Sydney Girls High School
2016

TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics

General Instructions
- Reading Time – 5 minutes
- Working time – 3 hours
- Write using black or blue pen, black pen is preferred
- Board-approved calculators may be used
- In Questions 11 – 16, show relevant mathematical reasoning and/or calculations
- A mathematics exam reference sheet is also provided

Total marks – 100

Section I Pages 3 – 6

10 Marks
- Attempt Questions 1 – 10
- Answer on the Multiple Choice answer sheet provided
- Allow about 15 minutes for this

Section II section Pages 7 – 16

90 Marks
- Attempt Questions 11 – 16
- Answer on the blank paper provided
- Begin a new page for each question
- Allow about 2 hours and 45 minutes for this section

Section I
10 marks
Attempt Questions 1 – 10
Allow about 15 minutes for this section
Use the multiple – choice answer sheet for Questions 1 – 10

(1) What are the coordinates of the midpoint of P(3,- 4) and Q(-1,2)?

(A) (-1,1)

(B) (1,-1)

(C) (1,1)

(D) (-1,-1)

(2) The value of the limit $\lim_{x\to 10}\frac{x^2-100}{x-10}$ is:

(A) Undefined

(B) 0

(C) 8

(D) 20

(3) A raffle consists of twenty tickets in which there are two prizes. Mike buys five tickets. First prize is two movie vouchers and second prize is one movie voucher. The probability that Mike wins at least one movie voucher is

(A) $\frac{17}{38}$

(B) $\frac{27}{76}$

(C) $\frac{7}{16}$

(D) $\frac{5}{20}$

(4) The curve $y = ax^2 - 6x + 3$ has a stationary point at $x = 1$. What is the value of a?

(A) 2

(B) -1

(C) 3

(D) -3

(5) Which of the following correctly represents the sum $1 + x + x^2 + x^3 + ... + x^n$?

(A) $\sum_{k=1}^{n} x^k$

(B) $\sum_{k=1}^{n+1} x^k$

(C) $\sum_{k=1}^{n} x^{k-1}$

(D) $\sum_{k=1}^{n+1} x^{k-1}$

(6) What is the derivative of $\frac{e^{-x}}{x}$?

(A) $\frac{-xe^{-x} - e^{-x}}{x^2}$

(B) $\frac{-xe^{-x} + e^{-x}}{x^2}$

(C) $\frac{e^{-x} + xe^{-x}}{x^2}$

(D) $\frac{e^{-x} - xe^{-x}}{x^2}$

(7) The quadratic equation $x^2 + 3x - 1 = 0$ has roots α and β.
The value of $\alpha\beta + (\alpha^2 + \beta^2)$ is:

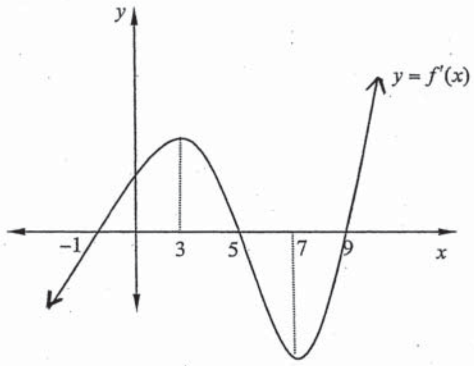
(A) -10

(B) 10

(C) -8

(D) 8

(8) The graph of the derivative $y = f'(x)$ is drawn below.



A maximum turning point on $y = f(x)$ occurs at:

(A) $x = -1$

(B) $x = 3$

(C) $x = 5$

(D) $x = 7$

(9) What is the value of $\int \frac{\sin x}{\cos x}\,dx$?

(A) $\sec^2 x + C$

(B) $\frac{1}{2}\tan^2 x + C$

(C) $\log_e \cos x + C$

(D) $\log_e \sec x + C$

(10) A water tank holds 800 litres of water. Water is let out of the tank at a rate of R litres per minute where $R = 100t$ after t minutes. How long does it take the tank to empty?

(A) 2 minutes

(B) 4 minutes

(C) 6 minutes

(D) 8 minutes

Section II

90 marks

Attempt Questions 11 – 16

Allow about 2 hours and 45 minutes for this section

Answer on the blank paper provided. Begin a new page for each question.
Your responses should include relevant mathematical reasoning and /or calculations.

Question 11 **(15 Marks)**

(a) Evaluate $\dfrac{2.1^3 - 29}{\sqrt{4.01^2 - 0.8^2}}$ to 3 significant figures. 1

(b) Factorise fully $3x^2 + 5x - 2$. 1

(c) Find the integers a and b such that: $\dfrac{4\sqrt{3}}{\sqrt{7}+\sqrt{3}} = a + b\sqrt{21}$ 2

(d) Solve $|2x-1| < 3$. 2

(e) Solve for x: $4^x - 5\times 2^x + 4 = 0$ 2

(f) Find the domain of the function $f(x) = \sqrt{x^2 + x - 6}$ 2

(g) Find the equation of the normal to $y = x\sin x$ at the point where $x = \frac{\pi}{2}$. 3

(h) Find the coordinates of the vertex of the parabola with equation 2
$$x^2 - 10x - 16y - 7 = 0$$

End of Question 11

Question 12 (Begin a new page) **(15 Marks)**

(a) Differentiate the following with respect to x:

(i) $(3x^2+1)^8$ 1

(ii) $e^{\tan 2x}$ 1

(iii) $\ln\dfrac{x}{2x+1}$ 2

(b) Find $\int e^x(e^x+1)dx$ 2

(c) Evaluate $\int_9^4 \sqrt{x}\,dx$ 2

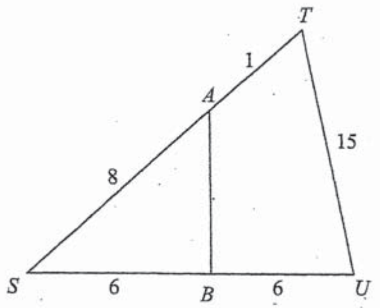
(d) The first three terms of an arithmetic series are 48, 41 and 34.

(i) Find an expression for the k^{th} term. 1

(ii) Find the 45^{th} term. 1

(iii) Find the sum of the first 45 terms. 1

(e)



NOT TO SCALE

(i) Prove triangle SAB is similar to triangle SUT. 3

(ii) Hence, find the length of AB. 1

End of Question 12

Question 13 (Begin a new page) **(15 Marks)**

(a) Solve $\log_3(2x-7)=2$. 2

(b) Given the parabola $(y+2)^2=-4(x+3)$ state the :

(i) co-ordinates of its focus. 1

(ii) equation of the directrix. 1

(c) Sketch, on the same graph, the intersection of the regions 3

$$y<x^2-4x+3 \text{ and } y\le x+3,$$

showing the x and y intercepts in the sketch.

(d) There are two groups of people at a party and Minh is blind-folded. In the first group there are 4 men, 3 women and 2 children. In the second group there are 7 men and 5 women.

Minh is spun around and asked to select one person at random.

(i) Find the probability that Minh approaches the first group and then selects a woman. 1

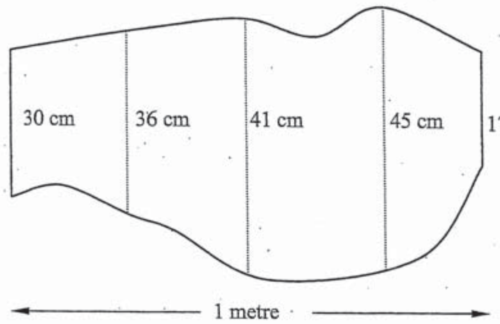
(ii) Find the probability that a woman from either group is selected. 2

Question 13(continued) **(15 Marks)**

(e) The area of the sector of a circle with radius 8 cm is $\frac{56\pi}{5}cm^2$. 2

Find the angle that is subtended at the centre of the sector.

(f) Sammy needs to estimate the area of the following hole in the wall.



NOT TO SCALE

(i) Copy and complete the table below. 1

Distance from left edge (cm)	0				100
Height of hole (cm)	30	36			

(ii) Use Simpson's Rule and all the values from the table to find an approximation for the area of the hole. 2

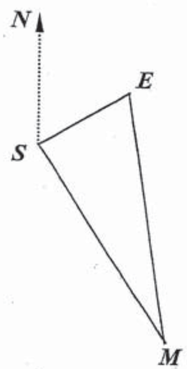
End of Question 13

Question 14(Begin a new page) **(15 Marks)**

(a) Show that $\frac{\tan\theta}{\sec\theta-1}-\frac{\tan\theta}{\sec\theta+1}=2\cot\theta$. 2

(b) Find the value of x if $\sum_{n=0}^{\infty}\frac{9}{x^{n+1}}=18$. 3

(c) Two cruise ships set sail from Sydney Harbour (S).
The *Elvis Presley Tribute Cruise* (E) sails at 18 km/h on a bearing of 049°
while the *Michael Jackson Tribute Cruise* (M) sails at 21 km/h along a bearing of 151°.

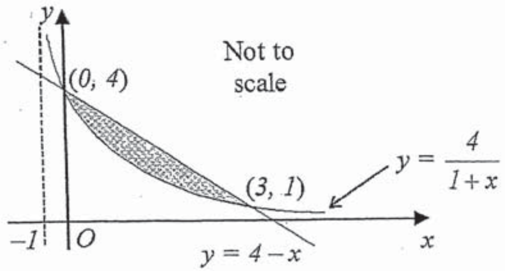


(i) Show that $\angle ESM = 102^\circ$. 1

(ii) Calculate the distance between the cruise ships to the nearest kilometre after 3 hours. 2

Question 14(continued) **(15 Marks)**

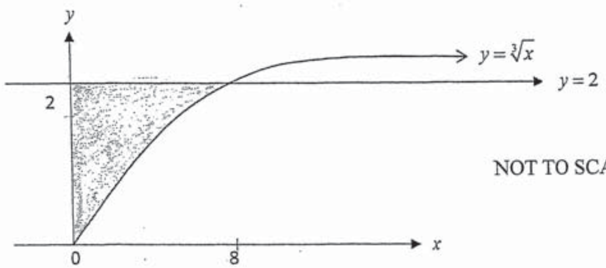
(d) The diagram below shows part of the hyperbola $y=\dfrac{4}{1+x}$ and the line $y=4-x$.



(i) Show that the line $y=4-x$ intersects the hyperbola $y=\dfrac{4}{1+x}$ at (0,4) and (3,1). 2

(ii) Hence calculate the exact area of the shaded region. 2

(e) The diagram below shows the region bounded by the curve $y=\sqrt[3]{x}$, the y-axis and the line $y=2$.



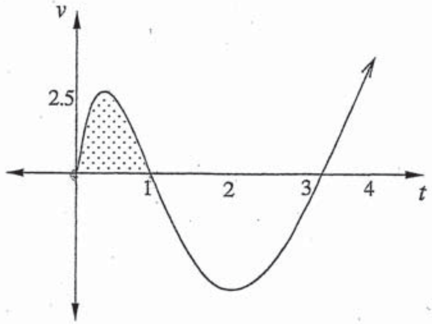
3

Given that the point of intersection of $y=\sqrt[3]{x}$ and $y=2$ is (8,2), find the <u>exact</u> volume of the solid formed when the region shown is rotated about the $x-$axis.

End of Question 14

Question 15(Begin a new page) **(15 Marks)**

(a) The graph below represents the velocity (v m/s) with respect to time (t sec) of a particle moving in a straight line.



(i) What is the velocity of the particle when $t=3$ seconds? 1

(ii) At what time(s) is the particle's acceleration zero? 1

(iii) What does the shaded area on the graph represent? 1

(b) Consider the curve $y=2x^3+3x^2-36x+4$ for $-5\le x\le 5$.

(i) Find the stationary points and determine their nature. 3

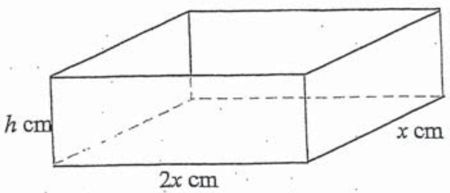
(ii) Find the point of inflexion. 2

(iii) Sketch the curve for $-5\le x\le 5$. 1

(iv) Find the maximum value in the domain given. 1

Question 15(continued) **(15 Marks)**

(c) Joe is building a small toy box with no lid. The box is twice as long as it is wide. The box has a total external surface area of $3750 cm^2$.



(i) Show that the height h of the toy box is given by $h = \frac{625}{x} - \frac{x}{3}$. 1

(ii) Find the dimensions of the box which gives a maximum volume. 2

(iii) Joe decides that the height of the box will be $10\frac{5}{6} cm$. 2

Find the new dimensions of the box and hence find its volume if the surface area is to remain at $3750 cm^2$.

End of Question 15

Question 16 (Begin a new page) **(15 Marks)**

(a) A particle is moving in a straight line. At time t seconds its displacement is x metres from the fixed point O on the line and its velocity $v\ ms^{-1}$ is given by $v = 3t^2 - 2t - 1$. Initially the particle is 1 metre to the right of O.

(i) Show that the particle is at rest after 1 second. 1

(ii) Find the displacement x in terms of t. 1

(iii) Find the distance travelled by the particle in the first 2 seconds of its motion. 2

(b) The University of Gauss offers scholarships to young Mathematicians. The fund is set up with a single investment of \$70 000. The fund earns interest at 8% p.a. compounded yearly. A scholarship, valued at \$10 000, is awarded each year by the university. The first scholarship is awarded 1 year after the investment is made.

Let F_n be the value of the fund after n years.

(i) Show that the value of the fund after 3 years is 2

$$F_3 = 70\,000(1.08)^3 - 10\,000(1.08)^2 - 10\,000(1.08) - 10\,000$$

(ii) Deduce that $F_n = 125\,000 - 55\,000(1.08)^n$. 2

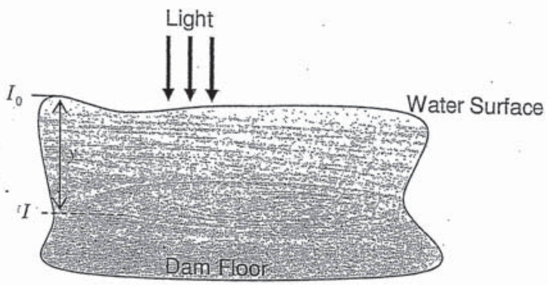
(iii) Calculate the numbers of years the full value of the scholarship can be awarded by the university. 2

Question 16(continued) **(15 Marks)**

(c) The light intensity, I units, passing through y metres of water is given by the equation:

$$I = I_{\circ}e^{-ky} \qquad y \geq 0$$

where $I_{\circ}$ units is the light intensity at the surface and k is a constant called the absorption coefficient.



Above is the cross-section of Warragamba Dam and in the table below are the readings of two light intensity measurements from the dam.

y metres	I Units
2	1.2
8	0.9

(i) Using the table show that $k = \frac{1}{6}\log_e\left(\frac{4}{3}\right)$, and hence find $I_{\circ}$ correct to 2 decimal places. 2

(ii) Sketch the graph $I = I_{\circ}e^{-ky}$ indicating the vertical intercept. 1

(iii) The fish that live in this dam require light in order to survive. One kind of fish requires light of an intensity that is no less than 35% of the intensity at the surface. Determine the maximum depth, correct to the nearest metre, at which this kind of fish can survive. 2

End of Trial Paper

Sydney Girls High School

Mathematics Faculty

Multiple Choice Answer Sheet
2016 Trial HSC Mathematics

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample $2 + 4 = ?$ (A) 2 (B) 6 (C) 8 (D) 9

A ○ B ● C ○ D ○

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ● B ✖ C ○ D ○

If you change your mind and have crossed out what you consider to be the correct answer, then indicate this by writing the word *correct* and drawing an arrow as follows:

correct

A ✖ B ✖ C ○ D ○

Student Number: ANSWERS

Completely fill the response oval representing the most correct answer.

1. A ○ B ● C ○ D ○
2. A ○ B ○ C ○ D ●
3. A ● B ○ C ○ D ○
4. A ○ B ○ C ● D ○
5. A ○ B ○ C ○ D ●
6. A ● B ○ C ○ D ○
7. A ○ B ● C ○ D ○
8. A ○ B ○ C ● D ○
9. A ○ B ○ C ○ D ●
10. A ○ B ● C ○ D ○

Q11

a) $\frac{2.1^3 - 29}{\sqrt{4.01^2 - 0.8^2}} = -5.02$

b) $3x^2 + 5x - 2$
$= (x+2)(3x-1)$

c) $\frac{4\sqrt{3}}{\sqrt{7}+\sqrt{3}} \times \frac{\sqrt{7}-\sqrt{3}}{\sqrt{7}-\sqrt{3}}$

$= \frac{4\sqrt{21}-12}{7-3}$

$= \frac{4(\sqrt{21}-3)}{4}$

$= \sqrt{21} - 3$

$\therefore a = -3, \quad b = 1$

d) $|2x-1| < 3$

$-3 < 2x-1 < 3$

$-2 < 2x < 4$

$-1 < x < 2$

The answer should be written in one statement and not separated (since x is in between -1 and 2.)

e) $4^x - 5 \times 2^x + 4 = 0$
$2^{2x} - 5 \times 2^x + 4 = 0$
let $a = 2^x$

$\therefore a^2 - 5a + 4 = 0$
$(a-4)(a-1) = 0$

$\therefore a = 4$ or $a = 1$
$\therefore 2^x = 2^2 \qquad 2^x = 2^0$
$x = 2 \qquad x = 0$

Q11

f) $f(x) = \sqrt{x^2 + x - 6}$
$= \sqrt{(x+3)(x-2)}$

$\therefore x \leq -3, \ x \geq 2$

The domain should include -3 and 2 since it is possible to square root zero.

g) $y = x \sin x$

$y' = x\cos x + \sin x$

The gradient of tangent at $x = \frac{\pi}{2}$

$m = \frac{\pi}{2}\cos\frac{\pi}{2} + \sin\frac{\pi}{2}$

$= 0 + 1$

$m = 1$

$\therefore$ the gradient of the normal is -1

at $x = \frac{\pi}{2} \therefore y = \frac{\pi}{2}\sin\frac{\pi}{2}$

$y = \frac{\pi}{2}$

The eq. of the normal is

$y - \frac{\pi}{2} = -(x - \frac{\pi}{2})$

$\therefore x + y - \pi = 0$

h) $x^2 - 10x - 16y - 7 = 0$
$(x-5)^2 = 16y + 7 + 25$

$(x-5)^2 = 16(y+2)$

$\therefore$ vertex is $(5-2)$

Q12

a) i) $\frac{d}{dx}(3x^2+1)^8$

$= 48x\,(3x^2+1)^7$

ii) $\frac{d}{dx}\left(e^{\tan 2x}\right)$

$= 2\sec^2 2x\; e^{\tan 2x}$

iii) $\frac{d}{dx}\left(\ln \frac{x}{2x+1}\right)$

The most efficient approach is to use the log laws to separate the fraction before differentiating.

$= \frac{d}{dx}\left(\ln x - \ln(2x+1)\right)$

$= \frac{1}{x} - \frac{2}{2x+1}$

or

$= \frac{1}{x(2x+1)}$

b) $\int e^x(e^x+1)\,dx$

$\int (e^{2x} + e^x)\,dx$

$= \frac{1}{2}e^{2x} + e^x + C$

or $\frac{(e^x+1)^2}{2} + C$ (ξ×1)

Q12

c) $\int_9^4 \sqrt{x}\;dx$

$= \int_9^4 x^{\frac{1}{2}}\,dx$

$= \left[\frac{2x^{3/2}}{3}\right]_9^4$

$= \frac{2}{3}\left(2^3 - 3^3\right)$

$= \frac{-38}{3}$

d) i) $d = -7 \quad a = 48$

$T_k = 48 + (k-1)(-7)$

$= 55 - 7k$

ii) $T_{45} = 55 - 7\times 45$

$= -260$

iii) $S_{45} = \frac{45}{2}\left[48 - 260\right]$

$= -4770$

e) i) $\frac{SB}{ST} = \frac{SA}{SU} = \frac{2}{3}$

$\angle S$ common

$\therefore \Delta SAB \,|||\, \Delta SUT$ two pairs of sides are in proportion and their included angles are equal.

ii) $\frac{AB}{15} = \frac{2}{3}$ $\therefore AB = 10$

YR 12 Mathematics TRIAL HSC 2016 SOLUTIONS.

Question 13.

a) $\log_3 (2x-7) = 2$

$$3^2 = 2x-7$$
$$2x = 16$$
$$\underline{x = 8}$$

* By definition: If $\log_a x = m$ Then $x = a^m$

Not many students are confident to use this definition to solve an equation.

b) $(y+2)^2 = -4(x+3)$

vertex $= (-3,-2)$ focal length $= 1$.

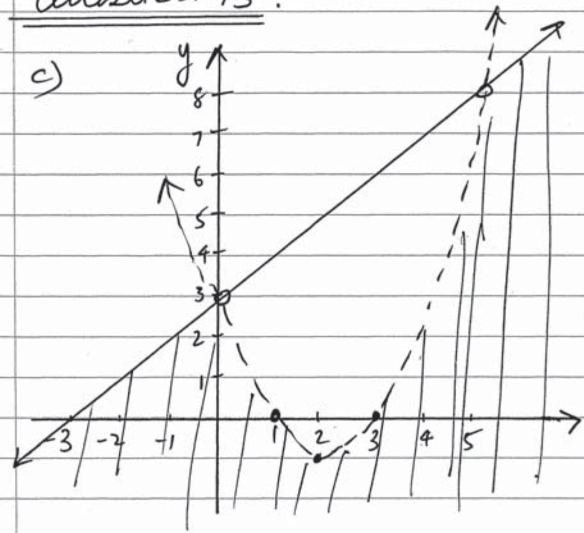
i) focus $= (-3-1, -2)$
$$= \underline{(-4,-2)}$$

ii) directrix: $x = -3+1$
$$\underline{x = -2}$$

* Many students are still indicating that the focal length is a negative value. Length is always positive.
* Due to the large variation of answers only the correct answers were awarded one mark for each part.

Question 13.

c)



Region:

$y < x^2 - 4x + 3$

$y \le x + 3$.

* POI: (0,3) and (5,8)
* · intercepts
  · dotted parabola
  · solid line
* · shaded region.

* Students needed to show the intersection of the region, which means finding points of intersection. This part needs to be revised by most students.

d).

- Group 1: 4M, 3W, 2C
- Group 2: 7M, 5W

i) $P(G1, W)$
$$= \frac{1}{2} \times \frac{3}{9}$$
$$= \frac{3}{18}$$
$$= \frac{1}{6}$$

ii) $P(G1, W)$ or $P(G2, W)$
$$= \frac{1}{2} \times \frac{3}{9} + \frac{1}{2} \times \frac{5}{12}$$
$$= \frac{1}{6} + \frac{5}{24}$$
$$= \frac{3}{8}.$$

* Most students did not answer this question well.

Question 13 :

e) $A = \frac{1}{2} r^2 \theta$

$$\frac{56\pi}{5} = \frac{1}{2} \times 8^2 \times \theta$$

$$32\theta = \frac{56\pi}{5}$$

$$\theta = \frac{56\pi}{160} = \frac{7\pi}{20}$$

* θ is always in radians.

f) i)

distance (x cm)	0	25	50	75	100
height (y cm)	30	36	41	45	17
weight	1	4	2	4	1
weight × y cm	30	144	82	180	17

ii) Area $\doteqdot \frac{h}{3} \times \Sigma(\text{weight} \times y\text{cm})$

$$= \frac{25}{3} \times 453$$

$$= 3775 \text{ cm}^2.$$

* Well done on this part of the question.

Question 14.

a) Show $\dfrac{\tan\theta}{\sec\theta - 1} - \dfrac{\tan\theta}{\sec\theta + 1} = 2\cot\theta$.

$$LHS = \frac{\tan\theta(\sec\theta+1) - \tan\theta(\sec\theta - 1)}{\sec^2\theta - 1}$$

$$= \frac{\cancel{\tan\theta\sec\theta} + \tan\theta - \cancel{\tan\theta\sec\theta} + \tan\theta}{\tan^2\theta}$$

$$= \frac{2\cancel{\tan\theta}}{\cancel{\tan\theta}\tan\theta}$$

$$= 2\cot\theta$$

$$= RHS.$$

* Work should be set out as $LHS \Rightarrow RHS$.

b) $\displaystyle\sum_{n=0}^{\infty} \frac{9}{x^{n+1}} = \frac{9}{x} + \frac{9}{x^2} + \frac{9}{x^3} + \cdots\cdots$

limiting sum $a = \frac{9}{x}$ $r = \frac{1}{x}$

$$S_\infty = \frac{a}{1-r}$$

$$18 = \frac{\frac{9}{x}}{1 - \frac{1}{x}}$$

$$18 - \frac{18}{x} = \frac{9}{x}$$

$$18 = \frac{27}{x}$$

$$\therefore x = \frac{27}{18} = \frac{3}{2}.$$

* Many students did not recognise this as a limiting sum and used the other G.P sum formula, with poor setting out.

Question 14:

c) i) $\angle ESM = \angle NSM - \angle NSE$

$= 151° - 49°$

$\therefore \angle ESM = 102°$

ii) $SE = 18\text{ km/h} \times 3\text{h} = 54\text{ km}$.

$SM = 21\text{ km/h} \times 3\text{h} = 63\text{ km}$.

$EM = ?$

$EM^2 = SE^2 + SM^2 - 2 \times SE \times SM \cos \angle ESM$

$= 54^2 + 63^2 - 2 \times 54 \times 63 \cos 102°$

$EM^2 = 6885 - 6804 \cos 102°$

$\therefore EM \doteqdot \sqrt{8299.631144}$

$EM \doteqdot 91$ km (nearest km).

d) i) $\dfrac{4}{1+x} = 4 - x$.

$4 = (4-x)(1+x)$

$4 = 4 + 4x - x - x^2$

$x^2 - 3x = 0$

$x(x-3) = 0$

$\therefore \begin{cases} x = 0 \\ y = 4 \end{cases}$ or $\begin{cases} x = 3 \\ y = 1 \end{cases}$

$\therefore$ The line $y = 4 - x$ and hyperbola $y = \dfrac{4}{1+x}$ intersect at the points $(0, 4)$ and $(3, 1)$.

Question 14:

d) ii) Area $= \displaystyle\int_0^3 \left[(4-x) - \frac{4}{1+x}\right] dx$

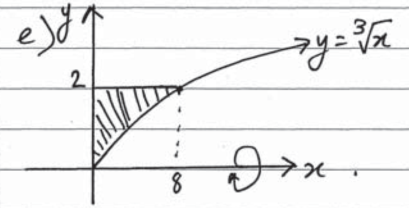
$= \left[4x - \dfrac{x^2}{2} - 4\ln(1+x)\right]_0^3$

$= \left(12 - \dfrac{9}{2} - 4\ln 4\right) - \left(0 - 0 - \underbrace{\cancel{4\ln 1}}_{0}\right)$

$= \left(\dfrac{15}{2} - 4\ln 4\right)$ units^2.

* Many students had trouble when integrating the log. function.

e)



$y = x^{1/3}$

$y^2 = x^{2/3}$

Volume (about x-axis)

= Volume of cylinder − Volume under curve.

$= \pi \times 2^2 \times 8 - \pi \displaystyle\int_0^8 y^2\, dx$

$= 32\pi - \pi \displaystyle\int_0^8 x^{2/3}\, dx$

$= 32\pi - \pi \left[\dfrac{3}{5} x^{5/3}\right]_0^8$

$= 32\pi - \dfrac{3\pi}{5}\left(8^{5/3} - 0\right)$

$= 32\pi - \dfrac{3\pi}{5} \times 32$

$= \dfrac{64\pi}{5}$ units^3.

* Students lost marks due to their indices calculations but generally understood the concept of volume about the x-axis.

Question 15 (15 marks) Mathematics

a) i) at $t=3$, $v = 0\,ms^{-1}$ (1)

(1 each) ii) at $t=\frac{1}{2}$, $t=2$ (1)

iii) The distance travelled in the first second. (1)

(i), ii) was completed well, iii) was completed poorly, most students didn't know).

b) i) $y = 2x^3 + 3x^2 - 36x + 4$ (3)

$\frac{dy}{dx} = 6x^2 + 6x - 36$

(This part was completed very well).

stationary point at $\frac{dy}{dx} = 0$

$x^2 + x - 6 = 0$

$(x+3)(x-2) = 0$

$x = -3$ or $x = 2$

at $x = -3$, $y = 85$ $P_1(-3, 85)$ (1)

at $x = 2$, $y = -40$ $P_2(2, -40)$

$\frac{d^2y}{dx^2} = 12x + 6$

at $x = -3$, $\frac{d^2y}{dx^2} < 0$ ∴ Maximum turning point

at $x = 2$, $\frac{d^2y}{dx^2} > 0$ ∴ Minimum turning point (1)

∴ $P_1(-3, 85)$ is a maximum turning point

$P_2(2, -40)$ is a minimum turning point. (1)

ii) Point of inflexion at $\frac{d^2y}{dx^2} = 0$ Test for change in concavity (2)

$\frac{d^2y}{dx^2} = 12x + 6$

$12x + 6 = 0$ (1)

$x = -\frac{1}{2}$ $y = 22\frac{1}{2}$

x	-1	$-\frac{1}{2}$	0
d^2y/dx^2	$-$	0	$+$

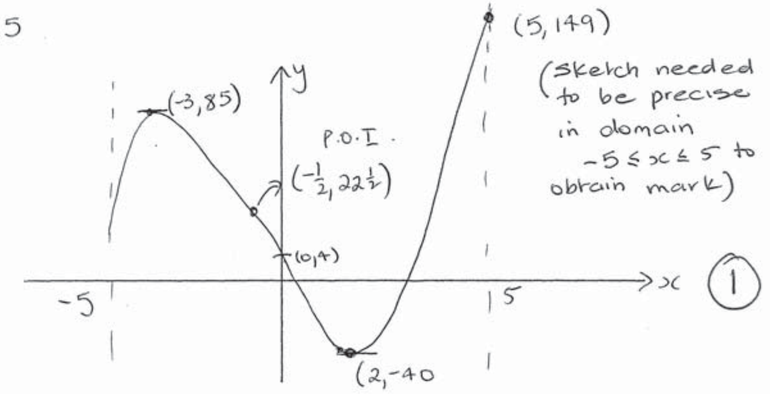
(1)

∴ Since there is a change in concavity $P(-\frac{1}{2}, 22\frac{1}{2})$ is a P.O.I

(Many students are still not testing P.O.I)

Question 15

b) iii) (1)



(1)

(sketch needed to be precise in domain $-5 \le x \le 5$ to obtain mark)

iv) Maximum value $= 149$, at $x = 5$. (1) (1)

(was completed well)

c) i) $SA = 2x^2 + 2(2x)h + 2xh$ (1)

$3750 = 2x^2 + 6xh$

$6xh = 3750 - 2x^2$

$h = \frac{3750}{6x} - \frac{2x^2}{6x}$ (1)

$h = \frac{625}{x} - \frac{x}{3}$

(This section was completed well)

ii) $V = 2x(x).h$ (2)

$= 2x^2\left(\frac{625}{x} - \frac{x}{3}\right)$

$V = 1250x - \frac{2x^3}{3}$

$V' = 1250 - 2x^2$

$V' = 0$

$1250 = 2x^2$

$x^2 = 625$ (1)

$x = 25$

$V'' = -4x$

at $x = 25$, $V'' < 0$

∴ Max Volume

$h = \frac{625}{x} - \frac{x}{3}$

$h = \frac{625}{25} - \frac{25}{3} = 16\frac{2}{3}$ (1)

∴ Dimensions: x, $2x$, h

: 25 cm, 50 cm and $16\frac{2}{3}$ cm

(Overall, section was completed well)

Question 15

c) iii) $h = \frac{625}{x} - \frac{x}{3}$

2

$10\frac{5}{6} = \frac{625}{x} - \frac{x}{3} \quad (\times 6x)$

$65x = 3750 - 2x^2$

$2x^2 + 65x - 3750 = 0$

$x = \frac{-65 \pm \sqrt{4225 + 30000}}{4}$

$x = \frac{-65 + 185}{4}$

$x = 30$ 1

(This section was completed poorly. Most failed to form a quadratic and then be able to use formula to solve and obtain dimensions)

$\therefore$ Dimensions: x, $2x$ and h

30 cm, 60 cm and $10\frac{5}{6}$ cm

$\therefore$ Volume $= 30 \times 60 \times 10\frac{5}{6}$

$= 19500 \text{ cm}^3$ 1

Mathematics 2016 Trial

Q16

a) i) $t = 1$

$V = 3(1) - 2(1) - 1$

$= 3 - 2 - 1$

$= 0$

particle at rest after 1 sec.

ii) $x = \int 3t^2 - 2t - 1 \, dt$

$= \frac{3t^3}{3} - \frac{2t^2}{2} - t + c$

at $t = 0$ $x = 1$

$1 = 0 - 0 - 0 + c$

$\therefore c = 1$

$\therefore \boxed{x = t^3 - t^2 - t + 1}$

Some students didn't find C.

iii)

$x = \left| \int_0^1 3t^2 - 2t - 1 \, dt \right| + \int_1^2 3t^2 - 2t - 1 \, dt$

$x = \left| \left[t^3 - t^2 - t \right]_0^1 \right| + \left[t^3 - t^2 - t \right]_1^2$

$= |1 - 1 - 1| + \left[(8 - 4 - 2) - (1 - 1 - 1) \right]$

$= 1 + 3$

$= 4$

Some students had no proper working so didn't get the full mark

b) i) $F_1 = 70000(1.08) - 10000$

$F_2 = [70000(1.08) - 10000]1.08 - 10000$

$= 70000(1.08)^2 - 1.08(10000) - 10000$

$F_3 = F_2 \times 1.08 - 10000$

$F_3 = 70000(1.08)^3 - 10000(1.08)^2 - 10000(1.08) - 10000$

* Some student still can't show this property

ii) $F_n = 70000(1.08)^n - 10000(1 + 1.08 + 1.08^2 + \ldots 1.08^{n-1})$

$= 70000(1.08)^n - 10000\left(\frac{1(1.08^n - 1)}{0.08}\right)$

$= 70000(1.08)^n - 125000(1.08^n - 1)$

$= 70000(1.08)^n - 125000(1.08)^n + 125000$

$= 125000 - 55000(1.08)^n$

* To get the full mark you should have had all the steps shown.

iii) $125000 - 55000(1.08)^n < 10000$

$-55000(1.08)^n < -115000$

$n \log 1.08 > \log \frac{115}{55}$

$n > 9.58$

$\therefore$ 10 years

* students should do this as an inequality

c) $I = I_0 e^{-ky}$

$1.2 = I_0 e^{-2k}$ — ①

$0.9 = I_0 e^{-8k}$ — ②

From ①

$$I_0 = \frac{1.2}{e^{-2k}}$$

$$0.9 = \frac{1.2}{e^{-2k}} \cdot e^{-8k}$$

$$0.9 = 1.2\, e^{-6k}$$

$$0.9 = \frac{1.2}{e^{6k}}$$

$$e^{6k} = \frac{1.2}{0.9}$$

$$6k = \ln\left(\frac{3}{4}\right)$$

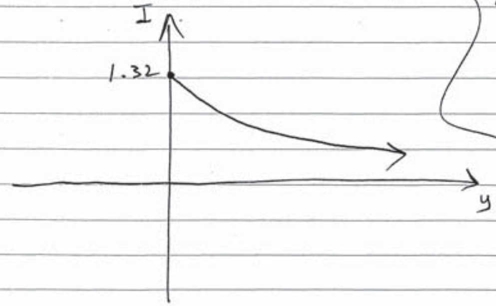
$$k = \frac{1}{6}\ln\left(\frac{3}{4}\right)$$

* all steps are required to get full mark as it is show question

$$\therefore I_0 = \frac{1.2}{e^{-2\left(\frac{1}{6}\ln\frac{4}{3}\right)}}$$

$$= \frac{1.2}{e^{\left(\ln\frac{4}{3}\right)^{-\frac{1}{3}}}}$$

$$\doteq 1.32$$



iii) $I > 0.35 I_0$

$$K \doteq 0.04795$$

$$\therefore \cancel{I_0} e^{-0.04795y} \geq 0.35 \cancel{I_0}$$

$$-0.04795y \geq \ln 0.35$$

$$y \leq \frac{\ln 0.35}{-0.04795}$$

$$y \leq 21.8$$

$\therefore$ max depth is

21 m

many students had problems with this question. Again they had to use inequality otherwise you couldn't get the correct answer.

Some also couldn't calculat k correctly.