SYDNEY GIRLS HIGH SCHOOL
TRIAL HIGHER SCHOOL CERTIFICATE

2000

MATHEMATICS

2 UNIT

Time Allowed – 3 hours
(Plus 5 minutes reading time)

DIRECTIONS TO CANDIDATES NAME

- Attempt ALL questions
- ALL questions are of equal value
- All necessary working should be shown in every question. Marks may be deducted for careless or badly arranged work.
- Board-approved calculators may be used
- Each question attempted should be started on a new sheet. Write on one side of the paper only.

This is a trial paper ONLY. It doe not necessarily reflect the format or the contents of the 2000 HSC Examination Paper on this subject

QUESTION 1

(a) Find, correct to two decimal places, the value of: [2]

$$\frac{9.65^2 - 3.43^3}{\sqrt{56.5}}$$

(b) Find the values of a and b given: [2]

$$\frac{2}{3-\sqrt{2}} = a + b\sqrt{2}$$

(c) Factorize the equation and solve for x [2]

$3x^2 - 2x - 5 = 0$

(d) Solve for x and mark the set of values on a number line: [2]

$|2x - 3| \leq 7$

(e) Express $0.4\dot{7}$ as an exact fraction in its simplest form. [2]

(f) Given $\frac{1}{P} = \frac{1}{Q} + \frac{1}{R}$

(i) make Q the subject of the formula

(ii) Hence, or otherwise, find Q when $P = 0.2$ and $R = 0.5$ [2]

QUESTION 2

(a) On a number plane, mark the origin O and the points A (4,1), B (6,3) and C (2,7). Join A to B, B to C and C to A [1]

(b) Find the gradient of line AB [1]

(c) Find the equation of line AB [1]

(d) Write down the acute angle (to the nearest degree) line AB makes with the x-axis. [1]

(e) Find, in exact form, distance AB [1]

(f) Prove ΔABC is right-angled [2]

(g) Find the mid-point of AC [1]

(h) Using part (g), or otherwise, find the co-ordinates of D which would form the rectangle ABCD. (Show all necessary working) [1]

(i) Find the exact area of rectangle ABCD [1]

(j) Find the shortest (perpendicular) distance from the origin to line AB (leave your answer in exact form). [2]

QUESTION 3

(a) Differentiate:

(i) $4x^3 - \frac{4}{x^3}$ [1]

(ii) $ln\left(\frac{x^2-5}{x+3}\right)$ [2]

(iii) $x^2.e^{\cos x}$ [2]

(b) Find: (i) $\int \frac{dx}{3x-1}$ [1]

(ii) $\int (2x+1)^9\, dx$ [1]

(c) Evaluate, leaving your answer in exact form

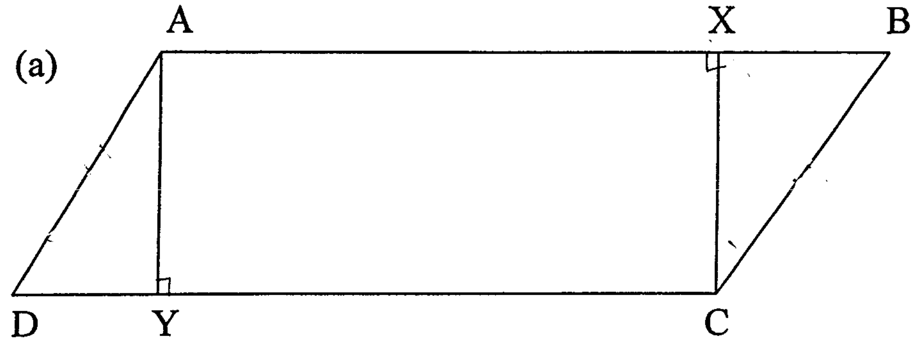
$\int_0^1 e^{4x}\, dx$ [2]

(d) (i) If $y = x\,.\,sin\,x + cosx$ find $\frac{dy}{dx}$ [1]

(ii) Using part (i), or otherwise, find the value of: $\int_{\frac{\pi}{2}}^{\pi} x.\cos x.dx$ [2]

QUESTION 4

(a)



ABCD is a parallelogram
$CX \perp AB$ and $AY \perp DC$

(i) Prove ∠ triangles ADY and CBX are congruent
(ii) Hence, or otherwise, prove AY = CX [4]

(b) A ship leaves Port A and travels 55km on bearing 067^{o} to reach Port B. It then travels on bearing 310^{o} for 25km until it reaches Port C (see figure below).

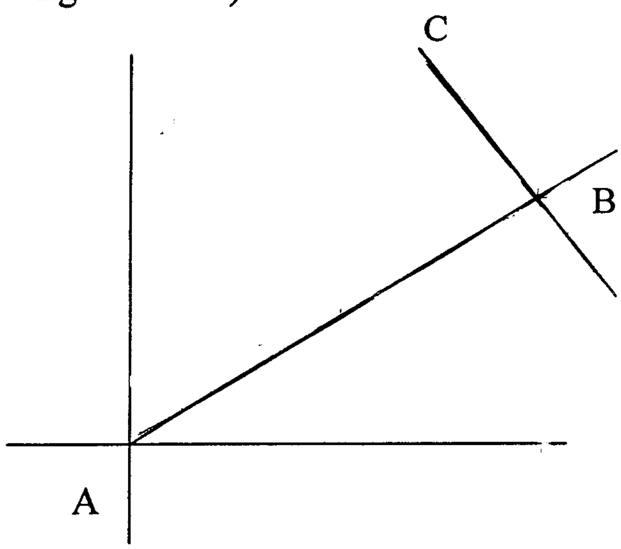


Figure not to scale [3]

(i) Find ∠ABC
(ii) Find distance AC (to the nearest whole km)

(c) A parabola has its focus at (3,3) and its directrix is the line $y = -1$
(i) Sketch the parabola showing the vertex, focus and directrix
(ii) Write down the equation of the axis of symmetry
(iii) Write down the equation of the parabola
(iv) Find the equation of the tangent to the parabola at the point where the parabola intersects the y- axis. [5]

QUESTION 5

(a) Consider the curve given by $y = 3x^2 - x^3$

(i) Find the stationary points and determine their nature.

(ii) Sketch the curve, indicating all essential features.

(iii) Find the equation of the normal to the curve where the curve crosses the x – axis [7]

(b) In an arithmetic progression the sum of the 3^{rd} and 4^{th} terms is 24. Further, the sum of the 5^{th} and 6^{th} terms is 32. Find.

(i) the first three terms [3]

(ii) the sum of the first 20 terms.

(c) For what values of m has the equation [2]

$4x^2 + (1+m)x + 1 = 0$ equal roots.

QUESTION 6.

(a) One hundred tickets are sold in a raffle which has two prizes. First prize is drawn and the winning ticket discarded. Second prize is then drawn. Elizabeth buys 5 tickets in the raffle. What is the probability

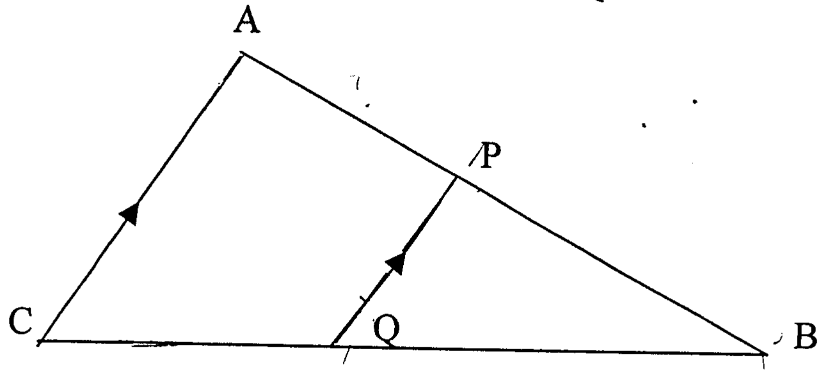
(i) Elizabeth wins first prize?

(ii) Elizabeth only wins first prize?

(iii) Elizabeth only wins second prize?

(iv) Elizabeth wins both prizes. [4]

(b) Given triangle ABC with line PQ//AC

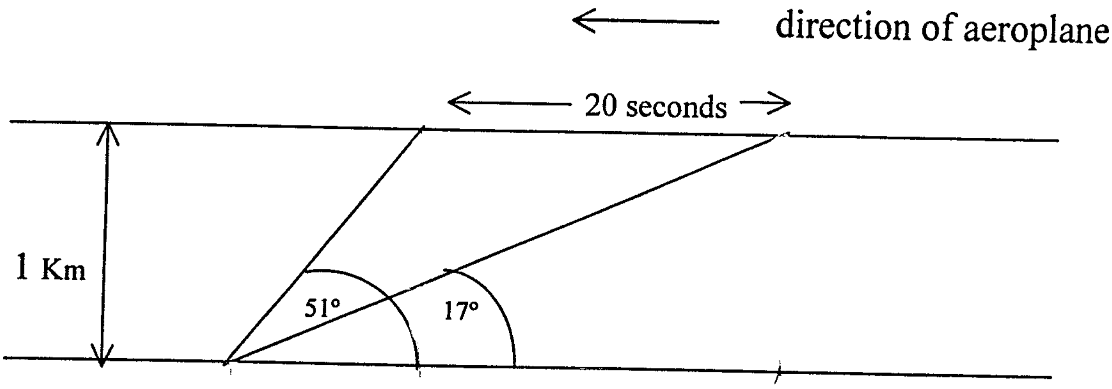


(i) Prove triangles ABC and PQB similar

(ii) Find the length of AP given PB = 8, QB = 10 and CQ = 4 [5]

(c) An aeroplane is flying at a level altitude of 1km above a flat plane. The angle of elevation of the aeroplane when it is first observed is 17°. Twenty seconds later the angle of elevation is 51°. What is the speed of the aeroplane in km/hr. (Correct to 3 significant figures)

[3]



QUESTION 7

(a) Given the roots to the equation

$2x^2 - 5x + 5 = 0$ are α and β

Write down the values of:

(i) $\alpha + \beta$

(ii) $\alpha \beta$

(iii) $\frac{1}{\alpha} + \frac{1}{\beta}$

(iv) $\alpha^2 + \beta^2$ [4]

(b) A particle moves along the x – axis with acceleration at time t equal to $12(t+3)^2$

i.e $\frac{d^2x}{dt^2} = 12(t+3)^2$

If the particle is initially at rest at the origin (ie when $t=0$, $x=0$ and $\frac{dx}{dt} = 0$), find the position of the particle when $t = 1$. [4]

(c) (i) Draw a neat sketch of the curves $y = x^2$ and $y = (x-2)^2$

(ii) Find the co-ordinates of the point of intersection of the two two curves

(iii) Find the area enclosed by the two curves and x-axis [4]

QUESTION 8

(a) (i) Write down an expression for the sum of the series
$1+3+5+$ for n terms

(ii) Write down an expression for the sum of the series
$2+4+6+ \ldots.$ for n terms

(iii) Find the sum of the first 2000 terms of the series [3]
$1 - 2 + 3 - 4 + 5 - 6 + \ldots.$

(b) Can there be an infinite geometric series with a limiting sum of $\frac{5}{8}$ and a first term of 2? (All working and reasoning must be shown) [2]

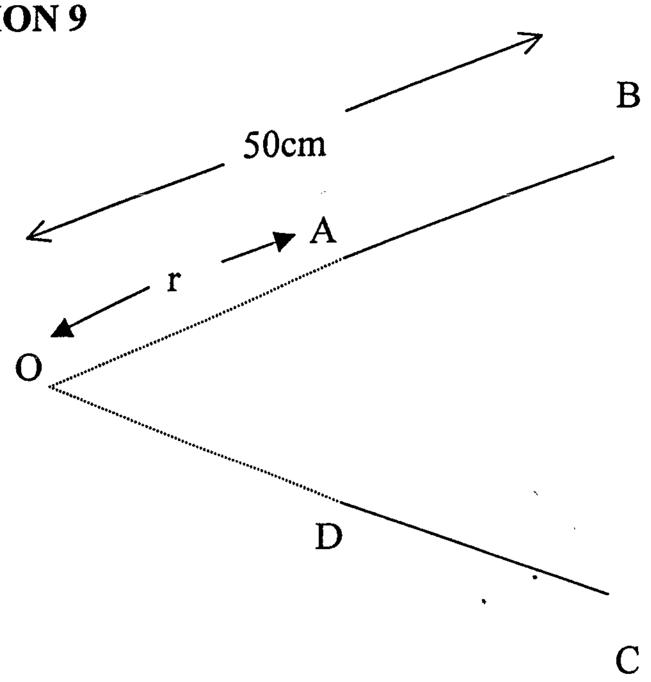
(c) Indicate, by shading on a diagram, the region in the positive quadrant bounded by the y-axis, the line $y = 2$ and the curve $y = e^x$. Calculate the area of this region. [4]

(d) The area under the curve $y = 4^x$ between $x = 0$ and $x = 2$ is rotated about the x-axis. Copy down and complete the table below. Use your results with Simpson's Rule to find an approximate value for the volume of rotation using 5 function values (correct to 1d.p) [3]

(correct to 1d.p.)

x	0	0.5	1	1.5	2
4^x					

QUESTION 9



OBC is a sector
AD and BC are arcs of concentric circles centre O

$\angle BOC = \Theta$ (where Θ is in radians)

OA = r cm
OB = 50 cm

The perimeter of figure ABCD is 60cm

(i) Show that $\Theta = \dfrac{2r-40}{r+50}$ [2]

(ii) Show that the area of ABCD is $= 70r - 1000 - r^2$ [2]

(iii) Find the values of r and Θ which maximize the area of figure ABCD and find this maximum area [5]

(b) Two people, John and Kim, play a dice game. John throws two dice and Kim throws one. Kim wins if John fails to beat his number with either one of his two dice. What is the probability that Kim wins the game? (Note: in the case of a draw Kim is declared the winner) [3]

QUESTION 10

(a) A certain bacteria is growing according to the formula

$B = B_o e^{kt}$ where B_0 is the original population of bacteria
t is time in hours
k is a constant
And B the population of bacteria at time t

If the initial bacterial population is 10000 and doubles in size in 1 hour find:

(i) the population of bacteria after $4\frac{1}{2}$ hours

(ii) how long it will take, in hours, and minutes, for the population to reach 3 million [4]

(b) Jennifer borrowed $100000 at 9% p.a reducible over 20 years to purchase a home unit. She repays $M each month immediately after the monthly interest has been calculated and added to the loan.

(i) Write down the balance owing at the end of the first month after interest has been added and a repayment deducted.

(ii) Write down a formula for the balance owing after n repayments.

(iii) Calculate the value of $M, the monthly repayment, if the loan is repaid at the end of 20 years (answer to the nearest whole dollar).

(iv) If Mary increases her repayment in part (iii) by $100 from the first repayment, by how many months does she reduce the life of the loan? (answer to correct 1 decimal place) [8]

Question One

12

a) 7.02 (2)

b) $\frac{2}{3-\sqrt{2}} = a + b\sqrt{2}$

$\frac{2}{3-\sqrt{2}} \times \frac{3+\sqrt{2}}{3+\sqrt{2}}$

$\frac{6+2\sqrt{2}}{9-2} = \frac{6}{7} + \frac{2}{7}\sqrt{2}$

$\therefore a = \frac{6}{7},\ b = \frac{2}{7}$ (2)

c) $3x^2 - 2x - 5 = 0$

$(3x-5)(x+1) = 0$

$\therefore x = \frac{5}{3}, -1$ (2)

d) $|2x-3| \le 7$

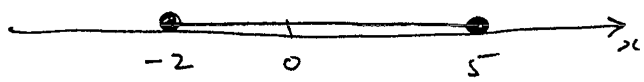
$2x - 3 \le 7 \qquad -(2x-3) \le 7$

$2x \le 7+3 \qquad -2x+3 \le 7$

$x \le 5 \qquad -4 \le 2x$

$-2 \le x$

$\therefore -2 \le x \le 5$ (2)



e) $0.4\dot{7}$

let $x = 0.4777\ldots$

$10x = 4.7777$

$9x = 4.3$

$x = \frac{43}{90}$ (2)

f)(i) $\frac{1}{P} = \frac{1}{Q} + \frac{1}{R}$

$\frac{1}{P} - \frac{1}{R} = \frac{1}{Q}$

$\frac{R-P}{PR} = \frac{1}{Q}$ (1)

$Q = \frac{PR}{R-P}$

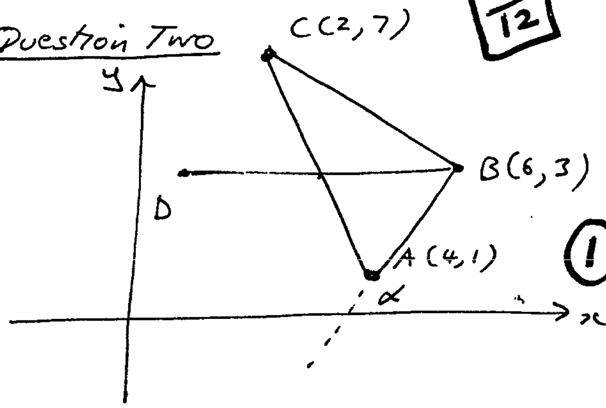
(ii) $Q = \frac{0.2 \times 0.5}{0.5 - 0.2}$ (1)

$Q = 0.\dot{3}$

Question Two

12

a)



(1)

b) Grad of AB $\Rightarrow m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3-1}{6-4}$

$m = 1$ * (1)

c) Eqn of AB $\quad y - y_1 = m(x - x_1)$

$y - 1 = 1(x - 4)$

$y = x - 3$ * $\quad x - y - 3 =$ (1)

d) $\tan\alpha = 1$

$\therefore \alpha = \frac{\pi}{4}$ or $\alpha = 45^\circ$ (1)

e) Distance AB $= \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$

$= \sqrt{(6-4)^2 + (3-1)^2}$

$= \sqrt{4+4} = \sqrt{8}$ (1)

$= 2\sqrt{2}$ units

f) Grad of BC $\quad m_2 = \frac{7-3}{2-6} = \frac{4}{-4} = -1$

Now AB ⊥ BC if $m_1 \times m_2 = -1$

$\Rightarrow -1 \times 1 = -1$ (2)

$\therefore AB \perp BC$

i.e. △ABC is right angled.

g) mid-point of AC.

$x = \frac{2+4}{2} = \frac{6}{2} = 3$ (1)

$y = \frac{7+1}{2} = \frac{8}{2} = 4. \qquad (3,4)$

h) Same mid-point for BD as AC.

Co-ords of D. $x = 3 = \frac{x_2 + 6}{2}$

$\therefore x_2 = 0$

$y = 4 = \frac{y_2 + 3}{2}$ (1)

$y_2 = 5$

$\therefore D(0,5)$

(i) Distance $BC = \sqrt{(6-2)^2 + (3-7)^2} = \sqrt{4^2 + \ldots}$

$= \sqrt{32}$

Area of rectangle $= 2\sqrt{2} \times 4\sqrt{2} = 16$ s.u

(j) $d = \left|\frac{ax_1 + by_1 + c}{\sqrt{a^2+b^2}}\right| = \left|\frac{0+0-3}{\sqrt{1^2+1^2}}\right| = \left|\frac{3}{\sqrt{2}}\right|$

$= \frac{3}{2}\sqrt{2}$ units (2)

Question Three [12]

a) (i) $y = 4x^3 - \frac{4}{x^3} = 4x^3 - 4x^{-3}$ (1)

$\frac{dy}{dx} = 12x^2 + 12x^{-4} = 12x^2 + \frac{12}{x^4}$

(ii) $y = \ln\left(\frac{x^2-5}{x+3}\right) = \ln(x^2-5) - \ln(x+3)$

$\frac{dy}{dx} = \frac{2x}{x^2-5} - \frac{1}{x+3}$

$= \frac{2x^2+6x-x^2+5}{(x^2-5)(x+3)} = \frac{x^2+6x+5}{(x^2-5)(x+3)}$ (2)

(iii) $y = x^2 . e^{\cos x}$

$\frac{dy}{dx} = 2x.e^{\cos x} + x^2. -\sin x . e^{\cos x}$

$= 2x.e^{\cos x} - \sin x . x^2 . e^{\cos x}$ (2)

b) (i) $\int \frac{dx}{3x-1} = \frac{1}{3}\int \frac{3.dx}{3x-1}$

$= \frac{1}{3}\ln(3x-1) + C$ (1)

(ii) $\int (2x+1)^9 dx.$

$= \frac{1}{10}.\frac{1}{2}.(2x+1)^{10} + C$

$= \frac{1}{20}(2x+1)^{10} + C$ (1)

c) $\int_0^1 e^{4x} . dx = \left[\frac{1}{4}e^{4x}\right]_0^1$

$= \left[\frac{1}{4}e^4 - \frac{1}{4}e^0\right]$

$= \frac{1}{4}(e^4 - 1)$ (2)

d) (i) $y = x.\sin x + \cos x$

$\frac{dy}{dx} = 1.\sin x + x.+\cos x - \sin x$

$= x.\cos x.$ (1)

(ii) $\int_{\frac{\pi}{2}}^{\pi} x.\cos x . dx$

$= \left[x.\sin x + \cos x\right]_{\frac{\pi}{2}}^{\pi}$ (from (i))

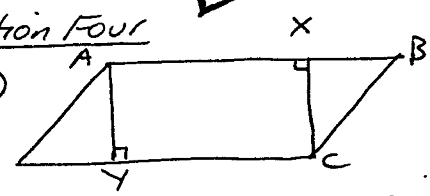
$= \left[\pi.\sin\pi + \cos\pi\right] - \left[\frac{\pi}{2}\sin\frac{\pi}{2} + \cos\frac{\pi}{2}\right]$

$= \left[\pi.0 - 1\right] - \left[\frac{\pi}{2}.1. + 0\right]$

$= -1 - \frac{\pi}{2} = -\left(1 + \frac{\pi}{2}\right)$ (2)

Question Four [12]

a)



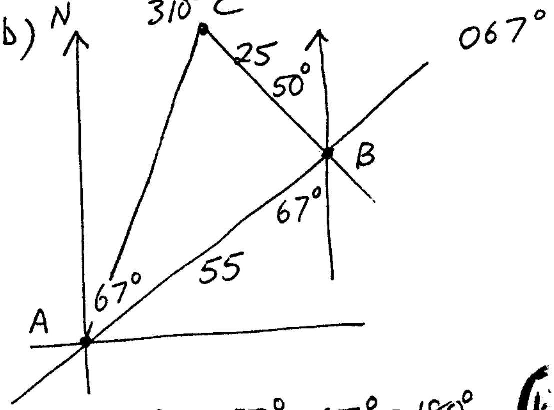
(i) R.T.P. $\triangle ADY = \triangle CBX$

1. $AD = BC$ (opp sides of parm equal)
2. $\angle ADY = \angle CBX$ (opp ∠'s of parm equal)
3. $\angle AYD = \angle CXB = 90°$ (data)

$\therefore \triangle ADY = \triangle CBX$ (AAS) (3)

(ii) $AY = CX$ (corres. sides of cong △'s)

b)



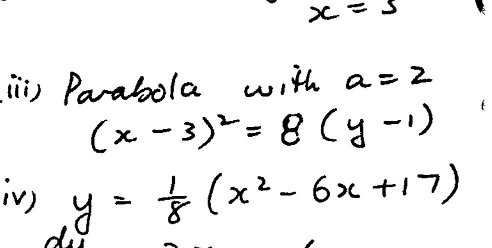
(i) $\angle ABC + 50° + 67° = 180°$

$\angle ABC = 63°$ (1)

(ii) $AC^2 = 55^2 + 25^2 - 2\times55\times25\times c$

$AC = 49$ (

c) (i)



(ii) axis of symmetry $x = 3$ (

(iii) Parabola with $a = 2$

$(x-3)^2 = 8(y-1)$

(iv) $y = \frac{1}{8}(x^2 - 6x + 17)$

$\frac{dy}{dx} = \frac{2x}{8} - \frac{6}{8}$

at $x = 0$ $\frac{dy}{dx} = -\frac{3}{4}$

$y = \frac{17}{8}$

Eqn of tangent $y - \frac{17}{8} = \frac{-3}{4}(x - c$

$y = \frac{-3}{4}x + \frac{17}{8}$ *

12

Question Five

a) $y = 3x^2 - x^3$

(i) $\frac{dy}{dx} = 6x - 3x^2$

st. pts $\frac{dy}{dx} = 0 \Rightarrow 6x - 3x^2 = 0$

$3x(2 - x) = 0$

$\therefore x = 0,\ x = 2 \qquad (0,0)$

$y = 0,\ y = 4 \qquad (2,4)$

$\frac{d^2y}{dx^2} = 6 - 6x$ (3)

at $x = 0$ $\frac{d^2y}{dx^2} > 0 \therefore$ MIN

at $x = 2$ $\frac{d^2y}{dx^2} < 0 \therefore$ MAX.

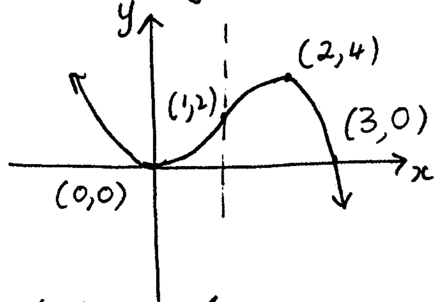
$\therefore$ (0,0) MIN (2,4) MAX

$\frac{d^2y}{dx^2} = 6 - 6x = 0 \Rightarrow x = 1$

$y = 2$

(1,2) change of concavity

(ii)



(2)

(iii) at (3,0) $\frac{dy}{dx} = 18 - 27 = -9$.

Grad of normal $= \frac{1}{9}$

Eqn of normal

$y - 0 = \frac{1}{9}(x - 3)$ (2)

$y = \frac{x}{9} - \frac{1}{3}$ *

b) $T_3 + T_4 = 24 \qquad T_5 + T_6 = 32$

$a + 2d + a + 3d = 24 \qquad a + 4d + a + 5d = 32$

$2a + 5d = 24$ —① $\qquad 2a + 9d = 32$ —②

② − ① $\Rightarrow 4d = 8$

$d = 2 \Rightarrow a = 7$

(i) 7, 9, 11 (2)

(ii) $= S_n = \frac{n}{2}\{2a + (n-1)d\}$

$S_{20} = \frac{20}{2}\{14 + (19) \times 2\}$ (1)

$= 520$

c) $4x^2 + (1 + m)x + 1 = 0$.

$b^2 - 4ac = 0 \Rightarrow (m+1)^2 - 4.4.1 = 0$

$\therefore (m+1)^2 = 16$ (2)

$m + 1 = \pm 4$

$m = 3, -5$

Question Six

a) (i) $P(\text{1st}) = \frac{5}{100} = \frac{1}{20}$ (1)

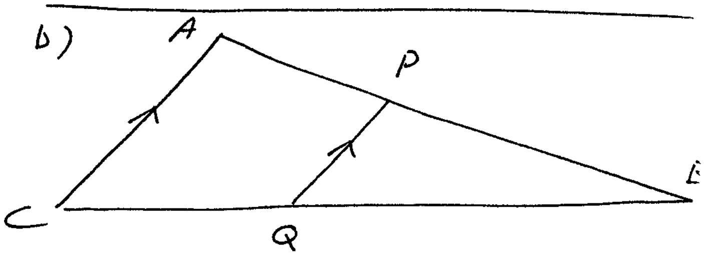
(ii) $P(\text{1st}/\text{Not Second}) = \frac{5}{100} \times \frac{95}{99}$

$= \frac{19}{396}$ (1)

(iii) $P(\text{Not 1st}/\text{2nd}) = \frac{95}{100} \times \frac{5}{99} = \frac{19}{396}$ (

(iv) $P(\text{1st}/\text{2nd}) = \frac{5}{100} \times \frac{4}{99} = \frac{1}{495}$ (

b)



(i) R.T.P. $\triangle ABC\ |||\ \triangle PBQ$

1. $\angle CAB = \angle QPB$ (corres $\angle$'s $AC \parallel PQ$)
2. $\angle ACB = \angle PQB$ (. . . .)
3. $\angle B$ is common. (3)

$\therefore \triangle ABC\ |||\ \triangle PBQ$ (equiangular).

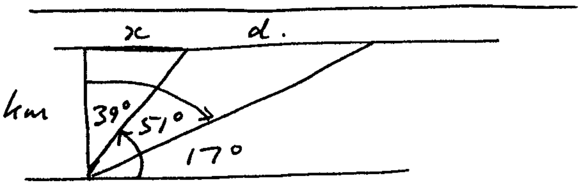
(ii) Now: $\frac{AB}{PB} = \frac{CB}{QB}$ (corres sides o[f] similar $\triangle$s a[re] in proportio[n])

$\therefore \frac{x + 8}{8} = \frac{10 + 4}{10}$ (2)

$10x + 80 = 112 \Rightarrow 10x = 32$

$x = 3.2$

c)



$\tan 39° = \frac{x}{1km} \Rightarrow x = 0.81$(

$\tan 73° = \frac{x + d}{1km} \Rightarrow x + d = 3.2$

$\therefore d = 3.27 - 0.81\ km$

$d = 2.461\ km$ in 20 secs

Speed $= 443$ km/hr (3)

$\tan 39 = \frac{x}{1} = x.$

$\tan 73° = \frac{A}{1} = A.$

$\therefore \tan 73° - \tan 39° = 2.46.$

$\therefore d = s \times t. \quad s = \frac{d}{t}.$

$s = \frac{2.46}{\frac{20/60}{60}} = 442.8$

$= 443$ km/hr. ✓

(12)

Q. SEVEN

7\. a. $2x^2 - 5x + 5 = 0 \quad \alpha, \beta.$

i. $\alpha + \beta = \frac{-b}{a} = \frac{5}{2}.$ ✓

ii. $\alpha\beta = \frac{c}{a} = \frac{5}{2}.$ ✓

iii. $\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha + \beta}{\alpha\beta} = \frac{5/2}{5/2} = 1$ ✓

iv. $a^2 + \beta^2 = (\alpha^2 + \beta^2 + 2\alpha\beta)$
$= (\alpha + \beta)^2 - 2\alpha\beta.$
$= \frac{25}{4} - \frac{10}{2} = \frac{5}{4}.$ ✓

b. $\frac{d^2x}{dt^2} = 12(t+3)^2$

$\frac{dx}{dt} = \frac{12(t+3)^3}{3} = 4(t+3)^3 + c.$

when $\frac{dx}{dt} = 0 \quad t = 0 \quad \therefore 0 = 27 \times 4 + c$
$c = -108.$ ✓

$\frac{dx}{dt} = 4(t+3)^3 - 108.$

$x = \frac{4(t+3)^4}{4} - 108t. + c.$

$x = (t+3)^4 - 108t + c.$

when $x = 0$, $t = 0$.

$0 = 3^4 + c$

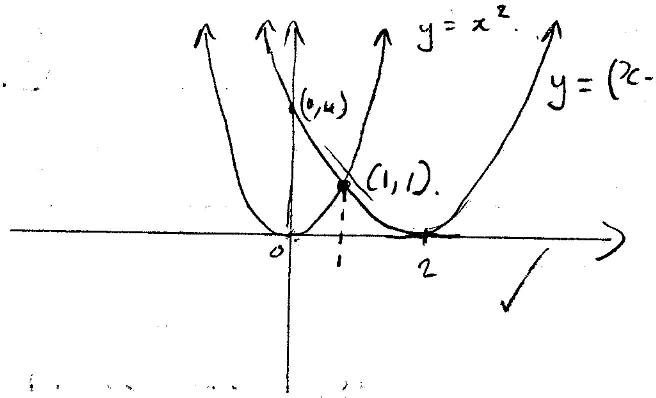
$c = -81$ ✓

$\therefore x = (t+3)^4 - 108t - 81.$

when $t = 1.$

$x = 4^4 - 108 - 81$
$= 67.$ ✓

c. i.



✓

$\therefore y = x^2, \; y = (x-2)^2.$

$x^2 = (x-2)^2$

$x^2 = x^2 - 4x + 4.$

$4x - 4 = 0$

$4x = 4.$

$x = 1.$ ✓

$\therefore y = 1$

(10)

iii. $\int_0^1 x^2 - (x-2)^2. \quad \int_0^1 x^2 dx + $

$\int_0^1 x^2 - (x^2 - 4x + 4) \quad \int_1^2 (x-2)^2$

$\int_0^1 4x - 4. \quad = \left[\frac{x^3}{3}\right]_0^1 + \left[\frac{(x-2)}{3}\right.$

$= \left[\frac{4x^2}{2} - 4x\right]_0^1$ ✗ $= \frac{1}{3} + \frac{1}{3} = \frac{2}{3}$ units

$= \left[2x^2 - 4x\right]_0^1$

$= [2 - 4] - [0]$

$= -2$

$\therefore$ Area $= 2$ ✗ since it cannot be a negative no.

QUESTION EIGHT

8. a.i. $S_n = \frac{n}{2}\left(2a + (n-1)d\right)$

$= \frac{n}{2}\left(2 + (n-1)2\right)$

$= \frac{n}{2}\left(2 + 2n - 2\right) = n^2$ ✓

ii. $S_n = \frac{n}{2}\left(2a + (n-1)d\right]$

$S_n = \frac{n}{2}\left(4 + (n-1)2\right)$

$= \frac{n}{2}\left(4 + 2n - 2\right)$

$= \frac{n}{2}\left(2 + 2n\right)$

$= n + n^2$ ✓

$= n^2 + n$

$S_{100} = S_{1000}(\text{odd}) - (S_{1000}(\text{Even}))$

iii. $= n^2 + n - (n^2) = n$.

$n = 1000$. $= n^2 - (n^2 + n)$

$= -n = \underline{-1000}$

(6)

$2 = e^x$

$\ln 2 = x$

$\therefore \int_0^{\ln 2} 2 - e^x$

$= \left[2x - \frac{1}{2}e^x\right]_0^{\ln 2}$ ✗

$= \left[2\ln 2 - \frac{2}{\ln 2}\right]$ $= \left[2\ln 2 - e^{\ln 2}\right] - [\cdot$

$= \ln 2^2 - \frac{2}{\ln 2}$ $= 2\ln 2 - 2 + 1$

units^2 $= \underline{2\ln 2 - 1}$

d= $y = 4^x$ $x = 0$, $x = 2$.

x	0	0.5	1	1.5	2
4^x	1	2.	4	8	16
$(4^x)^2$	1	4	16	64	256

✓

$A \doteqdot \frac{h}{3}($sum end + 4x first + 2x secon ordinates

$\doteqdot \frac{1/2}{3}\left(17 + 4(2) + 2(4) + 4(8)\right]$

$\doteqdot \frac{1}{6}\left(17 + 48\right)$

$\doteqdot$ ~~10.83~~ 10.8.

$Vol = \pi\int_0^2 y^2\,dx$

$= \pi\int_0^2 (4^x)^2\,dx$

$= \underline{293.7\ u^3}$

b. $S_\infty = \frac{a}{1-r}$ $|r| < 1$

$\frac{5}{8} = \frac{2}{1-r}$ or $\frac{5}{8} = \frac{2}{r-1}$

$5 - 5r = 16$ $5r - 5 = 16$

$5r = -11$ $5r = 21$

$r = \frac{-11}{5}$ ✓ $r = \frac{21}{5}$

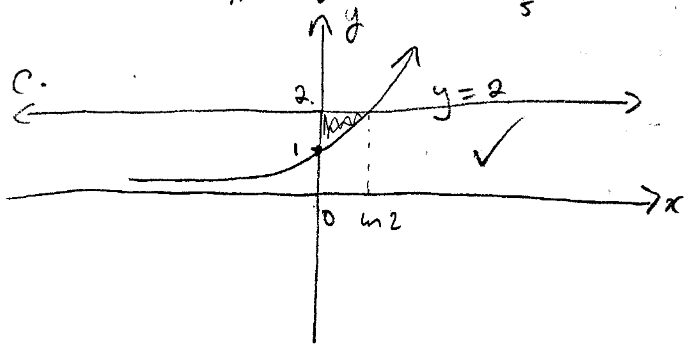
$r = -\frac{11}{5}$ since $|r| < 1$ $\therefore r = \frac{21}{5}$ ✗

$r > 1$.

No there is no limiting sum since $|r| > 1$.

$\frac{21}{5} > 1$.

c.



✓

9. i. Show $\theta = \frac{2r - 40}{r + 50}$

$l = r\theta$.

$BC = 50\theta$

$AD = r\theta$

$\therefore$ Perimeter.

$2(50 - r) + 50\theta + r\theta =$ ✓

$100 - 2r + \theta(50 + r) =$

$\theta(50 + r) = 2r - 40$

$\theta = \frac{2r - 40}{r + 50}$ ✓

✱ ii. Area of segment ABCD.

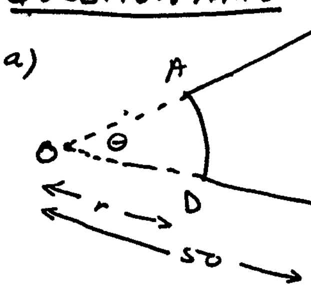
$= \frac{1}{2}r^2(\theta - \sin\theta)$

$= \frac{1}{2}\times$

Question Nine

(12)

a)



(i) $AB = 50 - r$
$CD = 50 - r$
$BC = 50\theta$
$AD = r\theta$.

$$\text{Perimeter} = 2(50-r) + \theta(r+50).$$

$$\therefore 60 = 100 - 2r + \theta(r+50)$$

$$2r - 40 = \theta(r+50)$$

$$* \quad \theta = \frac{2r-40}{r+50}$$

(2)

(ii) Area of ABCD

$$= \frac{1}{2}\theta(50^2 - r^2)$$

$$A = \frac{1}{2}\cdot\frac{2r-40}{r+50}(50-r)(50+r)$$

$$A = (r-20)(50-r)$$

$$* \quad A = 70r - 1000 - r^2$$

(2)

(iii) $A = 70r - 1000 - r^2$

$$\frac{dA}{dr} = 70 - 2r \quad \left(\text{For Max/Min put } \frac{dA}{dr} = 0\right)$$

$$\therefore 70 - 2r = 0$$

$$\Rightarrow r = 35.$$

$$\frac{d^2A}{dr^2} = -2 < 0 \quad \therefore \text{only max possible.}$$

Max area when $r = 35$

$$\text{Now } \theta = \frac{2\times35 - 40}{35+50} = \frac{30}{85}$$

$$\theta = \frac{6}{17} \text{ (radians)}$$

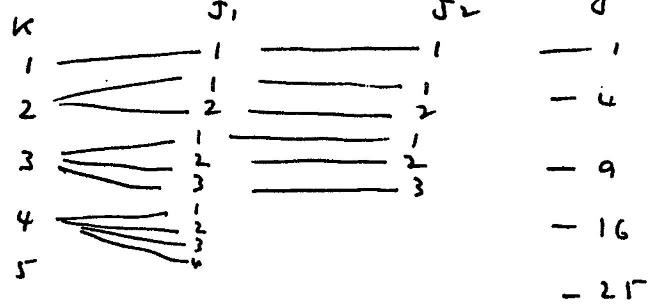
Now Maximum Area

$$A = \frac{1}{2}\cdot\frac{6}{17}(50^2 - 35^2)$$

$$A = 225\text{ cm}^2$$

(5)

b) Kim wins with a draw or larger



$$\text{Kim Wins} = \frac{91}{216}$$

(3)

Question Ten

(12)

a) $B = B_0 e^{kt}$ $\quad B_0 = 10000$

$\therefore 20000 = 10000e^k$ $\quad B = 20000 \; t =$

$$2 = e^k$$

$$k = \ln 2$$

(i) $B = 10000e^{t\ln 2}$ $\quad t = 4\frac{1}{2}$.

$$\therefore B = 10000e^{4\frac{1}{2}(\ln 2)} =$$

$$B = 226274.17$$

(2)

(ii) $3000000 = 10000e^{t\ln 2}$

$$300 = e^{t\ln 2}$$

$\therefore t = 8$ hours 14 minutes (2)

b)(i) Balance after 1 month.

$$B = \$100000 \times (1.0075) - \$M$$

(2)

(ii) $\text{Balance} = \$100000 \times (1.0075)^n - \$M\left(\frac{1.0075^n - 1}{1.0075 - 1}\right)$

(iii) for $n = 20$ Balance $= 0$.

$$\$100000\times(1.0075)^{240} = \$M\left(\frac{1.0075^{240}}{1.0075 - }\right.$$

$$\$M = \$899.73$$

$$\$M = \$900$$

(iv) Require n if $\$M = \$1000$

$$\therefore \$100000\times(1.0075)^n = \$1000\left(\frac{1.0075^n}{.007}\right.$$

$$100(1.0075)^n = \frac{(1.0075)^n - 1}{.0075}$$

$$.75(1.0075)^n = (1.0075)^n - 1.$$

$$1 = (1.0075)^n(1 - 0.$$

$$1 = (1.0075)^n(0.25)$$

$$\therefore (1.0075)^n = 4$$

$$n\ln(1.0075) = \ln 4$$

$$n = \frac{\ln 4}{\ln(1.0075)} = 185.5$$

(2)

$$\text{Months saved} = 240 - 185.5$$
$$= 55.5 \text{ months}$$