Sydney Girls High School
2017

TRIAL HIGHER SCHOOL CERTIFICATE
EXAMINATION

Mathematics

General Instructions

- Reading Time – 5 minutes
- Working time – 3 hours
- Write using black or blue pen
  Black pen is preferred
- Board-approved calculators may be used
- A table of standard integrals is provided at the back of this paper
- In Questions 11 – 16, show relevant mathematical reasoning and/or calculations

Total marks – 100

Section I Pages 3 – 5

10 Marks

- Attempt Questions 1 – 10
- Answer on the Multiple Choice answer sheet provided
- Allow about 15 minutes for this section

Section II Pages 7 – 17

90 Marks

- Attempt Questions 11 – 16
- Answer on the blank paper provided
- Begin a new page for each question
- Allow about 2 hours and 45 minutes for this section

Name: …………………………………………………………. Teacher: …………………………………………………….	**THIS IS A TRIAL PAPER ONLY** It does not necessarily reflect the format or the content of the 2017 HSC Examination Paper in this subject.

Section I

10 marks
Attempt Questions 1 – 10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1 – 10

(1) Solve the equation $\frac{3-4x}{5}=2-\frac{3x}{4}$.

(A) -2

(B) -28

(C) $\frac{-7}{9}$

(D) 28

(2) What is the period of the function $y=4\tan\left(\frac{x}{5}\right)$?

(A) $\frac{2\pi}{5}$

(B) $\frac{\pi}{5}$

(C) 5π

(D) 10π

(3) For what values of k does $x^2+kx+k+3=0$ have real roots?

(A) 28

(B) $-2\le k\le 6$

(C) $k\ge 6$, $k\le -2$

(D) $k\ge 2$, $k\le -6$

(4) The coordinates of the focus of the parabola $y^2 - 4y = 3x + 8$ are:

(A) $\left(2, -3\frac{1}{4}\right)$

(B) $\left(-3\frac{1}{4}, 2\right)$

(C) $\left(2\frac{3}{4}, -3\frac{1}{4}\right)$

(D) $\left(2\frac{3}{4}, -4\right)$

(5) The derivate of $\cos(e^{3x})$ is :

(A) $3e^{3x}\sin(e^{3x})$

(B) $e^{3x}\sin(e^{3x})$

(C) $-3e^{3x}\cos(e^{3x})$

(D) $-3e^{3x}\sin(e^{3x})$

(6) How many solutions are there in the equation $\sin 2x = 1 + 3x$, where $-\pi \le x \le \pi$?

(A) 0

(B) 1

(C) 2

(D) 3

(7) Solve $\ln(6-x) = 2\ln x$.

(A) $x=-3$ or $x=2$

(B) $x=6$

(C) $x=2$

(D) $x=2$ or $x=6$

(8) Given $a=e^x$, which expression below is equal to $\log_e \sqrt{a^3}$?

(A) $e^{\frac{3}{2}}$

(B) $\frac{3x}{2}$

(C) $\frac{2x}{3}$

(D) $\frac{1}{2}e^{3x}$

(9) $\sum_{r=5}^{\infty} \left(\frac{2}{3}\right)^{r-1} =$

(A) 3

(B) $\frac{16}{81}$

(C) 2

(D) $\frac{16}{27}$

(10) If $\int_3^{10} f(x)\,dx = \ln 3$, what is the exact value of $\int_3^{10} \left(f(x) - 3^x \right) dx$?

(A) $\dfrac{(\ln 3)^2 - 3^{10} + 27}{\ln 3}$

(B) $\dfrac{(\ln 3)^2 - \ln 3.3^{10} + 27}{\ln 3}$

(C) $(\ln 3) - \ln 3.3^{10} + \ln 3.27$

(D) $\ln 3 - 3^{11} + 3^4$

Section II

90 marks

Attempt Questions 11 – 16
Allow about 2 hours and 45 minutes for this section

Answer on the blank paper provided. Begin a new page for each question
Your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (Begin a New Page) (15marks)

a) If $t = 0.65$, evaluate $\dfrac{1-t^2}{1+t^2}$ correct to 3 significant figures. [2]

b) Simplify $\dfrac{4x+5}{3} - \dfrac{2-3x}{4}$. [2]

c) On a number line, graph the solution to $|3x-4| \le 5$. [2]

d) Factorise completely $x^2 - 9y^2 - x - 3y$. [2]

e) Simplify $\dfrac{m^{3-n} \times m^3}{m^{2n-5}}$. [2]

f) Find the primitive function of $(6+3x)^5$. [1]

g) Differentiate the following :

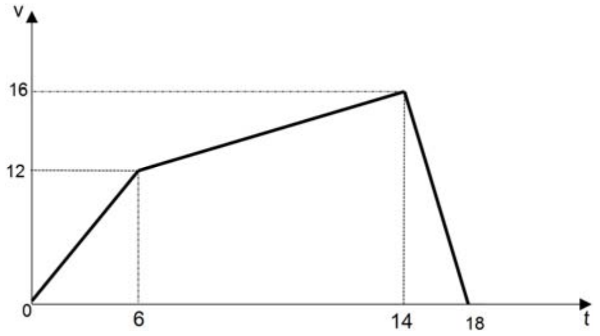
i) $y = \dfrac{e^{4x}}{x}$ [2]

ii) $y = \ln\sqrt{3x+4}$ [2]

End of Question 11

Question 12(Begin a New Page) (15marks)

a) The velocity – time graph of a particle moving on a straight line is shown below, where velocity is given in m/s and time in seconds :

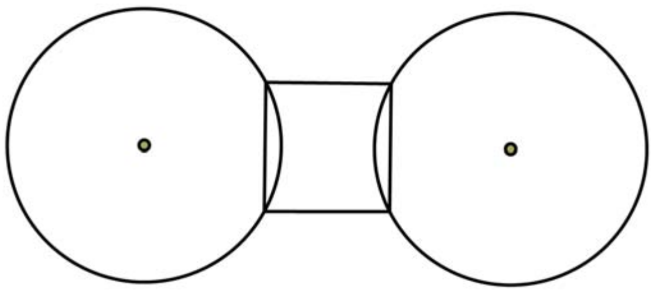


i) How far has the particle travelled in the time interval between $t=6$ and $t=14$? [2]

ii) What was the acceleration when $t=10$? [1]

b) Two circles of equal radii of 5 cm are joined by a square of sides 5 cm, as shown below :

Find the total area of the figure. (answer correct to 1 decimal place) [2]



c)

i) Differentiate $y = x\tan x$ [2]

ii) Hence find $\int x\sec^2 x\, dx$. [2]

Question 12 is continued on the next page

d) Let A and B be the points $(0,2)$ and $(-2,5)$ respectively.

i) Find the coordinates of the midpoint of AB . [1]

ii) Find the slope of line AB . [1]

iii) Find the equation of the perpendicular bisector of AB in general form. [2]

iv) C lies on the line $y = x + 1$ and is equidistant from A and B . Find the co-ordinates of C . [2]

End of Question 12

Question 13(Begin a New Page) (15marks)

a) ABCD is a rectangle in which $AB = 12\,cm$ and $AD = 8\,cm$. The points P , Q , R lie on AB, BC and CD respectively so that $PB = QC = DR = x\,cm.$

i) Show that the area of triangle PQR is given by $A = x^2 - 10x + 48$. [2]

ii) Find the least value of this area. [2]

iii) Show that the greatest value of the area of this triangle is $48\,cm^2$. [2]

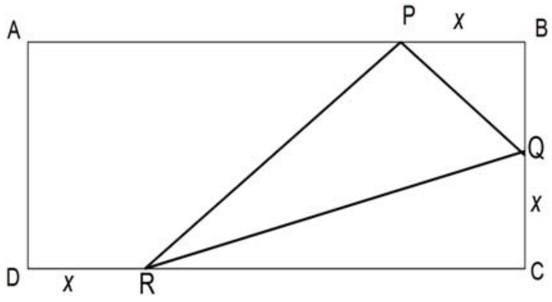


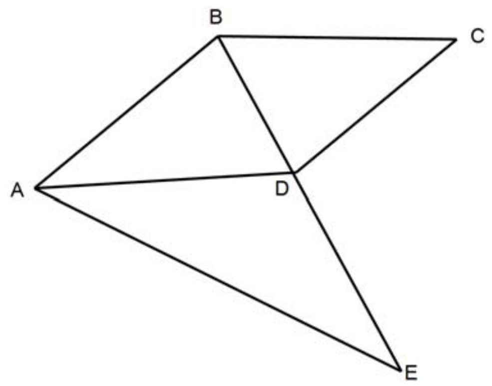
Figure not to scale

b) Find the sum of the first 51 terms in the series $7 + 3 + 14 + 6 + 21 + 12 + \ldots\ldots\ldots$ [3]

c) Use Simpson's Rule to approximate $\int_0^8 x\sqrt{x}\ dx$ using 3 function values.
(answer correct to 3 decimal places) [2]

Question 13 is continued on the next page

d) ABCD is a rhombus. BD is produced to E such that $AD = DE$.



(Figure not to scale)

i) Copy the diagram onto your answer sheet.

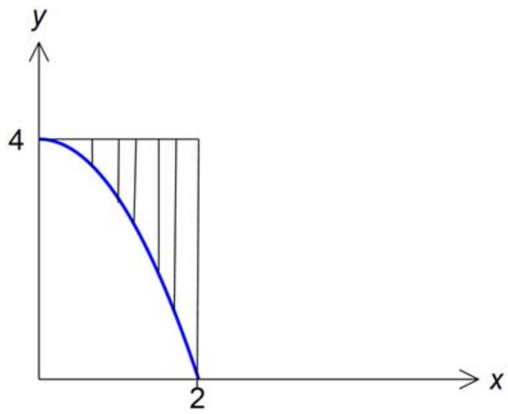
ii) Show that $\angle ABC = 4\angle AED$. [2]

iii) Show that D is the midpoint of BE, given that $\angle BAE = 90^\circ$. [2]

End of Question 13

Question 14(Begin a New Page) (15 marks)

a) The shaded area shown below is enclosed between the curve $y = 4 - x^2$ and the lines $x = 2$ and $y = 4$. Find the volume generated if this shaded area is rotated about the $x-$axis. [3]



b) Prove that $\dfrac{2^{2x}+1}{2^x}$ is an even function. [2]

c) Two schools P and Q have populations of 950 and 480 respectively. Students are leaving school P at a rate of 7% (ie $N = Ae^{-0.07t}$) and school Q is gaining students at a rate of 12%(ie $N = Be^{0.12t}$) where N is the number of students and t is in years.

i) Find the values of A and B. [2]

ii) After how many years will both schools have the same number of students? (answer correct to 3 significant figures) [2]

Question 14 is continued on the next page

d) The quadratic equation $x^2 - 3x - 13 = 0$ has roots α and β.

i) Write down the values of $(\alpha + \beta)$ and $\alpha\beta$. [2]

ii) What is the value of $\dfrac{1}{\alpha^2} + \dfrac{1}{\beta^2}$? [2]

iii) What is the value of $(6\beta - 2\beta^2)$? [2]

End of Question 14

Question 15(Begin a New Page) (15 marks)

a) In the beginning of the year 2013, Hagrid planted 5000 Dragon plants in the grounds of Pigwarts school. Since he will lose 40% of his plants every year, he decided to buy and plant an additional P Dragon plants at the beginning of each year. He bought the first lot of P plants at the beginning of 2014.

i) Show that only 648 of the original plants are left by the beginning of 2017. [1]

ii) Show that by the beginning of 2017, before he buys his new plants , he has $648+\dfrac{3P(1-0.6^3)}{2}$ plants alive in the ground. [2]

iii) How many plants should he buy each year to ensure he has at least 3000 plants left alive at the beginning of 2020.
(Note : no additional plants are purchaced in 2020) [2]

b) Find the exact value of $\int_{\frac{1}{3}}^{\frac{1}{2}} \sec^2 \dfrac{\pi x}{2}\, dx$. [2]

c) A Concord engine uses fuel at the rate of R litres per second. The rate of the fuel use t seconds after the engine starts operation is given by $R = 20+\dfrac{15}{1+t}$.

i) What is R when $t=9$? [1]

ii) What happens to R as t approaches ∞? Justify your answer . [1]

iii) Draw a sketch of R as a function of t. [2]

iv) Calculate the total amount of petrol used during the first 9 seconds . Give your answer correct to the nearest litre. [2]

d) Solve for $\sin\left(\theta-\dfrac{\pi}{2}\right)=\cos\left(\theta+\dfrac{\pi}{2}\right)$, given $0\le\theta\le 2\pi$. [2]

End of Question 15

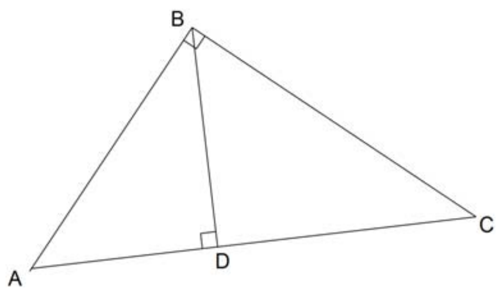
Question 16(Begin a New Page) (15 marks)

a) Donald and Bella are playing a game of tennis. They have 8 unused balls and 5 used balls.

i) If they chose two balls to play a match, find the probability that they choose two unused balls. [1]

ii) After the match the balls are returned to the box and Ivanka and Jacob randomly choose two balls to play their match. What is the probability that they play with unused balls? [2]

b) A triangle ABC is right angled at B. D is the point on AC such that BD is perpendicular to AC. Let $\angle BAC = \theta$.



i) Copy the diagram on your answer sheet.

ii) Given that $4(AD) - 3(BD) = 2(AC)$, show that

$$4\cos\theta - 3\sin\theta = 2\sec\theta$$ [2]

iii) Deduce that $2\cos^2\theta - 3\sin\theta\cos\theta - 2\sin^2\theta = 0$. [2]

iv) Find the value of θ, correct to the nearest degree. [2]

Question 16 is continued on the next page

c) The point $P(m,n)$ lies on the parabola $y^2 = 4ax$ and the line $kx + by = 1$.

i) Show that $kn^2 + 4abn - 4a = 0$ [2]

ii) Show that if the line is tangent to the parabola at *P*, then

$ab^2 + k = 0$. [2]

iii) Show that the equation of the tangent at P is $y = abx + \frac{1}{b}$. [2]

End of the paper

Sydney Girls High School

Mathematics Faculty

Multiple Choice Answer Sheet
Trial HSC Mathematics

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample 2 + 4 = ? (A) 2 (B) 6 (C) 8 (D) 9

A ○ B ● C ○ D ○

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ● B ~~●~~ C ○ D ○

If you change your mind and have crossed out what you consider to be the correct answer, then indicate this by writing the word *correct* and drawing an arrow as follows:

correct

A ~~●~~ B ~~●~~ C ○ D ○

Student Number: ANSWERS

Completely fill the response oval representing the most correct answer.

1. A ○ B ● C ○ D ○
2. A ○ B ○ C ● D ○
3. A ○ B ○ C ● D ○
4. A ○ B ● C ○ D ○
5. A ○ B ○ C ○ D ●
6. A ○ B ● C ○ D ○
7. A ○ B ○ C ● D ○
8. A ○ B ● C ○ D ○
9. A ○ B ○ C ○ D ●
10. A ● B ○ C ○ D ○

Question 11

a) $t = 0.65$

$$\frac{1-(0.65)^2}{1+(0.65)^2}$$

$\doteqdot 0.406$ (3 sig figs)

b) $$\frac{4x+5}{3} - \frac{2-3x}{4}$$

$$= \frac{4(4x+5) - 3(2-3x)}{12}$$

$$= \frac{16x+20-6+9x}{12}$$

$$= \frac{25x+14}{12}$$

c) $|3x-4| \leq 5$

$-5 \leq 3x-4 \leq 5$

$-1 \leq 3x \leq 9$

$-\frac{1}{3} \leq x \leq 3$

d) $x^2 - 9y^2 - x - 3y$

$= (x-3y)(x+3y) - 1(x+3y)$

$= (x+3y)(x-3y-1)$

* marks were not awarded for trying to complete the square as you were asked to factorise the expression.

(e) $$\frac{m^{3-n} \times m^3}{m^{2n-5}} = m^{6-n-(2n-5)}$$

$$= m^{11-3n} \quad \left(\text{or } \frac{m^{11}}{m^{3n}}\right)$$

(f) $\int (6+3x)^5 \, dx = \frac{1}{18}(6+3x)^6 + c$

(g) i) $y = \frac{e^{4x}}{x}$

$$\frac{dy}{dx} = \frac{x \cdot 4 \cdot e^{4x} - e^{4x} \cdot 1}{x^2}$$

$$= \frac{e^{4x}(4x-1)}{x^2}$$

ii) $y = \ln(3x+4)^{1/2}$

$$= \frac{1}{2} \ln(3x+4)$$

$$\frac{dy}{dx} = \frac{1}{2} \times \frac{3}{3x+4}$$

$$= \frac{3}{2(3x+4)}$$

Question 12

a) i) $D = \frac{h}{2}(x+y)$

$= \frac{8}{2}(12+16)$

$= 4 \times 28$

$= 112\text{m}$

ii) $a = \frac{4}{8} = \frac{1}{2}\text{m/s}^2$

(gradient of the line)

b) $\text{Area} = 2\pi r^2 + s^2 - \left(2 \times \frac{1}{2} r^2 (\theta - \sin\theta)\right)$

$= 2\pi(25) + 25 - \left(25\left(\frac{\pi}{3} - \sin\frac{\pi}{3}\right)\right)$

$\doteq 177.55\text{ cm}^2$

$\doteq 177.6\text{ cm}^2$ (1dp)

* 1 mark was awarded if you recognised the two segments needed to be deducted but your final answer was wrong

c) i) $y = x \cdot \tan x$

$\frac{dy}{dx} = x \cdot \sec^2 x + \tan x$

ii) $\int x \sec^2 x \, dx =$

$\int x \cdot \sec^2 x + \tan x = x \tan x$

$\int x \cdot \sec^2 x = x \cdot \tan x - \int \tan x \, dx$

$= x \cdot \tan x - \int \frac{\sin x}{\cos x} dx$

$= x \cdot \tan x + \ln \cos x + c$

d) i) $M = \left(\frac{0-2}{2}, \frac{2+5}{2} \right)$

$= \left(-1, \frac{7}{2}\right)$

ii) $m = \frac{5-2}{-2}$

$= -\frac{3}{2}$

iii) $m = \frac{2}{3}$ $\left(-1, \frac{7}{2}\right)$

$y - \frac{7}{2} = \frac{2}{3}(x+1)$

$6y - 21 = 4x + 4$

$4x - 6y + 25 = 0$

iv) $4x - 6y + 25 = 0$ (1)

$y = x + 1$ (2)

sub (2) in (1)

$4x - 6(x+1) + 25 = 0$

$4x - 6x - 6 + 25 = 0$

$-2x + 19 = 0$

$2x = 19$

$x = \frac{19}{2}$

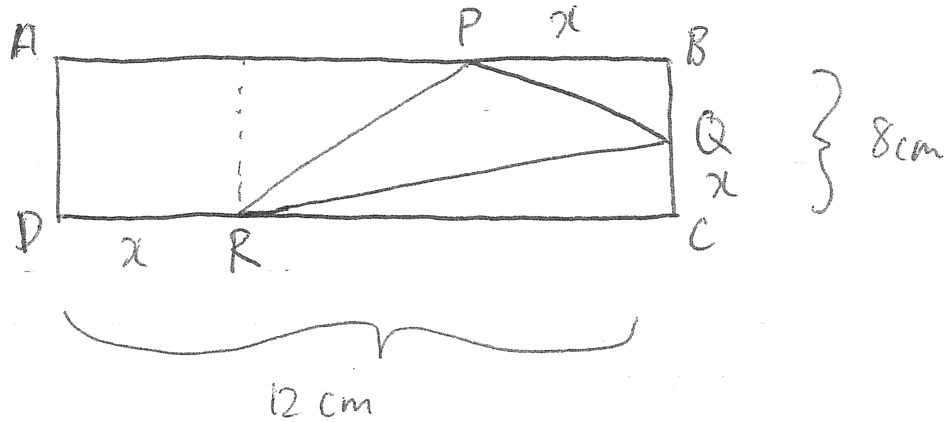
$= 9.5$

$y = 10.5$

$\therefore$ coordinates of C are $(9.5, 10.5)$

Question 13

13.(a)(i)



$$A_{PBCR} = \frac{8}{2}(12 - x + x)$$
$$= 4(12)$$
$$= 48$$

$$A_{PBQ} = \frac{1}{2}(8-x)(x)$$
$$= \frac{1}{2}(8x - x^2)$$
$$= 4x - \frac{1}{2}x^2$$

$$A_{RCQ} = \frac{1}{2}(x)(12-x)$$
$$= \frac{1}{2}(12x - x^2)$$
$$= 6x - \frac{1}{2}x^2$$

$A_{PBCR} - A_{PBQ} - A_{RCQ}$

$$\therefore A_{PQR} = \cancel{48 - (4x - \tfrac{1}{2}x^2)}$$

$$\therefore A_{PQR} = 48 - \left(4x - \frac{1}{2}x^2\right) - \left(6x - \frac{1}{2}x^2\right)$$
$$= 48 - 4x + \frac{1}{2}x^2 - 6x + \frac{1}{2}x^2$$
$$= x^2 - 10x + 48$$

13. (a)(ii)

$$A = x^2 - 10x + 48$$

$$\frac{dA}{dx} = 2x - 10$$

$$0 = 2x - 10$$

$$2x = 10$$

$$x = 5$$

x	4	5	6
$\frac{dA}{dx}$	-2	0	2

$\therefore$ minimum at $x=5$

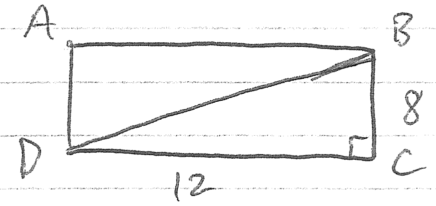
Now, when $x=5$,

$$A = 5^2 - 50 + 48$$

$$= -25 + 48$$

$$\therefore A = 23\text{cm}^2$$

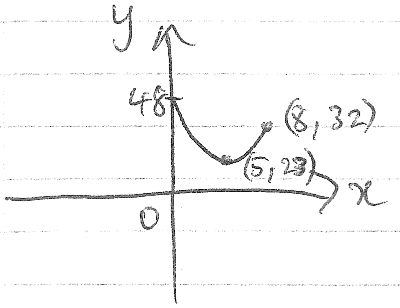
(iii) Greatest area for a triangle ~~is when~~ in a rectangle is when it is half the rectangle



$$A_{DBC} = \frac{1}{2}(8)(12)$$

$$= 48\text{cm}^2$$

OR



$x \neq 8$. $x < 8$ since $AD = 8$cm

from the graph, when $x=0$, $y=48$

$\therefore$ Greatest area is 48cm^2

13. (b) $7+3+14+6+21+12+\ldots$

$7+14+21+\ldots \rightarrow T_3 - T_2 = T_2 - T_1$

$7 = 7 \quad \therefore$ A.P.

$3+6+12+\ldots$

$$\frac{T_3}{T_2} = \frac{T_2}{T_1}$$

$$\frac{12}{6} = \frac{6}{3}$$

$$2 = 2 \quad \therefore \text{G.P.}$$

For A.P. there must be 26 terms since ~~th~~ series started with 7.

For A.P.:

$$S_{26} = \frac{26}{2}\left(2(7)+(26-1)7\right)$$

$$= 13(14+175)$$

$$= 2457$$

For G.P.:

$$S_{25} = \frac{3(2^{25}-1)}{2-1}$$

$$= 100663293$$

$$\Rightarrow S_{51} = S_{26} + S_{25}$$

$$= 100665750$$

13. (c) $\int_0^8 x\sqrt{x}\,dx$, 3 function values

$\Rightarrow$ 2 subintervals

$h = \frac{b-a}{n}$

$= \frac{8-0}{2}$

x	0	4	8
y	0	8	$8\sqrt{8}$
	y_0	y_1	y_2

$h = 4$

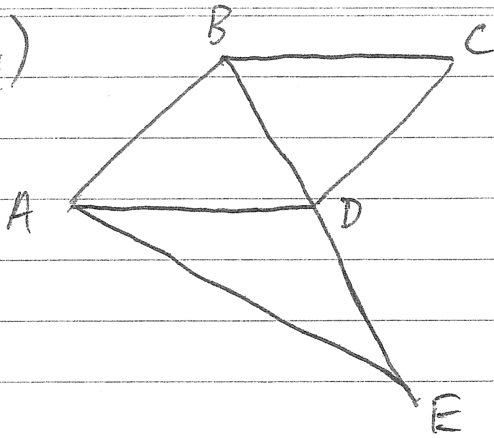
$\Rightarrow \frac{h}{3}$ (first + last + FOTE)

$= \frac{4}{3}\left(0 + 8\sqrt{8} + (4 \times 8)\right)$

$= \frac{4}{3}(8\sqrt{8} + 32)$

$= 72.837$ (3 d.p)

13. (d)(i)



(ii) let $\angle AED = x$

$AD = DE$ (given)

$\therefore \Delta ADE$ is isosceles

$\angle DAE = \angle AED = x$ (base $\angle$s of isosceles Δ)

$\angle ADE = 180 - 2x$ ($\angle$ sum of Δ is $180°$)

$\angle ADB = 180 - (180 - 2x)$ ($\angle$ sum of straight line is $180°$)

$= 2x$

$\angle CBD = 2x$ (alternate $\angle$s on $AD \parallel BC$ are equal)

$\angle ABD = 2x$ (diagonals bisect angles in a rhombus)

$\angle ABC = \angle ABD + \angle CBD$

$= 2x + 2x$

$= 4x$

$\therefore \angle ABC = 4\angle AED$

13.(d)(iii)

$\angle BAE = 90^\circ$

$\angle BAE + \angle AEB + \angle ABE = 180^\circ$ ($\angle$ sum of ΔABE is 180°)

$90^\circ + x + 2x = 180^\circ$

$3x = 90^\circ$

$\therefore x = 30^\circ$

Now,

$\angle ABD = \angle ADB = 2x$

$= 60^\circ$

$\therefore \Delta ABD$ is an equilateral Δ

$\therefore AB = BD = AD$

Since $AD = DE$, $BD = DE$

$\therefore D$ is the midpoint of BE

Question 14

14.(a)

$$V = V_{cylinder} - \pi \int_0^2 y^2 \, dx$$

$$V_{cylinder} = \pi r^2 h$$
$$= \pi (4)^2 \times 2$$
$$= 32\pi$$

$$y^2 = (4 - x^2)^2$$
$$= 16 - 8x^2 + x^4$$

$$\Rightarrow \pi \int_0^2 16 - 8x^2 + x^4 \, dx$$

$$= \pi \left[16x - \frac{8x^3}{3} + \frac{x^5}{5} \right]_0^2$$

$$= \pi \left[32 - \frac{64}{3} + \frac{32}{5} \right]$$

$$= \frac{256\pi}{15} \text{ units}^3$$

$$\therefore V = 32\pi - \frac{256\pi}{15}$$

$$V = \frac{224\pi}{15} \text{ units}^3$$

14-(b) let $f(x) = \dfrac{2^{2x}+1}{2^{x}}$

If even function,

$$f(x) = f(-x)$$

$$f(-x) = \frac{2^{2(-x)}+1}{2^{-x}}$$

$$= \frac{2^{-2x}+1}{2^{-x}}$$

$$= \frac{\frac{1}{2^{2x}}+1}{\frac{1}{2^{x}}}$$

$$= \frac{\frac{1+2^{2x}}{2^{2x}}}{\frac{1}{2^{x}}}$$

$$= \frac{(1+2^{2x})\,2^{\cancel{x}}}{2^{\cancel{2}x}}$$

$$= \frac{2^{2x}+1}{2^{x}}$$

$$= f(x)$$

$\therefore$ even function

14. (c)(i) $P: \quad N = Ae^{-0.07t}$ ①

$Q: \quad N = Be^{0.12t}$ ②

For P: when $t=0$, $N=950$

$950 = Ae^{0}$

$\therefore A = 950$

For Q: when $t=0$, $N=480$

$480 = Be^{0}$

$\therefore B = 480$

(ii) Sub ① into ②:

$~~A~~\ 950e^{-0.07t} = 480e^{0.12t}$

$$\frac{950}{480} = \frac{e^{0.12t}}{e^{-0.07t}}$$

$$\frac{95}{48} = e^{0.19t}$$

$$\ln\frac{95}{48} = 0.19t$$

$$t = \frac{\ln\frac{95}{48}}{0.19}$$

$\therefore t = 3.59$ years (3 sig figs)

14(d)(i) $x^2-3x-13=0$

$$\alpha+\beta=-\frac{b}{a}=-\frac{-3}{1}$$

$$=3$$

$$\alpha\beta=\frac{c}{a}$$

$$=-\frac{13}{1}$$

$$=-13$$

(ii) $\frac{1}{\alpha^2}+\frac{1}{\beta^2}$

$$=\frac{\beta^2+\alpha^2}{\alpha^2\beta^2}$$

$$=\frac{\alpha^2+\beta^2}{(\alpha\beta)^2}$$

$$=\frac{(\alpha+\beta)^2-2\alpha\beta}{(\alpha\beta)^2}$$

$$=\frac{3^2-2(-13)}{(-13)^2}$$

$$=\frac{35}{169}$$

(iii) $6\beta-2\beta^2$

$$=2\beta(3-\beta)$$

$$=2\beta(\alpha+\beta-\beta)\text{, since } \alpha+\beta=3$$

$$=2\beta(\alpha)$$

$$=2\alpha\beta$$

$$=2(-13)$$

$$=-26$$

OR

Since β is a root,

$$\beta^2-3\beta-13=0$$

$$\beta^2-3\beta=13$$

$$3\beta-\beta^2=-13$$

$$\therefore 6\beta-2\beta^2=-26$$

Marking Notes on Sydney Girls Trial Advanced Mathematics Exam (2U)

Question 13:

13. (a) (i) Most students answered this question correctly. There were various methods to show $A = x^2 - 10x + 48$ as you could split the areas into multiple shapes. However, regardless of your method it must have included A_{PBQ} and A_{RCQ} . Most students calculated the area of the whole rectangle and then subtracted the remaining areas which were in the shape of Trapeziums or Triangles.

Some common mistakes were assuming that ΔPQR was a right angled triangle and tried to use Pythagoras' theorem.

(ii) Most students recognised to differentiate and determine the nature of the stationary point to show that it was a minimum value at $x = 5$. Some common mistakes were not showing that it was a minimum or not subbing in $x = 5$ to find the least value of the area (since $x = 5$ is when the least value occurs).

(iii) This question needed justification as to why the greatest value occurs when $x = 0$. Some students drew a diagram to show their answer. If you think about a rectangle, the least amount of triangles that can be created in a triangle is 2. Thus, the greatest area of a triangle would be half of the area of the rectangle. Alternatively, students graphed $A = x^2 - 10x + 48$. However, the justification needed was to understand that $x < 8$ since $AD = 8cm$. So although $x = 10$ gives $A = 48$, it is impossible for x to be equal to 10.

(b) Some students were unable to recognise the pattern within this series. If you look at every odd term (i.e. $T_1, T_3, T_5, \dots$) and every even term then you will notice that the series can be split into two. After proving the series, you will notice that $T_1, T_3, T_5, \dots$ creates an arithmetic progression and $T_2, T_4, T_6, \dots$ creates a geometric progression.

Most students could recognise some sort of pattern but used the same formula for both series. It was important to notice that the sum of the arithmetic progression should be S_{26} and the sum of the geometric progression is S_{25}. Hence, the sum of 51 terms would be $S_{26} + S_{25}$

(c) Some students used the incorrect method or formula to answer this question, mixing it up with the trapezoidal rule. Also, some students incorrectly calculated the value of h in $h = \frac{b-a}{n}$.

(d) (ii) There were many different methods to show this. Most common methods were to use algebra (most common was to let $\angle AED = x$, or some other pronumeral) or prove congruent triangles. Many students need to remember to provide reasoning even for the most 'obvious' answers. Most responses involved base angles of isosceles triangles, exterior angle is equal to sum of two opposite interior angles, straight angle is equal to 180°, opposite angles in a rhombus are equal and diagonals bisect angles in a rhombus.

(iii) The most common response was to utilise $\angle BAE = 90°$. Find the angle sum of ΔABE and you would be able to solve that $\angle AEB = 30°$ and thus $\angle ABE = 60°$. This would mean that ΔABD is an equilateral triangle and thus $AB = AD = BD$ and given that $AD = DE$, that would mean that $BD = DE$, thus showing that D is the midpoint of BE.

Some other students used Circle Geometry Theorems but it is recommended that these methods should not be the first thought in an Advanced Mathematics Exam.

Question 14

14. (a) Some students tried to find the area instead of volume. Many students were able to do the first step of finding the volume by creating an integral. Students needed to notice that to find the volume of the shaded area was to find the volume of the cylinder and subtract it from the volume of the integral.

(b) To prove an even function is to prove $f(x) = f(-x)$. There were many different methods to prove this but the most common response was to change the numbers with a negative index into fractions.

(c) (i) Most students were able to find A and B by letting $t = 0$ for the initial value.

(ii) Most responses were able to equate both schools with one another to achieve 1 mark. However, students failed to simplify $\frac{e^{0.12t}}{e^{-0.07t}}$ or used the log / ln method incorrectly by failing to move the constants to one side first.

(d) (i) Most responses were correct. Just remember that $\alpha + \beta = -\frac{b}{a}$ and $\alpha\beta = \frac{c}{a}$

(ii) Some students either substituted values incorrectly, created a combined fraction incorrectly or forgot that $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$.

(iii) There were various responses to this question. The quickest response was to notice that $\alpha + \beta = 3$ and thus $\alpha = 3 - \beta$. If you factorise: $(6\beta - 2\beta^2) = 2\beta(3 - \beta)$ then you will be able to substitute $\alpha = 3 - \beta$.

Trial 2017 Q15 2U

i)

$A_4 = 5000(1-0.4)^4$

$= 648$

ii) $A_1 = 5000 \times 0.6$

$A_2 = (5000(0.6) + P) \times 0.6$

$A_2 = 5000(0.6)^2 + 0.6P$

$A_3 = (5000(0.6)^2 + 0.6P + P)\,0.6$

$= 5000(0.6)^3 + 0.6^2P + 0.6P$

$A_4 = 5000(0.6)^4 + 0.6^3P + 0.6^2P + 0.6P$

$= 648 + P(0.6 + 0.6^2 + 0.6^3)$

$= 648 + P\left(\dfrac{0.6(1-0.6^3)}{0.4}\right)$

$= 648 + \dfrac{3P(1-0.6^3)}{2}$

Many students couldn't do this question

iii) $A_7 = 5000(0.6)^7 + \dfrac{3P}{2}(1-0.6^6)$

$5000(0.6)^7 + \dfrac{3P}{2}(1-0.6^6) \geq 3000$

$3P(1-0.6^6) \geq 5720.064$

students used the wrong indices.

$P \geq 2000$

$P = 2000$

b) $\int \sec^2 \dfrac{\pi x}{2}\, dx$

$\left[\dfrac{2\tan\frac{\pi x}{2}}{\pi}\right]_{\frac{1}{3}}^{\frac{1}{2}}$

$= \dfrac{2\tan\frac{\pi}{4} - 2\tan\frac{\pi}{6}}{\pi}$

$= \dfrac{6 - 2\sqrt{3}}{3\pi}$

c) i) $t = 9$

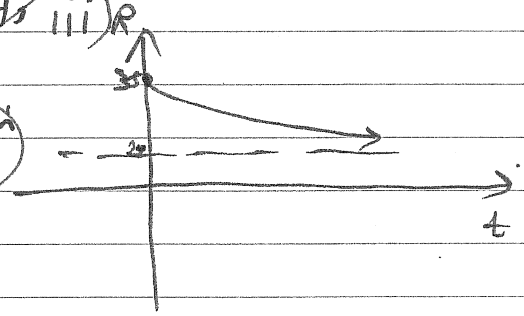
$R = 20 + \dfrac{15}{10}$

$= 21.5$ L/s

ii) $t \to \infty \quad \dfrac{15}{t+1} \to 0$

$\therefore R \to 20$ L/s

iii)



iv) $\int_0^9 20 + \dfrac{15}{1+t}\, dt$

$V = \left[20t + 15\ln(1+t)\right]_0^9$

$V = 180 + 15\ln 10$

$= 215$ L

d) $\sin\left(-\left(\frac{\pi}{2} - \theta\right)\right)$ Many students used the wrong formula

$= -\sin\left(\frac{\pi}{2} - \theta\right)$

$= -\cos\theta$

$\cos(\pi - \theta) = \cos\left(\theta + \frac{\pi}{2}\right)$

$\pi - \theta = \theta + \frac{\pi}{2}$

$2\theta = \frac{\pi}{2}$

$\theta = \frac{\pi}{4}, \frac{5\pi}{4}$

Hsc ~~A~~ Mathematics Trial 2017

16

a)

i) $$\frac{8\times7}{13\times12} = \frac{56}{156} = \frac{14}{39}$$

(ii) Probability to choose 2 not previously used balls from 13 when 8 are not previously used.

$$\frac{8\times7}{13\times12}$$

conditionally on this, probability that these 2 balls were not played with because they would have been chosen in the first phase

$$\frac{11\times10}{13\times12}$$

Thus the desired probability is

$$\frac{11\times10\times8\times7}{13\times12\times13\times12} = \frac{385}{1521}$$

Majority of students got it wrong. The credit was given even if some working out was provided.

16
b
IV

Deduced above

$2\cos^2\theta - 3\sin\theta\cos\theta - 2\sin^2\theta = 0$

$(\cos\theta - 2\sin\theta)(2\cos\theta + \sin\theta) = 0$

either $\cos\theta - 2\sin\theta = 0$ or $2\cos\theta + \sin\theta = 0$

$\cos\theta = 2\sin\theta$

$\frac{\sin\theta}{\cos\theta} = \frac{1}{2}$

$\tan\theta = \frac{1}{2}$

$\theta = \tan^{-1}(\frac{1}{2})$

$\theta = 27^\circ$ nearest

other values of θ rejected as it is an actue angled ~~right~~ $\triangle$.

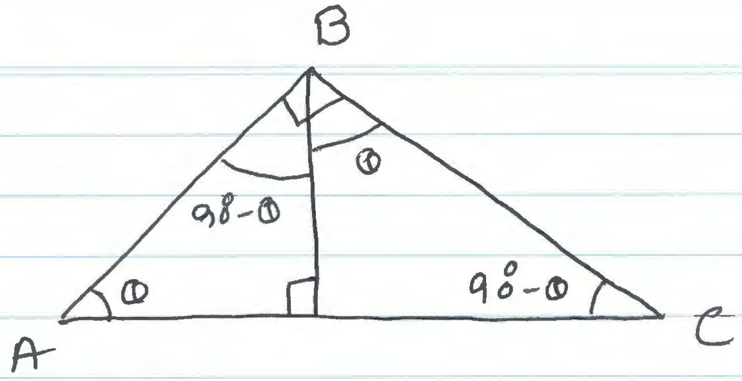
$\sin\theta = -2\cos\theta$

$\frac{\sin\theta}{\cos\theta} = -2$

$\tan\theta = -2$

most students got it correct

16 b)



i) ~~Given~~

$$4\,AD - 3\,BD = 2\,AC$$

$\div AB$

$$4\frac{AD}{AB} - \frac{3BD}{AB} = 2\frac{AC}{AB}$$

$$4\cos\theta - 3\sin\theta = 2\sec\theta.$$

most students got this question correct. Some did put unnecessary working out. others provided a very complicated solution.

16 b (iii) ~~Given.~~ Proved above

$$4\cos\theta - 3\sin\theta = 2\sec\theta.$$

$$4\cos\theta - 3\sin\theta = \frac{2}{\cos\theta}$$

$$4\cos^2\theta - 3\sin\theta\cos\theta = 2$$

$$4\cos^2\theta - 3\sin\theta\cos\theta = 2\sin^2\theta + 2\cos^2\theta$$

$$\cancel{4}\ 2\cos^2\theta - 3\sin\theta\cos\theta - 2\sin^2\theta = 0$$

Hence deduced.

Some students provided very complicated solutions.

16

c)

i) $y^2 = 4ax$ — ①

Given $kx + by = 1$

$$x = \frac{1-by}{k}$$

Sub in ①

$$y^2 = \frac{4a(1-by)}{k}$$

$$ky^2 = 4a - 4aby$$

$$ky^2 + 4aby - 4a = 0$$

As Point (m,n) lies on it.

$$kn^2 + 4abn - 4a = 0$$

(ii)

$\cancel{y^2 = 4ax}$

~~m-gradient of tangent~~

$\cancel{\frac{dy}{dx}} = \cancel{4ax}$

$$kn^2 + 4abn - 4a = 0$$

$\Delta = 0$ as the line is tangent to the parabola at P. so

$$16a^2b^2 + 16ak = 0$$

$$a^2b^2 + ak = 0$$

$\div a$

$$ab^2 + k = 0$$

Most students got it correct.

16

c)

(iii) As line $kx+by=1$ is tangent to the parabola $y^2=4ax$

$$by = 1-kx$$

$$y = \frac{1-kx}{b}$$

$$y = \frac{1}{b} - \frac{k}{b}x \quad —— (1)$$

we deduced in part (ii) $ab^2+k=0$

$$k = -ab^2$$

Sub in (1)

$$y = \frac{1}{b} + \frac{ab^{\cancel{2}}}{\cancel{b}}x$$

$$y = abx + \frac{1}{b}$$

Hence shown.

majority of the students got it correct, however, some followed very complicated path.