



Sydney Girls High School 2015

TRIAL HIGHER SCHOOL CERTIFICATE
EXAMINATION

Mathematics

General Instructions

- Reading Time – 5 minutes
- Working time – 3 hours
- Write using black or blue pen
Black pen is preferred
- Board-approved calculators may be used
- A table of standard integrals is provided at the back of this paper
- In Questions 11 – 16, show relevant mathematical reasoning and/or calculations

Total marks – 100

Section I Pages 3 – 6

10 Marks

- Attempt Questions 1 – 10
- Answer on the Multiple Choice answer sheet provided
- Allow about 15 minutes for this section

Section II Pages 7 – 15

90 Marks

- Attempt Questions 11 – 16
- Answer on the blank paper provided
- Begin a new page for each question
- Allow about 2 hours and 45 minutes for this section

Name:

Teacher:

THIS IS A TRIAL PAPER ONLY

It does not necessarily reflect the format or the content of the 2015 HSC Examination Paper in this subject.

Section I

10 marks

Attempt Questions 1 – 10

Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1 – 10

(1) What is the correct factorisation of $2x^2 + 9x - 5$?

(A) $(x-5)(2x+1)$

(B) $(2x-1)(x+5)$

(C) $(2x-1)(x-5)$

(D) $(x-1)(2x+5)$

(2) $\frac{20\sqrt{15}}{4\sqrt{5}} =$

(A) $5\sqrt{3}$

(B) $20\sqrt{3}$

(C) $80\sqrt{75}$

(D) $5 + \sqrt{3}$

(3) What is the domain of the function $f(x) = \sqrt{2x+4}$?

(A) All real x such that $x \leq -2$

(B) All real x such that $x > -2$

(C) All real x such that $x < -2$

(D) All real x such that $x \geq -2$

(4) What is the distance between $A(-2,3)$ and $B(-5,-1)$?

(A) $\sqrt{53}$

(B) 5

(C) $\sqrt{18}$

(D) 7

(5) The line which is perpendicular to $2x + y - 5 = 0$ with a y intercept at 3 has the equation:

(A) $y = 2x + 3$

(B) $y = -\frac{1}{2}x + 3$

(C) $y = \frac{1}{2}x + 3$

(D) $y = -2x + 3$

(6) What is the derivative of $e^{-\sin 2x}$?

(A) $-2 \cos 2x e^{-\sin 2x}$

(B) $2 \cos 2x e^{-\sin 2x}$

(C) $-2x e^{-\sin 2x}$

(D) $-2 \sin 2x e^{-\sin 2x}$

(7) A geometric series with a common ratio r will have a limiting sum if:

- (A) $r < 1$
- (B) $r > 1$
- (C) $|r| < 1$
- (D) $|r| > 1$

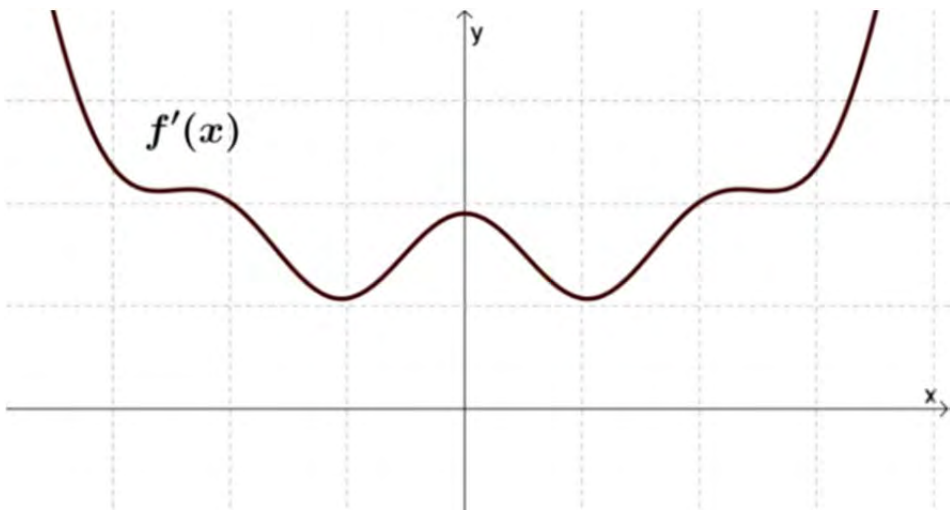
(8) Which of the following expressions is the correct simplification of $\frac{\operatorname{cosec} \theta \sec \theta}{\tan \theta}$?

- (A) $\sin^2 \theta$
- (B) $\cos^2 \theta$
- (C) $\operatorname{cosec}^2 \theta$
- (D) $\sec^2 \theta$

(9) Which of the following expressions represents $\int \frac{x}{3x^2} dx$?

- (A) $\ln 3x + c$
- (B) $3 \ln x + c$
- (C) $\frac{1}{3} \ln 3x^2 + c$
- (D) $\frac{1}{3} \ln x + c$

(10)



Looking at the graph of $y = f'(x)$ above, which of the following could be true?

- (A) $f(x)$ is an odd function with a local maximum at $f(0)$
- (B) $f(x)$ is an even function with a local maximum at $f(0)$
- (C) $f(x)$ is an even function with a point of inflexion at $f(0)$
- (D) $f(x)$ is an odd function with a point of inflexion at $f(0)$

Section II

90 marks

Attempt Questions 11 – 16

Allow about 2 hours and 45 minutes for this section

Answer on the blank paper provided. Begin a new page for each question

Your responses should include relevant mathematical reasoning and/or calculations.

Question 11

(15 Marks)

- (a) Factorise $2x^2 - 32$. [2]
- (b) Solve $|5x - 1| \leq 9$. [2]
- (c) Find the equation of the tangent to the curve $y = 3x^2 - x$ at the point where $x = 1$. [2]
- (d) Differentiate $(2x + \tan 7x)^4$. [2]
- (e) Given the points $A(2,1)$ and $B(-3,-2)$, find the equation of the locus of point $P(x, y)$ as it moves so that the distance PA is always twice the distance PB . [3]
- (f) Solve for x , $5^x = 20$. [2]
- (g) Find $\int \tan^2 2x dx$. [2]

End of Question 11

Question 12 (Begin a New Page)

(15 Marks)

(a) Differentiate with respect to x :

(i) $y = (x + 5) \ln 5x$ [2]

(ii) $f(x) = \frac{\sin x}{x+1}$ [2]

(b) Find $\int \frac{5x}{x^2 + 4} dx$. [2]

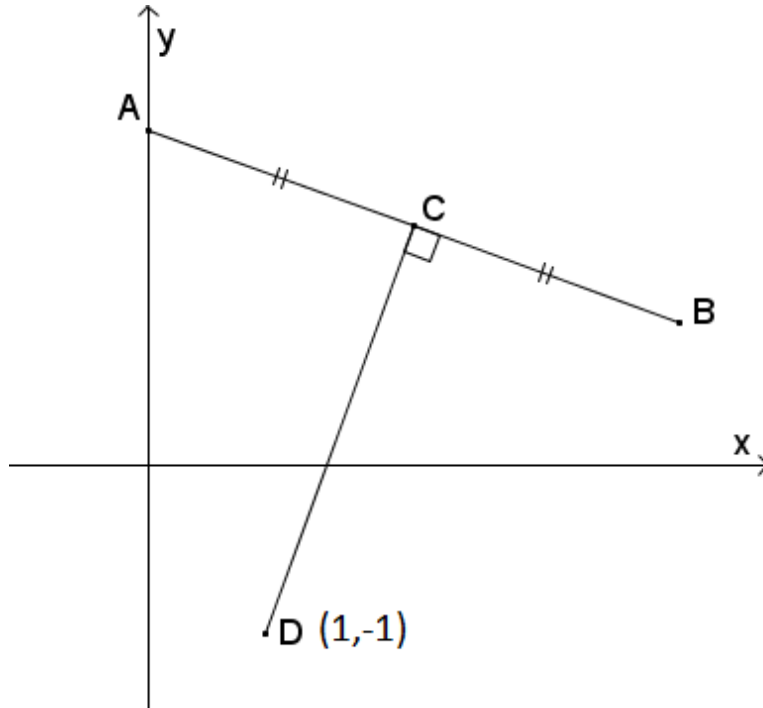
(c) Show that $\sqrt{\left(\frac{1}{\tan^2 \theta} - \frac{1}{\sec^2 \theta}\right)} = \cos \theta \cot \theta$. [2]

(d) Use the trapezoidal rule to find an approximation to $\int_0^2 xe^x dx$ using 5 function values. Give your answer correct to 3 significant figures. [2]

Question 12 continues on the next page

Question 12 (Continued)

- (e) The diagram shows points A , B and C lying on the line $x + 3y = 18$. The point A lies on the y axis and $AC = CB$. The line from $D(1, -1)$ to C is perpendicular to AB .



NOT TO SCALE.

- (i) Find the coordinates of A . [1]
- (ii) Find the equation of the line CD . [2]
- (iii) Find the coordinates of B . [2]

End of Question 12

Question 13 (Begin a New Page)

(a) Evaluate the arithmetic series $3 + 7 + 11 + 15 + \dots + 4003$. [2]

(b) A bag contains 8 blue marbles and 4 yellow marbles. Two marbles are selected at random without replacement.

(i) Draw a tree diagram to show all possible outcomes. Include the probability on each branch. [2]

(ii) What is the probability that the two marbles are of different colours? [1]

(c) The graphs of $y = -x^2 + 9$ and $y = 3x + 5$ intersect at the points A and B .

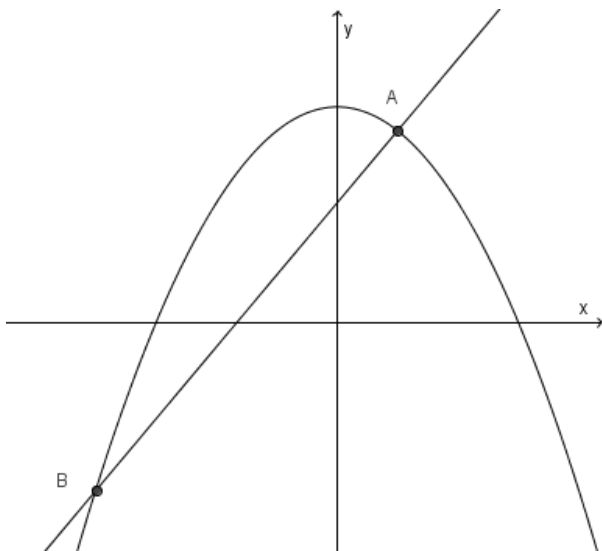


Diagram not to scale

(i) Find the x values of the points A and B . [2]

(ii) Find the area bounded by the curve $y = -x^2 + 9$ and the line $y = 3x + 5$. [2]

Question 13 (Continued)

(d)

(i) Show that $\frac{d(\tan^2 x)}{dx} = 2 \tan x + 2 \tan^3 x$. [2]

(ii) Hence or otherwise find $\frac{d(\sec^2 x)}{dx}$. [1]

(iii) Use part (i) to show that $\int_0^{\frac{\pi}{3}} 2 \tan^3 x dx = 3 - 2 \ln 2$. [3]

End of Question 13

Question 14 (Begin a New Page)

(15 Marks)

- (a) Solve $(\log_3 x)^2 - \log_3 x^4 - 12 = 0$, for $x > 0$. [3]
- (b) On 1st July 2015, Jessica invested \$18 000 in a bank account that paid interest at a rate of 5% p.a, compounded annually.
- (i) How much would be in the account after the payment of interest on 1st July 2025 if no additional deposits were made? [1]
- (ii) Consider if Jessica made additional deposits of \$1500 to her account on the 1st July each year, beginning on 1st July 2016. After the payment of interest and her deposit on 1st July 2025, how much was in her account? [3]
- (c) Consider the function $y = x^3 + 6x^2 - 135x$.
- (i) Show that $x^3 + 6x^2 - 135x = x(x + 15)(x - 9)$. [1]
- (ii) Find the stationary points and determine their nature. [2]
- (iii) Find any point(s) of inflexion. [1]
- (iv) Sketch the curve showing the x and y intercepts, the turning points and point(s) of inflexion. [2]
- (d) Sketch $y = 2 \cos(2x - \frac{\pi}{3})$ for $0 \leq x \leq \pi$. [2]

End of Question 14

(15 Marks)

Question 15 (Begin a New Page)

(a) Solve $4\sin^2 x - 3 = 0$ for $-\pi \leq x \leq \pi$ [3]

(b) $v = 6\cos 2t$ is the equation of a particle's velocity in m/s, where t is in seconds. The particle was initially 2 metres to the left of the origin.

(i) Find the equations for the displacement and acceleration of the particle. [2]

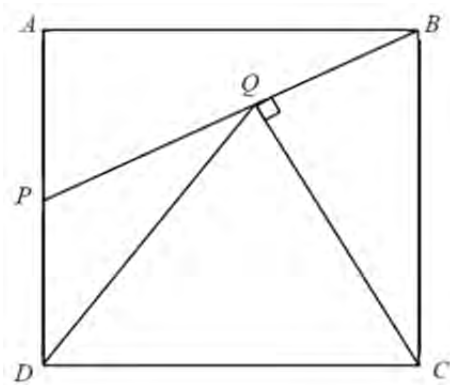
(ii) When does the particle first reach the origin? Answer correct to 2 decimal places. [2]

(iii) When does the particle first come to rest. [1]

(iv) Find the total distance travelled by the particle during the period $0 \leq t \leq \frac{\pi}{2}$ [2]

(c) ABCD is a square of side length 2 units. P is the midpoint of AD.

CQ is drawn perpendicular to PB and $\angle APB = x^\circ$.



(i) Prove $\angle APB = \angle QBC$. [1]

(ii) Hence, or otherwise show $QC = \frac{4}{\sqrt{5}}$ units. [2]

(iii) Show $QD = CD$. [2]

End of Question 15

Question 16 (Begin a New Page)

(15 Marks)

(a) Sketch $y = \log_{10}(x + 3)$, showing the x and the y intercepts.

[2]

(b) A bacteria culture of N bacteria is increasing exponentially so that after 10

minutes $\frac{dN}{dt} = kN$. If the number of bacteria increases from 100 to 400 in

2 minutes, find:

(i) The value of constant k .

[2]

(ii) The number of bacteria present after 10 minutes.

[1]

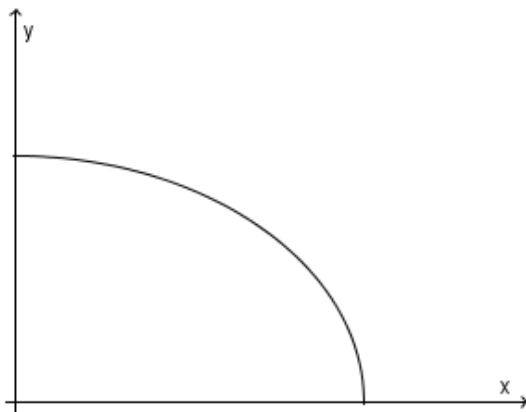
(iii) When there will be 1000 bacteria.

[2]

(c) The area bounded by the curve $\frac{x^2}{2} + y^2 = 8$ and the x and y axis in the

[3]

first quadrant is rotated about the x axis. Find the volume of the solid of revolution.



Question 16 (Continued)

(d) Prove $\int_e^{e^2} \frac{1 + \ln x}{x \ln x} dx = 1 + \ln 2$ [2]

(e) Find the largest vertical distance between graphs $y = \frac{1}{2}x$ and $y = \sin x$ in [3]

the interval $0 \leq x \leq 2\pi$.

End of paper.



Sydney Girls High School

Mathematics Faculty

Multiple Choice Answer Sheet

2015 Trial HSC Mathematics Extension

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample $2 + 4 = ?$ (A) 2 (B) 6 (C) 8 (D) 9

A B C D

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A B C D

If you change your mind and have crossed out what you consider to be the correct answer, then indicate this by writing the word *correct* and drawing an arrow as follows:

A B C D
correct
↑

Student Number: Answers

Completely fill the response oval representing the most correct answer.

1. A B C D
2. A B C D
3. A B C D
4. A B C D
5. A B C D
6. A B C D
7. A B C D
8. A B C D
9. A B C D
10. A B C D

Trial 2015

Q.11

$$\begin{aligned} \text{a) } 2x^2 - 32 \\ &= 2(x^2 - 16) \\ &= 2(x+4)(x-4) \end{aligned}$$

$$\text{b) } |5x - 1| \leq 9$$

$$-9 \leq 5x - 1 \leq 9$$

$$-8 \leq 5x \leq 10$$

$$-\frac{8}{5} \leq x \leq 2$$

$$\begin{aligned} \text{c) at } x=1 \\ y &= 3(1)^2 - (1) \\ y &= 2 \end{aligned}$$

$$y = 3x^2 - x$$

$$\frac{dy}{dx} = 6x - 1$$

$$\text{at } x=1$$

The gradient of tangent
is 5

$$y - 2 = 5(x - 1)$$

$$y - 2 = 5x - 5$$

$$y = 5x - 3$$

$$d) \frac{dy}{dx} = 4(2x + \tan 7x)^3 \times (2 + 7 \sec^2 7x)$$

$$e) PA = 2PB$$

$$\sqrt{(x-2)^2 + (y-1)^2} = 2\sqrt{(x+3)^2 + (y+2)^2}$$

$$x^2 - 4x + 4 + y^2 - 2y + 1 = 4(x^2 + 6x + 9 + y^2 + 4y + 4)$$

$$x^2 - 4x + 4 + y^2 - 2y + 1 = 4x^2 + 24x + 36 + 4y^2 + 16y + 16$$

$$3x^2 + 28x + 3y^2 + 18y + 47 = 0$$

$$f) 5^x = 20$$

$$\log_e 5^x = \log_e 20$$

$$x \log_e 5 = \log_e 20$$

$$x = \frac{\log_e 20}{\log_e 5}$$

$$g) \int \tan^2 2x \, dx$$

$$= \int (\sec^2 2x - 1) \, dx$$

$$= \frac{1}{2} \tan 2x - x + c$$

Common errors included
 $2PA=PB$ and $2PA^2=PB^2$

Some student unsuccessfully
 attempted to use substitution
 (Ext1) method.

Q12

a) i) $y = (x+5) \ln 5x$

$$y' = \frac{(x+5)}{5} + \ln 5x$$

ii) $f(x) = \frac{\sin x}{x+1}$

$$f'(x) = \frac{(x+1) \cos x - \sin x}{(x+1)^2}$$

b) $\int \frac{5x}{x^2+4} dx$

$$= \frac{5}{2} \ln(x^2+4) + c$$

c) $\sqrt{\frac{1}{\tan^2 \theta} - \frac{1}{\sec^2 \theta}}$

$$= \sqrt{\frac{\sec^2 \theta - \tan^2 \theta}{\tan^2 \theta \sec^2 \theta}}$$

$$= \sqrt{\frac{1}{\tan^2 \theta \sec^2 \theta}}$$

$$= \cot \theta \cos \theta$$

d) $h = 0.5$

Some students use the wrong formula

$$A \doteq \frac{0.5}{2} \left\{ 0 + 2 \times 0.5 e^{0.5} + 2 e^1 + 2 \times 1.5 e^{1.5} + 2 e^2 \right\}$$

$$\doteq 8.83 \quad 3 \text{ sig. fig.}$$

e) i)

$$x = 0$$

$$\therefore 3y = 18$$

$$y = 6$$

$$A: (0, 6)$$

ii) $x + 3y = 18$

$$y = -\frac{1}{3}x + 6$$

$$m = -\frac{1}{3}$$

\therefore m for CD is 3

$$y - 1 = 3(x - 1)$$

$$y + 1 = 3x - 3$$

$$y = 3x - 4$$

iii)

$$x + 3y = 18 \quad \dots \textcircled{1}$$

$$y = 3x - 4 \quad \dots \textcircled{2}$$

From eq ① & eq ②

$$x + 3(3x - 4) = 18$$

$$x + 9x - 12 = 18$$

$$10x = 30$$

$$x = 3$$

$$\therefore y = 5$$

using the midpoint

Many students used the distance formula and this was not the best method

$$3 = \frac{x+0}{2}$$

$$\therefore x = 6$$

$$\text{and } \frac{y+6}{2} = 5$$

$$y = 4$$

$$\therefore B(6, 4)$$

Question 13

a) $a = 3, d = 4, l = 4003$

$$T_n = a + (n-1)d$$

$$4003 = 3 + (n-1)4$$

$$4003 = 4n - 1$$

$$4n = 4004$$

$$n = 1001$$

$$S_n = \frac{n}{2}(a+l)$$

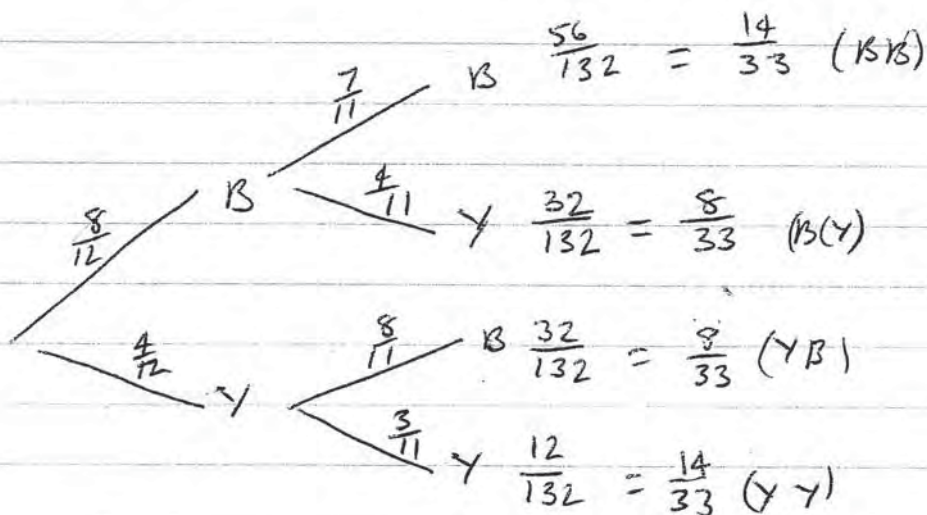
$$= \frac{1001}{2}(3+4003)$$

$$= 2005003$$

2

b)

i)



2

ii) $P(B|Y \text{ or } Y|B) = \frac{8}{33} + \frac{8}{33}$
 $= \frac{16}{33}$

$$c) \quad y = -x^2 + 9, \quad y = 3x + 5$$

$$i) \quad 3x + 5 = -x^2 + 9$$

$$x^2 + 3x - 4 = 0$$

$$(x+4)(x-1) = 0$$

$$x = 1, \quad x = -4$$

2

$$ii) \quad A = \int_{-4}^1 (-x^2 + 9) - (3x + 5) dx$$

$$= \int_{-4}^1 -x^2 - 3x + 4 dx$$

$$= \left[-\frac{x^3}{3} - \frac{3x^2}{2} + 4x \right]_{-4}^1$$

$$= \left(-\frac{1}{3} - \frac{3}{2} + 4 \right) - \left(\frac{64}{3} - 24 - 16 \right)$$

$$= \frac{13}{6} - \frac{56}{3}$$

$$= \frac{125}{6} \quad \left(20\frac{5}{6} \right) \text{ units}^2$$

$$d) \quad i) \quad \frac{d}{dx} (\tan^2 x) dx = 2 \tan x \cdot \sec^2 x$$

$$= 2 \tan x (1 + \tan^2 x)$$

2

$$= 2 \tan x + 2 \tan^3 x$$

$$ii) \quad \frac{d}{dx} (\sec^2 x) = \frac{d}{dx} (1 + \tan^2 x)$$

$$= 2 \tan x + 2 \tan^3 x$$

1

$$iii) \quad \int_0^{\frac{\pi}{3}} 2 \tan^3 x dx$$

$$\frac{d}{dx} (\tan^2 x) = 2 \tan x + 2 \tan^3 x$$

$$\frac{d}{dx} (\tan^2 x) - 2 \tan x = 2 \tan^3 x$$

$$\begin{aligned} 1. \quad \int_0^{\frac{\pi}{3}} 2 \tan^3 x dx &= [\tan^2 x - \int 2 \tan x dx]_{\frac{\pi}{3}}^0 \\ &= [\tan^2 x - 2 \int \frac{\sin x}{\cos x} dx]_{\frac{\pi}{3}}^0 \\ &= [\tan^2 x - 2 \ln |\cos x|]_{\frac{\pi}{3}}^0 \\ &= (\sqrt{3})^2 - [2 \ln(\frac{1}{2}) - 2 \ln(1)] \\ &= 3 + 2 \ln(\frac{1}{2}) \\ &= 3 - 2 \ln 2 \end{aligned}$$

Q14

$$a) (\log_3 x)^2 - 4\log_3 x - 12 = 0 \quad \checkmark$$

$$\text{Let } u = \log_3 x$$

$$u^2 - 4u - 12 = 0$$

$$\begin{cases} u = 6 & \dots \log_3 x = 6 \\ u = -2 & \dots \log_3 x = -2 \end{cases} \quad \begin{cases} x = 729 \quad \checkmark \\ x = \frac{1}{9} \quad \checkmark \end{cases}$$

[The majority of students did well in this question.]

$$b) r = 5\%$$

$$i) n = 10$$

$$p = 18000$$

$$A = p(1+r)^n$$

$$A = 18000 \left(1 + \frac{5}{100}\right)^{10} \\ = \$29320.10 \quad \checkmark$$

ii)

$$A_1 = 1500(1.05)^9$$

$$A_2 = 1500(1.05)^8$$

$$A_3 = 1500(1.05)^7$$

...

$$A_{10} = 1500$$

$$A = A_1 + A_2 + A_3 + \dots + A_{10}$$

$$= 1500(1 + 1.05 + 1.05^2 + \dots + 1.05^9) \quad \checkmark$$

$$= 1500 \left(\frac{1(1.05^{10} - 1)}{1.05 - 1} \right)$$

$$= \$18866.84 \quad \checkmark$$

Thus the total amount

$$= 29320.10 + 18866.84$$

$$= \$48186.94 \quad \checkmark$$

Note: on the last deposit, there will be no interest earned. There is a number of students could not solve this question completely.

Q14 c) $y = x^3 + 6x^2 - 135x$

i) $= x(x^2 + 6x - 135)$
 $= x(x+15)(x-9) \checkmark$

ii) $y' = 3x^2 + 12x - 135$

$y' = 0 \therefore 3(x^2 + 4x - 45) = 0$

$\begin{cases} x = -9 \rightarrow y = 972 \\ x = 5 \rightarrow y = -400 \end{cases}$

$y'' = 6x + 12$ [Some students forgot to test for min/max points] \checkmark

$f''(-9) < 0$ Concave down, Max point at $(-9, 972) \checkmark$

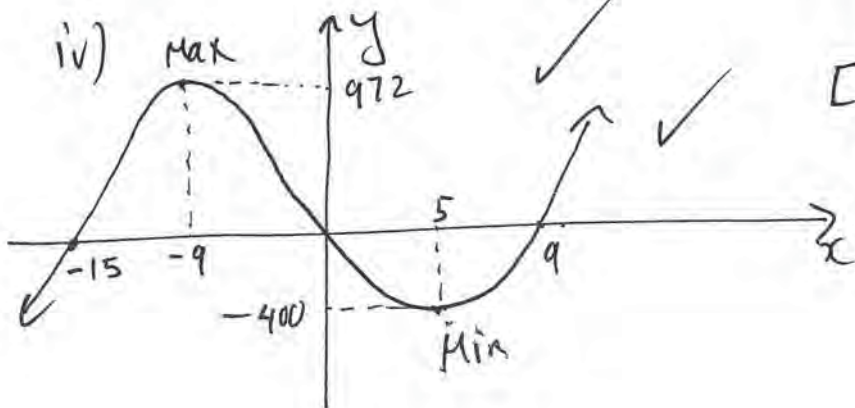
$f''(5) > 0$ Concave up, Min point at $(5, -400)$

iii) $f''(x) = 0 \therefore 6x + 12 = 0$
 $x = -2$

x	-2^{\ominus}	-2	-2^{\oplus}
f''	$-$	0	$+$

y'' changes sign \rightarrow point of inflexion

at $x = -2, y = 286$ p.o.i $(-2, 286) \checkmark$



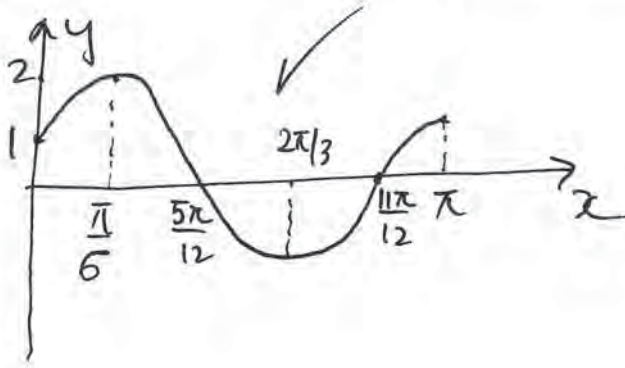
[Most students sketched well for this question.]

d) $y = 2 \cos\left(2x - \frac{\pi}{3}\right)$ for $0 \leq x \leq \pi$

$$y = 2 \cos 2\left(x - \frac{\pi}{6}\right)$$

amplitude = 2 ✓

period = $\frac{2\pi}{2} = \pi$



Note:

The graph is shifted by $\frac{\pi}{6}$ to the RHS. It is important to find the amplitude and period before graph.

Q15 2U 2015

a) $\sin x = \pm \frac{\sqrt{3}}{2}$

$x = \frac{\pi}{3}, \frac{2\pi}{3}, -\frac{\pi}{3}, -\frac{2\pi}{3}$

* many students only gave 2 answers.

b) $v = 6 \cos 2t$

i) $\frac{dv}{dt} = -12 \sin 2t$

$x = \int 6 \cos 2t dt$

$x = 3 \sin 2t + c$

at $t=0$ $x=-2$

$-2 = 0 + c$

$\therefore c = -2$

$x = 3 \sin 2t - 2$

* Many students put $x=2$ at $t=0$ \therefore had the incorrect answer

ii) $0 = 3 \sin 2t - 2$

$\sin 2t = \frac{2}{3}$

$2t = 0.729 \dots$

$t = 0.365$

* many students didn't use radian mode

iii) $6 \cos 2t = 0$

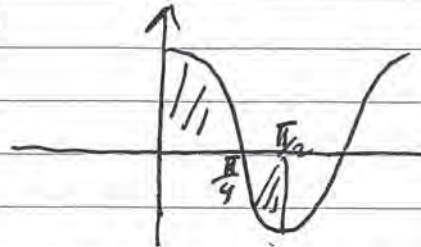
$2t = 1.57 \dots$

$t = 0.79 \text{ sec}$

iv)

$x = \int_0^{\frac{\pi}{4}} 6 \cos 2t dt \Big| \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} 6 \cos 2t dt$

$= 6m$

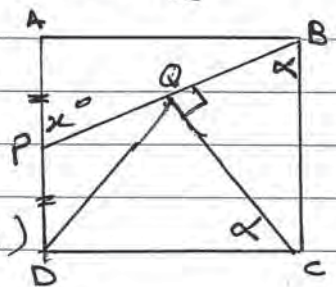


* Some students used different method. As long as there was clear working out I awarded the mark. But no marks for no working.

c)

i) $x = \alpha$

(alt \angle s, $AD \parallel BC$)



ii) In Δ 's BAP & QBC

$\angle BAP = \angle BQC = 90^\circ$

$\angle APB = \angle QBC$

$\therefore \Delta ABP \cong \Delta QBC$ (equiangular)

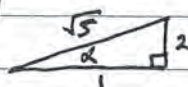
$PB^2 = 2^2 + 1^2$

$PB = \sqrt{5}$

$\frac{QC}{2} = \frac{2}{\sqrt{5}} \therefore QC = \frac{4}{\sqrt{5}}$

* or use Trig

iii)



$\sin \alpha = \frac{4}{\sqrt{5}}$

$= \frac{2}{\sqrt{5}}$

$\therefore \cos \alpha = \frac{1}{\sqrt{5}}$

$QD^2 = 2^2 + \left(\frac{4}{\sqrt{5}}\right)^2 - 2 \times 2 \left(\frac{4}{\sqrt{5}}\right) \left(\frac{1}{\sqrt{5}}\right)$

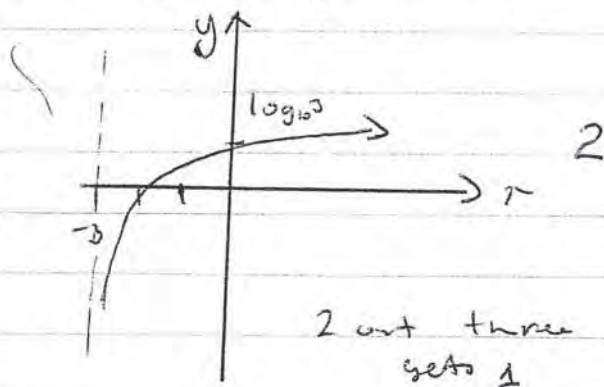
$QD = 2$

$= DC$

Many students used the wrong side or assumed isosceles Δ .

Question 16

a) $y = \log_e(x+3)$



b) i) $N = Ae^{kt}$

when $t=0$, $N=A=100$

ie $N = 100e^{kt}$

when $t=2$, $N=400$

$400 = 100e^{2k}$

$4 = e^{2k}$ 2

$k = \frac{\log_e 4}{2}$ (≈ 0.69)

0.6931471806

ii) when $t=10$, $N = 100e^{10k}$
 $= 102400$ 1

iii) $1000 = 100e^{kt}$

$10 = e^{kt}$

$kt = \log_e 10$

$k = \frac{\log_e 10}{k} = 3 \text{ min } 19 \text{ s}$

c) $y^2 = 8 - \frac{x^2}{2}$ on the x axis $y=0$ $x^2=16$
 $x=4$

$V = \pi \int_0^4 \left(8 - \frac{x^2}{2}\right) dx$

$= \pi \left[8x - \frac{x^3}{6}\right]_0^4$

$= \pi \left[32 - \frac{64}{6}\right]$

$= \frac{64\pi}{3} \text{ units}^3$ (≈ 67)

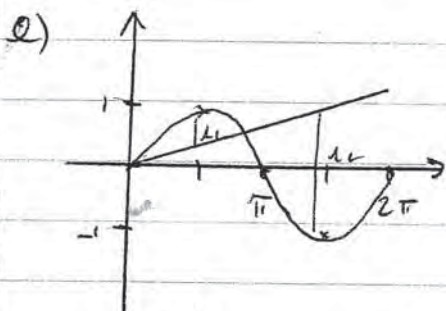
$$d) I = \int_2^{e^2} \frac{1 + \ln x}{x \ln x} dx$$

$$\text{Now } \frac{d}{dx} (x \ln x) = (x) \left(\frac{1}{x}\right) + 1(\ln x) \\ = 1 + \ln x$$

$$\therefore \int \frac{1 + \ln x}{x \ln x} dx = \ln(x \ln x)$$

$$I = \left[\ln(x \ln x) \right]_2^{e^2} \\ = \ln(e^2 \ln e^2) - \ln(2 \ln 2) \\ = \ln 2e^2 - \ln 2 \\ = \ln 2 + \ln e^2 - 1 \\ = \ln 2 + 2 \ln e - 1 \\ = \ln 2 + 2 - 1 \\ = \ln 2 + 1 \quad \checkmark$$

2



$$L_1 = \sin x - \frac{1}{2}x \\ L_2 = \frac{1}{2}x - \sin x$$

$$\frac{dL_1}{dx} = \cos x - \frac{1}{2}$$

$$\frac{dL_2}{dx} = \frac{1}{2} - \cos x$$

for a max/min $\frac{dL}{dx} = 0$

$$\cos x = \frac{1}{2} \\ x = \frac{\pi}{3} \text{ in quad 1}$$

$$\cos x = +\frac{1}{2} \\ x = \frac{5\pi}{3} \text{ in quad 4}$$

$$\frac{d^2L_1}{dx^2} = -\sin x \\ > 0$$

$$\frac{d^2L_2}{dx^2} = -\sin x \\ < 0 \therefore \text{max}$$

$$\therefore L_2 = \frac{5\pi}{3} \times \frac{1}{2} - \sin \frac{5\pi}{3} \\ = \frac{5\pi}{6} - \left(-\frac{\sqrt{3}}{2}\right) \\ = \frac{5\pi}{6} + \frac{\sqrt{3}}{2}$$