SYDNEY GIRLS HIGH SCHOOL

HIGHER SCHOOL CERTIFICATE

TRIAL EXAMINATION

Mathematics 2014

General Instructions

- Reading time – 5 minutes
- Working time – 3 hours
- Write using black or blue pen
- Board-approved calculators may be used
- A table of standard integrals is provided at the back of this paper which can be detached and used throughout the paper

Name:...

Teacher:..

Total marks – 100

SECTION I - 10 marks

- Attempt Questions 1–10
- Answer on the Multiple Choice answer sheet provided
- Allow about 15 minutes for this section

SECTION II - 90 marks

- Attempt Questions 11–16
- Answer on the blank paper provided
- Allow about 2 hours and 45 minutes for this section

This is a Trial paper ONLY.
It does not necessarily reflect the format or the contents of the 2014 HSC Examination Paper in this subject.

Section I

10 marks
Attempt Questions 1–10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1–10.

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1 Which of the following is equal to $\dfrac{1}{2\sqrt{3}+\sqrt{2}}$?

(A) $\dfrac{2\sqrt{3}-\sqrt{2}}{14}$

(B) $\dfrac{2\sqrt{3}+\sqrt{2}}{14}$

(C) $\dfrac{2\sqrt{3}-\sqrt{2}}{10}$

(D) $\dfrac{2\sqrt{3}+\sqrt{2}}{10}$

2 The fourth term of an arithmetic series is 27 and the seventh term is 12.
What is the common difference?

(A) 5

(B) -5

(C) 13

(D) -13

3 The quadratic equation $x^2-3x+5=0$ has roots α and β.
What is the value of $2\alpha^2\beta+2\alpha\beta^2$?

(A) 15

(B) -15

(C) 30

(D) -30

4 The primitive function of $x^{-2} - 2$ is:

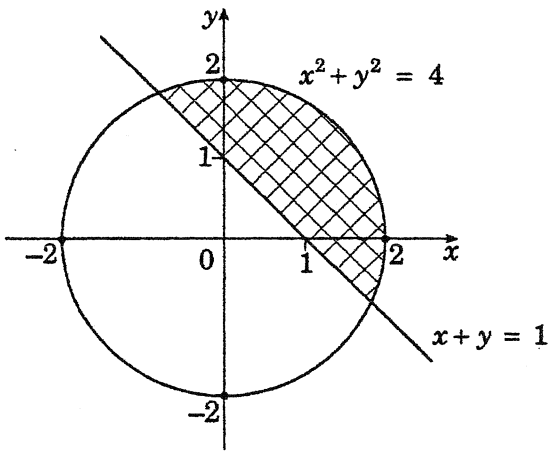
(A) $\frac{1}{x} - 2x + C$

(B) $-\frac{1}{x} - 2x + C$

(C) $\frac{1}{3x^3} - 2x + C$

(D) $-\frac{1}{3x^3} - 2x + C$

5 The diagram shows the region enclosed by $x^2 + y^2 = 4$ and $x + y = 1$



Which of the following pairs of inequalities describes the shaded region in the diagram?

(A) $x^2 + y^2 \le 4$ and $x + y \le 1$

(B) $x^2 + y^2 \le 4$ and $x + y \ge 1$

(C) $x^2 + y^2 \ge 4$ and $x + y \le 1$

(D) $x^2 + y^2 \ge 4$ and $x + y \ge 1$

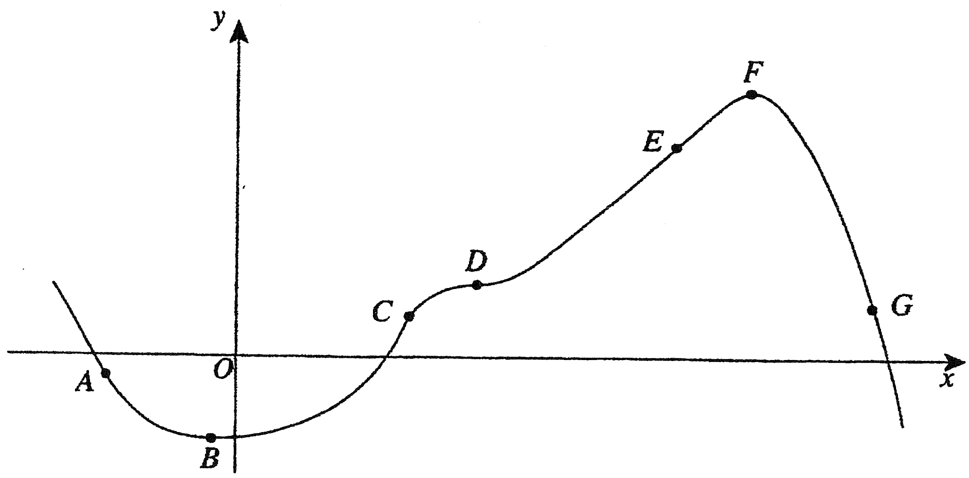
6 A cupboard contains 7 white mugs and 4 black mugs. A mug is drawn at random from this cupboard, and then returned to the cupboard after its colour has been noted. A second mug is then drawn at random from the cupboard. What is the probability both mugs are the same colour?

(A) $\dfrac{28}{121}$ (B) $\dfrac{49}{121}$

(C) $\dfrac{56}{121}$ (D) $\dfrac{65}{121}$

7 In the diagram, the points A, B, C, D, E, F and G lie on the curve $y = f(x)$.

Points B, D and F are stationary points, and points C, D and E are points of inflexion.



Which point corresponds to the description $y > 0,\ \dfrac{dy}{dx} = 0,\ \dfrac{d^2y}{dx^2} < 0$?

(A) B (B) C (C) D (D) F

8 Boxes are stacked in layers, where each layer contains one box less than the layer below. There are six boxes in the top layer, seven boxes in the next layer and so on. There are n layers altogether. Which of the following is the correct expression for the number of boxes in the bottom layer?

(A) $n+5$

(B) $n+6$

(C) $6n-1$

(D) $6n-5$

9 What is the derivative of $\frac{e^x}{x^2}$?

(A) $\frac{e^x}{2x}$

(B) $\frac{3e^x}{x^3}$

(C) $\frac{e^x(x-2)}{x^3}$

(D) $\frac{e^x(x^2-2x)}{x^2}$

10 What are the solutions of $\cos 2x = \frac{1}{2}$ for $-\pi \le x \le \pi$?

(A) $x = \frac{\pi}{6}, \frac{5\pi}{6}, -\frac{5\pi}{6}, -\frac{\pi}{6}$

(B) $x = \frac{\pi}{12}, \frac{11\pi}{12}, -\frac{11\pi}{12}, -\frac{\pi}{12}$

(C) $x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

(D) $x = \frac{\pi}{12}, \frac{11\pi}{12}, \frac{13\pi}{12}, \frac{23\pi}{12}$

Section II

90 marks
Attempt Questions 11–16
Allow about 2 hours and 45 minutes for this section

Answer each question on the writing paper provided.

Question 11 (15 marks)

a) Factorise $3x^2 - 2x - 5$. **2**

b) Solve $|2x-3| \leq 7$. **2**

c) Find the equation of the normal to the curve $y = 2x^2 - 5x + 1$ at the point where $x = 2$. **3**

d) Simplify $\dfrac{a^2 \times a^{x-4}}{a^{1-x}}$. **2**

e) Differentiate $(2x^2 - 5)^7$. **2**

f) Differentiate $\dfrac{\tan x}{x}$. **2**

g) Solve $\log_4 32 = x$. **2**

Question 12 (15 marks)

a) Differentiate the following:

i) x^2e^{2x} 2

ii) $\ln\left(\dfrac{x^2-5}{x+3}\right)$ 2

b) Simplify $\dfrac{\sin\left(\dfrac{\pi}{2}-\theta\right)}{\sin(\pi-\theta)}$ 2

c) Find an expression for the limiting sum of the geometric series given below. 2
Express your answer in simplest form.

$$\sin^2 x+\sin^4 x+\sin^6 x+\ldots\ldots,\ \text{for}\ \ 0<x<\frac{\pi}{2}$$

d) Plot the points $A(3,2), B(-1,-1)$ and $C(0,3)$.

i) Show the equation of the line through *C* and parallel to *AB* is $3x-4y+12=0$. 2

ii) Find the co-ordinates of *D* , the point where the line in (i) meets the x-axis. 1

iii) Prove that *ABDC* is a parallelogram. 2

iv) Find the perpendicular distance from *B* to the line *CD*. 1

v) Hence, or otherwise find the area of the parallelogram *ABDC*. 1

Question 13 (15 marks)

a) Evaluate $\sum_{n=1}^{10} 2\times 3^{n-1}$. 2

b) Consider the function $f(x) = x\sin x$.

x	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π
$f(x)$	0	0.555	1.571		0

i) Copy and complete the table above on your writing paper. 1
Values of $f(x)$ are given correct to three decimal places where appropriate.

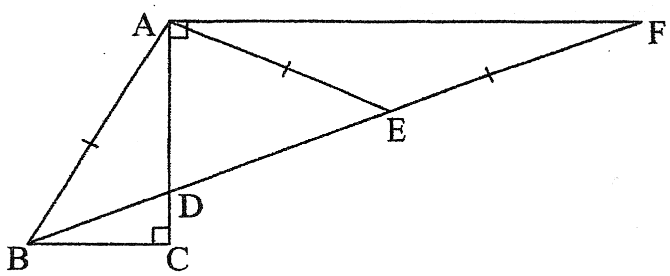
ii) Use Simpson's Rule with five function values to evaluate $\int_0^{\pi} x\sin x\, dx$,

correct to two decimal places. 2

c) A function $y = f(x)$ has $\frac{d^2y}{dx^2} = 6x - 2$ and a stationary point at (3,10).

Find $f(x)$. 2

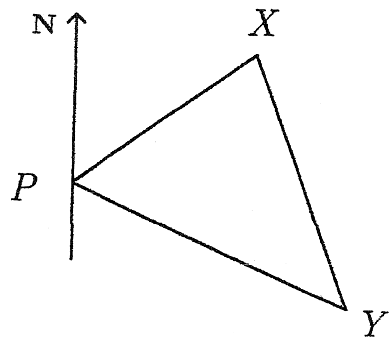
d) In the diagram, $\angle BCA = \angle CAF = 90°$ and $AB = AE = EF$.



i) Copy the diagram onto your answer sheet.
ii) Prove that $\angle ABD = 2\angle DBC$. 3

Question 13 (continued)

e) Ship X is 30 nautical miles from port P and is on a bearing of 065°.
Ship Y is 40 nautical miles from port P and is on a bearing of 125°.



i) Show that $\angle XPY = 60^\circ$. 1

ii) Determine the distance between the two ships, correct to one decimal place. 2

iii) Find the bearing of ship X from ship Y, to the nearest degree. 2

Question 14 (15 marks)

a) Liam, Harry and Zach work independently on a problem. If the respective probabilities that they will solve it are $\frac{3}{4},\frac{1}{2}$ and $\frac{2}{5}$, find the probability that the problem will be solved. **2**

b) Given the equation of the parabola $y=x^2+6x+6$

i) Find the coordinates of the vertex. **2**

ii) Find the coordinates of the focus. **1**

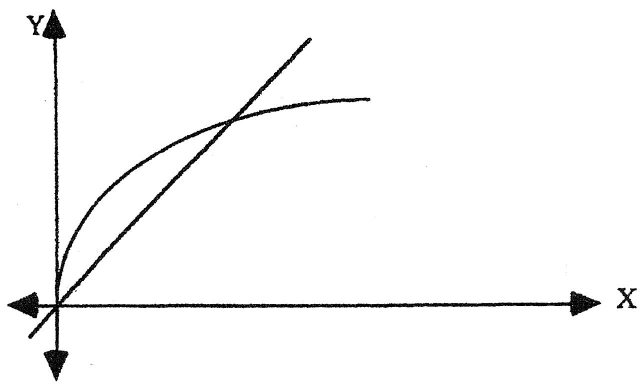
iii) Find the equation of the directrix. **1**

c) Find the value of m for which the equation $(m-1)x^2+3x-3=0$ has one root twice the other. **3**

d) The region which lies between the curve $y=2\sqrt{x}$ and $y=\frac{x}{2}$ is rotated about the x axis to form a solid.

i) Find their points of intersection. **2**

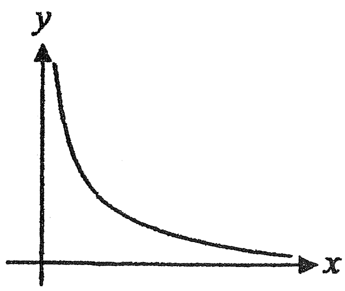
ii) Find the volume of the solid. **2**



e) The diagram below shows the graph of $y=f'(x)$ for $x>0$. **2**

For this graph $f'(x)=\frac{1}{ax}$ where a is a positive constant, $f(1)=1$ and $f(e^4)=3$.

Find the value of a.



Question 15 (15 marks)

a) Solve $(\log_{10} x^3)(\log_{10} x) + \log_{10} x^4 - 7 = 0$ 3

b) i) If $y = 2x\sin 2x + \cos 2x$ find $\frac{dy}{dx}$ 1

ii) Hence, or otherwise, find the value of $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} x\cos 2x\, dx$. 2

c) Consider the function $f(x) = \frac{x}{4} + \frac{1}{x}$.

i) Show that the function is odd. 1

ii) Show that there is no value of x for which $f(x) = 0$. 1

iii) State the vertical asymptote of $y = f(x)$. 1

iv) Find the stationary point/s. 2

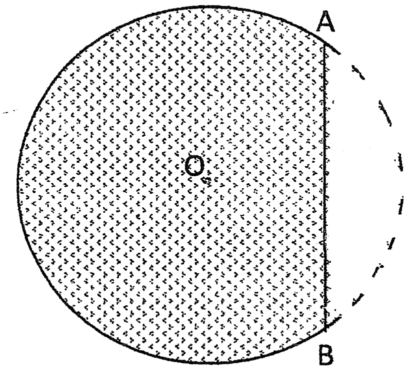
v) Determine the nature of the stationary point/s. 1

vi) Sketch the graph of $y = f(x)$. 2

vii) State the range of $y = f(x)$. 1

Question 16 (15 marks)

a) A garden bed is in the shape of a circle with a minor segment removed as shown. 3
The circle has centre 0 and radius 5 metres.
The length of the straight edge $AB = 5\sqrt{3}$ metres.
Find the exact area of the garden bed.



b) Michelle wants to save \$20 000 as a deposit for a car.
She banks \$250 at the beginning of every month.
Interest is paid at the rate of 1% per month compounded monthly.

i) Show that after n months the value of her investment is given by: 2

$$S_n = 25\ 250(1.01^n - 1)$$

ii) How many months will it take for Michelle to achieve her goal? 2

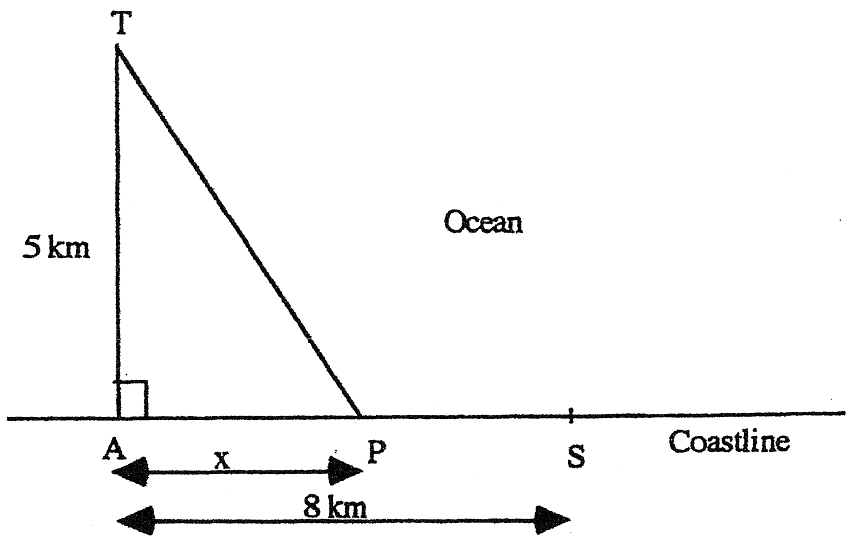
c) i) Sketch the curve $y = \cos\left(x + \frac{\pi}{2}\right)$ in the domain $-2\pi \le x \le 2\pi$. 2

ii) Hence, determine the number of solutions to the equation 1

$$\cos\left(x + \frac{\pi}{2}\right) - \frac{x}{2\pi} = 0 \ .$$

Question 16 (continued)

d) A natural gas pipeline is to be built connecting a coastal city S to an offshore island T which is 5 km from the closest coastline point A. The distance between A and the city S is 8 km. The pipeline is to run from S to a point P then underwater to T. The cost of laying the pipeline is $75 000 per km on land and $100 000 per km underwater.



Let $AP = x$ km.

i) Show that the length of the pipeline is $\sqrt{x^2+25}+(8-x)$. 1

ii) Find an expression for the cost C of building the pipeline. 1

iii) Find where P should be located to minimise the cost of the pipeline, 3

correct to 2 decimal places.

THE END

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STANDARD INTEGRALS

$$\int x^n \, dx = \frac{1}{n+1} x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x} \, dx = \ln x, \quad x > 0$$

$$\int e^{ax} \, dx = \frac{1}{a} e^{ax}, \quad a \neq 0$$

$$\int \cos ax \, dx = \frac{1}{a} \sin ax, \quad a \neq 0$$

$$\int \sin ax \, dx = -\frac{1}{a} \cos ax, \quad a \neq 0$$

$$\int \sec^2 ax \, dx = \frac{1}{a} \tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax \, dx = \frac{1}{a} \sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2 + x^2} \, dx = \frac{1}{a} \tan^{-1} \frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} \, dx = \sin^{-1} \frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} \, dx = \ln\left(x + \sqrt{x^2 - a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} \, dx = \ln\left(x + \sqrt{x^2 + a^2}\right)$$

NOTE: $\ln x = \log_e x, \quad x > 0$

Sydney Girls High School

Mathematics Faculty

Multiple Choice Answer Sheet
Mathematics

Completely fill the response oval representing the most correct answer.

1.	A ○	B ○	C ●	D ○
2.	A ○	B ●	C ○	D ○
3.	A ○	B ○	C ●	D ○
4.	A ○	B ●	C ○	D ○
5.	A ○	B ●	C ○	D ○
6.	A ○	B ○	C ○	D ●
7.	A ○	B ○	C ○	D ●
8.	A ●	B ○	C ○	D ○
9.	A ○	B ○	C ●	D ○
10.	A ●	B ○	C ○	D ○

TRIAL 2014 ANSWERS

11.

Feed back

a) $3x^2-2x-5$

$=(x+1)(3x-5)$

Some students solved for x .However it is not an equation and the question said to factorise .the equation ask to factorise

b) $|2x-3|\le 7$

$-7\le 2x-3\le 7$

$-4\le 2x\le 10$

$-2\le x\le 5$

The answer must be in the form $-2\le x\le 5$

c) $y=2x^2-5x+1$

when $x=2\therefore\ y=-1$

$\frac{dy}{dx}=4x-5$

at $x=2$

$\frac{dy}{dx}=3$

The gradient of the normal is $\frac{-1}{3}$

The equation of the normal is

$y+1=\frac{-1}{3}(x-2)$

$\therefore\ 3y+3=-x+2$

$\therefore\ x+3y+1=0$

d) $\frac{a^2\times a^{x-4}}{a^{1-x}}$

$=\frac{a^{2+x-4}}{a^{1-x}}$

$=a^{x-2-(1-x)}$

$=a^{2x-3}$

You must simplify fully and not leave your answer in the form $\frac{a^{x-2}}{a^{1-x}}$.

e) $\frac{d}{dx}(2x^2-5)^7$

$=7(2x-5)^6\times 4x$

$=28x(2x-5)^6$

f)

$y=\frac{\tan x}{x}$

$\frac{dy}{dx}=\frac{x\sec^2 x-\tan x}{x^2}$

g) $\log_4 32=x$

$4^x=32$

$2^{2x}=2^5$

$\therefore 2x=5$

$x=\frac{5}{2}$

12. a) i)

$$\frac{d}{dx}\left(x^2e^{2x}\right)$$
$$= 2x^2e^{2x} + 2xe^{2x}$$
$$\text{or } = 2xe^{2x}(x+1)$$

ii)

$$\frac{d}{dx}\left(\ln\left(\frac{x^2-5}{x+3}\right)\right)$$
$$= \frac{d}{dx}\left(\ln\left(x^2-5\right) - \ln\left(x+3\right)\right)$$
$$= \frac{2x}{x^2-5} - \frac{1}{x+3}$$

b)

$$\frac{\sin\left(\frac{\pi}{2}-\theta\right)}{\sin\left(\pi-\theta\right)}$$
$$= \frac{\cos\theta}{\sin\theta}$$
$$= \cot\theta$$

Some students did not complete the question and left their answers as $\frac{\cos\theta}{\sin\theta}$. Also not that $\sin(\pi-\theta) =- \sin\theta$

c)

$$\sin^2\theta + \sin^4\theta + ...$$
$$a = \sin^2\theta \quad and \quad r = \sin^2\theta$$
$$S_\infty = \frac{\sin^2\theta}{1-\sin^2\theta}$$
$$= \frac{\sin^2\theta}{\cos^2\theta}$$
$$= \tan^2\theta$$

d) i) The gradient of *AB* is

$$\frac{-1-2}{-1-3}$$
$$= \frac{3}{4}$$

The equation of the line pass through *C* and paralleled to *AB* is

$$y - 3 = \frac{3}{4}(x-0)$$
$$4(y-3) = 3x$$
$$3x - 4y + 12 = 0$$

ii) To find the co-ordinates of D substitute $y=0$ into

$$3x-4y+12=0$$

$$\therefore D(-4,0)$$

iii) From (i) the pair of lines are parallel i.e. $AB//DC$

We need to find the distance AB and DC

The distance AB = $\sqrt{(3+1)^2+(2+1)^2}$

$= 5\,units$

The distance DC = $\sqrt{(0--4)^2+(3-0)^2}$

$= 5\,units$

$\therefore ABCD$ is a parallelogram .

iv) The perpendicular distance from B to CD is

$(-1,-1)\ \ 3x-4y+12=0$

$$d=\frac{|3\times(-1)-4\times(-1)+12|}{\sqrt{3^2+4^2}}$$

$$=\frac{13}{5}$$

v) From (iii) and (iv) the area is $\frac{13}{5}\times 5=13\ u^2$

Some students incorrectly used use $\frac{1}{2}bh$ for the parallelogram.

Yr 12 2U Mathematics Trial 2014

13) a)

$2+6+18+\dots$

$S_{10} = \frac{2(3^{10}-1)}{3-1}$

$= 59048$

* Some students didn't recognise this as G.P.

b i) $x = \frac{3\pi}{4}$ $f(x) = 1.666$

ii)

x	$f(x)$	w	$wf(x)$
0	0	1	0
$\frac{\pi}{4}$	0.555	4	2.22
$\frac{\pi}{2}$	1.571	2	3.142
$\frac{3\pi}{4}$	1.666	4	6.664
π	0	1	0

$A \doteq \frac{\pi}{12} \sum f(x).w$

$\doteq 3.15 \ u^2$

Some students didn't use the correct h when calculating the area.

c) $y' = 3x^2 - 2x + c$

$0 = 27 - 6 + c$

$c = -21$

$f(x) = x^3 - x^2 - 21x + c$

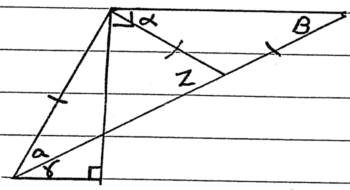
$10 = 27 - 9 - 63 + c$

$c = 55$

$f(x) = x^3 - x^2 - 21x + 55$

Some students didn't use (3, 10) in the correct equation.

d)



$\alpha = B$ (base ∠'s of an isosceles Δ)

$Z = 2B$ (ext ∠)

$Z = a$ (base ∠'s of an isosceles Δ)

$\therefore a = 2B$

$\gamma = B$ (Alt ∠'s, // lines)

$\therefore a = 2\gamma$

Many students have problem using their diagram properly to reduce the working and stop wasting time

e) i)

$\alpha = 125^\circ - 65^\circ$

$= 60^\circ$

ii) $XY^2 = 30^2 + 40^2 - 2 \times 30 \times 40 \cos 6$

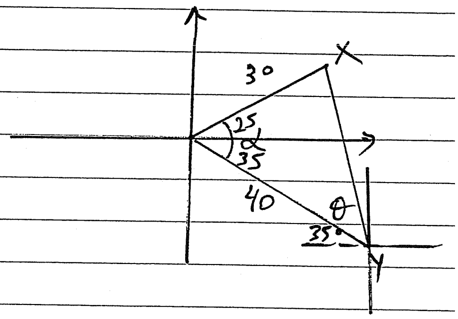
$XY = 36.1$

iii) $\frac{\sin\theta}{30} = \frac{\sin 60}{36.1}$

$\theta = 46^\circ$

Bearing $= 270 + 46 + 35$

$= 351^\circ T$



Some students didn't get the correct angle for finding the bearing

Question 14 - 15 marks - Mathematics - 2014 - Trials

a) Probability at least one solves problem

$= 1 -$ Probability that no one solves problem

$P = 1 - \left(\frac{1}{4} \times \frac{1}{2} \times 3/5\right)$

$= 1 - \frac{3}{40}$

$P = 37/40$ (2 marks)

→ This question was done well overall. Some over complicated and missed the point.

b) i) $y = x^2 + 6x + 6$

$y = x^2 + 6x + 9 - 3$

$y = (x+3)^2 - 3$

$y + 3 = (x+3)^2$

OR/ $(x+3)^2 = y + 3$

$(x-h)^2 = 4a(y-k)$

$\therefore$ Vertex $(-h, -k)$

$(-3, -3)$ (2 marks)

→ Overall question was answered very well. Some students forgot to complete the square and were unable to put in the form $(x-h)^2 = 4a(y-k)$ and hence were unable to get focus and directrix

ii) Focal length: $4a = 1$

$a = \frac{1}{4}$

$\therefore$ Focus $(-3, -2\frac{3}{4})$ (1 mark)

iii) Directrix $y = -3\frac{1}{4}$ (1 mark)

c) $a = m-1$, $b = 3$ and $c = -3$

$\alpha + \beta = \frac{-3}{m-1}$ and $\alpha\beta = \frac{-3}{m-1}$

Let $\beta = 2\alpha$

$\alpha + 2\alpha = \frac{-3}{m-1}$

$3\alpha = \frac{-3}{m-1}$

$\therefore \alpha = \frac{-1}{m-1}$ .. ①

and $2\alpha^2 = \frac{-3}{m-1}$. ②

$\therefore 2\left(\frac{-1}{m-1}\right)^2 = \frac{-3}{m-1}$

$\frac{2}{(m-1)^2} = \frac{-3}{m-1}$

$\frac{\cancel{(m-1)}}{(m-1)^{\cancel{2}}} = -\frac{3}{2}$

$\frac{1}{m-1} = -\frac{3}{2}$

$2 = -3(m-1)$

$2 = -3m + 3$

$-1 = -3m$

$\therefore \frac{1}{3} = m$

(3 marks)

Most students were able to obtain sum and product of roots. They also were able to substitute correctly, however, many were unable to solve for m.

d) i) Point of intersection

$$2\sqrt{x} = \frac{x}{2}$$

$$4\sqrt{x} = x$$

$$16x = x^2$$

$$x^2 - 16x = 0$$

$$x(x-16) = 0$$

$x = 0$ $\quad x = 16$

$y = 0$ $\quad y = 8$

$\therefore P_1(0,0)$ and $P_2(16,8)$ (2marks)

Most students were able to find the point of intersection correctly, some only found x-values. You need both x & y-values for P.O.I.

ii) $$V = \pi \int_0^{16} (2\sqrt{x})^2 - \left(\frac{x}{2}\right)^2 dx$$

$$= \pi \int_0^{16} 4x - \frac{x^2}{4}\, dx$$

$$= \pi \left[2x^2 - \frac{x^3}{12} \right]_0^{16}$$

$$= \pi \left[2(16)^2 - \frac{(16)^3}{12} - 0 \right]$$

$$= 170\tfrac{2}{3}\pi \text{ or } \frac{512}{3}\pi \text{ units}^3$$

(2 marks)

Many students made the error $\left(2\sqrt{x} - \frac{x}{2}\right)^2$. Some didn't integrate & substitute correctly.

e) $$f'(x) = \frac{1}{ax}$$

$$\therefore f(x) = \frac{1}{a}\int \frac{1}{x}\, dx$$

$$= \frac{1}{a}\ln x + C$$

But $f(1) = 1$, $\therefore 1 = \frac{1}{a}\ln 1 + C$

$$\therefore C = 1$$

$$f(x) = \frac{1}{a}\ln x + 1$$

But $f(e^4) = 3$

$$\therefore 3 = \frac{1}{a}\ln e^4 + 1$$

$$3 = \frac{4}{a}\ln e + 1$$

$$2 = \frac{4}{a}$$

$$\therefore a = 2$$

- Most common error was not realising $f(x) = \frac{1}{a}\int \frac{1}{x}dx$

Question 15.

a) $\left(\log_{10} x^3\right)\left(\log_{10} x\right) + \log_{10} x^4 - 7 = 0$

$3\log_{10} x + 4\log_{10} x - 7 = 0$ ✓

Let $U = \log_{10} x$

$\therefore 3U^2 + 4U - 7 = 0$ ✓

$U = 1$ $\therefore \log_{10} x = 1$ $\therefore x = 10$ ✓

$U = \frac{-7}{3}$ $\therefore \log_{10} x = -\frac{7}{3}$ $\therefore x = 10^{-7/3}$

★ A number of students wrongly eliminated $x = 10^{-7/3}$
Note: $\log_a x$ is true if $x > 0$ but $\log_a x$ can be negative.

b) i) $y = 2x\sin 2x + \cos 2x$

$\frac{dy}{dx} = 2\sin x + 4x\cos 2x - 2\sin 2x$

$= 4x\cos 2x$ ✓

ii) $\int_{\pi/4}^{\pi/2} x\cos 2x\,dx = \frac{1}{4}\int 4x\cos 2x\,dx$

$= \frac{1}{4}\left[2x\sin 2x + \cos 2x\right]_{\pi/4}^{\pi/2}$ ✓

$= \frac{1}{4}\left[-1 - \frac{\pi}{2}\right] = -\frac{1}{8}\left[2 + \pi\right]$ ✓

b/i) For some reasons, many students could not differentiate the function correctly. This is a quite simple ~~question~~ but very common kind of question

c) i) $f(x) = \frac{x}{4} + \frac{1}{x} = \frac{x^2 + 4}{4x}$

$f(-x) = \frac{(-x)^2 + 4}{4(-x)} = \frac{x^2 + 4}{-4x} = -\frac{x^2+4}{4x}$ ✓

$= -f(x) \therefore$ odd fn.

Everyone did well in this question.

c) ii) $f(x) = 0$ $\therefore x^2 + 4 = 0$

$x^2 = -4$. This is not true.

$\therefore$ No Solutions

★ When you are asked to proved, make sure that you should show some mathematical/logical provings/works. Not just assuming or writting in words.

iii) Vertical Asymptope: $4x = 0$

$x = 0$ ✓

iv) Stationary pts: $f(x) = \frac{x^2 + 4}{4x}$

$f'(x) = \frac{2x(4x) - 4(x^2 + 4)}{16x^2}$

$= \frac{4x^2 - 16}{16x^2}$ ✓

$f'(x) = 0$ $\therefore 4(x^2 - 4) = 0$

$x = 2 \rightarrow y = 1$

$x = -2 \rightarrow y = -1$ ✓

$\therefore$ Stationary points: $(2, 1)$, $(-2, -1)$

★ A number of Students stopped at finding the x-values. Note, a point must have both x and y values.

v) $f'(x) = \frac{x^2 - 4}{4x^2}$

$f''(x) = \frac{2x(4x^2) - 8x(x^2 - 4)}{16x^4}$

$= \frac{32x}{16x^4}$

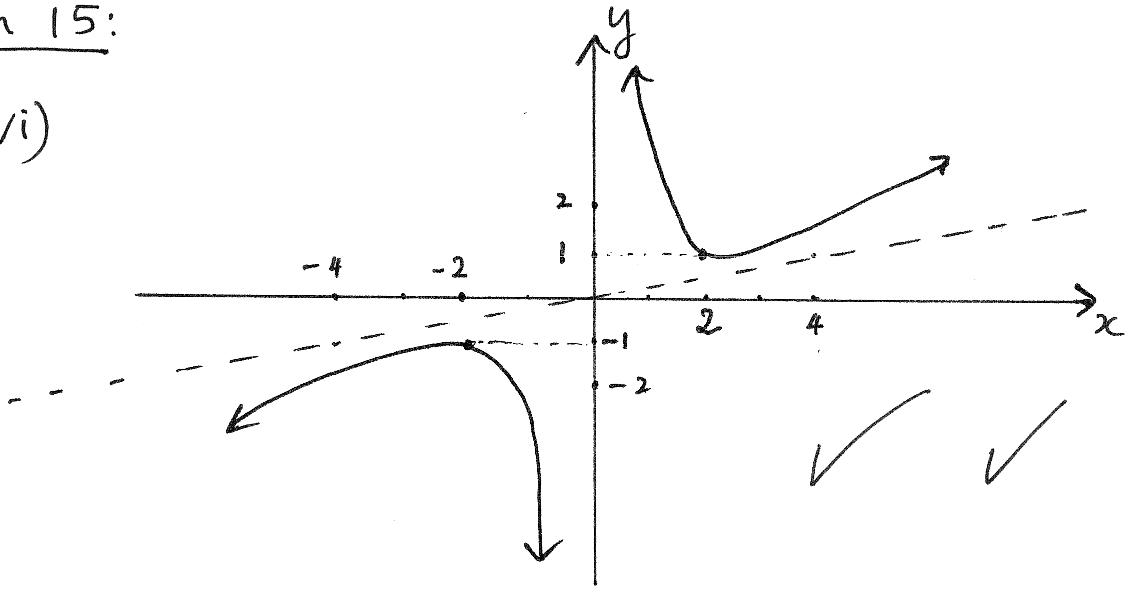
$f''(2) = \frac{1}{4} > 0 \therefore$ Min point $(2, 1)$

$f''(-2) = -\frac{1}{4} < 0 \therefore$ Max point $(-2, -1)$

★ Everyone did well in this question.

Question 15:

c) vi)



vii) Range: $y \geq 1$, $y \leq -1$

☆ A number of students sketched as $x \to \infty$ $y \to 0$ which is incorrect.

Question 16

a) $\cos\theta = \dfrac{5^2 + 5^2 - (5\sqrt{3})^2}{2(5)(5)}$

$\theta = 120^\circ$ OR $\theta = \dfrac{2\pi}{3}$ ✓

- Area of the circle $= \pi r^2$
  $= 25\pi$
- Sector Area $= \frac{1}{2} r^2 \theta$
  $= \frac{1}{2} \times 5^2 \times \sin\frac{2\pi}{3}$
  $= \dfrac{25\sqrt{3}}{4}$
- Area of the Triangle $= \frac{1}{2} ab \sin\frac{2\pi}{3}$
  $= \frac{1}{2} \times 5^2 \times \frac{\sqrt{3}}{2}$
  $= \dfrac{25\sqrt{3}}{4}$ ✓
- Area of garden $= 25\pi - \dfrac{25\pi}{3} + \dfrac{25\sqrt{3}}{4}$
  $= \dfrac{200\pi + 75\sqrt{3}}{12}$ m^2 ✓

Some students just took the Area of the Circle minus the Area of the Sector and some students used the wrong formula for calculating Sector Area.

b) At the end of first month

$A_1 = 250(1.01)$

$A_2 = 250(1.01)^2 + 250(1.01)$

$A_n = 250(1.01)^n + 250(1.01)^{n-1} + \dots 250(1.01)$

$= 250(1.01 + 1.01^2 + 1.01^3 + \dots 1.01^n)$ ✓

$= \dfrac{250(1.01)(1.01^n - 1)}{0.01}$

$\therefore S_n = 25250(1.01^n - 1)$ ✓

ii) $20000 = 25250(1.01^n - 1)$

$1.01^n = \dfrac{20000}{25250} + 1$ ✓

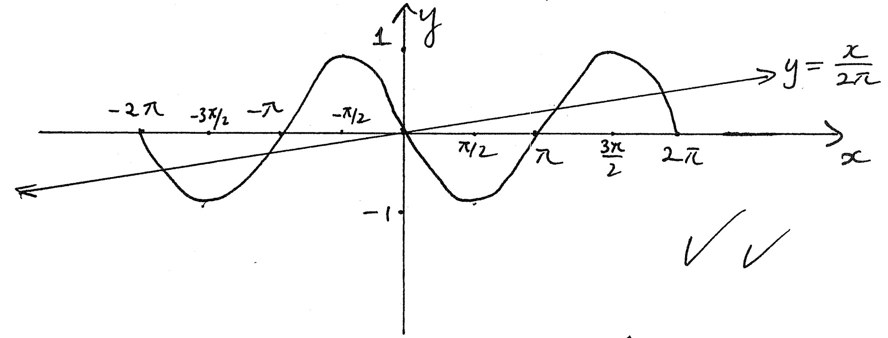
$1.01^n = 1.79$

$n = \dfrac{\ln(1.79)}{\ln(1.01)} = 58.629$

$n = 59$ months. ✓

i) Some students do not know how to write/develop into a geometric series. Any proving question requires step by step and logical processes.

c) i)



✓✓

ii) There are 5 solutions ✓

The majority of students did well in this question.

Question 16

d) $TP = \sqrt{5^2 + x^2}$

i) $PS = 8 - x$ ✓

Length of pipeline $= TP + PS$

$$= \sqrt{x^2 + 25} + 8 - x$$

ii) $\text{Cost} = 100\,000\sqrt{x^2 + 25} + 75000(8 - x)$ ✓

iii) $$\frac{dc}{dx} = \frac{100\,000x}{\sqrt{x^2+25}} - 75\,000$$

$\frac{dc}{dx} = 0 \quad \therefore \quad 100\,000x = 75000\sqrt{x^2+25}$

$4x = 3\sqrt{x^2+25}$ ✓

$16x^2 = 9(x^2+25)$

$7x^2 = 225$

$x = \pm\frac{15}{\sqrt{7}} = \pm\frac{15\sqrt{7}}{7}$

$x = -\frac{15\sqrt{7}}{7}$ (Rejected)

★ Some students understood the question but experienced difficulties to ~~get~~ solve $\frac{dc}{dx} = 0$ ~~so~~ Algebraic skill problems.

Test:

x	$\frac{14\sqrt{7}}{7}$	$\frac{15\sqrt{7}}{7}$	$\frac{16\sqrt{7}}{7}$
$\frac{dc}{dx}$	$-$	0	+

✓

Min

Min cost when $x = \frac{15\sqrt{7}}{7}$ km ✓

OR P is located $\frac{15\sqrt{7}}{7}$ or 5.67 (2dp) km from A.