ $c = -1$ $\therefore f'(x) = 3x^2 - 2x - 1$ Integrate $f'(x)$ with respect to x to obtain $f(x)$: $f(x) = \int 3x^2 - 2x - 1\, dx = \frac{3x^3}{3} - \frac{2x^2}{2} - x + c = x^3 - x^2 - x + c$ A stationary point at $(1, 2)$ implies $f(1) = 2$. $\therefore 2 = 1 - 1 - 1 + c$ $c = 3$ $\therefore f(x) = x^3 - x^2 - x + 3$	Common error(s): • Substituting $f'(1) = 2$. Students should take more care in their mental arithmetic and/or calculator work, as many failed to correctly evaluate the constants of integration.

B. The coordinates of any point(s) of inflection. 2

Solution	Comments
Inflection point implies $f''(x) = 0$. $\therefore 0 = 6x - 2$ $x = \frac{2}{6} = \frac{1}{3}$ $f\left(\frac{1}{3}\right) = \left(\frac{1}{3}\right)^3 - \left(\frac{1}{3}\right)^2 - \frac{1}{3} + 3 = \frac{70}{27}$ x: 0, $\frac{1}{3}$, 1 $f''(x)$: -2, 0, 4 The change in sign of $f''(x)$ shows that $\left(\frac{1}{3}, \frac{70}{27}\right)$ is an inflection point.	Common error(s): • Not showing the change in concavity through use of a table of x and $f''(x)$, a sketch of $y = f''(x)$ etc. • Not providing numerical values for $f''(x)$ when using a table to illustrate the change in concavity. • Not simplifying $\frac{2}{6}$.

2020 SBHS Mathematics Advanced Trial HSC: Section II, Part B

Question 18 — 7 marks

A. Differentiate $y = e^{\sin x}$. — 1

Solution	Comments
$\frac{d}{dx}e^{f(x)} = f'(x)e^{f(x)}$ $\therefore y' = \cos x\, e^{\sin x}$	Students generally performed well in this question.

B. Show that $y = e^{\sin x}$ has 2 stationary points for $0 \leq x \leq 2\pi$ and find the coordinates of these stationary points in simplest exact form. — 2

Solution	Comments
Stationary point implies $y' = 0$. $0 = \cos x\, e^{\sin x}$ $\therefore \cos x = 0, e^{\sin x} = 0$ But $e^{\sin x} > 0$, so there is no solution to $e^{\sin x} = 0$. For $\cos x = 0$, where $0 \leq x \leq 2\pi$: $x = \cos^{-1} 0$ $= \frac{\pi}{2}, \frac{3\pi}{2}$ Substitute $x = \frac{\pi}{2}$: $y = e^{\sin\frac{\pi}{2}} = e$ Substitute $x = \frac{3\pi}{2}$: $y = e^{\sin\frac{3\pi}{2}} = \frac{1}{e}$ Hence, stationary points are $\left(\frac{\pi}{2}, e\right)$ and $\left(\frac{3\pi}{2}, \frac{1}{e}\right)$.	Students who didn't express their coordinates in simplest exact form could not score full marks. Most students risked being penalised for not explicitly stating that $e^{\sin x} = 0$ has no solution.

C. Find the nature of these stationary points. — 2

Solution:

x	0	$\frac{\pi}{2}$	π
y'	1	0	-1

The change in sign of y' shows that $\left(\frac{\pi}{2}, e\right)$ is a local maximum.

x	π	$\frac{3\pi}{2}$	2π
y'	-1	0	1

The change in sign of y' shows that $\left(\frac{3\pi}{2}, \frac{1}{e}\right)$ is a local minimum.

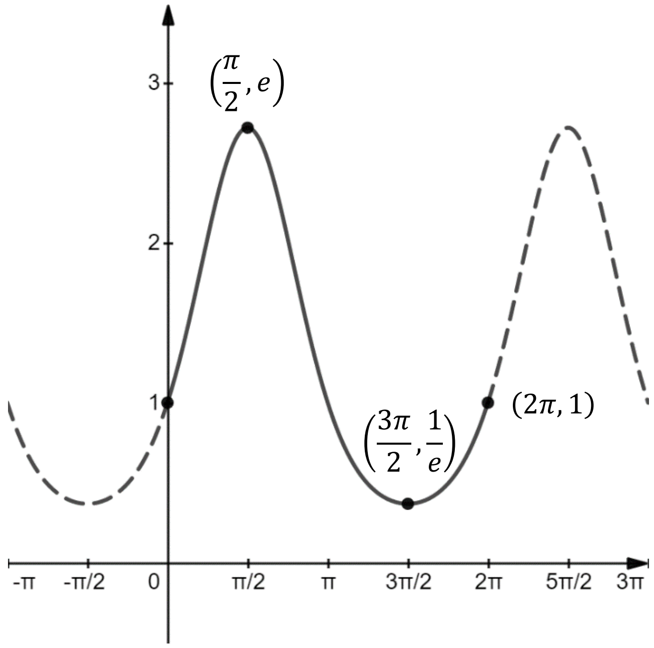
Comments:

Alternate solution:

- Find the second derivative, then substitute $x = \frac{\pi}{2}$ and $x = \frac{3\pi}{2}$ to show that y'' evaluates to a negative and positive value respectively.

Students who chose to use a table of x and y' to find the nature of the stationary points could not score full marks if they failed to provide a numerical value for y'.

D. Sketch $y = e^{\sin x}$ for $0 \le x \le 2\pi$. 2

Solution	Comments
	Graphs drawn in pencil will NOT be remarked. Many students risked being penalised by not stating coordinates of the stationary points and endpoints, while also not using dashed lines to indicate the coordinates. Some students mistook e^{-1} to be a negative value and lost 0.5 marks as a result. Students who made errors in Parts B and/or C must correctly incorporate these errors in their graphs. In some cases, errors from Parts B and/or C made sketching impossible and students were unable to score full marks as a result.

Question 19 4 marks

Given $y = x\sqrt{x+1}$:

A. Show that $\frac{dy}{dx} = \frac{3x+2}{2\sqrt{x+1}}$. 2

Solution	Comments
By the product rule: $\frac{dy}{dx} = \left(x \times \frac{1}{2}(x+1)^{-\frac{1}{2}}\right) + \left(\sqrt{x+1} \times 1\right)$ $= \frac{x}{2\sqrt{x+1}} + \sqrt{x+1}$ $= \frac{x + 2(x+1)}{2\sqrt{x+1}}$ $= \frac{3x+2}{2\sqrt{x+1}}$	Students generally performed well in this part of the question.

B. Hence or otherwise, evaluate $\int_3^8 \frac{3x+2}{\sqrt{x+1}}\,dx$. 2

Solution	Comments
$\int_3^8 \frac{3x+2}{\sqrt{x+1}}dx = 2\int_3^8 \frac{3x+2}{2\sqrt{x+1}}dx$ $= 2\left[x\sqrt{x+1}\right]_3^8$ $= 2\left(8\sqrt{9} - 3\sqrt{4}\right)$ $= 36$	Common errors: • Dividing the integral by 2 instead of multiplying. Students who evaluated the integral incorrectly and provided insufficient working could only score a maximum of 0.5 marks.

Part C

Question 20

$\int_{-4}^{4} x^3 \, dx = 0$, as $f(x) = x^3$ is an odd function integrated between 4 and –4.

1 mark for correct answer, regardless of whether the student stated that $f(x)$ is an odd function.

OR

$\int_{-4}^{4} x^3 \, dx = \left[\frac{x^4}{4}\right]_{-4}^{4}$ $\left[\frac{1}{2} \text{ mark for getting to this stage, though with incorrect anwser}\right]$

$= 0$ $\left[1 \text{ mark for correct answer}\right]$

The majority of students found the answer through direct integration, a few students used the fact that $f(x)$ is an odd function.

Question 21

$g(-3) = (-3)^2 + 5 = 14$ ½ mark

$f\big(g(-3)\big) = 2(14) + 1 = 29$ ½ mark

Most students got this correct.

Question 22

y is a positive definite quadratic function if $\Delta < 0$ and $a > 0$.

$$\Delta = b^2 - 4ac$$
$$= (2+k)^2 - 12(k+1) \qquad \text{1 mark}$$
$$< 0$$

$$k^2 + 4k - 12k - 12 < 0$$

1 mark

$$k^2 - 8k - 12 < 0$$

Note: $k^2 - 8k - 8 = 0 \Rightarrow k = \dfrac{8 \pm \sqrt{64+32}}{2} = 4 \pm 2\sqrt{6}$

$$\therefore k^2 - 8k - 8 < 0 \Rightarrow 4 - 2\sqrt{6} < k < 4 + 2\sqrt{6} \qquad \text{1 mark}$$

Note: $a > 0$ means that $k + 1 > 0$ i.e. $k > -1$

However, $4 - 2\sqrt{6} < k < 4 + 2\sqrt{6} \cap k > -1 = 4 - 2\sqrt{6} < k < 4 + 2\sqrt{6}$

A minority of students were awarded full marks for this question.

Common errors were:

- Saying that the discriminant > 0 is a condition for positive definite
- Incorrectly solving the inequality

No marks were deducted for not stating the condition $k > -1$, as this condition is already satisfied by the solution.

If students used discriminant > 0 but made no other mistakes, half a mark was deducted.

Question 23

a)

$$\log_e\left(\frac{x+4}{x-3}\right)=\log_e(x+4)-(x-3)$$ 1 mark

$$\therefore\frac{dy}{dx}=\frac{1}{x+4}-\frac{1}{x-3}$$ 1 mark

The majority of students got full marks.

Some only got to the first line.

Alternatively - using chain rule:

$$\log_e\left(\frac{x+4}{x-3}\right)$$

$$u=\left(\frac{x+4}{x-3}\right)$$

$$\frac{du}{dx}=\frac{x-3-(x+4)}{(x-3)^2}$$

1 mark

$$\frac{d}{dx}\left[\log_e u\right]$$

$$=\frac{1}{\left(\frac{x+4}{x-3}\right)}.\left[\frac{-7}{(x-3)^2}\right]$$

$$=\frac{-7}{(x+4)(x-3)}$$

1 mark

b)

$u=8\sin x \qquad v=\ln x$

$u'=8\cos x \qquad v'=\frac{1}{x}$

½ a mark for each of u' and v'.

$$\frac{d}{dx}(uv)=uv'+vu'$$

$$=\frac{8\sin x}{x}+8\cos x\ln x$$

1 mark for correct answer

Most students got this question correct or partially correct.

If students got u' or v' .wrong, no marks were deducted for error carried forward, if the product rule was used correctly.

Common errors:

- Differentiating sin*x* to sin*x*.
- Many students attempted to simplify their answer.
  No marks were deducted if they made errors in doing so.

Question 24

$$\int 1+\tan^2\theta \, d\theta = \int \sec^2\theta \, d\theta \qquad \left[\tfrac{1}{2}\right]$$

$$= \tan\theta + C \qquad \left[\tfrac{1}{2}\right]$$

No marks were deducted if the constant term C was not added.

Question 25

a) Body is at rest at $v = 0$

$0 = 3t - t^2$

$0 = t(3-t)$

$t = 3$

½ mark for $t = 3$ i.e. after 3 seconds (No marks deducted for absence of units)

$x = \int v dt = \int 3t - t^2 dt$

$x = \frac{3t^2}{2} - \frac{t^3}{3} + C$

½ mark for correct $x(t)$

$0 = 0 + C, C = 0$

$\therefore x(3) = 4.5$

The body travelled 4.5 m

1 mark for correct answer

Many students found the time $t = 3$, though did not find the displacement x.

b) $a = \frac{dv}{dt} = 3 - 2t$

1 mark for getting to this line and then substituting $t = 3$ and getting $a = -3$

No marks were deducted for trivial calculation errors.

<u>‘Interpret your answer’</u>

The body has slowed down, coming to rest. It then changes direction and speeds up.

Few students got full marks for this question. Many students just mentioned that a was negative but did not go further.

Students should not use the word ‘accelerating’ as a synonym for ‘speeding up’. Even when a body is slowing down, it is ‘accelerating’. Also avoid ‘decelerating’.

If students did not mention that the body will change direction then they could not get the full mark.

Part D

General Comment:

- Generally well done by majority of the candidates. However, there were many finer details that candidates should be aware of and improve on in their solutions, especially those with no marks being penalised.

Solution	**Marking scheme**	**Marker's comments**
Question 26 **a)** $\text{LHS}=\frac{\sqrt{3}}{3}$ $\text{RHS}=\frac{\sin\frac{\pi}{3}}{1+\cos\frac{\pi}{3}}$ $=\frac{\frac{\sqrt{3}}{2}}{1+\frac{1}{2}}$ $=\frac{\frac{\sqrt{3}}{2}}{\frac{3}{2}}$ $=\frac{\sqrt{3}}{2}\times\frac{2}{3}$ $=\frac{\sqrt{3}}{3}$ $=\text{LHS}$ $\therefore \left(\frac{\pi}{3},\frac{\sqrt{3}}{3}\right)$ lies on the curve.	**0.5 mark** for the substitution of the point into the equation of the curve. **0.5 mark** for evaluating the numerator and denominator with actual exact values.	- Significant number of candidates did not achieve full mark due to not providing the relevant step of evaluating the exact values. Since this is a SHOW question, ALL relevant steps must be provided even if it is only 1 mark. - Whilst not penalised, many candidates should use LHS and RHS to verify if the point lies on a curve.
b) Let $u=\sin x$ and $v=1+\cos x$ $\therefore\ u'=\cos x\ and\ v'=-\sin x$ $\frac{dy}{dx}=\frac{u'v-v'u}{v^2}$ $=\frac{\cos x\,(1+\cos x)-(-\sin x)\sin x}{(1+\cos x)^2}$ $=\frac{\cos x+\cos^2 x+\sin^2 x}{(1+\cos x)^2}$ $=\frac{1+\cos x}{(1+\cos x)^2}$ As $\cos^2 x+\sin^2 x=1$ $=\frac{1}{1+\cos x}$	**1 mark** for correctly using the quotient rule. **1 mark** for simplifying the result, showing ALL relevant steps to obtain the result.	- The quotient rule part was done well by majority of the candidates. - Some candidates got penalised as they didn't show all the steps i.e. $\frac{\cos x+\cos^2 x+\sin^2 x}{(1+\cos x)^2}$ $=\frac{1+\cos x}{(1+\cos x)^2}$ - Show questions means all steps need to be written down to verify the result.

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c)

At $x=\frac{\pi}{3}$,

$$m_{\text{Tangent}}=\frac{1}{1+\cos\frac{\pi}{3}}$$
$$=\frac{1}{1+\frac{1}{2}}$$
$$=\frac{1}{\frac{3}{2}}$$
$$=\frac{2}{3}$$

$At\left(\frac{\pi}{3},\frac{\sqrt{3}}{3}\right)$

$$y-\frac{\sqrt{3}}{3}=\frac{2}{3}\left(x-\frac{\pi}{3}\right)$$
$$y=\frac{2}{3}x-\frac{2\pi}{9}+\frac{\sqrt{3}}{3}$$

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1 mark for the finding the correct gradient of the tangent.

1 mark for correctly working out the equation of the tangent.

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- Some candidates did not evaluate the gradient at $x=\frac{\pi}{3}$ and used the non-simplified form as part of their equation i.e. $m=\frac{1}{1+\cos x}$ rather than $m=\frac{2}{3}$. Marks were penalised.
- Some candidates may see the acronym ISE which stands for Ignore Subsequent Error. This is for candidates who wrote the correct answer but then simplified into an incorrect answer i.e. $y=\frac{2}{3}x-\frac{2\pi}{9}+\frac{\sqrt{3}}{3}$ $\therefore y=\frac{2x}{3}-\frac{2\pi+3\sqrt{3}}{9}$

(Should be a minus rather than a plus). No marks deducted.

- Significant number of candidates should take more care in how they write a proper equation, whether in gradient intercept form or general form.

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Question 27

$$\log_2\left(\frac{1}{x}\right)+\log_2\left(\frac{1}{x^2}\right)+\log_2\left(\frac{1}{x^3}\right)+\ldots$$
$$=-\log_2(x)-2\log_2(x)-3\log_2(x)-\ldots$$

$\therefore$This series is an A.P. with common difference = $-\log_2 x$

Also,

$a=-\log_2 x$

$l=-10\log_2 x$

$n=10$

$$S_n=\frac{n}{2}(a+l)$$
$$-110=\frac{10}{2}(-\log_2 x-10\log_2 x)$$
$$-110=-5(\log_2 x+10\log_2 x)$$
$$22=11\log_2 x$$
$$2=\log_2 x$$
$$x=2^2=4$$

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1 mark for identifying the series is AP and find the common difference.

1 mark for substituting into the correct formula and showing correct steps.

1 mark for correct answer.

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- Significant number of candidates stated that the series is a GP. This is NOT a GP series. Maximum of 0.5 of a mark was awarded for those whom used GP formula. Candidates should check if it has a common difference or a common ratio.
- Significant number of candidates did the question alternatively without explicitly using AP series formula as *n* is relatively small. Whilst full marks was given if full correct solution was shown, candidates are encouraged to use the AP and GP series formulas especially with large *n*.

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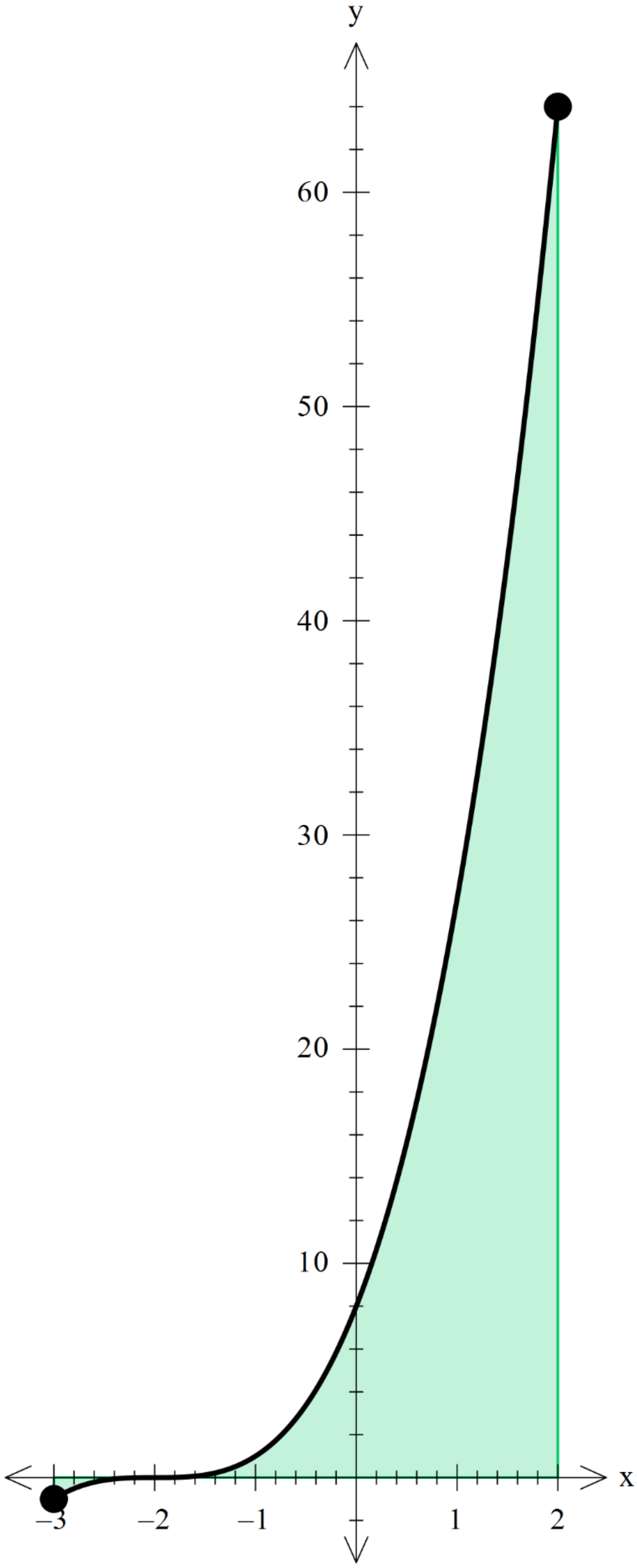
Question 28 **a)** P(late) = P(wake up late and late to work) + P(wake up on time and late to work) $\text{P(late)} = 0.6 \times 0.85 + 0.4 \times 0.4$ $= 0.67$	**1 mark** for correct working out. **1 mark** for correct answer.	- Candidates should write in the probability notation as the subject of their result i.e. **P(late)** = …
b) $\text{P(wakes up late} \mid \text{arrives late)} = \dfrac{\text{P(wakes up late} \cap \text{arrive late)}}{\text{P(late)}}$ $= \dfrac{0.6 \times 0.85}{0.67}$ $= \dfrac{51}{67}$	**1 mark** for correct working out. **1 mark** for correct answer.	- Majority of the candidates who lost marks in this question did not understand that this was a conditional probability question as the boss knows she has arrived late (is the given part). - Candidates who wrote a bold answer of 0.6 did not get any marks. - Whilst not penalised, a concern is the number of candidates that wrote $\dfrac{0.6 \times 0.85}{0.67} = 0.76$. That is not true as $\dfrac{0.6 \times 0.85}{0.67} \approx 0.76\ (2d.p.)$ and candidates should state the exact value (i.e. $\dfrac{51}{67}$) since it exists and more importantly is equal to the result.
Question 29 $V = \int \dfrac{1}{t+1}\,dt$ $V = \ln(t+1) + c \quad (as\ t \geq 0)$ At $t = 3$ and V = ln 2 ln 2 = ln 4 + c c = ln 2 - ln 4 $= \ln\left(\dfrac{2}{4}\right) = \ln\left(\dfrac{1}{2}\right)$ $\therefore V = \ln(t+1) + \ln\left(\dfrac{1}{2}\right)$ At $t = 8$ $V = \ln(9) + \ln\left(\dfrac{1}{2}\right)$ $\approx 1.50\ m^3 \quad (3\ s.f.)$	**1 mark** for the correct primitive, V. **1 mark** for correct value for c. **1 mark** the correct answer to 3 significant figures.	- Common errors included: • Doing c = ln 4 -ln 2 rather than the other way around. • $\ln 2 - \ln 4 \neq \dfrac{1}{2}$ • Not writing answer to 3 significant figures. • The question does not state anywhere that the vessel is empty initially. - Candidates whom used definite integral were able to show the correct answer if they went from t = 3 to t = 8. Not the most successful method used.

Question 30 (2 marks)

Find the area bounded by the curve $y=(x+2)^3$ and the x-axis between $x=-3$ and $x=2$. **2**

The graph of the above function looks exactly like $y=x^2$ except that it has been shifted to the left by 2 units. From the left, the graph crosses the x-axis from to below at $x = -2$. Taking the integral of the function from -2 to 2 will give us a positive Area for that interval. Taking the integral from -3 to -2 will give us a negative Area. To get a positive area in this interval one can either swap the boundaries, take the absolute value of the integral or subtract the integral within that interval. The method below subtracts the integral for the interval in which the curve is below the x-axis.



$$\int (x+2)^3\,dx = \frac{(x+2)^4}{4}+C$$

Hence:

$$\int_{-3}^{2}(x+2)^3\,dx = \int_{-2}^{2}(x+2)^3\,dx - \int_{-3}^{-2}(x+2)^3\,dx$$

$$= \left[\frac{(x+2)^4}{4}\right]_{-2}^{2} - \left[\frac{(x+2)^4}{4}\right]_{-3}^{-2}$$

$$= \frac{(2+2)^4}{4} - \left(-\frac{(-3+2)^4}{4}\right)$$

$$= 64+\frac{1}{4}$$

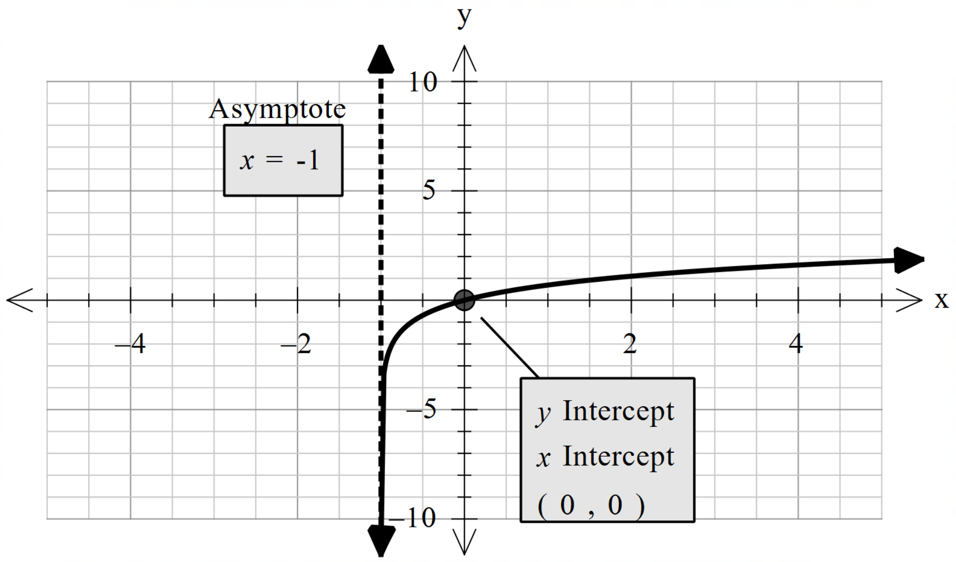
$$= \frac{257}{4} \text{ or } 64\frac{1}{4}$$

NOTES: At least half of all students did not take into consideration the fact that the curve crosses the x-axis. This results in part of the area between the curve and the x-axis having a negative value and just integrating across the given boundary did not give the correct answer.

A small amount of students did not correctly integrate the integrand and a handful made arithmetic errors if they either: 1) evaluated the correct integral or 2) expanded the integrand and then found the primitive.

Question 31 (4 marks)

(a) Sketch the curve of $y = \log_e(x+1)$ including intercept(s) and asymptotes. **1**



NOTE: There were three things I was looking for. The correct asymptote, the right general shape for a logarithm and the curve going through the origin. This was generally well done. A small handful of students had a horizontal asymptote (or two asymptotes) or the asymptote was indicated on the *y*-axis.

(b) Using the trapezoidal rule with 4 sub-intervals, estimate $\int_1^2 \ln(x+1)\,dx$, **2**

correct to 3 decimal places.

Trapezoidal Rule (*):

$\int_a^b f(x)dx \approx \frac{h}{2}\left[\left(f(a)+f(a+h)\right)+\left(f(a+h)+f(a+2h)\right)+\ldots+\left(f(b-h)+f(b)\right)\right]$, where h is $\frac{b-a}{n}$, and n is the number of sub-intervals. There will be n pairs of terms (in brackets) and $n+1$ unique terms if the brackets were taken away.

Let $f(x) = \ln(x+1)$. Four equal subintervals from 1 to 2 will have width $\frac{1}{4}$ (h).

Then $\int_1^2 f(x)dx \approx \frac{\frac{1}{4}}{2}\left[\left(f(1)+f\left(\frac{5}{4}\right)\right)+\left(f\left(\frac{5}{4}\right)+f\left(\frac{3}{2}\right)\right)+\left(f\left(\frac{3}{2}\right)+f\left(\frac{7}{4}\right)\right)+\left(f\left(\frac{7}{4}\right)+f(2)\right)\right]$

$$\approx \frac{1}{8}\left[f(1)+2\left(f\left(\frac{5}{4}\right)+f\left(\frac{3}{2}\right)+f\left(\frac{7}{4}\right)\right)+f(2)\right]$$

$$\approx \frac{1}{8}\left[\ln(1+1)+2\left(\ln\left(\frac{5}{4}+1\right)+\ln\left(\frac{3}{2}+1\right)+\ln\left(\frac{7}{4}+1\right)\right)+\ln(2+1)\right]$$

≈ 0.909 to the 3rd decimal place.

NOTE: Many students failed to get the formula (*) above correct. Usually, they got the value for $\frac{h}{2}$ wrong. Many students did not realise that 4 sub-intervals meant 4 equally spaced regions over the indicated boundary. This was interpreted as using only 4 values, but some ended up using 5 intervals for some reason - which I am unable to explain. When inputting function values, many students used $\ln(x)$ rather than $\ln(x + 1)$, why… I guess this was an easy mistake to make actually.

(c) Would the trapezoidal rule used in this instance provide a value that is greater than or less than $\int_1^2 \ln(x+1)\,dx$? Explain your answer. **1**

The estimation, using the trapezoidal rule would be less than the exact value since over the interval, the curve is concave down.

NOTE: If students did not write something like the solution above, they had to do LOT of convincing to get this mark. Visual representations of what was going on made my job easier.

Question 32 (5 marks)

The population of a town can be estimated by using the formula $P = P_0 e^{kt}$, where t is the time in years after the year 2020 and P_0 and k are constants.

The population at the beginning of the year 2020 was 300, and at the beginning of 2024 it was 450.

(a) Find the annual growth rate, k, of the population correct to 3 significant figures. **1**

When $t = 0$, $P = 300$, which means $P_0 = 300$.

When $t = 4$, $P = 450$.

$450 = 300e^{4k}$

$\frac{3}{2} = e^{4k}$

$\ln\left(\frac{3}{2}\right) = 4k$

Hence, $k = \frac{\ln 3 - \ln 2}{4}$ which is 0.101 to 3 significant figures.

NOTE: Almost everyone got to the expression for k in the last line above. A small handful of students did not feel the need to do what the question asked and give the value for k to 3 significant figures. In this case, I did not feel the need to award full marks.

(b) In what year will the town's population be double that of the year 2020? **2**

The population doubled from the year 2020 is 600 people.

$600 = 300e^{\frac{\ln 3 - \ln 2}{4}t}$, solve this for t.

$2 = e^{\frac{\ln 3 - \ln 2}{4}t}$

$\ln 2 = \frac{\ln 3 - \ln 2}{4}t$

$t = \frac{4\ln 2}{\ln 3 - \ln 2}$, which is approximately 6.84 years (to two decimal places).

Hence towards the end of the year 2026, the town will have double the population of the year 2020.

NOTE: Almost all student found the correct value for t as being 6.84 years. Most then gave the correct year. A popular response was 2027. The first year was from the start of 2020 to the start of 2021, hence the 7th year was the start of 2026 to the start of 2027. Therefore the year 2027 was the 8th year and incorrect.

Question 33 (3 marks)

The water height in an estuary fluctuates according to the tide. Flooding will occur when the water height reaches 1.5 m. One day the high tide of 1.6 m occurred at 3:00 am and the following low tide at 9:30 am. Assume that the following high tide was also 1.6 m and the height of the tide is modelled by the equation **3**

$$h = 1 + 0.6\sin\left(\frac{2\pi}{13}(t + 0.25)\right),$$

where h is the height in metres and t is the time in hours after midnight.
Find the time when the next flooding would be expected to occur, to the nearest minute.

Flooding occurs when the tide reaches 1.5 m. A high tide of 1.6 m is expected around 6 and a half hours after low tide, this is at 4 pm. So, sometime before 4 pm flooding will occur.

Solve the above equation for h = 1.5 m.

$$1.5 = 1 + \frac{3}{5}\sin\left(\frac{2\pi}{13}\left(t + \frac{1}{4}\right)\right)$$
$$\frac{1}{2} = \frac{3}{5}\sin\left(\frac{2\pi t}{13} + \frac{\pi}{26}\right)$$
$$\frac{5}{6} = \sin\left(\frac{2\pi t}{13} + \frac{\pi}{26}\right)$$
$$t \approx 1.7882$$

The question does not directly state that the high and low tides given are 3 hrs and 9 and a half hours respectively after the particular 'midnight' that the equation relates to. It may be assumed that the midnight in question is the one directly before the times stated and a quick check for the heights 1.6 m and 0.4 m would have verified this assumption.

The period T, of the function $\sin(Ax + b)$, is equal to $\frac{2\pi}{A}$. In this case $A = \frac{2\pi}{13}$. Thus, $T = 13$, which is equivalent to 13 hrs (noticing the time difference between high and low tides would have given the same value, since the difference in time between high and low tide is half a period). Hence adding 13 hrs to 1:47 am would have given the more reasonable (and correct) answer of 2:47 pm.

Alternative solution using the general solution for Sine.

The general solution for $\sin\theta = \sin\alpha$ is $\theta = (-1)^k \alpha + k\pi$, where k is an integer.

Here $\theta = \frac{2\pi t}{13} + \frac{\pi}{26}$ and $\alpha = \sin^{-1}\left(\frac{5}{6}\right)$, where $-\frac{\pi}{2} \le \alpha \le \frac{\pi}{2}$.

$$\frac{2\pi t}{13} - \frac{\pi}{26} = (-1)^k \sin^{-1}\left(\frac{5}{6}\right) + k\pi$$
$$t = \frac{13}{2\pi}\left((-1)^k \sin^{-1}\left(\frac{5}{6}\right) + k\pi - \frac{\pi}{26}\right)$$
$$t = \frac{13}{2\pi}(-1)^k \sin^{-1}\left(\frac{5}{6}\right) + \frac{13k}{2} - \frac{1}{4}$$

We can list the times (from midnight) that the tide reaches 1.5 m and choose the appropriate value for t after 9:30 am. Students need to choose values from zero onwards (this will result in positive solutions). Negative values for k will give a negative time and do not contextually fit the question (t is in hours after midnight).

When $k = 0$, $t \approx 1.7882$ in radians, which is equivalent to 1 hr and 47 minutes past midnight (to the nearest minute). The next value for t ($k = 1$) will be when the tide is decreasing, so we will ignore that. When $k = 2$, $t \approx 14.7882$ which is 14 hours and 47 minutes past midnight. This last time is after the 9 hrs and 30 minutes of the low tide and therefore must be the appropriate time we want.
Thus the time that the next flooding occurs is 14:47 or 2:47 pm.

NOTE: There are an infinite number of ways this question could go wrong, but happily, about 30% of students got it correct or at least very close. Many students only worked out a time for the high tide. There was no need to go to so much trouble to work out when the next high tide will be (one should be able to do this in one's head), but I generously award a mark for this. Many students arrived at the correct time of 1 hr 47 mins for the first period but failed to recognise that this time made NO SENSE AT ALL in the context of the question. Many students used degrees to solve this, mostly did it successfully. However; it made their lives harder. Students were more successful when they recognised that the use of radians was the appropriate choice for their working out.

NESA Number	S	O	L	U	T	I	O	N	S		Q34-36	

YEAR Mathematics Advanced

TERM 3 Cohort Task #3 (THSC)

Part F

Sample Solutions

Question 34 (6 marks)

Michael borrows \$650 000 to buy a unit at 6% per annum with interest compounded monthly.
The loan is to be repaid with equal monthly repayments of \$4200.
Let $\$A_n$ be the amount owing after n months.

(a) Show that $A_n = 840000 - 190000 \times 1.005^n$ **3**

6% p.a. = 0.005 per month

$A_0 = 650\,000$

$A_n = A_{n-1} \times 1.005 - 4200$

$A_1 = 650\,000 \times 1.005 - 4200$

$A_2 = A_1 \times 1.005 - 4200$

$= (650\,000 \times 1.005 - 4200) \times 1.005 - 4200$

$= 650\,000 \times 1.005^2 - 4200(1 + 1.005)$

$A_3 = A_2 \times 1.005 - 4200$

$= \left(650\,000 \times 1.005^2 - 4200(1 + 1.005)\right) \times 1.005 - 4200$

$= 650\,000 \times 1.005^3 - 4200(1 + 1.005 + 1.005^2)$

$$\therefore A_n = 650\,000 \times 1.005^n - 4200 \times (\underbrace{1 + 1.005 + 1.005^2 + \ldots + 1.005^{n-1}}_{\text{GP: } a=1,\, r=1.005,\, n \text{ terms}}) \qquad (*)$$

$$= 650\,000 \times 1.005^n - 4200 \times \frac{1.005^n - 1}{1.005 - 1}$$

$$= 650\,000 \times 1.005^n - 840\,000(1.005^n - 1)$$

$$= 840\,000 - 190\,000 \times 1.005^n$$

Comment:

This is a "Show that" question.
This is practically the same comment from all the other tasks that have had a similar question.
This is proof that students do not read the comments, either when studying or when receiving their exams back. More than likely, no one will read this except me ☹.

Students are expected to show the development of the formulae up to A_3.
For those students who were trying to set out their proof properly, they needed to have (*), or equivalent, in order to gain full marks.
That said, those students who chose to ignore this, e.g. writing out A_2 and A_3 without any development, were lucky to receive half the maximum marks (if at all).

(b)	After 15 years, the amount owing is $\$373722$ to the nearest dollar. At this time Michael borrows a further \$200 000 to build an extension. He adds this onto his previous mortgage. If the monthly repayments remain the same, what is the minimum remaining number of months it will take him to pay off the balance of the loan?	**3**

$A_{180} = 373\,722$

Now Michael's new loan is worth $\$(373\,722 + 200\,000) = \$573\,722$

Let \$ B_n be the amount owing after n months

Using the development of the formulae from part (a):

$$B_n = 573\,722 \times 1.005^n - 840\,000 \times (1.005^n - 1)$$
$$= 840\,000 - 266\,278 \times 1.005^n$$

When will $B_n = 0$?

$$840\,000 - 266\,278 \times 1.005^n = 0$$

$$\therefore 1.005^n = \frac{840\,000}{266\,278}$$

$$\therefore n \ln 1.005 = \ln\left(\frac{840\,000}{266\,278}\right)$$

$$\therefore n = \frac{\ln\left(\frac{840\,000}{266\,278}\right)}{\ln 1.005}$$

$$\doteqdot 230.3461562$$

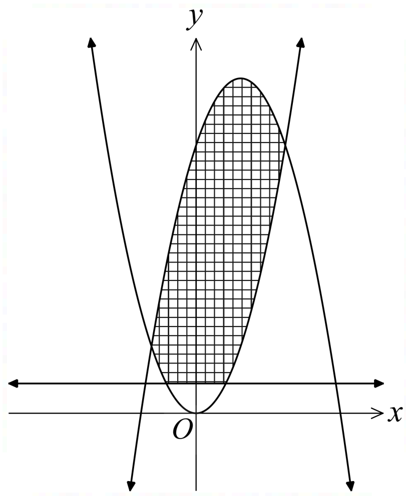
$\therefore$ 231 months is needed to pay off the loan.

Common:

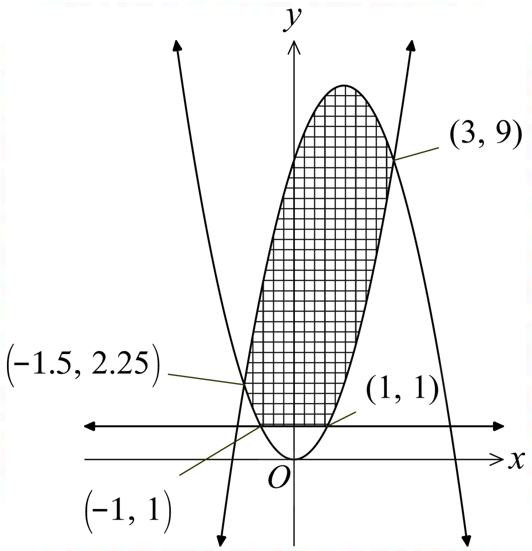
Many students couldn't make the connection with part (a) and wrote down weird and wonderful equations. These students received 1 mark if they successfully solved their equation.
This is not 'ecf', but simply 'e'.

Question 35 (4 marks)

Find the shaded area enclosed by the curves $y = x^2$ and $y = 9 + 3x - x^2$ and the line $y = 1$. **4**



1 Where do the graphs intersect?

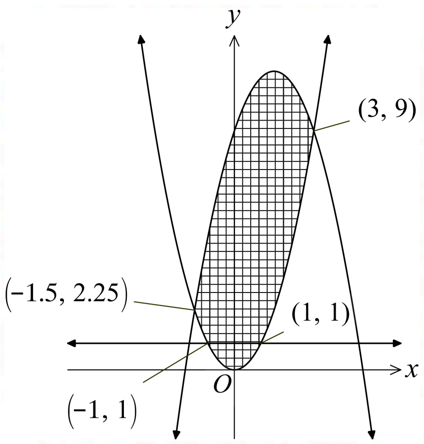


$$x^2 = 9 + 3x - x^2$$
$$2x^2 - 3x - 9 = 0$$
$$(2x+3)(x-3) = 0$$
$$\therefore x = -1.5, 3$$

And when $y = 1$, then $x^2 = 1$

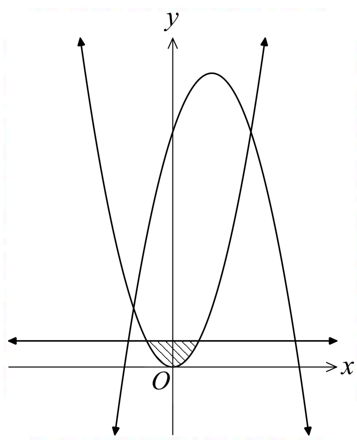
$\therefore x = \pm 1$

2 First find



$$\text{Area} = \int_{-1.5}^{3} 9 + 3x - x^2 - x^2 \; dx$$

3 Then subtract this area



$$\text{Area} = \int_{-1}^{1} 1 - x^2 \; dx$$

Question 35 (continued)

$$\text{Area} = \int_{-1.5}^{3} 9+3x-x^2-x^2\ dx - \int_{-1}^{1} 1-x^2\ dx$$

$$= \int_{-1.5}^{3} 9+3x-2x^2\ dx - 2\times\left[x-\frac{1}{3}x^3\right]_0^1$$

$$= \left[9x+\frac{3}{2}x^2-\frac{2}{3}x^3\right]_{-1.5}^{3} - \frac{4}{3}$$

$$= \left(27+\frac{3}{2}\times 9-\frac{2}{3}\times 27\right) - \left(-9\times 1.5+\frac{3}{2}\times 2.25+\frac{2}{3}\times 3.375\right) - \frac{4}{3}$$

$$= 29\frac{1}{24}$$

So the area is $29\frac{1}{24}$ u^2

Comment:

Many students struggled to simply find the points of intersection.

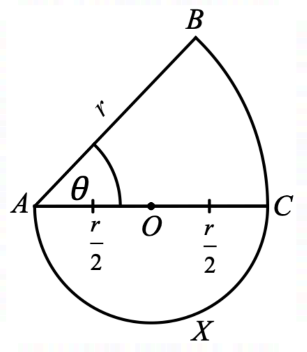
Many students successfully found straight forward ways to work out this problem, with a minimum of integrals.

Students are encouraged to simplify/combine their integrals where possible. Less substitutions leads to less likely making a mistake i.e avoid calculations like this:

$$\text{Area} = \int_{-1.5}^{3} 9+3x-x^2\ dx - \int_{-1.5}^{3} x^2\ dx - \int_{-1}^{1} 1-x^2\ dx$$

Question 36 (6 marks)

A rotating camera is formed with a cross-section as shown in the diagram.



The cross-section consists of a semi-circle *AXC* with centre *O* and radius $\frac{r}{2}$,

and a sector *ABC* of radius *r*, centre *A* and angle θ.

(a) Show that the perimeter, *P*, of the cross-section *ABCX* is given by $P = r\left(\frac{\pi}{2} + \theta + 1\right)$. **1**

Perimeter of semi-circle = $r + \frac{\pi r}{2}$ and perimeter of sector *ACB* = $r + r\theta$

$$\therefore P = \frac{\pi r}{2} + r + r\theta$$
$$= r\left(\frac{\pi}{2} + 1 + \theta\right)$$

Comment:

This is also a "Show that" question of which many students successfully expanded the given formula and then re-factorised it. Students who did this (without any other supporting evidence) did not score any marks.

The question didn't specify radians per se, but any student who has practised questions like this know that to get the desired result you have to work in radians.

Some students successfully converted to radians from degrees, but many of these when they came to the next problem, promptly forgot this and re-derived the results.

So many students made a mess of a relatively simple problem.

The formulae are in the Reference sheet.

A Terminology **NOTE:**

(*) *BC* refers to a straight line/segment.

(*) Arc *BC* refers to the arc subtended by $\angle BAC$.
Notations like $\overset{\frown}{BC}$ are old-school – avoid them.

(*) *AC* can only be a straight line in this context.
AXC either refers to a straight line or a triangle.
'Semi-circle *AXC*' is the correct way.

(*) Δ is the only geometrical shape that can be used as an abbreviation, e.g. Δ *ABC*.

(b)	If the area of cross-section *ABCX* is 1 square unit, show that $P = \frac{2}{r} + r\left(1 + \frac{\pi}{4}\right)$.	**2**

$$\text{Area} = \frac{1}{2} \times \pi \left(\frac{r}{2}\right)^2 + \frac{1}{2} r^2 \theta$$

$$= \frac{1}{8} r^2 (\pi + 4\theta)$$

$$\text{Area} = 1 \Rightarrow \frac{1}{8} r^2 (\pi + 4\theta) = 1$$

$$\therefore \pi + 4\theta = \frac{8}{r^2}$$

$$\therefore 4\theta = \frac{8}{r^2} - \pi$$

$$\therefore \theta = \frac{2}{r^2} - \frac{\pi}{4}$$

$$P = r\left(\frac{\pi}{2} + 1 + \theta\right)$$

$$= r\left(\frac{\pi}{2} + 1 + \frac{2}{r^2} - \frac{\pi}{4}\right)$$

$$= \frac{2}{r} + r\left(\frac{\pi}{4} + 1\right)$$

Comment

The same comment as for the last question applies here, except that some people didn't realise that the area of the semicircle is $\frac{1}{2}\pi\left(\frac{r}{2}\right)^2$.

Many students' algebra skills prevented them from simply showing this result, if at all.

(c)	Show that the least perimeter occurs when $r^2 = \dfrac{8}{\pi+4}$ and calculate θ. Write the size of this angle to the nearest degree.	**3**

$$P = 2r^{-1} + r\left(\frac{\pi}{4}+1\right)$$

$$\frac{dP}{dr} = -2r^{-2} + \frac{\pi}{4} + 1$$

$$= -\frac{2}{r^2} + \frac{\pi}{4} + 1$$

$$\frac{d^2P}{dr^2} = 4r^{-3}$$

Least perimeter when $\dfrac{dP}{dr} = 0$

$$\therefore -\frac{2}{r^2} + \frac{\pi+4}{4} = 0$$

$$\therefore \frac{2}{r^2} = \frac{\pi+4}{4}$$

$$\therefore \frac{r^2}{2} = \frac{4}{\pi+4}$$

$$\therefore r^2 = \frac{8}{\pi+4}$$

For $r > 0$ then $\dfrac{d^2P}{dr^2} = 4r^{-3} > 0$ and so P has a minimum (least value) when $r^2 = \dfrac{8}{\pi+4}$.

From (b): $\theta = \dfrac{2}{r^2} - \dfrac{\pi}{4}$

From above, $\dfrac{2}{r^2} = \dfrac{\pi}{4} + 1$

$$\theta = \frac{\pi}{4} + 1 - \frac{\pi}{4} = 1$$

$$\therefore \theta = 1^c \doteqdot 57°$$

Comment:

Students have been reminded many times that they have to give a numerical (or other justification) for the nature of a stationary point (or point of inflexion/inflection).
Many chose the numerical path and came up with exotic fractions rather than use the second derivative.

However, as $\dfrac{8}{\pi+4} \doteqdot 1.12$ then it would be simpler to test $r = 1$ and $r = 1.2$ (or even $r = 2$).
Even $r^2 = 1$ or $r^2 = 2$.

In the angle calculation part, the only half marks given were if a student left the answer as 1 radian or assumed it was 1°.