Sydney Girls High School
2013

TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics

General Instructions

- Reading Time – 5 minutes
- Working time – 3 hours
- Write using black or blue pen
  Black pen is preferred
- Board-approved calculators may be used
- A table of standard integrals is provided at the back of this paper
- In Questions 11 – 16, show relevant mathematical reasoning and/or calculations

Total marks – 100

Section I Pages 3 – 5

10 Marks

- Attempt Questions 1 – 10
- Answer on the Multiple Choice answer sheet provided
- Allow about 15 minutes for this section

Section II Pages 7 – 17

90 Marks

- Attempt Questions 11 – 16
- Answer on the blank paper provided
- Begin a new page for each question
- Allow about 2 hours and 45 minutes for this section

Name: .... Teacher:	**THIS IS A TRIAL PAPER ONLY** It does not necessarily reflect the format or the content of the 2013 HSC Examination Paper in this subject.

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Section I

10 marks

Attempt Questions 1 – 10

Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1 – 10

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(1) Which one of the following is equivalent to 3.507×10^{-3} ?

(A) 3507×1000

(B) 35.07×100

(C) $0.3507 \div 100$

(D) $3507 \div 100$

(2) $\sqrt{2} \times \sqrt{27} =$

(A) $3\sqrt{2} \times \sqrt{3}$

(B) $\sqrt{2+27}$

(C) $9\sqrt{2} \times \sqrt{3}$

(D) $\sqrt{4 \times 27 \times 27}$

(3) If A is the degree measure of an acute angle and $\sin A = 0.8$ then $\cos(90 - A) =$

(A) 0.2

(B) 0.4

(C) 0.6

(D) 0.8

(4) Which of the following is an equation whose graph is the set of points equidistant from the points $(0,0)$ and $(0,4)$?

(A) $x = 2$

(B) $y = 2$

(C) $x = 2y$

(D) $y = 2x$

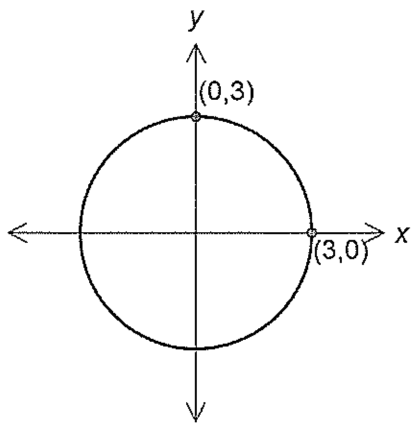
(5) Which one of the following rules describes the graph shown?

(A) $x^2 - y^2 = 9$

(B) $y^2 = 9 - x^2$

(C) $(x+y)^2 = 9$

(D) $y = \sqrt{9 - x^2}$



(6) If $\log_a b = 0.5$ then $\log_a \sqrt{b} =$

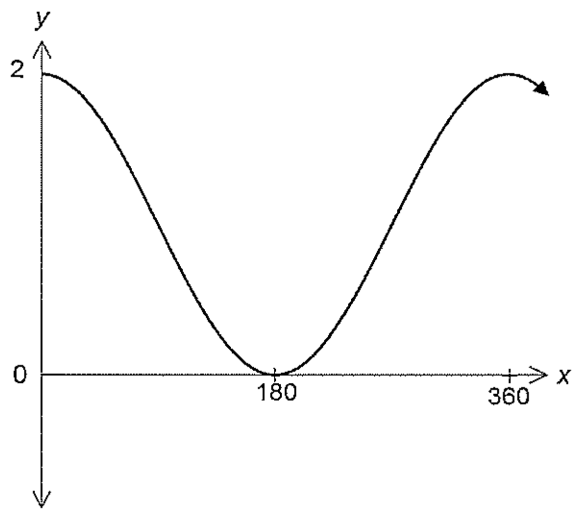
(A) 0.125

(B) 0.250

(C) 0.707

(D) 1.000

(7)



The graph above represents the equation:

(A) $y = \cos x^\circ + 1$

(B) $y = \cos x^\circ + 2$

(C) $y = 2\cos x^\circ$

(D) $y = \cos 2x^\circ$

(8) The parabola with equation $(x-3)^2 = 12y$ has:

(A) vertex = $(0,-3)$ and focus = $(3,-3)$

(B) vertex = $(-3,0)$ and focus = $(-3,3)$

(C) vertex = $(3,0)$ and focus = $(3,3)$

(D) vertex = $(0,3)$ and focus = $(3,3)$

(9) The quadratic equation with roots $2+\sqrt{3}$ and $2-\sqrt{3}$ is given by:

(A) x^2-4x-1

(B) x^2-4x+1

(C) x^2-4x+5

(D) x^2-4x-5

(10) What is the sum of the infinite geometric series: $\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\frac{1}{32}+....?$

(A) 1

(B) 2

(C) $\frac{1}{2}$

(D) $\frac{3}{2}$

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Section II

90 marks

Attempt Questions 11 – 16
Allow about 2 hours and 45 minutes for this section

Answer on the blank paper provided. Begin a new page for each question
Your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 Marks)

(a) Factorise $5y^3-40$ [2]

(b) Solve the inequality $|2x+1|\le 5$ and graph the solution on a number line. [2]

(c) Evaluate $\dfrac{x^3+y^4}{y^2}$ if $x=\left(\dfrac{2}{3}\right)^{\frac{1}{3}}$ and $y=\left(\dfrac{3}{5}\right)^{\frac{1}{2}}$. [2]
Give your answer in fractional form.

(d) Find the exact value of $\tan\dfrac{\pi}{3}+\operatorname{cosec}\dfrac{\pi}{4}$. [2]

(e) Show that $f(x)=x^5-x^3$ is an odd function. [2]

(f) Sketch the graph of $y=|8-2x|$ showing any intercepts. [2]

(g) For what values of k, will kx^2+5x+k be positive definite? [3]

End of Question 11

Question 12 (Begin a New Page) (15 Marks)

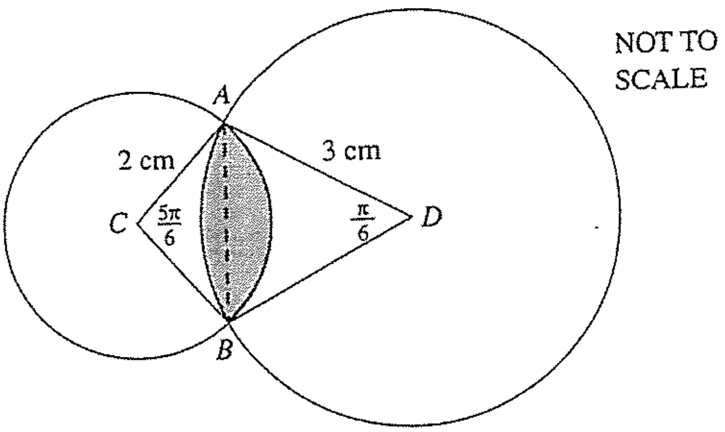
(a) Differentiate with respect to x:

i) $y=\left(e^{x}+3\right)^{4}$. [2]

ii) $f(x)=\dfrac{x^{2}}{\tan x}$. [2]

(b) Find the equation of the normal to $y=e^{\cos x}$ at the point where $x=\dfrac{\pi}{2}$. [2]

(c)



In the diagram, two circles with centres C and D intersect at A and B where $AD=3$ cm, $AC=2$ cm, $\angle ACB=\dfrac{5\pi}{6}$ and $\angle ADB=\dfrac{\pi}{6}$.

The shaded region represents the common region of the two circles.

i) Calculate the perimeter of the shaded region. [2]

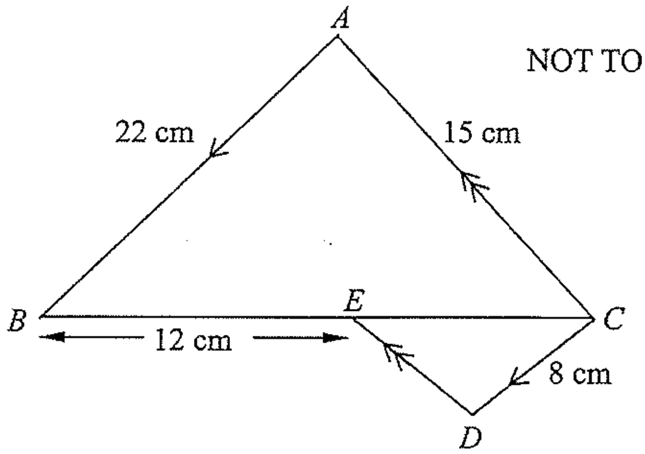
ii) Calculate the exact area of the shaded region. [3]

Question 12 continues on the next page

Question 12 (Continued)

(d) In the diagram, CD is parallel to AB and DE is parallel to CA.

$AC = 15$ cm, $AB = 22$ cm, $CD = 8$ cm and $BE = 12$ cm.



i) Prove triangle ABC is similar to triangle DCE. [2]

ii) Hence find the length of BC. [2]

End of Question 12

Question 13 (Begin a New Page) (15 Marks)

(a) Find $\int \frac{2x^2}{x^3-5}\,dx$. [2]

(b) Evaluate $\int_0^{\frac{\pi}{3}} \cos 3x\,dx$. [2]

(c) Two identical bags are placed on a table. One bag contains 5 black and 3 white balls. The other bag contains 4 black and 6 white balls. One bag is selected at random and two balls are taken from that bag without replacement.

i) Draw a tree diagram to show all possible outcomes. [2]

ii) Find the probability that the two balls are different colours. [2]

(d) Paint cans are stacked such that there are 38 cans on the bottom row, 35 cans on the next row and 32 on the next row and so on until a total of 253 cans are stacked.

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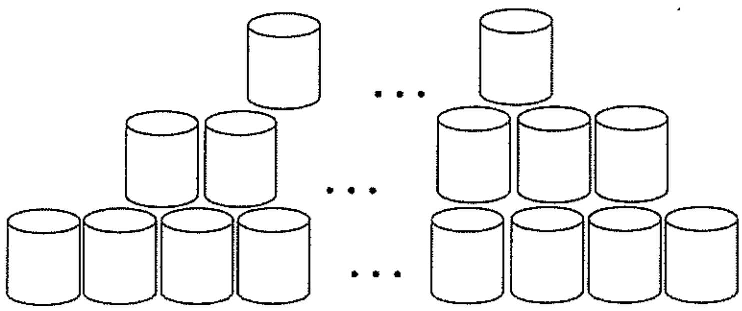


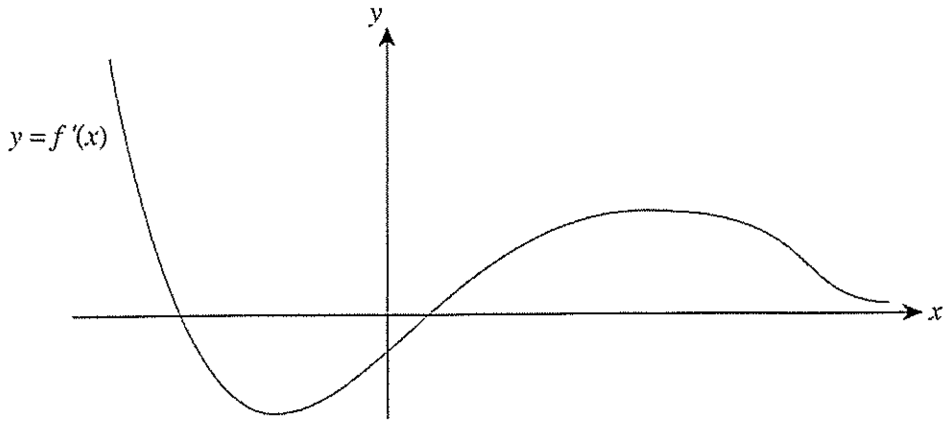
i) Write down a formula for the number of cans in the nth row. [1]

ii) How many rows are there in this stack? [2]

iii) How many cans are there in the final row of this stack? [1]

Question 13 continues on the next page

(e) The diagram shows the gradient function $y = f'(x)$.
Copy or trace the diagram into your answer booklet.



The curve $y = f(x)$ passes through the origin. [3]

Sketch the function $y = f(x)$ on the same set of axes.

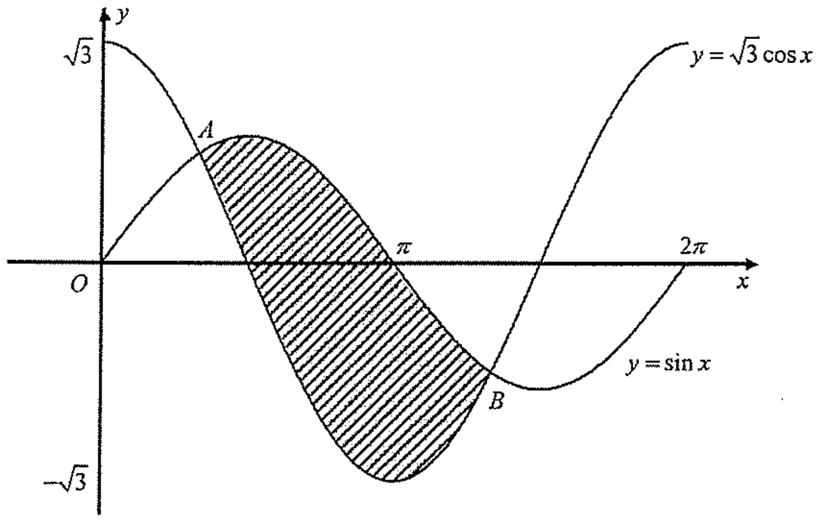
Clearly indicate any turning points, points of inflexion, and the behaviour of the graph for very large positive and negative values of x.

End of Question 13

Question 14 (Begin a New Page) (15 Marks)

(a) Solve the following equation: $\log_2 x + \log_2(x+7) = 3$, for $x > 0$. [3]

(b)



The diagram shows the graphs $y = \sin x$ and $y = \sqrt{3}\cos x,\ 0 \le x \le 2\pi$.
The graphs intersect at points A and B.

i) Show that point A has coordinates $\left(\dfrac{\pi}{3}, \dfrac{\sqrt{3}}{2}\right)$ and find the coordinates of B. [2]

ii) Find the area enclosed by the two graphs. [3]

(c) The probability that Jill will pass a Maths test is 0.8, an English test is 0.7 and a Science test is 0.9.
When she sits for the three tests, find the probability that Jill passes:

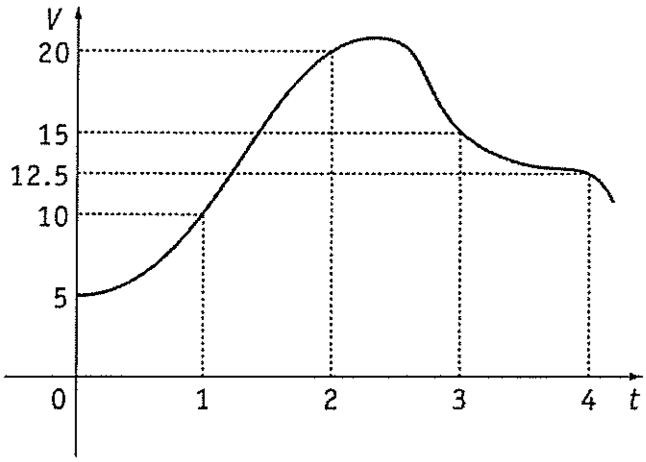
i) Exactly one of the three tests. [2]

ii) At least one of the three tests. [2]

Question 14 continues on the next page

Question 14 (Continued)

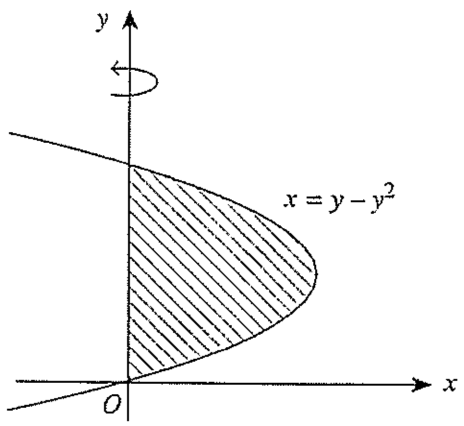
(d) Use Simpson's Rule with 5 function values to approximate the area enclosed by the curve $y = V(t)$, the t-axis and the lines $t = 0$ and $t = 4$. [3]



End of Question 14

Question 15 (Begin a New Page) (15 Marks)

(a) The diagram shows the graph of $x = y - y^2$



NOT TO SCALE

The shaded region, bounded by $x = y - y^2$ and the y-axis, is rotated about the y-axis to form a solid. Find the volume of this solid. [3]

(b) Alice has borrowed \$17000 to buy a new car. The interest on the loan is 18% per annum paid monthly. The loan is to be repaid in equal monthly instalments of $\$P$ over a term of 5 years.

Let the amount owing on the loan after n months be $\$A_n$.

i) Show that the amount $\$A_1$ owing after one month is given by: [1]

$\$A_1 = (17000 \times 1.015) - P.$

ii) Show that the amount $\$A_3$ owing after three months is given by: [2]

$\$A_3 = (17000 \times 1.015^3) - P(1 + 1.015 + 1.015^2).$

iii) Write down a similar expression for the amount owing after 5 years. [1]

iv) Calculate the monthly instalment $\$P$ paid on the loan. [2]

v) How much would Alice have saved by paying cash for the car? [1]

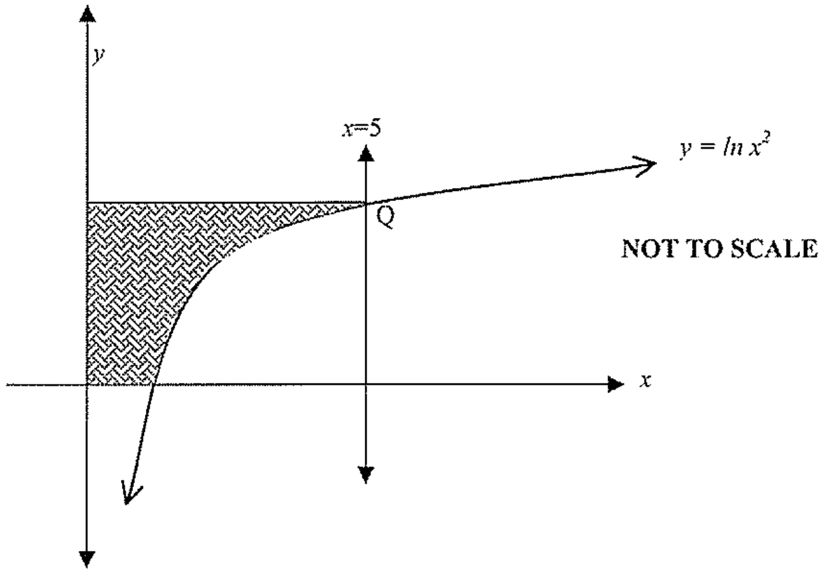
Question 15 continues on the next page

Question 15 (Continued)

(c) i) Show that $\frac{d}{dx}\left(x\ln x - x\right) = \ln x.$ [1]

ii) Hence, or otherwise, find $\int \ln x^2 dx.$ [1]

iii) The graph shows the curve $y = \ln x^2, (x > 0)$ which meets the line $x = 5$ at Q. Using your answers from i) and ii), or otherwise, find the area of the shaded region. [3]

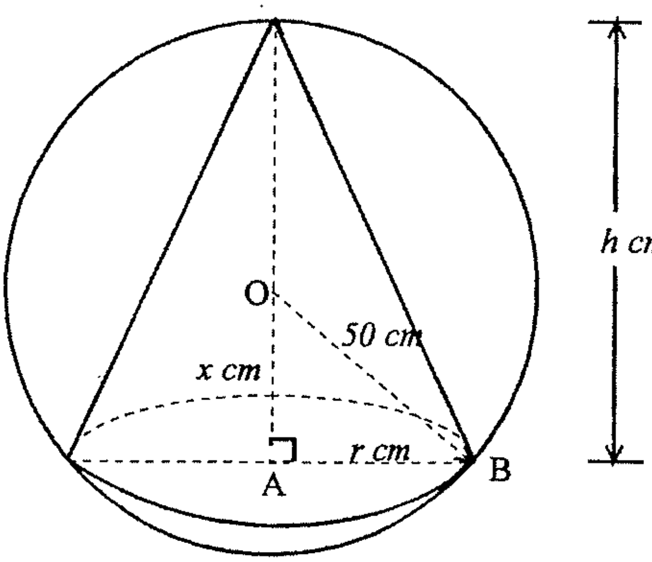


End of Question 15

Question 16 (Begin a New Page) (15 Marks)

(a) Evaluate: $\sum_{x=0}^{4}\left(\sin\frac{\pi x}{4}\right)$. [2]

(b)



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The diagram shows a cone of base radius r cm and height h cm inscribed in a sphere of radius 50 cm.
The centre of the sphere is O and $\angle OAB = 90°$. Let OA = x cm.

i) Show that $r=\sqrt{2500-x^2}$. [1]

ii) Hence show that the volume, V cm^3, of the cone is given by: [2]

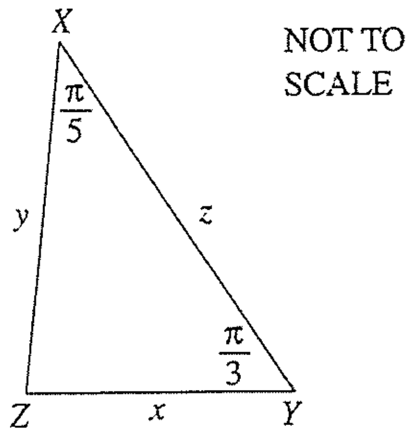
$$V=\frac{\pi}{3}(2500-x^2)(50+x).$$

iii) Find the radius of the largest cone which can be inscribed in the sphere. [3]
(Give your answer to the nearest mm.)

Question 16 continues on the next page

Question 16 (Continued)

(c)



In the diagram, XYZ is a triangle with $\angle ZXY = \frac{\pi}{5}$ and $\angle XYZ = \frac{\pi}{3}$.

The area of the triangle is A.

i) Show that $\sin\frac{\pi}{5} = \frac{2A}{yz}$. [1]

ii) Use the cosine rule to express $\cos\frac{\pi}{5}$ in terms of x, y and z. [1]

iii) Hence, show that $\cot\frac{\pi}{5} = \frac{y^2 + z^2 - x^2}{4A}$. [2]

iv) Deduce an expression for A in terms of z, $\cot\frac{\pi}{5}$ and $\cot\frac{\pi}{3}$. [3]

End of paper

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STANDARD INTEGRALS

$$\int x^n \, dx = \frac{1}{n+1} x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x} \, dx = \ln x, \quad x > 0$$

$$\int e^{ax} \, dx = \frac{1}{a} e^{ax}, \quad a \neq 0$$

$$\int \cos ax \, dx = \frac{1}{a} \sin ax, \quad a \neq 0$$

$$\int \sin ax \, dx = -\frac{1}{a} \cos ax, \quad a \neq 0$$

$$\int \sec^2 ax \, dx = \frac{1}{a} \tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax \, dx = \frac{1}{a} \sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2 + x^2} \, dx = \frac{1}{a} \tan^{-1} \frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} \, dx = \sin^{-1} \frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} \, dx = \ln\left(x + \sqrt{x^2 - a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} \, dx = \ln\left(x + \sqrt{x^2 + a^2}\right)$$

NOTE : $\ln x = \log_e x, \quad x > 0$

Sydney Girls High School Mathematics Faculty

Multiple Choice Answer Sheet –Trial HSC 2013
Mathematics

Student Number: MULITIPLE CHOICE ANSWERS .

Completely fill the response oval representing the most correct answer.

1.	A ○	B ○	C ●	D ○
2.	A ●	B ○	C ○	D ○
3.	A ○	B ○	C ○	D ●
4.	A ○	B ●	C ○	D ○
5.	A ○	B ●	C ○	D ○
6.	A ○	B ●	C ○	D ○
7.	A ●	B ○	C ○	D ○
8.	A ○	B ○	C ●	D ○
9.	A ○	B ●	C ○	D ○
10.	A ○	B ○	C ●	D ○

Question 11:

a) $5y^3 - 40$

$= 5(y^3 - 8)$

$= 5(y^3 - 2^3)$

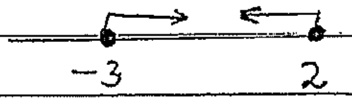
$= 5(y-2)(y^2 + 2y + 4)$

b) $|2x+1| \le 5$

$-5 \le 2x + 1 \le 5$

$-6 \le 2x \le 4$

$-3 \le x \le 2$



c) $\dfrac{x^3 + y^4}{y^2} = \dfrac{\left[\left(\frac{2}{3}\right)^{\frac{1}{3}}\right]^3 + \left[\left(\frac{3}{5}\right)^{\frac{1}{2}}\right]^4}{\left[\left(\frac{3}{5}\right)^{\frac{1}{2}}\right]^2}$

$= \dfrac{\frac{2}{3}}{\frac{3}{5}} + \dfrac{3}{5}$

$= \dfrac{77}{45}$

d) $\tan\dfrac{\pi}{3} + \operatorname{cosec}\dfrac{\pi}{4} - \tan 60^\circ + \dfrac{1}{\sin 45^\circ}$

$= \sqrt{3} + \sqrt{2}$

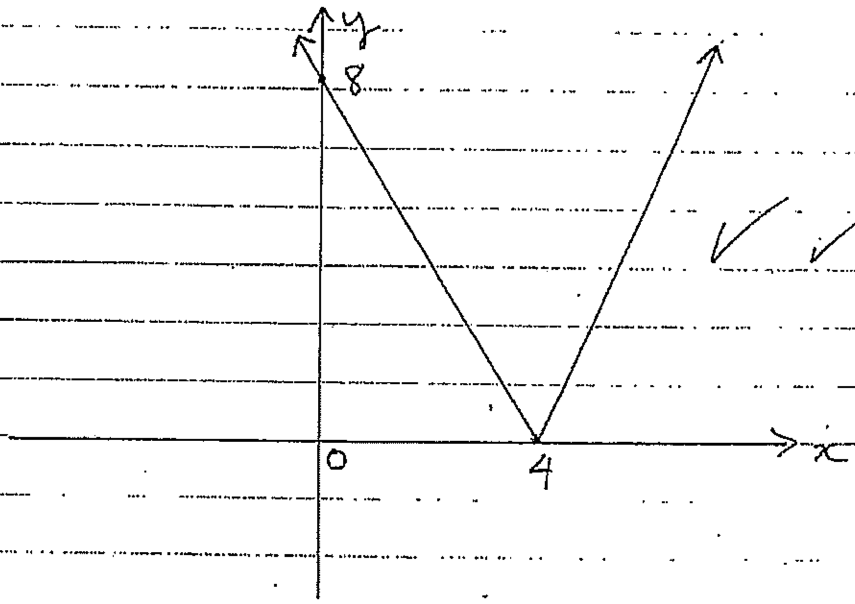
e) $f(x) = x^5 - x^3$

$f(-x) = (-x)^5 - (-x)^3$

$= -x^5 + x^3$

$= -(x^5 - x^3) = -f(x)$ $\therefore$ odd function f). $y = |8 - 2x|$

- $8 - 2x > 0 \quad \therefore x < 4 \quad \therefore y = -2x + 8$
- $8 - 2x < 0 \quad \therefore x > 4 \quad \therefore y = 2x - 8$



g) $kx^2 + 5x + k$

$\Delta = 5^2 - 4(k)(k)$

$\Delta = 25 - 4k^2$

- positive definite if $\begin{cases} a > 0 \\ \Delta < 0 \end{cases}$

$a = k$

$\Delta = 25 - 4k^2$

$\begin{cases} k > 0 \\ 25 - 4k^2 < 0 \end{cases} \quad \therefore \begin{cases} k > 0 \\ -\left(k + \frac{5}{2}\right)\left(k - \frac{5}{2}\right) < 0 \end{cases}$

$\therefore k > \frac{5}{2}$

Question 12

a)

i) $y = (e^x + 3)^4$

$y' = 4(e^x+3)^3(e^x)$ ✓

$= 4e^x(e^x+3)^3$ ✓

ii) $f(x) = \dfrac{x^2}{\tan x}$

$f'(x) = \dfrac{2x\tan x - x^2\sec^2 x}{\tan^2 x}$ ✓

$= \dfrac{x(2\tan x - x\sec^2 x)}{\tan^2 x}$ ✓

OR $2x\cot x - x^2\operatorname{cosec}^2 x$

b) $y = e^{\cos x}$ when $x = \frac{\pi}{2}$ $\therefore$ $y = 1$

$y' = -\sin x\, e^{\cos x}$ ✓

At $x = \frac{\pi}{2}$ $\therefore$ $y' = -1 . e^0 = -1 = m_1$

$m_1 \times m_2 = -1$ (m_2 = gradient of the Normal)

$m_2 = \frac{-1}{-1} = 1$

$y - y_1 = m_2(x - x_1)$

$y - 1 = 1\left(x - \frac{\pi}{2}\right)$

$y = x + 1 - \frac{\pi}{2}$ ✓

c) i) $l = \theta \times r$

Arc length of the smaller circle

$$l_{small} = \frac{5\pi}{6} \times 2 = \frac{5\pi}{3}$$

Arc length of the larger circle

$$l_{Large} = \frac{\pi}{6} \times 3 = \frac{\pi}{2}$$

Perimeter of the shaded $= l_{small} + l_{Large}$

$$= \frac{5\pi}{3} + \frac{\pi}{2} \checkmark$$

$$= \frac{13\pi}{6} \text{ cm} \checkmark$$

ii) ~~$A =$ Segment (small circle) + Segment (Larger circle)~~

$$A = \frac{1}{2} \times 2^2 \times \left[\frac{5\pi}{6} - \sin\frac{5\pi}{6}\right] + \frac{1}{2} \times 3^2 \left[\frac{\pi}{6} - \sin\frac{\pi}{6}\right]$$

$$A = 2\left(\frac{5\pi}{6} - \sin\frac{5\pi}{6}\right) + \frac{9}{2}\left[\frac{\pi}{6} - \sin\frac{\pi}{6}\right] \checkmark$$

$$= 2\left(\frac{5\pi}{6} - \frac{1}{2}\right) + \frac{9}{2}\left[\frac{\pi}{6} - \frac{1}{2}\right]$$

$$= \frac{29\pi - 39}{12} \text{ cm}^2 \checkmark \checkmark$$

d)

i) Prove $\triangle ABC \ ||| \ \triangle DCE$

$\angle ABC = \angle DCE$ (Alternate angles $AB \parallel CD$)

$\angle ACB = \angle CED$ (Alternate angles $AC \parallel DE$)

Thus $\angle BAC = \angle CDE$ (Sum angles of a Triangle)

Hence $\triangle ABC \ ||| \ \triangle DCE$ (AAA)

ii) $\dfrac{EC}{12 + EC} = \dfrac{DC}{AB}$

$$\frac{EC}{12+EC} = \frac{8}{22} = \frac{4}{11}$$

$$11\,EC = 4(12 + EC)$$

$$11\,EC = 48 + 4\,EC$$

$$7EC = 48$$

$$EC = \frac{48}{7}$$

Thus $BC = 12 + \dfrac{48}{7} = \dfrac{132}{7}$ cm

Q.13

a) $\int \frac{2x^2}{x^3-5}\,dx$

$= \frac{2}{3}\ln(x^3-5)+c$

b) $\int_0^{\pi/3} \cos 3x\,dx$

$= \left[\frac{1}{3}\sin 3x\right]_0^{\pi/3}$

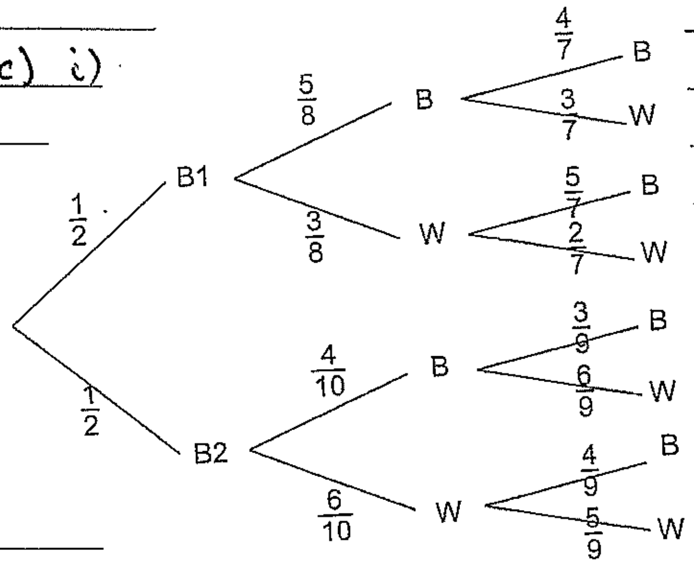
$= \frac{1}{3}\left[\sin 3\times\frac{\pi}{3} - \sin 0\right]$

$= \frac{1}{3}\left[\sin \pi - \sin 0\right]$

$= \frac{1}{3}\left[0 - 0\right]$

$= 0$

c) i)



ii) P(different collours) $= \frac{1}{2}\times\frac{5}{8}\times\frac{3}{7} + \frac{1}{2}\times\frac{3}{8}\times\frac{5}{7} + \frac{1}{2}\times\frac{4}{10}\times\frac{6}{9}$

$+ \frac{1}{2}\times\frac{6}{10}\times\frac{4}{9} = \frac{449}{840} = 0.5345...$

13 d) i) $38, 35, 32, \ldots$ $\therefore a = 38$

$d = -3$

$T_n = a + (n-1)d$

$= 38 + (n-1)(-3)$

$T_n = 41 - 3n$

ii) $S_n = \frac{n}{2}[2a + (n-1)d]$

$253 = \frac{n}{2}[2 \times 38 + (n-1)(-3)]$

$506 = n[76 - 3n + 3]$

$\therefore 3n^2 - 79n + 506 = 0$

$(3n - 46)(n - 11) = 0$

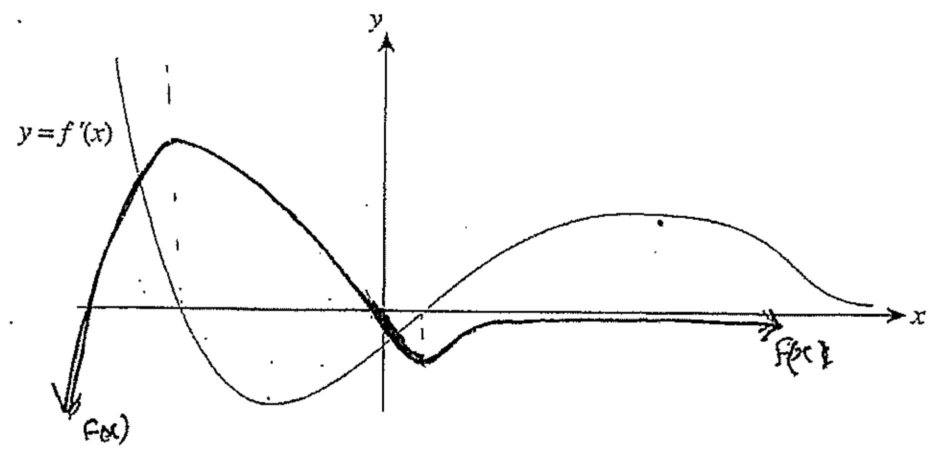
$n = \frac{46}{3}$ x or $n = 11$

$\therefore n = 11$

iii) $T_{11} = 41 - 3 \times 11$

$= 8$ cans.

e)



Q_{14}.

a) $\log_2 x + \log_2 (x+7) = 3$

$\therefore \log_2 x(x+7) = 3$

$2^3 = x(x+7)$

$8 = x^2 + 7x$

$x^2 + 7x - 8 = 0$

$(x+8)(x-1) = 0$

$x = -8 \quad \text{or} \quad x = 1$

we reject $x = -8$ as $x > 0$

b) i) $\sqrt{3} \cos x = \sin x$

$\therefore \frac{\sin x}{\cos x} = \sqrt{3}$

$\tan x = \sqrt{3}$

tan +ve in 1st and 3rd Quadrant.

$\therefore x = \frac{\pi}{3}$ and $\pi + \frac{\pi}{3}$

$x = \frac{\pi}{3}$ and $\frac{4\pi}{3}$

at $x = \frac{\pi}{3}$ $y = \sin \frac{\pi}{3}$

$y = \frac{\sqrt{3}}{2}$

$\therefore A$ is $\left(\frac{\pi}{3}, \frac{\sqrt{3}}{2}\right)$, $\quad y = \sin \frac{4\pi}{3}$

$= -\frac{\sqrt{3}}{2}$

$\therefore B$ is $\left(\frac{4\pi}{3}, -\frac{\sqrt{3}}{2}\right)$

Q14

b) ii) $$\text{Area} = \int_{\pi/3}^{4\pi/3} (\sin x - \sqrt{3}\cos x)\, dx$$

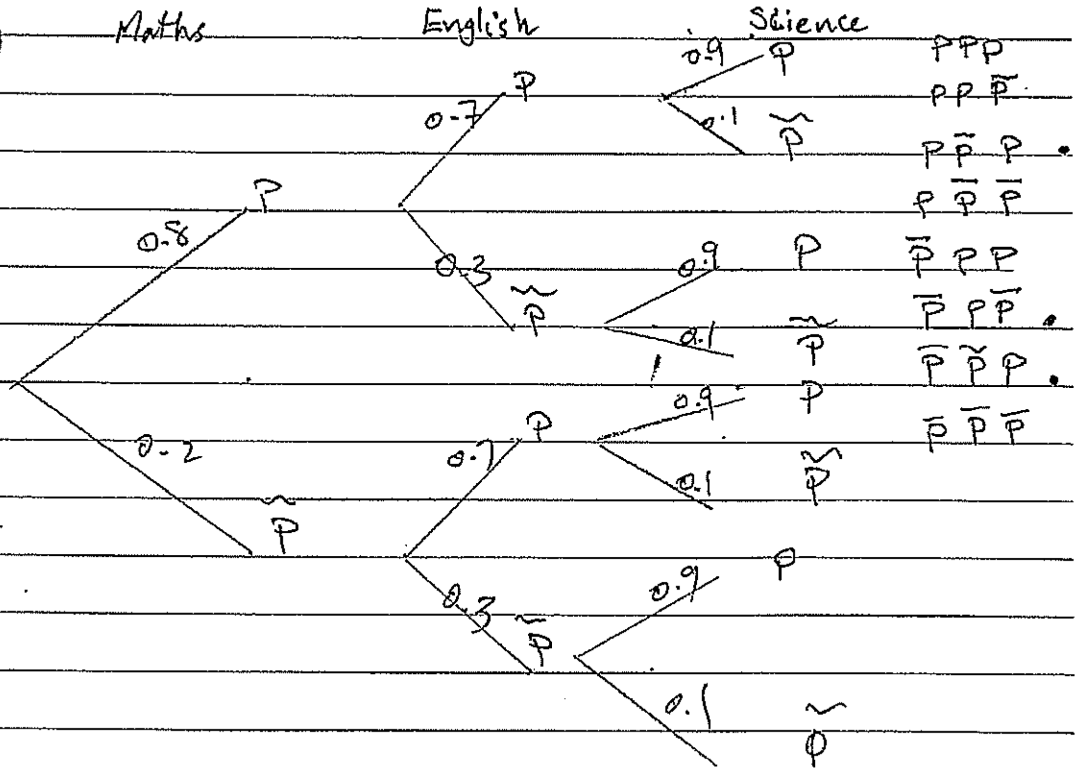
$$= \Big[-\cos x - \sqrt{3}\sin x\Big]_{\pi/3}^{4\pi/3}$$

$$= \left(-\cos\left(\tfrac{4\pi}{3}\right) - \sqrt{3}\sin\left(\tfrac{4\pi}{3}\right)\right) - \left(-\cos\left(\tfrac{\pi}{3}\right) - \sqrt{3}\sin\left(\tfrac{\pi}{3}\right)\right)$$

$$= \left(-\left(-\tfrac{1}{2}\right) - \sqrt{3}\left(\tfrac{-\sqrt{3}}{2}\right)\right) - \left(-\tfrac{1}{2} - \sqrt{3} \times \tfrac{\sqrt{3}}{2}\right)$$

$$= 4\ u^2$$

14) c)



i) P(Pass exactly one of the three test)

$$= P\tilde{P}\bar{P} + \tilde{P}P\tilde{P} + \tilde{P}\tilde{P}P$$

$$= 0.8 \times 0.3 \times 0.1 + 0.2 \times 0.7 \times 0.1 + 0.2 \times 0.3 \times 0.9$$

$$= \frac{23}{250} \text{ or } 0.092$$

14 c) II

$P(\bar{P}\bar{P}\bar{P}) = 0.2 \times 0.3 \times 0.1$

$= \frac{3}{500}$ or 0.006

$P(\text{at least one pass}) = 1 - P(\bar{P}\bar{P}\bar{P})$

$= 1 - 0.006$

$= \frac{497}{500}$ or 0.994 ...

Q14 d)

t	V	W	W x V
0	5	1	5
1	10	4	40
2	20	2	40
3	15	4	60
4	12.5	1	12.5
		Total	157.5

$\text{Area} = \frac{1}{3} \times 157.5$

$= 52.5 \ u^2$

Trial Exam

Question 15

a) $V = \pi \int_a^b x^2 \, dy$

$x = y - y^2$

$x = y(1-y)$

$\therefore$ cuts y-axis at $y=0$ and $y=1$

$V = \pi \int_0^1 (y-y^2)^2 \, dy$ 1

$= \pi \int_0^1 y^2 - 2y^3 + y^4 \, dy$

$= \pi \left[\frac{y^3}{3} - \frac{2y^4}{4} + \frac{y^5}{5} \right]_0^1$

$= \pi \left[\frac{y^3}{3} - \frac{y^4}{2} + \frac{y^5}{5} \right]$ 1

3

$= \pi \left[\left(\frac{1}{3} - \frac{1}{2} + \frac{1}{5} \right) - 0 \right]$

$= \frac{\pi}{30}$ units^3 1

b) $r = 18\%$ p.a $\div 12$ $P = 17000$

$r = 1.5\%$ per month

$r = 1.015$

$n = 5$ years $\times 12$

$n = 60$ months.

i) $A = P\left(1 + \frac{r}{100}\right)^n$

$A_1 = 17000\left(1 + \frac{1.5}{100}\right)^1 - \$P$ 1

$A_1 = 17000(1.015) - \$P$

ii) $A_2 = A_1 \times 1.015 - \$P$

$= \left[17000(1.015) - \$P\right] \times 1.015 - \$P$

$= 17000(1.015)^2 - \$P(1 + 1.015)$ 1

$A_3 = A_2 \times 1.015 - \$P$ (2)

$= \left[17000(1.015)^2 - \$P(1 + 1.015)\right] \times 1.015 - \$P$

$= 17000(1.015)^3 - \$P(1 + 1.015 + 1.015^2)$ 1

$\vdots$

iii) $A_n = 17000(1.015)^n - \$P(1 + 1.015 + 1.015^2 + \ldots + 1.015^{n-1})$

$A_{60} = 17000(1.015)^{60} - \$P(1 + 1.015 + 1.015^2 + \ldots + 1.015^{59})$ (1)

iv) $A_{60} = 0$

$\$P(1 + 1.015 + 1.015^2 + \ldots + 1.015^{59}) = 17000(1.015)^{60}$.

$$\$P = \frac{17000(1.015)^{60}}{\underbrace{1 + 1.015 + 1.015^2 + \ldots + 1.015^{59}}_{G.S.}}$$

$a = 1$

$r = 1.015$

$n = 60$.

$S_n = \frac{a(r^n - 1)}{r - 1}$

$S_{60} = \frac{1(1.015^{60} - 1)}{1.015 - 1}$

$= \frac{1.015^{60} - 1}{0.015}$

$\therefore \$P = 17000 \times (1.015)^{60} \times \frac{0.015}{1.015^{60} - 1}$ 1 (2)

$\$P = \$431.69$ per month. 1

v) Total Paid $= 431.69 \times 60$

$= \$25\,901.30$ (1)

$\therefore$ If paid cash saved $= 25901.30 - 17000$

$= \$8901.30$

c) i) let $y = x\ln x - x$

$u = x, \quad v = \ln x$

$u' = 1 \quad v' = \frac{1}{x}$

$\therefore \frac{dy}{dx} = \ln x + 1 - 1$

$= \ln x$ 1

$\therefore \frac{d}{dx}(x\ln x - x) = \ln x$

ii) $\int \ln x^2 \, dx = 2\int \ln x \, dx$

$= 2[x\ln x - x] + c$ 1

$= 2x\ln x - 2x + c$

iii) Shaded Area = Rectangle $- \int_1^5 \ln x^2 \, dx$

$= [5 \times \ln 5^2] - 2\int_1^5 \ln x \, dx$ 1

$= [5 \times 2\ln 5] - 2[x\ln x - x]_1^5$

$= 10\ln 5 - 2[5\ln 5 - 5 - (\ln 1 - 1)]$

$= 10\ln 5 - 2[5\ln 5 - 5 - 0 + 1]$ 1

$= 10\ln 5 - 2[5\ln 5 - 4]$

$= 10\ln 5 - 10\ln 5 + 8$

$= 8$ units^3. 1

OR/

3

iii) $\text{Area} = \int_0^{\ln 5^2} x \, dy$

$y = \ln x^2$

$y = a^x$

$\log_a y = x$

$= \int_0^{2\ln 5} e^{1/2 y} \, dy$ 1

$x^2 = e^y$

$x = \sqrt{e^y}$

$= (e^y)^{1/2}$

$= e^{1/2 y}$

$= 2e^{1/2 y} \Big]_0^{2\ln 5}$

$= 2e^{\cancel{1/2(2\ln 5)}} - 2e^0$ 1

$= 2e^{\ln 5} - 2$

$= 10 - 2$

$= 8 \text{ units}^3.$ 1

3

Question 16

a) $\sum_{x=0}^{4} \left(\sin \frac{\pi x}{4}\right) = \sin(0) + \sin\frac{\pi}{4} + \sin\frac{\pi}{2} + \sin\frac{3\pi}{4} + \sin\pi$ ✓

$= 0 + \frac{1}{\sqrt{2}} + 1 + \frac{1}{\sqrt{2}} + 0$

$= 1 + \frac{2}{\sqrt{2}}$ or $1+\sqrt{2}$ ✓

b) i) In $\triangle OAB$

$x^2 + r^2 = 50^2$

$r = \sqrt{2500 - x^2}$ ✓

ii) $V = \frac{\pi}{3} r^2 h$ ✓

$h = x + OB$

$= x + 50$ ✓

$\therefore V = \frac{\pi}{3}(2500 - x^2)(50 + x)$

iii) $V = \frac{\pi}{3}[125000 + 2500x - 50x^2 - x^3]$

$\frac{dV}{dx} = \frac{\pi}{3}[2500 - 100x - 3x^2]$ ✓

for a max $\frac{dV}{dx} = 0$

ie $2500 - 100x - 3x^2 = 0$

$3x^2 + 100x - 2500 = 0$

$(3x - 50)(x + 50) = 0$

$x = \frac{50}{3}$ or $x = -50$ (not a soln)

when $x = \frac{50}{3}$ ✓

x	$\frac{50}{3}^-$	$\frac{50}{3}$	$\frac{50}{3}^+$
$\frac{dv}{dx}$	+	0	−

$\therefore$ max at $x = \frac{50}{3}$ ✓

and $r = \sqrt{2500 - \left(\frac{50}{3}\right)^2}$

$= 47.14$ cm

c) i) $A = \frac{1}{2} ab \sin C$

$= \frac{1}{2} yz \sin \frac{\pi}{5}$

$\sin \frac{\pi}{5} = \frac{2A}{yz}$

ii) $\cos \frac{\pi}{5} = \frac{y^2 + z^2 - x^2}{2yz}$

iii) $\cot \frac{\pi}{5} = \frac{\cos \frac{\pi}{5}}{\sin \frac{\pi}{5}}$

$= \frac{y^2 + z^2 - x^2}{2\cancel{yz}} \times \frac{\cancel{yz}}{2A}$

$= \frac{y^2 + z^2 - x^2}{4A}$ ①

iv In a similar manner to iii) above

$\cot \frac{\pi}{3} = \frac{z^2 + x^2 - y^2}{4A}$ ②

① + ② $\cot \frac{\pi}{5} + \cot \frac{\pi}{3} = \frac{\cancel{2}z^2}{2\cancel{4}A}$

$\cot \frac{\pi}{5} + \cot \frac{\pi}{3} = \frac{z^2}{2A}$

$A = \frac{z^2}{2\left(\cot \frac{\pi}{5} + \cot \frac{\pi}{3}\right)}$

Sydney Girls High School

Mathematics Faculty

Years 12 HSC Mathematics Advanced

2013 TRIAL

Question	Marker's Comment
11	f) This kind of question is very common in the HSC exam. You should divide into two cases when 8 - 2x > 0 and 8 – 2x < 0 and show all intercepts. g) Many students forgot the conditions for 'positive definite': a>0 and $\Delta < 0$, not $\Delta > 0$.
12	b) If the question asks to find the equation of the Normal. You should have worked out the gradient of the Normal first before using the point gradient formula: $y - y_1 = m_N(x - x_1)$. Some students used the gradient of the Tangent instead. Be careful!
13	e) Some students forgot to sketch the graph through the origin. Sketching the ends of the graph, as *x* gets large, was not done very well.
14	b) This is a typical HSC question. Some students had problems with finding the limits to find the area enclosed by the two graphs d) Some students used the wrong formula or used the wrong weights for Simpson's Rule. This is also a typical HSC question. The correct formula must be known.
15	a) Some students didn't find correct y-intercepts, y=0 and y=1 and didn't expand correctly. b) i) A few students just rewrote question and didn't 'show' the result. The rest of the parts were completed very well except for a few students in part iii) where the amount after 5 years was A_{60} not A_5. c) Was completed very poorly overall and most students lost their marks in this section.
16	b)ii) Many students failed to explain that the height of the cone was 'x plus the radius of the cylinder'. c) This question was well done with many students correctly answering all four parts. • Students need to be careful about multiple attempts. The general rule is you don't get two bites at the cherry if the attempts are in conflict with each other. Leave your best attempt and cross out the others. • Note that marks (not full) can be awarded for work crossed out. • Under no circumstances use liquid paper.