Sydney
Girls
High
School

2018	TRIAL HIGHER SCHOOL CERTIFICATE **EXAMINATION**

Mathematics

General Instructions

- Reading time – 5 minutes
- Working time – 3 hours
- Write using black pen
- NESA approved calculators may be used
- A reference sheet is provided
- In Questions 11 – 16, show relevant mathematical reasoning and/or calculations

Total marks : 100

Section I – 10 marks (pages 3 – 6)

- Attempt Questions 1 – 10
- Answer on the Multiple Choice answer sheet provided
- Allow about 15 minutes for this section

Section II – 90 marks (pages **7 – 15**)

- Attempt Questions **11 – 16**
- Answer on the blank paper provided
- Begin a new page for each question
- Allow about 2 hours and 45 minutes for this section

Name: ……………………………………………………...	**THIS IS A TRIAL PAPER ONLY**
Teacher: ……………………………………………………	It does not necessarily reflect the format or the content of the 2018 HSC Examination Paper in this subject.

Section I

10 marks
Attempt Questions 1 – 10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1 – 10

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1. The first three terms of a geometric series are $\sqrt{2}$, 2 and $2\sqrt{2}$.
   What is the 21st term of this series ?

   (A) $21\sqrt{2}$

   (B) $22\sqrt{2}$

   (C) $1024\sqrt{2}$

   (D) 2048

2. The solution of the inequality $3x - 1 > 5 - x$ is :

   (A) $x > \frac{3}{2}$

   (B) $x > 1$

   (C) $x > 2$

   (D) $x > 3$

3. The equation $(p - 1)x^2 + 4x = 5 - p$ has no real roots for x when :

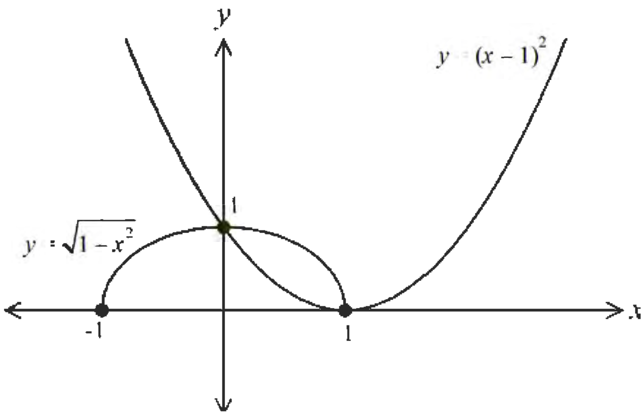
   (A) $p^2 - 6p + 6 < 0$

   (B) $p^2 - 6p + 1 > 0$

   (C) $p^2 - 6p + 1 < 0$

   (D) $p^2 - 6p + 6 > 0$

4. The diagram below shows the graphs of $y = \sqrt{1-x^2}$ and $y = (x-1)^2$.



The solution to $\sqrt{1-x^2} \geq (x-1)^2$ is :

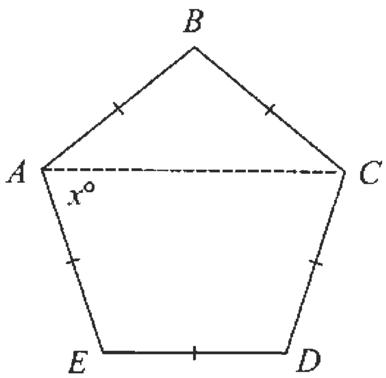
(A) $0 \leq x \leq 1$

(B) $-1 \leq x \leq 0$

(C) $x \geq 1$

(D) $x \leq -1$

5. $ABCDE$ is a regular pentagon.



What is the size of $\angle CAE$?

(A) $36°$

(B) $72°$

(C) $84°$

(D) $108°$

6. For the curve $y = x^2 - 5$, the tangent to the curve will be parallel to the line connecting the positive x −intercept and the y −intercept of the curve when x is equal to :

(A) $\sqrt{5}$

(B) 5

(C) $\frac{\sqrt{5}}{2}$

(D) $\frac{1}{\sqrt{5}}$

7. A box contains six yellow marbles and four green marbles. Two marbles are drawn from the box, without replacement.
The probability that they are the same colour is:

(A) $\frac{1}{3}$

(B) $\frac{28}{45}$

(C) $\frac{7}{15}$

(D) $\frac{13}{25}$

8. Which expression is equivalent to $\sin\theta \tan\theta$?

(A) $\sec\theta$

(B) $\cos\theta$

(C) $\frac{1-\cos^2\theta}{\cos\theta}$

(D) $\frac{1-\cos\theta}{\cos\theta}$

9. If $\sin\alpha + \cos\alpha = 1.2$, then the value of $\sin^3\alpha + \cos^3\alpha$ is :

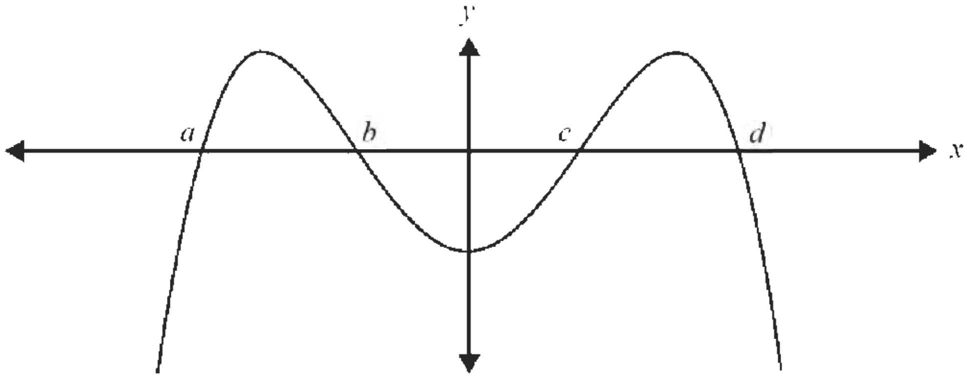
(A) 1.2

(B) 0.936

(C) 0

(D) 0.432

10. The graph of an even function $y = f\ (x)$ is shown below.



The graph has x −intercepts at $(a, 0)$, $(b, 0)$, $(c, 0)$ and $(d, 0)$ only.

The area bound by the curve and the x −axis for $a \le x \le d$ is given by :

(A) $\int_a^d f(x)\,dx$

(B) $\int_a^b f(x)\,dx - \int_c^b f(x)\,dx + \int_c^d f(x)\,dx$

(C) $2\int_a^b f(x)\,dx + \int_b^c f(x)\,dx$

(D) $2\int_a^b f(x)\,dx - 2\int_b^{b+c} f(x)\,dx$

Section II

90 marks

Attempt Questions 11 – 16
Allow about 2 hours and 45 minutes for this section

Answer on the blank paper provided. Begin a new page for each question.
Your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks)

(a) Expand and simplify :

$$(x+3)(x-3)+(x+5)^2$$ **2**

(b) Express $\frac{3-\sqrt{8}}{3+\sqrt{8}}$ in the form $a-\sqrt{b}$ where a and b are rational. **2**

(c) Differentiate the following with respect to x :

(i) $y = x^2 - 9x$ **1**

(ii) $y = 6\tan(x)$ **1**

(iii) $y = (e^{8x}+1)^5$ **1**

(d) State the domain and range for the function : **2**

$$f(x) = \sqrt{9-x^2}$$

(e) Find the gradient of the tangent to the curve $y = \frac{x^3}{2x-3}$ at the point where $x = 1$. **2**

(f) Evaluate the following definite integral and express your answer in simplest exact form.

$$\int_1^3 \frac{x}{x^2+1}\,dx$$ **2**

(g) Find the equation of the directrix for the parabola $y = \frac{x^2}{8} - 3$. **2**

Question 12 **(15 marks)** Begin a new page.

(a) The quadratic equation $3x^2+2x-6=0$ has roots α and β.

Find the value of :

(i) $\alpha\beta$ **1**

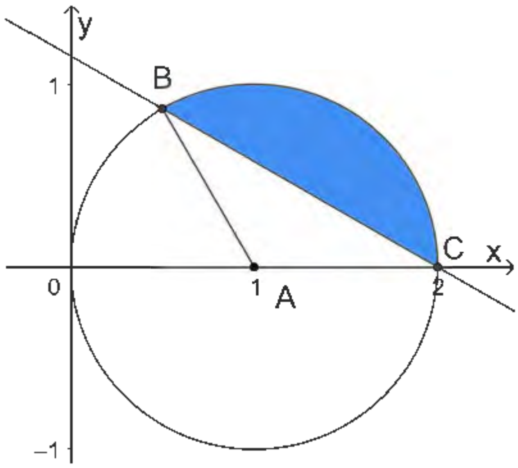
(ii) $(1-3\alpha)(1-3\beta)$ **2**

(b) Solve $3\tan^2 x - 1 = 0$ for $0 \le x \le 2\pi$. **3**

(c) In the diagram below, the centre of the circle is located at $A(1,0)$

and the circle passes through the points O , B and $C(2,0)$.

The line BC has the equation $x+\sqrt{3}y-2=0$.



(i) Find the size of $\angle BCA$ in radians. **2**

(ii) Giving reasons, find the size of $\angle BAC$ in radians. **2**

(iii) Hence, or otherwise, find the area of the shaded segment. Express your answer in simplest exact form. **2**

(d) Find all solutions of the equation : **3**

$$|4x| = 5 - x^2$$

Question 13 **(15 marks)** Begin a new page.

(a) Consider the function $f(x) = x^3 - 3x^2 + 4$.

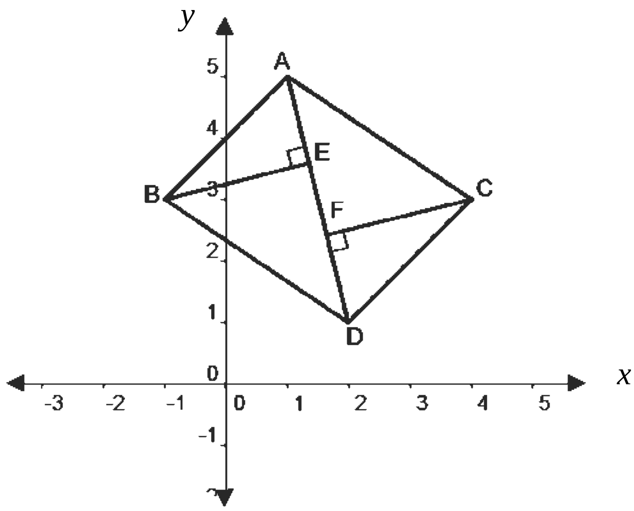
(i) Prove that the function $f(x)$ is neither odd nor even. **1**

(ii) Find the stationary points for the curve $y = f(x)$ and determine their nature. **3**

(iii) Sketch the curve $y = f(x)$ for $-2 \le x \le 2$, labelling the stationary points and all intercepts. **2**

(iv) Hence, find the range of the curve for $-2 \le x \le 2$. **1**

(b) In the diagram below, $A(1,5), B(-1,3), C(4,3)$ and $D(2,1)$ are the vertices of a parallelogram.



(i) Prove that $\Delta BAE \equiv \Delta CDF$. **3**

(ii) Find the equation of the line AD. Express the equation of the line in general form. **2**

(iii) Find the exact length of AD. **1**

(iv) Hence, or otherwise, calculate the area of $ABDC$. **2**

Question 14 **(15 marks)** Begin a new page.

(a) A theatre has 20 seats in the first row, 26 seats in the second row and 32 seats in the third row. Each row continues to have six more seats than the previous row. There are 30 rows in the theatre.

(i) How many seats are there in the final row ? **2**

(ii) Find the total number of seats in the last ten rows of the theatre. **2**

(b) Andy and Mindy agree to meet for coffee at their favourite cafe.
The cafe (C) is situated on a bearing of $110°$ from Andy (A).
From Mindy (M), the bearing to the cafe is $168°$.

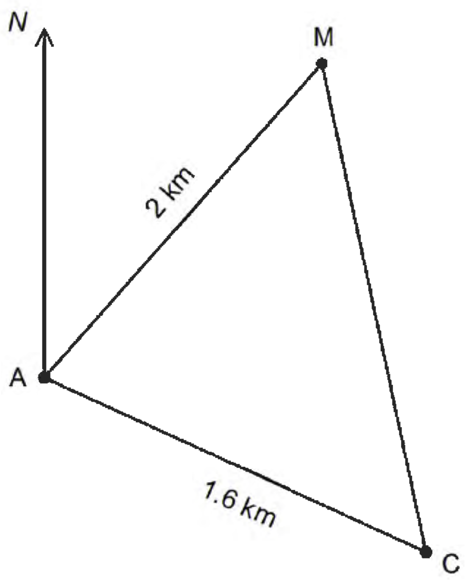


Diagram not to scale.

The distance between Andy and Mindy is 2 km.
The distance from Andy to the cafe is 1.6 km.

(i) Find the size of $\angle MCA$. **1**

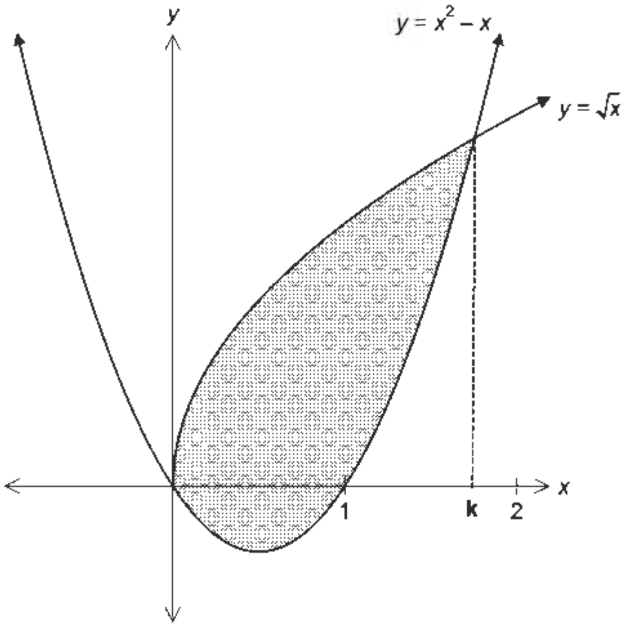
(ii) What is the bearing from Andy to Mindy ? **2**

(iii) Calculate the distance between Mindy and the café. Give your answer correct to 1 decimal place. **2**

Question 14 continues on the next page.

(c) In the diagram below the curves $y = x^2 - x$ and $y = \sqrt{x}$ intersect at $x = 0$ and $x = k$ where $1 < k < 2$.

Show that the shaded region enclosed by the curves $y = x^2 - x$ and $y = \sqrt{x}$ is given by $A = \frac{2k^3 - k^2}{6}$. **3**



(d) Find the solutions to the equation : **3**

$$e^x \log_x 2 - 3e^x = 4\log_x 2 - 12$$

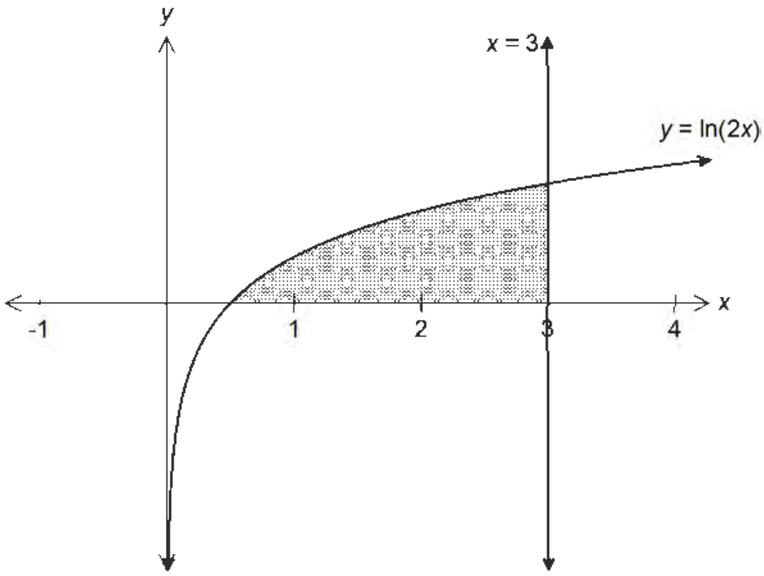
Express the solutions in exact form.

End of Question 14

Question 15 **(15 marks)** Begin a new page.

(a) Use Simpson's rule with five function values to estimate the value of $\int_{\frac{\pi}{3}}^{\frac{5\pi}{3}} \sec x \, dx$. Express your estimate in simplest exact form. **3**

(b) In the diagram below, the shaded region is bounded by the graph $y = \ln(2x)$, the x −axis and the line $x = 3$.



The shaded region is rotated about the y −axis to form a solid. Find the volume of this solid, giving your answer correct to 3 significant figures. **3**

(c) The angles of a triangle are in the ratio $4 : 3 : 2$.

(i) Find the size of the largest angle. **1**

(ii) Determine the length of the largest side of the triangle given its area is 100 m^2.

Give your answer correct to 2 decimal places. **2**

Question 15 continues on the next page.

Question 15 (continued)

(d) The points S and T are located on the respective sides PQ and PR of an equilateral triangle PQR so that $ST = TR$ and ST is perpendicular to PQ. Given that the length of QR is 1 m, find the exact length of ST. **2**

(e) The faces of a ten-sided die are labelled with the numbers 1 to 10 and all faces are equally likely to appear when the die is rolled. Dianne plays a game by rolling the ten-sided die until she wins or loses the game.

- If she rolls a 9 or 10 at any time she wins the game.
- If she rolls a 1 at any time she loses the game.

Otherwise, she must keep rolling the die.

(i) What is the probability that Dianne rolls the die less than three times ? **2**

(ii) What is the probability that Dianne wins the game at any time ? **2**

End of Question 15

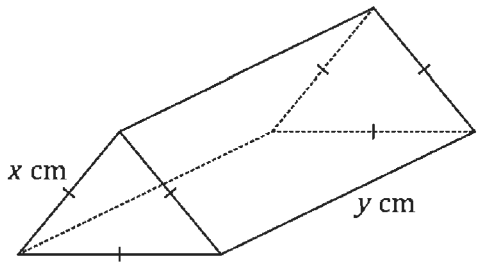
Question 16 **(15 marks)** Begin a new page.

(a) Benny invests $\$P$ into an account with interest to be calculated at the end of each month at a rate of 3% per annum. He intends to withdraw \$100 at the end of each month, immediately after the interest has been paid. He wishes to be able to do this for exactly 10 years. Let $\$A_n$ be the amount remaining in the account at the end of the nth month, after the nth withdrawal.

(i) Write an expression for A_2 in terms of P. **2**

(ii) Calculate the value of P which leaves his account empty at the end of 10 years. **3**

(b) A triangular prism has a base that is an equilateral triangle with a side length of x cm. The length of the triangular prism is y cm. The volume of the prism is 1000 cm^3.



(i) Find an expression for y in terms of x. **2**

(ii) Show that the surface area of the prism, A cm^2, is given by : **2**

$$\mathrm{A} = \frac{4000\sqrt{3}}{x} + \frac{\sqrt{3}x^2}{2}$$

(iii) Find the value of x such that the surface area of the prism will be minimised. **3**

Question 16 continues on the next page.

Question 16 (continued)

(c) The function $f(x) = x^3 - 3bx^2 + 3cx + d$ has a local maximum at $A(x_1, y_1)$ and a local minimum at $B(x_2, y_2)$. Prove that there is a point of inflexion when $x = \frac{x_1+x_2}{2}$. **3**

End of paper

Sydney Girls High School

Mathematics Faculty

Multiple Choice Answer Sheet
Trial HSC Mathematics

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample 2 + 4 = ? (A) 2 (B) 6 (C) 8 (D) 9

A ○ B ● C ○ D ○

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ● B ~~●~~ C ○ D ○

If you change your mind and have crossed out what you consider to be the correct answer, then indicate this by writing the word *correct* and drawing an arrow as follows:

correct

A ~~●~~ B ~~●~~ C ○ D ○

Student Number: Answers

Completely fill the response oval representing the most correct answer.

1. A ○ B ○ C ● D ○
2. A ● B ○ C ○ D ○
3. A ○ B ● C ○ D ○
4. A ● B ○ C ○ D ○
5. A ○ B ● C ○ D ○
6. A ○ B ○ C ● D ○
7. A ○ B ○ C ● D ○
8. A ○ B ○ C ● D ○
9. A ○ B ● C ○ D ○
10. A ○ B ○ C ○ D ●

2018 2U Trial (Mathematics) : SOLUTIONS

SECTION 1

1. $a = \sqrt{2}$, $r = \sqrt{2}$

$T_{21} = \sqrt{2} \times (\sqrt{2})^{20}$

$= 1024\sqrt{2}$ (C)

2. $3x - 1 > 5 - x$

$4x > 6$

$x > \frac{3}{2}$ (A)

3. $\Delta = 4^2 - 4(p-1)(p-5)$

$= 16 - 4(p^2 - 6p + 5)$

$= -4p^2 + 24p - 4$

$\Delta < 0 \qquad -4p^2 + 24p - 4 < 0$

$p^2 - 6p + 1 > 0$

(B)

4. From the diagram given : $0 \le x \le 1$ (A)

5. Angle sum $= 3 \times 180$

Int. $\angle = \frac{3 \times 180}{5}$

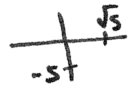
$= 108^\circ$

$\angle BAC = \frac{180 - 108}{2} = 36^\circ$

$x = 108 - 36$ (B)

$= 72$

6. x-int. $= \sqrt{5}$ y-int. $= -5$



$m = \frac{5}{\sqrt{5}} = \sqrt{5}$

$y' = 2x \qquad 2x = \sqrt{5}$

when $x = \frac{\sqrt{5}}{2}$ (C)

7. P (2 Yellow or 2 Green)

$= \frac{6}{10} \times \frac{5}{9} + \frac{4}{10} \times \frac{3}{9}$

$= \frac{7}{15}$ (C)

8. $\sin\theta \tan\theta = \sin\theta \times \frac{\sin\theta}{\cos\theta}$

$= \frac{\sin^2\theta}{\cos\theta}$

$= \frac{1 - \cos^2\theta}{\cos\theta}$ (C)

9. $\sin^3\alpha + \cos^3\alpha = (\sin\alpha + \cos\alpha)(\sin^2\alpha - \sin\alpha\cos\alpha + \cos^2\alpha)$

$= 1.2 \times (1 - \sin\alpha\cos\alpha)$

$(\sin\alpha + \cos\alpha)^2 = \sin^2\alpha + 2\sin\alpha\cos\alpha + \cos^2\alpha$

$2\sin\alpha\cos\alpha = 1.2^2 - 1$

$\therefore \sin^3\alpha + \cos^3\alpha = 1.2\left(1 - \frac{1.2^2 - 1}{2}\right)$

$= 0.936$ (B)

10. Since the function is even

$b = -c \quad \therefore b + c = 0$.

Hence, the correct choice is : (D)

Yr 12

Q11

a) $(x+3)(x-3)+(x+5)^2$

$= x^2 - 9 + x^2 + 10x + 25$

$= 2x^2 + 10x + 16$

b) $\dfrac{3-\sqrt{8}}{3+\sqrt{8}} \times \dfrac{3-\sqrt{8}}{3-\sqrt{8}}$

$= \dfrac{9 - 6\sqrt{8} + 8}{9-8}$

$= 17 - \sqrt{288}$

c) i) $y = x^2 - 9x$

$y' = 2x - 9$

ii) $y = 6\tan(x)$

$y' = 6\sec^2(x)$

iii) $y = (e^{8x}+1)^5$

$y' = 5(e^{8x}+1)^4 \times 8e^{8x}$

$= 40e^{8x}(e^{8x}+1)^4$

d)

Domain $-3 \leq x \leq 3$

Range $0 \leq y \leq 3$

e) $y = \frac{x^3}{2x-3}$

$y' = \frac{(2x-3) \times 3x^2 - 2x^3}{(2x-3)^2}$

at $x=1$

$m_T = \frac{[2(1)-3] \times 3(1)^2 - 2(1)^3}{(2\times1-3)^2}$

$m_T = -5$

f) $\int_1^3 \frac{x}{x^2+1} \, dx$

$= \frac{1}{2} \int_1^3 \frac{2x}{x^2+1} \, dx$

$= \frac{1}{2} \left[\ln(x^2+1) \right]_1^3$

$= \frac{1}{2} \left[\ln 10 - \ln 2 \right]$

$= \frac{1}{2} \ln 5$

$= \ln \sqrt{5}$

g) $y = \frac{x^2}{8} - 3$

$x^2 = 8y + 24$

$x^2 = 8(y+3)$

$\therefore a = 2$, $V(0, -3)$

directrix $y = -5$

Q12

a) i) $a=3,\ b=2,\ c=-6$

$\alpha\beta = \frac{-6}{3}$

$\alpha\beta = -2$

ii) $(1-3\alpha)(1-3\beta)$ $\qquad \alpha+\beta = \frac{-2}{3}$

$= 1-3(\alpha+\beta)+9\alpha\beta$

$= 1-3\left(\frac{-2}{3}\right)+9\times -2$

$= -15$

b) $3\tan^2 x - 1 = 0$

$\tan^2 x = \frac{1}{3}$

$\tan x = \pm\frac{1}{\sqrt{3}}$

$x = \frac{\pi}{6}$

$\therefore x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

c) i) $x+\sqrt{3}y-2=0$

$y = \frac{-1}{\sqrt{3}}x + \frac{2}{\sqrt{3}}$

$m = \frac{-1}{\sqrt{3}}$, $\tan\theta = \frac{-1}{\sqrt{3}}$

$\therefore \angle BCX = \frac{5\pi}{6}$

$\therefore \angle BCA = \frac{\pi}{6}$ (or $0.5235\ldots$)

c) II) $\angle BAC = \pi - \frac{2\pi}{6}$ (isso. Δ, $<$ sum Δ)

$= \frac{2\pi}{3}$ (or 2.09439...)

III)

shaded Area $= \frac{1}{2}r^2\theta - \frac{1}{2}r^2\sin\theta$

$= \frac{1}{2}(1)^2 \times \frac{2\pi}{3} - \frac{1}{2}(1)^2 \sin\left(\frac{2\pi}{3}\right)$

$= \frac{\pi}{3} - \frac{1}{2}\sin\left(\frac{2\pi}{3}\right)$

$= \frac{\pi}{3} - \frac{\sqrt{3}}{4}$

$= \frac{4\pi - 3\sqrt{3}}{12}$

d) $|4x| = 5 - x^2$

$4x = 5 - x^2$ or $4x = -(5 - x^2)$

$x^2 + 4x - 5 = 0$ $\quad$ $x^2 - 4x - 5 = 0$

$(x+5)(x-1) = 0$ $\quad$ $(x-5)(x+1) = 0$

$x = -5$ or $x = 1$ $\quad$ $x = 5$ or $x = -1$

Test

at $x = -5$ $\therefore$ $5 - x^2 < 0$ no solution.

at $x = 1$ $\quad$ $5 - x^2 > 0$ it is a solution.

at $x = 5$ $\quad$ $5 - x^2 < 0$ No solution.

at $x = -1$ $\quad$ $5 - x^2 > 0$ it is a solution.

Q13 Trial 2018 Advanced Mathematics

a)

i) 1 mark given to every one

$f(-x) \neq f(x)$

$\therefore$ not odd

ii) $y = x^3 - 3x^2 + 4$

$y' = 3x^2 - 6x$

For st. pts $y' = 0$

$3x(x-2) = 0$

$x = 0$ or 2

$y = 4$ $\quad y = 0$

$(0, 4)$ $\quad (2, 0)$

$y'' = 6x - 6$

at $x = 0 \rightarrow y'' = -6 < 0$

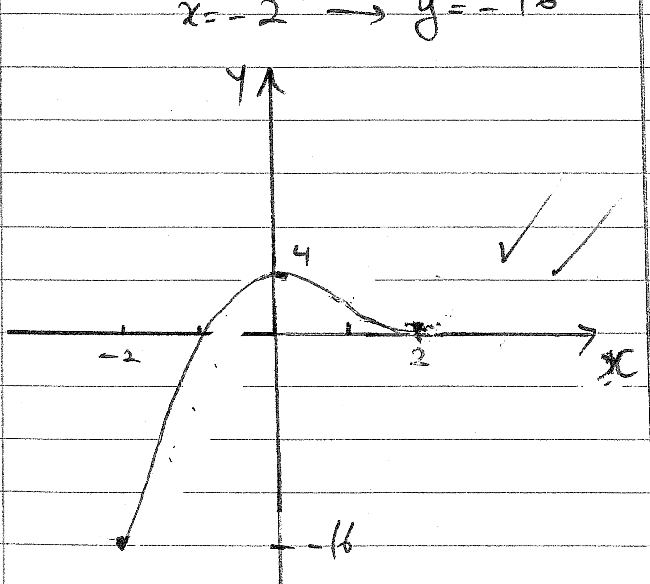
$\therefore (0,4)$ max. t.p

at $x = 2 \rightarrow y'' = 6 > 0$

$(2,0)$ min t.p.

iii) at $x = 2 \rightarrow y = 0$

$x = -2 \rightarrow y = -16$



if the students didn't show the curvature between 0 and 2 they lost a mark

iv) Range $-16 \leq y \leq 4$

b) In Δ's BAE & CDF

$\angle E = \angle F = 90$ (given)

$AB = CD$ (opp. sides of a parallelogram)

$\angle BAE = \angle CDF$ (alt. $\angle$'s $AB \parallel CD$)

$\therefore \Delta BAE \equiv \Delta CDF$ (AAS)

ii) $m = \frac{5-1}{1-2}$

$= -4$

$y - 1 = -4(x-2)$

$y = -4x + 8 + 1$

$y = -4x + 9$

$4x + y - 9 = 0$

iii) $AD = \sqrt{(1-2)^2 + (5-1)^2}$

$= \sqrt{17}$

iv) $d = \frac{|4(4) + 1(3) - 9|}{\sqrt{17}}$

$= \frac{10}{\sqrt{17}}$

iv) $A = \left(\frac{1}{2}\sqrt{17} \times \frac{10}{\sqrt{17}}\right) \times 2$

$= 10\ u^2$

* many students couldn't find the area

Q14 2018 Trial Advanced Mathematics

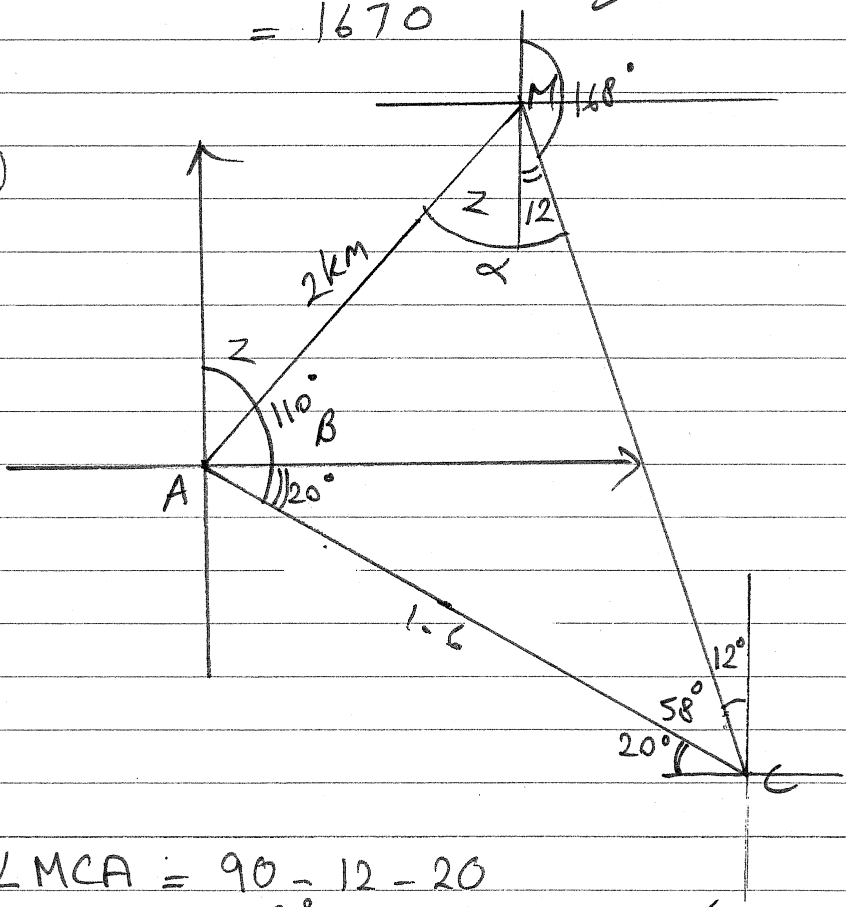
a) i)

$20 + 26 + 32 + \ldots$ ✓

$T_{30} = 20 + 29 \times 6$ ✓
$= 194$

ii) $S_{30} - S_{20} = \frac{30}{2}(40 + 29 \times 6)$
$- \frac{20}{2}(40 + 19 \times 6)$ ✓
$= 1670$ ✓

b)



i) $\angle MCA = 90 - 12 - 20$
$= 58°$ ✓

ii) $\frac{\sin \alpha}{1.6} = \frac{\sin 58}{2}$ ✓✓

$\alpha = 43°$ Bearing $= 031°T$

$z = 31°$

iii) $MC^2 = 2^2 + 1.6^2 - 2 \times 1.6 \times \cos 79°$ ✓✓
$MC = 2.3$ km

c) $A = \int_0^k \sqrt{x} - x^2 + x \, dx$

$= \left[\frac{2x^{\frac{3}{2}}}{3} - \frac{x^3}{3} + \frac{x^2}{2} \right]_0^k$

when $x = k$

$y = \sqrt{k}$

$\sqrt{k} = k^2 - k$

$= \frac{2k\sqrt{k}}{3} - \frac{k^3}{3} + \frac{k^2}{2}$

$= \frac{2k(k^2 - k)}{3} - \frac{k^3}{3} + \frac{k^2}{2}$ ✓

$= \frac{2k^3 - 2k^2}{3} - \frac{k^3}{3} + \frac{k^2}{2}$ ✓

$= \frac{4k^3 - 4k^2 - 2k^3 + 3k^2}{6}$ ✓

$= \frac{2k^3 - k^2}{6}$

Many students couldn't simplify their answer

d) $e^x \log_x 2 - 4\log_x 2 = 3e^x - 12$

$\log_x 2 (e^x - 4) = 3(e^x - 4)$ ✓

$\log_x 2 (e^x - 4) - 3(e^x - 4) = 0$

$(e^x - 4)(\log_x 2 - 3) = 0$

$e^x - 4 = 0 \longrightarrow e^x = 4$ ✓

$x \ln e = \ln 4$ ✓

$\boxed{x = \ln 4}$

$\log_x 2 = 3 \quad \therefore x^3 = 2 \quad \therefore x = \sqrt[3]{2}$

Many students only found one answer for this question

Q15

a) $\displaystyle\int_{\frac{\pi}{3}}^{\frac{5\pi}{3}} \sec x \, dx$

$h = \dfrac{\frac{5\pi}{3} - \frac{\pi}{3}}{4}$

$h = \dfrac{4\pi/3}{4} = \dfrac{\pi}{3}$ ✓

x	$\frac{\pi}{3}$	$\frac{2\pi}{3}$	π	$4\pi/3$	$\frac{5\pi}{3}$
y	2	-2	-1	-2	2
	y_1	y_2	y_3	y_4	y_5

$A \doteqdot \frac{h}{3}\left[y_1 + y_5 + 4(y_2+y_4) + 2(y_3)\right]$

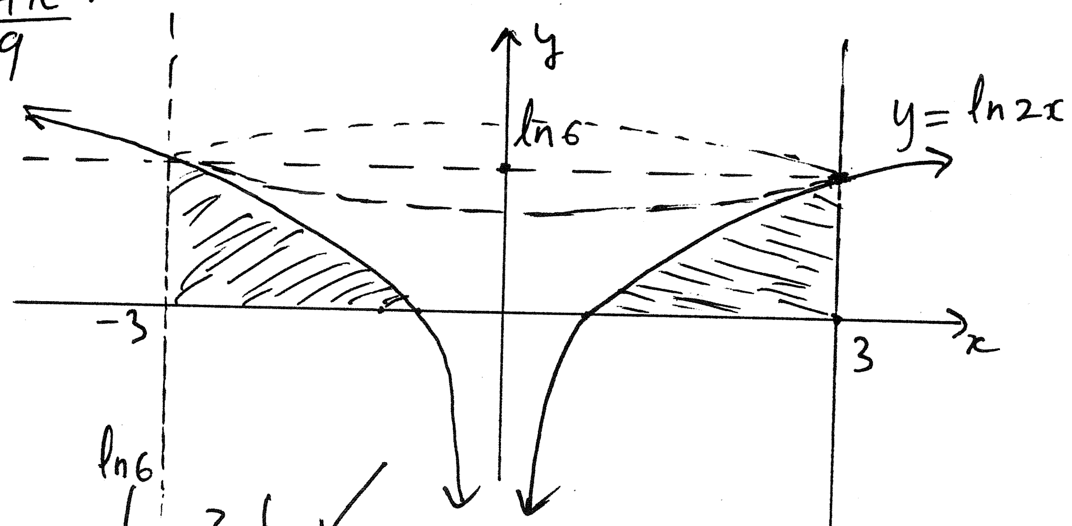
$\doteqdot \frac{\pi}{9}\left[2+2 + 4(-4) + 2(-1)\right]$ ✓

$\doteqdot -\frac{14\pi}{9}$ ✓

b)

$y = \ln 2x$

$x = \frac{1}{2}e^y$



$V = \pi r^2 h - \pi \displaystyle\int_0^{\ln 6} x^2 \, dy$ ✓

$= \pi(3)^2 \ln 6 - \pi \displaystyle\int_0^{\ln 6} \frac{1}{4} e^{2y} \, dy$ ✓

A number of students incorrectly identified the Vol of the solid formed.

$= \pi 9 \ln 6 - \frac{\pi}{4}\left[\frac{1}{2} e^{2y}\right]_0^{\ln 6}$

$= \pi 9 \ln 6 - \frac{\pi}{8}\left(e^{2\ln 6} - 1\right) = 36.9$ ✓

Q15

c) $4 : 3 : 2$

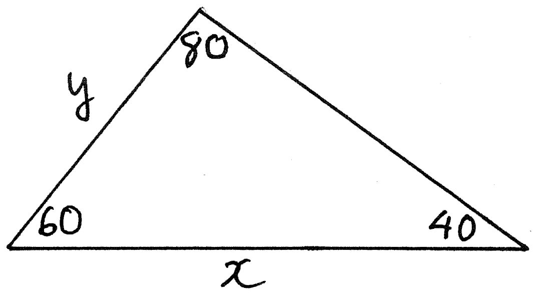
i) Largest angle $= \dfrac{4 \times 180}{9} = 80^\circ$ ✓

ii) The two smaller angles are

$4 : 3$ $\qquad$ $4 : 2$

$80 : a$ $\qquad$ $80 : b$

$a = \dfrac{240}{4} = 60^\circ$ $\qquad$ $b = \dfrac{160}{4} = 40^\circ$



Let the longest side be x

$$\left[\begin{array}{l} \dfrac{xy\sin 60^\circ}{2} = 100 \\[2ex] \dfrac{x}{\sin 80} = \dfrac{y}{\sin 40} \end{array}\right. \quad \therefore \quad \left[\begin{array}{ll} xy\sin 60^\circ = 200 & (1) \\[2ex] y = \dfrac{x\sin 40^\circ}{\sin 80} & (2) \end{array}\right.$$ ✓

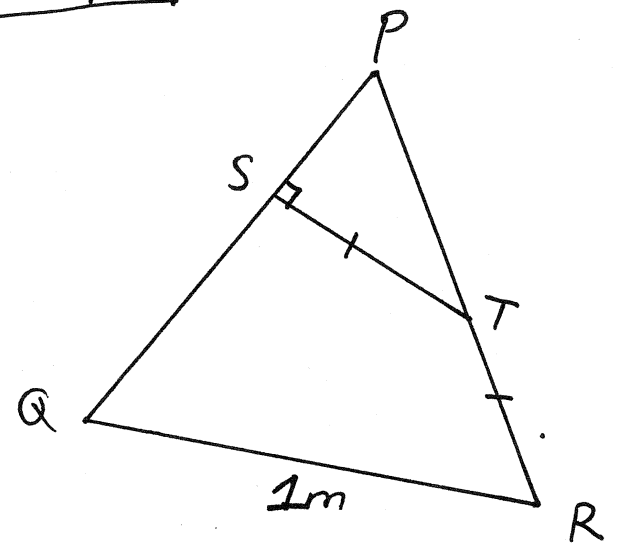
Subs (2) into (1): $x\left(\dfrac{x\sin 40}{\sin 80}\right)\sin 60^\circ = 200$

$$x^2 \sin 40 \sin 60 = 200\sin 80^\circ$$

$$x = \sqrt{\dfrac{200\sin 80}{\sin 40 . \sin 60}}$$

$x = 18.81$ m ✓

Q15/d



A number of students could not establish the equations relate to ST.

Let $ST = TR = x$

$PT = 1 - x$

In $\triangle PST$

$$\sin 60^\circ = \frac{ST}{PT}$$

$$\frac{\sqrt{3}}{2} = \frac{x}{1-x} \checkmark$$

$$\sqrt{3}(1-x) = 2x$$

$$\sqrt{3} - \sqrt{3}x = 2x$$

$$\sqrt{3} = (2+\sqrt{3})x$$

$$x = \frac{\sqrt{3}}{2+\sqrt{3}}$$

$$x = \frac{\sqrt{3}(2-\sqrt{3})}{4-3} \checkmark$$

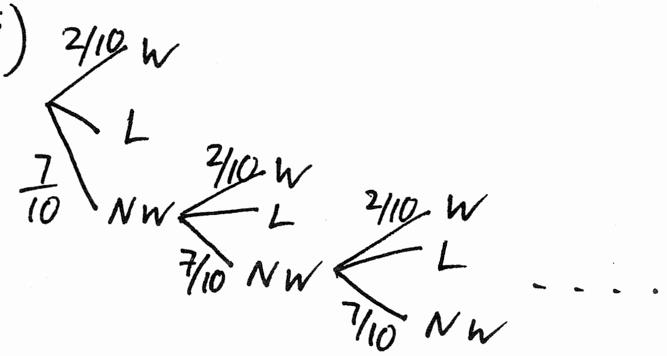
$$\boxed{x = 2\sqrt{3} - 3}\ \text{m}$$

Q15

e) i) $\frac{2}{10} + \frac{1}{10} + \frac{7}{10}\left(\frac{2}{10} + \frac{1}{10}\right)$ ✓

$= \frac{51}{100}$ ✓

ii)



$$P(\text{win}) = \frac{2}{10} + \frac{7}{10} \times \frac{2}{10} + \left(\frac{7}{10}\right)^2 \times \frac{2}{10} + \left(\frac{7}{10}\right)^3 \times \frac{2}{10} \ldots$$ ✓

This is a Geo Serie

$$S_\infty = \frac{a}{1-r} = \frac{2/10}{1 - 7/10} = \frac{2}{3}$$ ✓

H.sc Mathematics 2018
Trial

Q.16
a) (i)

$A_1 = P(1+\frac{3}{12}\%) - 100 = P(1.0025) - 100$

$A_2 = (P(1.0025) - 100)1.0025 - 100$

$A_2 = P(1.0025)^2 - 100(1 + 1.0025)$

$A_3 = P(1.0025)^3 - 100(1 + 1.0025)^2$

Most students got A_2 correct.

a) (ii)

$A_n = P(1.0025)^n - 100(1 + 1.0025 + \dots + 1.0025^{n-1})$

At the end of 10 years, his account will be empty so $A_{120} = 0$ as $n = 12 \times 10 = 120$ month

$0 = P(1.0025)^{120} - 100(1 + 1.0025 + \dots + 1.0025^{119})$

$P = \frac{100(1.0025^{120} - 1)}{1.0025 - 1} \times \frac{1}{1.0025^{120}}$

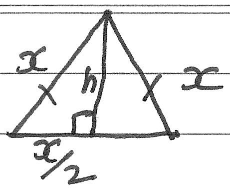
$P = \frac{100(1.0025^{120} - 1)}{0.0025} \times \frac{1}{1.0025^{120}}$

$P = \$10356.18$

Some students got the value of n as 10 which is wrong as interest is calculated at the end of each month, hence calculate the no. of months in 10 years which is 120.

HSC Mathematics Trial – 2018

16 b) (i)



$$h = \sqrt{x^2 - \left(\frac{x}{2}\right)^2} = \sqrt{\frac{3x^2}{4}} = \frac{\sqrt{3}x}{2}$$

Many students were confused with this step and did not pay attention to the fact that the base is an equilateral $\triangle$.

$$\text{Area of base} = \frac{1}{2} \times x \times \frac{\sqrt{3}x}{2} = \frac{\sqrt{3}x^2}{4}$$

$$\text{volume} = 1000\ cm^3 \text{ (given)}$$

$$1000 = \frac{\sqrt{3}x^2}{4} \times y$$

$$y = \frac{4000}{\sqrt{3}x^2}$$

Many students made the error while manipulating algebra in this equation and obtained wrong expression for y.

b (ii) $$\text{Surface Area} = 3 \times x \times y + 2 \times \text{Area of base}$$

$$= 3 \times \cancel{x} \times \frac{4000}{\sqrt{3}x^{\cancel{2}}} + 2 \times \frac{\sqrt{3}x^2}{4}$$

$$A = \frac{4000\sqrt{3}}{x} + \frac{\sqrt{3}x^2}{2}$$

As some students got incorrect expression for y, so they were unable to show the area of prism equal to the given expression.

(iii) $$\frac{dA}{dx} = 4000\sqrt{3}(-x^{-2}) + \frac{\sqrt{3}}{2} \times 2x$$

For minimum area $\frac{dA}{dx} = 0$

$$-x^{-2} \times 4000\cancel{\sqrt{3}} + \cancel{\sqrt{3}}x = 0$$

$$-\frac{4000}{x^2} = -x$$

$$x^3 = 4000$$

$$x = (4000)^{1/3} = 15.87\text{cm (2d.p)}$$

To prove surface area is minimised

$$\frac{d^2A}{dx^2} = \frac{4000\sqrt{3} \times 2}{x^3} + \sqrt{3}$$

As x is distance, so will be positive, hence $\frac{d^2A}{dx^2} > 0$

So obtained value of x will ensure

Most students got this correct, that however they did not show value of x for which A is minimum.

HSC Trial Mathematics – 2018

16 c)

$$f(x) = x^3 - 3bx^2 + 3cx + d$$

$$f'(x) = 3x^2 - 6bx + 3c$$

$$f''(x) = 6x - 6b$$

Given (x_1, y_1) is local maximum, so

$3x_1^2 - 6bx_1 + 3c = 0$ and $f''(x_1) < 0$ — ①

Also (x_2, y_2) is local minimum

$3x_2^2 - 6bx_2 + 3c = 0$ and $f''(x_2) > 0$ — ②

From ① and ②

$$3(x_1^2 - x_2^2) - 6b(x_1 - x_2) = 0$$

$$(x_1 + x_2) = 2b$$

$$b = \frac{x_1 + x_2}{2} \quad — ③$$

As $x = \frac{x_1+x_2}{2}$ is the mid point of AB, so $x_1 < x < x_2$.

For point $x = \frac{x_1+x_2}{2}$ to be point of inflexion, $f''\left(\frac{x_1+x_2}{2}\right)$ should be equal to zero and concavity should changes at this point

$$f''\left(\frac{x_1+x_2}{2}\right) = 6\frac{(x_1+x_2)}{2} - 6b$$

use ③

$$f''\left(\frac{x_1+x_2}{2}\right) = 3(x_1+x_2) - \cancel{6}^{3} \times \frac{x_1+x_2}{\cancel{2}} = 0$$

x_1	$\frac{x_1+x_2}{2}$	x_2
$f''(x_1) < 0$	0	$f''(x_2) > 0$ from ②

Hence $x = \frac{x_1+x_2}{2}$ is point of inflexion.

This question was done correctly by most students. A small minority did not show the change in concavity so lost