Sydney Girls High School

2019	TRIAL HIGHER SCHOOL CERTIFICATE **EXAMINATION**

Mathematics

General Instructions

- Reading time – 5 minutes
- Working time – 3 hours
- Write using black or blue pen
- NESA approved calculators may be used
- A reference sheet is provided
- In Questions 11 – 16, show relevant mathematical reasoning and/or calculations. A correct answer without working will be awarded a maximum of 1 mark.

Total marks : 100

Section 1 – 10 marks (pages **2 – 5**)

- Attempt Questions 1 – 10
- Answer on the Multiple Choice answer sheet provided
- Allow about 15 minutes for this section

Section II – 90 marks (pages **6 – 15**)

- Attempt Questions **11 – 16**
- Answer on the blank paper provided
- Begin a new page for each question
- Allow about 2 hours and 45 minutes for this section

Name: ……………………………………………………... Teacher: ……………………………………………………	**THIS IS A TRIAL PAPER ONLY** It does not necessarily reflect the format or the content of the 2019 HSC Examination Paper in this subject.

Section I

10 marks
Attempt Questions 1 – 10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1 – 10

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1. Which is a simplified expression for $\frac{2}{a}-\frac{1}{a+1}$?

   (A) $\frac{a+2}{a^2+1}$

   (B) 1

   (C) $\frac{a+2}{a(a+1)}$

   (D) $\frac{1}{a(a+1)}$

2. Expand and simplify $(\tan\theta-1)^2$.

   (A) $\sec^2\theta$

   (B) $\operatorname{cosec}^2\theta-2\tan\theta$

   (C) $\cot^2\theta-2\tan\theta$

   (D) $\sec^2\theta-2\tan\theta$

3. The angle of a sector in a circle of radius 18 cm is $\frac{\pi}{9}$ radians.
   What is the perimeter of the sector?

   (A) $2\pi+18$ (B) $2\pi+36$

   (C) $\pi+18$ (D) $\pi+36$

4. The probability that there will be rain in Sydney this weekend is $\frac{1}{3}$.

The probability that there will be rain in Newcastle this weekend is $\frac{2}{5}$.

What is the probability that there will be no rain in Sydney and no rain in Newcastle this weekend?

(A) $\frac{2}{3}$

(B) $\frac{3}{5}$

(C) $\frac{2}{5}$

(D) $\frac{2}{15}$

5. The first three terms of an arithmetic series are 5, 9 and 13.

What is the 15th term of this series?

(A) 61

(B) 66

(C) 495

(D) 585

6. Differentiate $(x^2 + \ln 2)^3$.

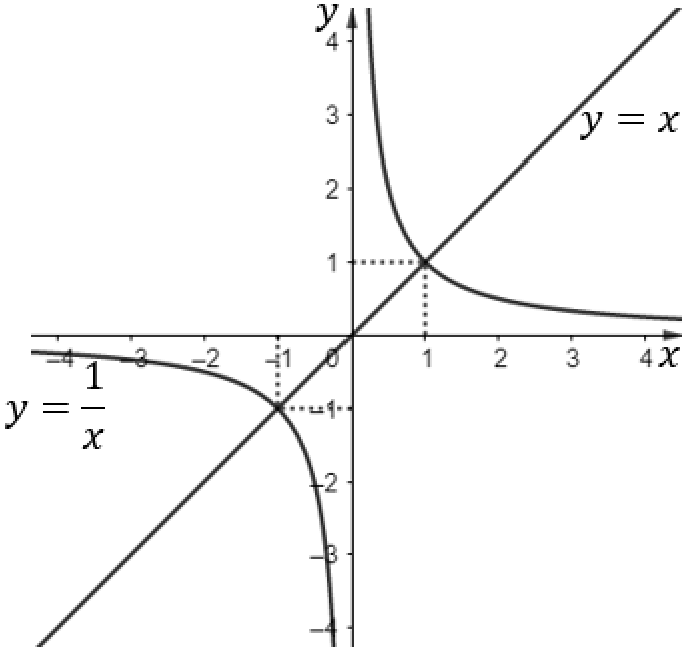
(A) $3 \times (x^2 + \ln 2)^2$

(B) $3 \times 2x \times \frac{1}{2}(x^2 + \ln 2)^2$

(C) $3 \times 2x(x^2 + \ln 2)^2$

(D) $3 \times \left(2x + \frac{1}{2}\right)(x^2 + \ln 2)^2$

7. Use the diagram below to solve the inequation $\frac{1}{x} < x$.



(A) $-1 < x < 0$ or $x > 1$.

(B) $x < -1$ or $0 < x < 1$.

(C) $-1 < x < 1$.

(D) $x < 0$ or $x > 1$.

8. A particle is moving in a straight line. At time t seconds its displacement from a fixed point O on the line is $x = t^2 - 2t$ metres. What distance is travelled by the particle in the first 3 seconds of its motion?

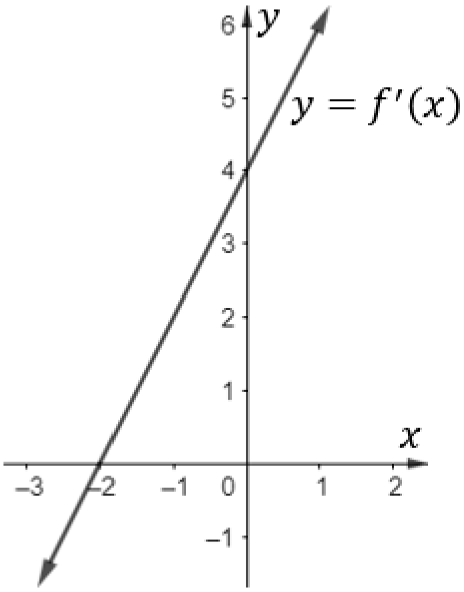
(A) 3 metres

(B) 4 metres

(C) 5 metres

(D) 6 metres

9. The graph of $y = f'(x)$ is shown below.



The curve $y = f(x)$ has a minimum value of 6.

What is the equation of the curve?

(A) $y = x^2 - 4x + 2$

(B) $y = x^2 - 4x + 10$

(C) $y = x^2 + 4x + 2$

(D) $y = x^2 + 4x + 10$

10. For $\lambda > 1$, what is the limiting value of $\int_0^n \frac{1}{\lambda} e^{-\lambda x}\, dx$ as $n \to \infty$?

(A) 0

(B) $\frac{1}{\lambda^2}$

(C) $\frac{1}{\lambda}$

(D) 1

Section II

90 marks

Attempt Questions 11 – 16
Allow about 2 hours and 45 minutes for this section

Answer on the blank paper provided. Begin a new page for each question.
Your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks)

(a) Evaluate $\dfrac{e^{\pi}}{\ln \pi}$ correct to 3 significant figures. **1**

(b) Simplify $\dfrac{x^2y - xy^2}{x^2 - y^2}$. **2**

(c) Express $\dfrac{5}{\sqrt{3}+2}$ in the form $a\sqrt{3}+b$. **2**

(d) Solve $|2x-1|<3$. **2**

(e) If $g'(t)=6t^2-1$ and $g(-1)=2$, find an expression for $g(t)$. **2**

(f) Evaluate $\displaystyle\sum_{n=1}^{5}\frac{1}{2^n}$. **2**

(g) Differentiate $\dfrac{3x^5}{\cos x}$. **2**

(h) Evaluate $\displaystyle\int_0^{\frac{\pi}{4}}(\sin 2x + \sec^2 x)\,dx$. **2**

End of Question 11.

Question 12 **(15 marks)** Begin a new page.

(a) Differentiate with respect to x

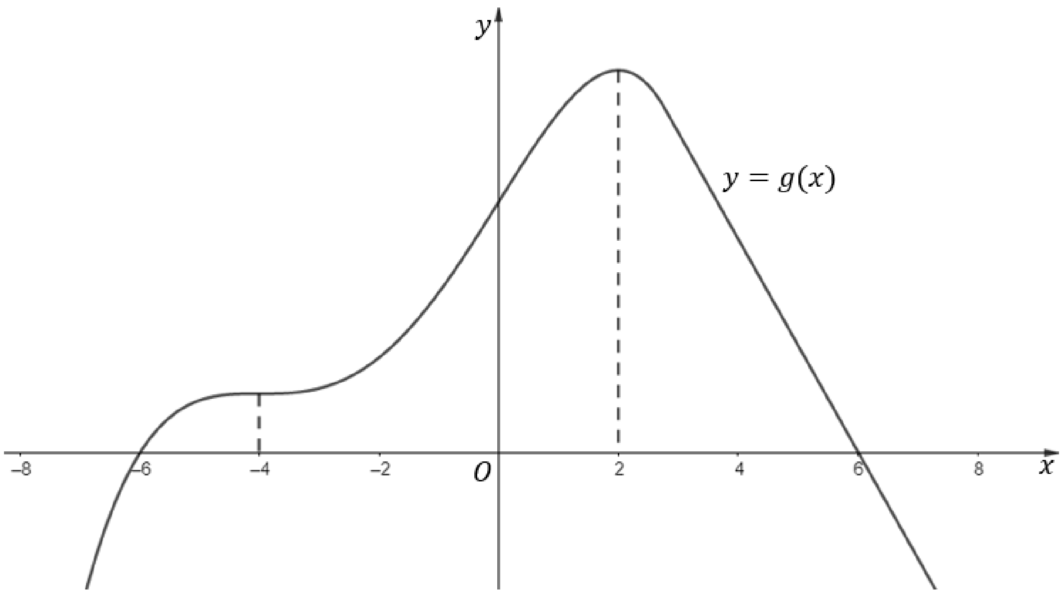
(i) $e^{x^2} \tan x$ **2**

(ii) $\dfrac{\ln x}{x^2}$ **2**

(b) The quadratic equation $2x^2 + 5x - 3 = 0$ has roots α and β. **2**

Find the value of $\alpha^2 + \beta^2$.

(c) The graph shows the function $y = g(x)$.



There is a horizontal point of inflexion at $x = -4$.

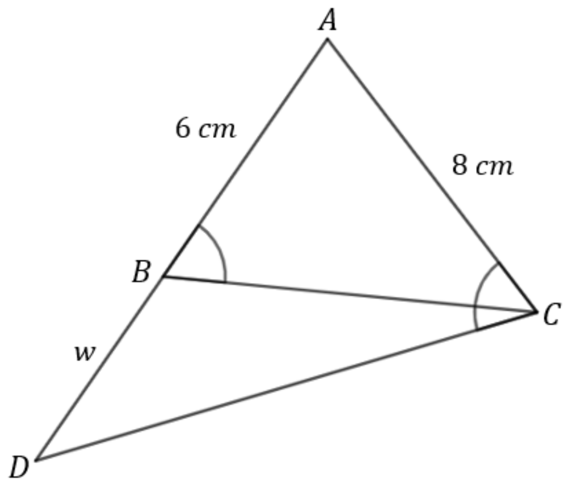
(i) For what values of x is the curve stationary? **1**

(ii) For what values of x is the curve decreasing? **1**

(d) Solve for x : $2\log_2(x-1) + \log_2 x - \log_2 4x = 0$ **3**

Question 12 (continued)

(e) Consider ΔABC in the diagram below.
AB is extended to point D forming triangle ACD.
$\angle ABC = \angle ACD$, $AB = 6$ cm, $AC = 8$ cm and $BD = w$.



(i) Prove that triangle ABC is similar to triangle ACD. **2**

(ii) Find the value of w, giving reasons. **2**

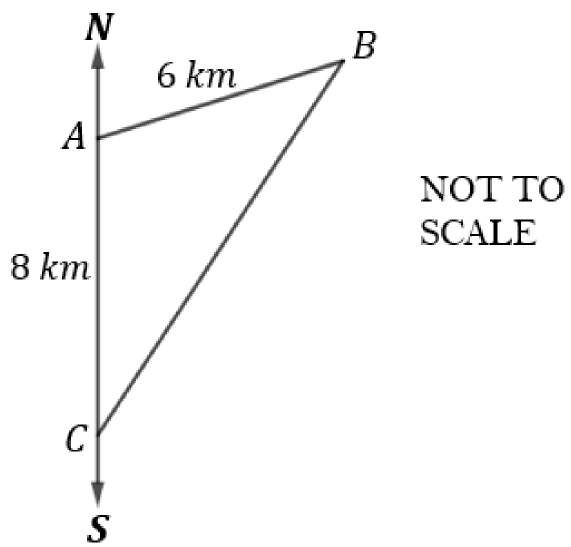
End of Question 12.

Question 13 **(15 marks)** Begin a new page.

(a) Find the domain of the function $f(x)=\dfrac{1}{\sqrt{4x^2-1}}$. **2**

(b) Find the equation of the tangent to the curve $f(x)=e^{1-2x}$ at the point where $x=\dfrac{1}{2}$. **2**

(c) In the diagram a drone leaves point A and flies in a straight line for 6 km on a bearing of 070° to point B. It then flies in a straight line to point C which is 8 km due south of A.

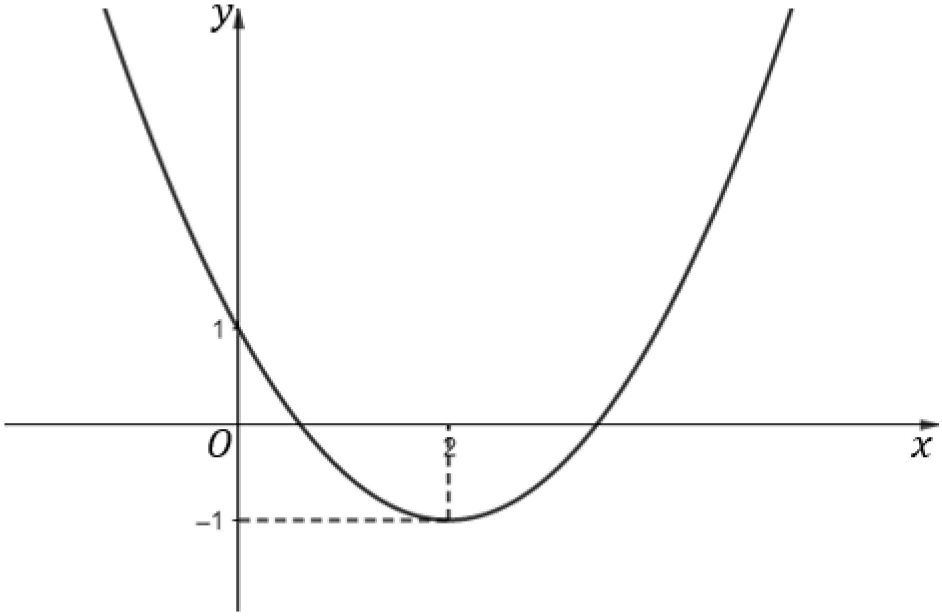


(i) Find the distance from B to C in kilometres, correct to 1 decimal place. **2**

(ii) Find the bearing of C from B, correct to the nearest degree. **2**

Question 13 (continued)

(d) The diagram shows the graph of a parabola passing through the points (0,1) and (2,-1).



(i) Show that the equation of the parabola is given by

$$(x-2)^2 = 2y+2.$$ **2**

(ii) Write down the coordinates of the focus and the equation of the directrix. **2**

(e) Consider the function $y = \left(\frac{1}{2}\right)^{-x}$.

(i) Copy and complete the following table of values onto your writing paper. **1**

x	−2	−1	0	1	2
y					

(ii) Hence, use Simpson's rule with 5 function values to find an approximation to the value of $\int_{-2}^{2} \left(\frac{1}{2}\right)^{-x} dx$. **2**

End of Question 13.

Question 14 **(15 marks)** Begin a new page.

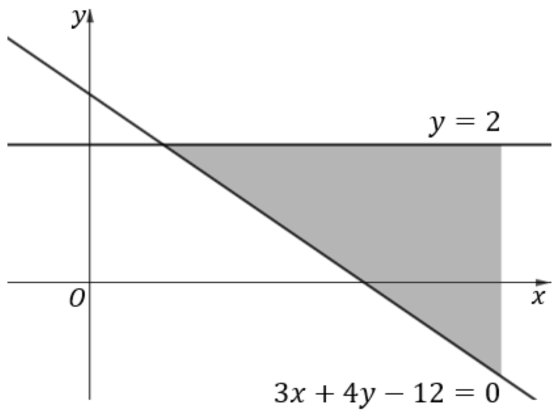
(a) Consider the curve $y = 1 + 3x - x^3$, for $-2 \le x \le 3$.

(i) Find the stationary points and determine their nature. **3**

(ii) Find the point of inflexion. **1**

(iii) Sketch the curve for $-2 \le x \le 3$. **2**

(b) The point $P(x, y)$ is equidistant from the lines $y = 2$ and $3x + 4y - 12 = 0$. Given that the point P lies in the shaded region of the diagram below, find the equation of the locus of P. **2**



(c) A dodecagon has 12 sides. The angles of a dodecagon are in an arithmetic progression.

(i) Given that the size of the smallest angle is 62°, find the common difference. **2**

(ii) How many of these angles are obtuse? Justify your answer. **1**

(d) On the 1st of January 2014, \$15 000 was deposited into an account to enable Emily to save for her university tuition. On the first day of January of each of the following years, a further \$2000 is deposited into the account. Interest of 8% per annum is paid into the account at the end of each quarter.

(i) What was the balance in the account at the end of 2014? **1**

(ii) Emily hopes to commence her university course in 2020. How much would be in the account after the final interest payment is made on 31 December 2019? **3**

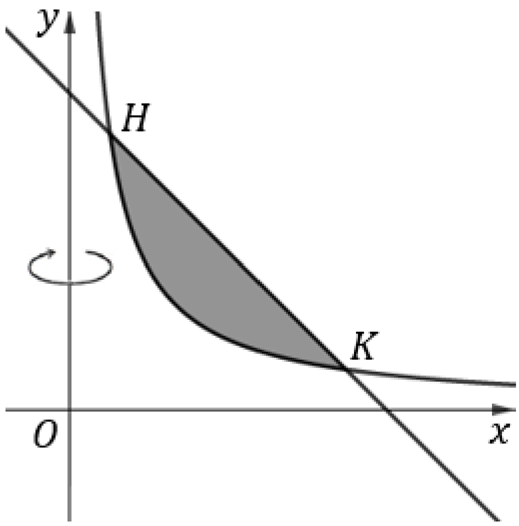
End of Question 14.

Question 15 **(15 marks)** Begin a new page.

(a) The diagram below shows the graphs of $y = \dfrac{2}{x}$ and $y = 3 - x$ for $x > 0$.

The shaded area is enclosed between the two graphs and their points

of intersection H and K, as shown.



(i) Find the coordinates of the points H and K. **2**

(ii) The shaded area is rotated about the $y-$axis.

Find the exact volume of the solid formed. **3**

Question 15 (continued)

(b) The number N of bacteria in a mouldy loaf of bread at time t hours is given by the equation $N = 21e^{kt}$.
After 7 hours the number of bacteria present is 30.

(i) Find the value of k, correct to 3 decimal places. **1**

(ii) Determine the number of bacteria after 1 day. **1**

(iii) At what rate is the number of bacteria increasing after 1 day? **1**

(iv) Mouldy bread is considered unsafe to eat when the number of bacteria present reaches 3000.
For how many days can the bread be considered safe to eat? **2**

(c) Lola is obsessed by the colour of her hair. On any given day there is an 80% chance she will change the colour of her hair for the next day.
Her hair is blond 40% of the time, brown 30% , red 20% and purple for the remainder.
Given Lola has red hair on Friday, what is the probability that :

(i) tomorrow her hair is red ? **1**

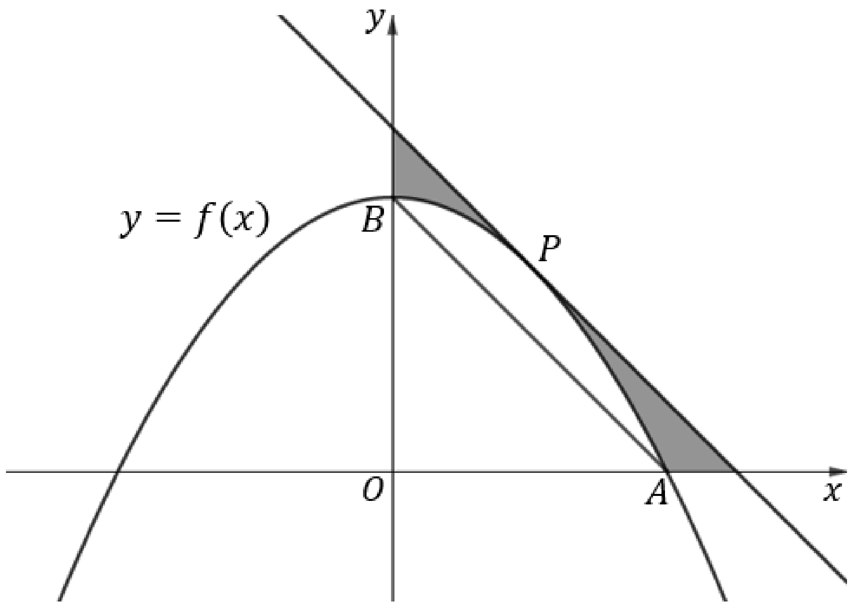
(ii) tomorrow her hair is brown ? **2**

(iii) her hair is not red on Saturday and Sunday AND her hair is a different colour on Saturday and Sunday. **2**

End of Question 15.

Question 16 **(15 marks)** Begin a new page.

(a) The graph of the function $f(x) = \dfrac{9 - x^2}{3}$ is shown below.



The graph intersects the $x-$axis and the $y-$axis at the point A and B respectively.

The tangent to the graph at point P is parallel to the line AB.

The coordinates of B are (0,3).

(i) Find the coordinates of the point A. **1**

(ii) Show that the coordinates of the point P are $\left(1\frac{1}{2}, 2\frac{1}{4}\right)$. **3**

(iii) Find the equation of the tangent at the point P. **1**

(iv) The shaded region shown in the diagram above is bounded by the curve $y = f(x)$, the tangent at P, the $x-$axis and $y-$axis. **3**

Find the area of this shaded region.

Question 16 (continued)

(b) The velocity, $\dot{x}$, in m/s of a particle moving in a straight line is given by

$\dot{x} = 3 - \dfrac{9}{t-2}$ for $t > 2$, where t is the time in seconds.

(i) In which direction is the particle travelling when $t = 3$? **1**

(ii) Find the time when the particle changes direction during its motion. **1**

(iii) Hence, or otherwise, find the distance travelled by the particle between $t = 3$ and $t = 7$. **2**

Give your answer correct to 2 decimal places.

(c) Use calculus to show that the sum of a positive number and its reciprocal is never less than 2. **3**

End of paper

Sydney Girls High School

Mathematics Faculty

Multiple Choice Answer Sheet

Trial HSC Mathematics

Select the alternative A, B, C or D that best answers the question. Fill in the response completely.

Sample 2 + = ? (A) 2 (B) 6 (C) 8 (D) 9

B ● C ○ D

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ● B ~~●~~ C ○ D ○

If you change your mind and have crossed out what you consider to be the correct answer, then indicate this the word *correct* and drawing an arrow as follows:

correct

A ~~●~~ B ~~●~~ C ○ D ○

Student Number: SOLUTIONS

Completely fill the response oval representing the most correct answer.

1. A ○ B ○ C ● D ○
2. A ○ B ○ C ○ D ●
3. A ○ B ● C ○ D ○
4. A ○ B ○ C ● D ○
5. A ● B ○ C ○ D ○
6. A ○ B ○ C ● D ○
7. A ● B ○ C ○ D ○
8. A ○ B ○ C ● D ○
9. A ○ B ○ C ○ D ●
10. A ○ B ● C ○ D ○

2 unit Trial Exam.

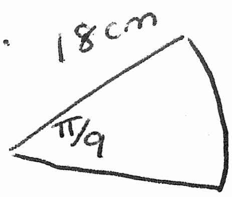
1. $\dfrac{2}{a} - \dfrac{1}{a+1} = \dfrac{2(a+1)-a}{a(a+1)}$

$= \dfrac{2a+2-a}{a(a+1)}$ (C)

$= \dfrac{a+2}{a(a+1)}$

2. $(\tan\theta - 1)^2 = (\tan^2\theta - 2\tan\theta + 1)$ (D)

$= \sec^2\theta - 2\tan\theta$

3. 

$l = r\theta$

$= 18 \times \frac{\pi}{9}$

$= 2\pi$

$\therefore$ Perimeter $= 2\pi + 2(18)$

$= 2\pi + 36$

(B)

4. $\tilde{S} = 1 - \frac{1}{3}$ $\qquad$ $\tilde{N} = 1 - 2/5$

$= \frac{2}{3}$ $\qquad$ $= 3/5$

$\therefore$ P(no rain in either) $= \frac{2}{3} \times 3/5$ (C)

$= \frac{2}{5}$

5. $a = 5$

$d = T_2 - T_1 = T_3 - T_2$

$d = 9-5 = 13-9$

$d = 4$

$T_n = a + (n-1)d$

$T_{15} = 5 + (15-1) \times 4$

$= 5 + 14 \times 4$

$= 61$ (A)

10. $\int_0^n \frac{1}{\lambda} e^{-\lambda x} = -\frac{1}{\lambda^2} e^{-\lambda x}\Big]_0^n$

$= \frac{-1}{\lambda^2} e^{-\lambda n} - \left[-\frac{1}{\lambda^2} e^0\right]$

since $n \to \infty$

$= 0 + \frac{1}{\lambda^2}$

$= \frac{1}{\lambda^2}$ B.

2019. SGHS – Trial
Advance Mathematics
Question 11 Solutions.

a) $\dfrac{e^{\pi}}{\ln \pi} = 20.2$ (3 sig. fig) 1

(to obtain mark 3 sig fig. had to be given) ie/ 20.2 as an answer was only accepted.

b) $\dfrac{x^2y - xy^2}{x^2 - y^2} = \dfrac{xy(\cancel{x-y})}{(\cancel{x-y})(x+y)}$

$= \dfrac{xy}{x+y}$ 2

c) $\dfrac{5}{\sqrt{3}+2} \times \dfrac{\sqrt{3}-2}{\sqrt{3}-2} = \dfrac{5(\sqrt{3}-2)}{(\sqrt{3})^2 - 2^2}$

$= \dfrac{5\sqrt{3} - 10}{3 - 4}$

$= \dfrac{5\sqrt{3} - 10}{-1}$ 2

$= 10 - 5\sqrt{3}$

$\therefore a\sqrt{3} + b = -5\sqrt{3} + 10.$

(Completed very well with most students getting this correct)

d) $|2x-1| < 3$

$-3 < 2x-1 < 3$

$-2 < 2x < 4$

$-1 < x < 2$ 2

(Final solution had to be written as $-1 < x < 2$
1 mark was deducted if $x < 2$ and $x > -1$)

e) $g'(t) = 6t^2 - 1$

$$g(t) = \frac{6t^3}{3} - t + C$$

$$g(t) = 2t^3 - t + C$$

$$2 = 2(-1)^3 - (-1) + C$$

$$2 = -2 + 1 + C$$

$$2 = -1 + C$$

$$\therefore C = 3$$ 2

$$\therefore g(t) = 2t^3 - t + 3$$

f) $$\sum_{n=1}^{5} \frac{1}{2^n} = \frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + \frac{1}{2^4} + \frac{1}{2^5}$$

$$= \frac{31}{32}$$ 2

g) let $u = 3x^5$, $v = \cos x$

$u' = 15x^4$ $v' = -\sin x$

$$\therefore \frac{d}{dx}\left(\frac{3x^5}{\cos x}\right) = \frac{vu' - uv'}{v^2}$$

$$= \frac{15x^4\cos x + 3x^5\sin x}{\cos^2 x}$$

$$= \frac{3x^4(5\cos x + x\sin x)}{\cos^2 x}$$ 2

h) $\displaystyle\int_0^{\pi/4} (\sin 2x + \sec^2 x)\,dx$

$$= \left[-\frac{1}{2}\cos 2x + \tan x\right]_0^{\pi/4}$$

$$= \left[-\frac{1}{2}\cos 2\left(\frac{\pi}{4}\right) + \tan\frac{\pi}{4}\right] - \left[-\frac{1}{2}\cos 0 + \tan 0\right]$$

$$= [0 + 1] - \left[-\frac{1}{2} + 0\right]$$

$$= 1\frac{1}{2}$$

2

Overall, question was completed extremely well.
Most obtaining full marks.

Question 12

a) i) $e^{x^2}.\tan x$

let $u = e^{x^2}$ $\quad v = \tan x$

$u' = 2xe^{x^2}$ $\quad v' = \sec^2 x$

(Most students answered very well)

$$\therefore \frac{d}{dx}(e^{x^2}.\tan x) = e^{x^2}.\sec^2 x + 2xe^{x^2}\tan x$$

$$= e^{x^2}(\sec^2 x + 2x\tan x) \quad 2$$

ii) $\frac{\ln x}{x^2} = x^{-2}.\ln x$

let $u = x^{-2}$ $\quad v = \ln x$

$u' = -2x^{-3}$ $\quad v' = \frac{1}{x}$

$$\therefore \frac{d}{dx}\left(\frac{\ln x}{x^2}\right) = -2x^{-3}.\ln x + x^{-2}.\frac{1}{x}$$

(Very Well Completed)

$$= \frac{-2\ln x}{x^3} + \frac{1}{x^3} \quad 2$$

$$= \frac{1-2\ln x}{x^3}$$

b) $\alpha^2+\beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$ $\quad \therefore \alpha^2+\beta^2 = \left(-\frac{5}{2}\right)^2 - 2\left(-\frac{3}{2}\right)$

$\alpha+\beta = -\frac{b}{a}$ $\quad \alpha\beta = \frac{c}{a}$ $\quad = \frac{25}{4} + \frac{6}{2}$

$= -\frac{5}{2}$ $\quad = \frac{-3}{2}$ $\quad = \frac{37}{4}. \quad 2$

(Nearly all students got this correct)

c) i) Stationary at $x = -4$ and $x = 2$ (a lot of students only got $x=2$) 1

ii) Decreasing for $x > 2$ 1

(most students didn't get this correct).

d) $\log_2 (x-1)^2 + \log_2 x - \log_2 4x = \log_2 1$

$$\frac{\cancel{x}(x-1)^2}{4\cancel{x}} = 1$$

(Overall question completed well. Most common error was elimination of $x=-1$ didn't occur.)

$$(x-1)^2 = 4$$

$$x^2 - 2x + 1 = 4$$

$$x^2 - 2x - 3 = 0$$

$$(x-3)(x+1) = 0$$

$\therefore \; x = 3$ or $x = -1$

however, $x \neq -1$ $\therefore$ only solution: $x = 3$. 3

e) i) In ΔABC and ΔACD

(AAS was not an acceptable reason. Some forgot to give a reason) 2

$\angle ABC = \angle ACD$ (given)

$\angle A$ is common

$\therefore \; \Delta ABC \; ||| \; \Delta ACD$ (equiangular)

ii) $\dfrac{6}{8} = \dfrac{8}{6+w}$ (corresponding sides in similar Δ's)

$$6(6+w) = 64$$

(Most common error was some students forgot to give a reason.) 2

$$36 + 6w = 64$$

$$6w = 28$$

$$w = \frac{14}{3}.$$

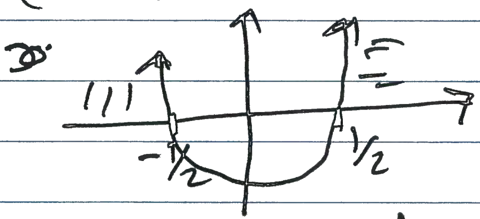
HSC Trial – 2019

Mathematics

Q.13

a) $4x^2 - 1 > 0$

$(2x+1)(2x-1) > 0$



$x < -\frac{1}{2} \quad x > \frac{1}{2}$

This question was fairly done by majority of students.

b) $y = e^{1-2x}$

$\frac{dy}{dx} = e^{1-2x}(-2)$

gradient of tangent $m_{x=\frac{1}{2}} = e^{1-\cancel{2}\times\frac{1}{\cancel{2}}}(-2)$

$= -2$

$x = \frac{1}{2} \quad y = 1$

so using gradient point form, the equation of tangent to the given curve is

$y - 1 = -2(x - \frac{1}{2})$

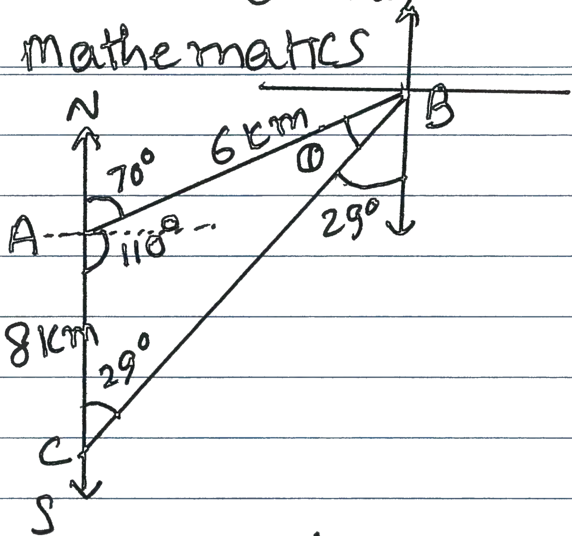
$y - 1 = -2x + 1$

$2x + y - 2 = 0$

majority got this question correctly. Few got confused with either subbing $x = \frac{1}{2}$ in the tangent's gradient or finding y-coordinate of the point.

HSC 2019 (Trial)
Mathematics

13 c)



i) using cosine rule.

$$BC = \sqrt{8^2+6^2-2\times8\times6\cos110^\circ} = 11.5\text{ km (1 d.p)}$$

Majority of students got this part correct.

(ii) using sine rule

$$\frac{\sin\theta}{8} = \frac{\sin110^\circ}{11.5}$$

$$\theta = \sin^{-1}\left(\frac{8\sin110^\circ}{11.5}\right) = 41^\circ \text{ (nearest degree)}$$

Bearing of C from B $= 180^\circ + 29^\circ$

$= 209^\circ$

Most students got it correct with slight variation in the end result.

13 d) i) From diagram vertex (2,−1) and point (0,1) lying on it

$$y = a(x-h)^2 + k$$

$$y = a(x-2)^2 + (-1)$$

sub (0,1) in eq.

$$1 = a(0-2)^2 - 1$$

$$a = \tfrac{1}{2}$$

a → focal length

Hence eq. becomes

$$y = \tfrac{1}{2}(x-2)^2 - 1$$

$$2y = (x-2)^2 - 2$$

$$2y + 2 = (x-2)^2$$

or

$$(x-2)^2 = 2y + 2$$

This question was fairly done by students, however with different approaches.

HSC Trial - 2019
Mathematics

Q13 d (ii)

coordinate of focus $(2, -\frac{1}{2})$

Directrix $y = -\frac{3}{2}$

Some students were confused about this part. Remember you are to use coordinates of vertex and focal length of parabola 'a' to determine the coordinates of focus and equation of directrix.

13 e) i

$y = \left(\frac{1}{2}\right)^{-x}$

x	-2	-1	0	1	2
y	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4

This is a simple question, where you could use calculator and defi of the function and given values of x to determine corresponding values of y.

13 e(ii)

$$\int_{-2}^{2} \left(\frac{1}{2}\right)^{-x} = \frac{h}{3}\left[\underbrace{\sum w}_{\text{weighting}} f(x)\right]$$

$h = \frac{2--2}{4}$

$h = 1$

x	$f(x)$	w	$wf(x)$
-2	$\frac{1}{4}$	1	$\frac{1}{4}$
-1	$\frac{1}{2}$	4	2
0	1	2	2
1	2	4	8
2	4	1	4
			$\sum f(x)w = \frac{65}{4}$

Some people were confused about 'w' i.e. the sequence 1, 4, 2, 4, 1

So $\int_{-2}^{2} \left(\frac{1}{2}\right)^{-x} = \frac{1}{3} \times \frac{65}{4} = \frac{65}{12}$

$= 5\frac{5}{12}$ unit^2

You are to be careful when subbing values in calculator.

HSC Trial - 2019

14

a) i) $y = 1 + 3x - x^3$

To find stationary points, find the first derivative and equate it to zero

$y' = 3 - 3x^2$

Now $3 - 3x^2 = 0$

$3(1 - x^2) = 0$

$1 - x = 0$	$1 + x = 0$
$x = 1$	$x = -1$
$y = 1 + 3 - 1$	$y = 1 - 3 + 1$
$= 3$	$y = -1$

So To determine the nature of stationary points, find the second derivative.

$y'' = -6x$

stationary points $(1,3)$ has $y'' = -6 < 0$ So we have $\cup$ and minima	Stationary point $(-1,-1)$ has $y'' = 6$ i.e. > 0 so $\cap$ and maxima.

Some students did not find the nature of stationary point correctly and so lost marks.

14 a (ii) To find point of inflexion, equate $y'' = 0$

which gives $x = 0$, $y = 1$

No check whether this is a point of inflexion, check as follows

x	-0.1	0	0.1
y''	$>$	0	$<$

As concavity changes So $(0,1)$ is the point of inflexion.

This is either right or wrong question a i it carries 1 mark only

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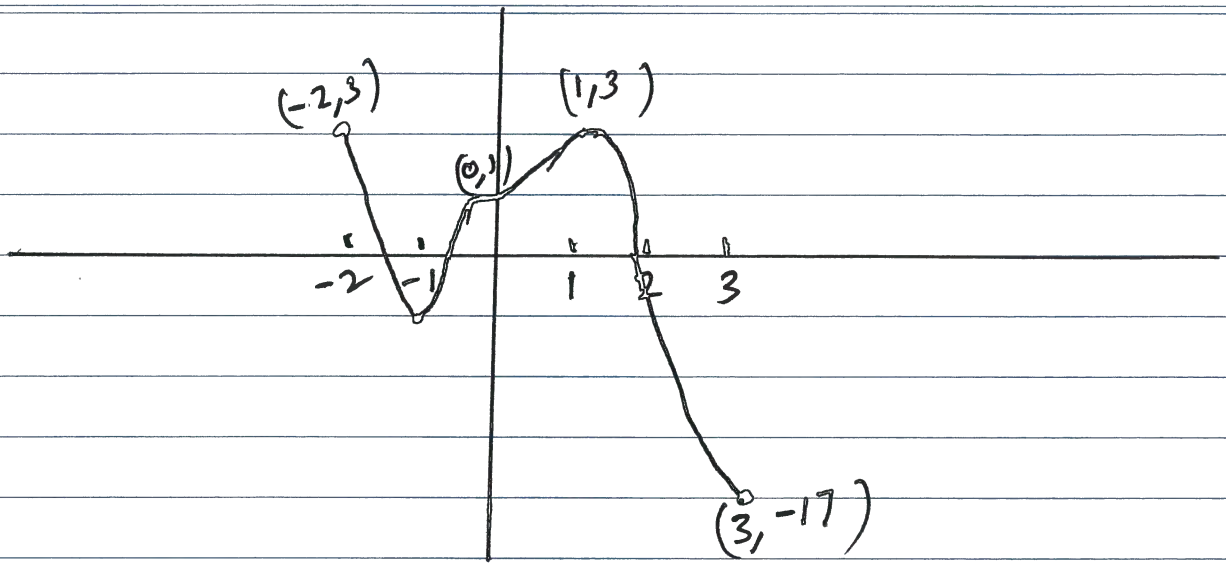
14 a (iii)



Use information obtained in previous parts of the Q and values of y at $x=-2$ and 3 to sketch the group. Some students were confused about ~~and~~ point of inflexion and hence lost marks.

14 b)

If x lies on $y=2$
then $P(x,2)$

Distance of point P from $y=2$ is $2-y$
and distance of P from $3x+4y-12=0$
is $\dfrac{3x+4y-12}{5}$

As given in the question

$$2-y=\frac{3x+4y-12}{5}$$

$$(2-y)5=3x+4y-12$$

$$10-5y=3x+4y-12$$

$$3x+9y-22=0$$

This is the required equation of Parabola.

$y=-\frac{3}{9}x+\frac{22}{9}$ is the another form

Also $y=2$, so $x=\frac{4}{3}$ satisfies the shaded region

Some students made mistake with their calculations, and got answers different than $3x+9y-22=0$, hence lost mark.

HSC Trial - 2019

Mathematics

14 c) We ~~kno~~ know angle sum of dodecagon $= 180(12-2) = 1800^\circ$

i) We know angles form A.P. so using sum of A.P formula

$$S_n = \frac{n}{2}(2a + (n-1)d)$$

$$1800 = \frac{12}{2}[2\times 62 + 11d]$$

$$11d = 176$$

common difference. $d = 16^\circ$

This question was done fairly by most students.

(ii) using common difference and smallest ∠, we have angles which are $62^\circ, 78^\circ, 94^\circ, 110^\circ, 126^\circ, \cancel{148^\circ}\ 142^\circ, 158^\circ, 174^\circ, 190^\circ, 222^\circ, 238^\circ$

obtuse angles are $94^\circ, 110^\circ, 126^\circ, 142^\circ, 158^\circ, 174^\circ$

So 6 is the answer

Most students made errors while identifying obtuse angles, hence lost marks in this part.

HSC Trial – 2019

Mathematics

14 d) i)

Balance at the end of 2014

$= 15000 \times 1.02^4$

$= \$16236.49$

Most students got this part correct. Note that interest is calculated for four quarters at the end of each quarter. So 2% per quarter is the interest rate.

(ii)

Amount At the end of 2019 $= 15000 \times 1.02^{24} + 2000 \times 1.02^{20}$
$+ 2000 \times 1.02^{16} + 2000 \times 1.02^{12} + 2000 \times 1.02^{8}$
$+ 2000 \times 1.02^{4}$

$= \$36888.70$

Some students kept the interest rate 8% per annum. Some miscalculate the span of time money was in the account and calculated wrong amount. So ~~mar~~ they were penalised.

Q15 Advanced

a) i) $y = \frac{2}{x}$, $y = 3 - x$

$\frac{2}{x} = 3 - x$

$2 = 3x - x^2$

$x^2 - 3x + 2 = 0$

$(x-1)(x-2) = 0$

$x = 1$ or $x = 2$

$y = 2$ or $y = 1$

$M(1,2)$ $K(2,1)$

* This question was done well

ii) $V = \pi \int_1^2 (3-y)^2 - \left(\frac{2}{y}\right)^2 dy$

$= \pi \left[\frac{(3-y)^3}{3 \times -1} - \frac{4y^{-1}}{-1} \right]_1^2$

$= \pi \left[\frac{(3-y)^3}{-3} + \frac{4}{y} \right]_1^2$

$= \pi \left[-\frac{1}{3} + 2 - \left(\frac{8}{-3} + 4 \right) \right]$

$= \pi \left[-2 + \frac{7}{3} \right]$

$= \frac{\pi}{3} u^3$

* some students had problems with this question

b) i) $30 = 21 e^{7k}$

$\frac{30}{21} = e^{7k}$

$\ln \frac{10}{7} = 7k$

$k \doteq 0.051$

ii) $N = 21 e^{0.051 \times 24}$

$= 71$ ✓

iii) $\frac{dN}{dt} = 21 \times 0.051 \times e^{0.051 \times 24}$

$= 3.64$ b/day ✓

* a few students couldn't do This question

iv) $3000 = 21 e^{0.051 t}$

$\ln \frac{3000}{21} = 0.051 t$

$t \doteq 97.291$ hours

$= 4$ days ✓✓

* many students had 97 days as answer.

C) i) 0.2 ✓

ii) $0.8 \times \frac{3}{8}$ ✓

$= 0.3$ ✓

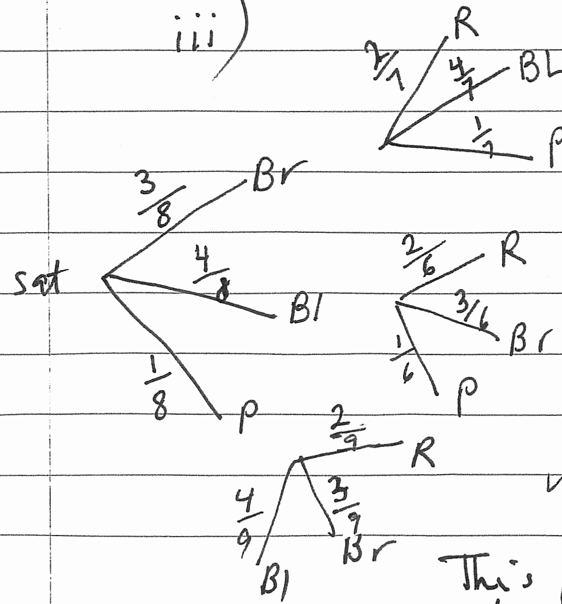
oR

$P(nR) = 0.8 \times 0.8 \left(\frac{4}{8} \times \frac{4}{6} + \frac{3}{8} \times \frac{5}{7} + \frac{1}{8} \times \frac{7}{9}\right)$

$\doteq 0.447$

many students had made a mistake in This question

iii)



$P(nR) = 0.8 \times \frac{3}{8} \times 0.8 \times \frac{4}{7} +$

$0.8 \times \frac{3}{8} \times 0.8 \times \frac{1}{7} + 0.8 \times \frac{4}{8} \times 0.8 \times \frac{3}{6}$

$+ 0.8 \times \frac{4}{8} \times 0.8 \times \frac{1}{6} + 0.8 \times \frac{1}{8} \times 0.8 \times \frac{3}{9}$

$+ 0.8 \times \frac{1}{8} \times 0.8 \times \frac{4}{9}$

2 ✓✓

$= 0.8 \times 0.8 \left(\frac{3}{8} \times \frac{4}{7} + \frac{3}{8} \times \frac{1}{7} + \frac{4}{8} \times \frac{3}{6} + \frac{4}{8} \times \frac{1}{6}\right.$

$\left. + \left(\frac{1}{8} \times \frac{3}{9} + \frac{1}{8} \times \frac{4}{9}\right)\right) \doteq 0.447$

This part was done poorly.

$\boxed{Q16}$

a) $A(x,0)$ is on $f(x)$

$$0 = \frac{9-x^2}{3} \quad \therefore \quad x^2 = 9$$

$$x = 3 \quad (x\text{-coordinate of } A \text{ is positive})$$

i) $A(3,0)$ ✓ $B(0,3)$

ii) $$mAB = \frac{3-0}{0-3} = -1$$

$$f'(x) = \frac{1}{3}(-2x) = -\frac{2x}{3}$$ ✓

At $P(x_1, y_1)$: $$mT = -\frac{2x_1}{3} = -1$$ ✓

$$2x_1 = 3 \quad \therefore \quad x_1 = 1\tfrac{1}{2}$$

But P is on $f(x)$

$$y = \frac{9 - \left(\frac{3}{2}\right)^2}{3} = \frac{9}{4} = 2\tfrac{1}{4}$$

Thus $P\left(1\tfrac{1}{2}, 2\tfrac{1}{4}\right)$ ✓

iii) $$y - y_1 = mT(x - x_1)$$

$$y - \frac{9}{4} = -1\left(x - \frac{3}{2}\right)$$

$$y - \frac{9}{4} = -x + \frac{3}{2}$$

$$4y - 9 = -4x + 6$$

$$4x + 4y - 15 = 0$$ ✓

Q16

a/iv) Tangent $4x + 4y - 15 = 0$

At $x = 0 \longrightarrow y = 15/4$

At $y = 0 \longrightarrow x = 15/4$

$$\text{Shaded Area} = \frac{\frac{15}{4} \times \frac{15}{4}}{2} - \int_0^3 \left(\frac{9 - x^2}{3}\right) dx$$

$$= \frac{225}{32} - \frac{1}{3}\left[9x - \frac{x^3}{3}\right]_0^3$$

$$= \frac{225}{32} - \frac{1}{3}\left[29 - 9 - 0\right]$$

$$= \frac{33}{32} \text{ units}^2$$

b) $\dot{x} = 3 - \dfrac{9}{t-2}$

i) $t = 3 \quad \therefore \dot{x} = 3 - 9 = -6 < 0$

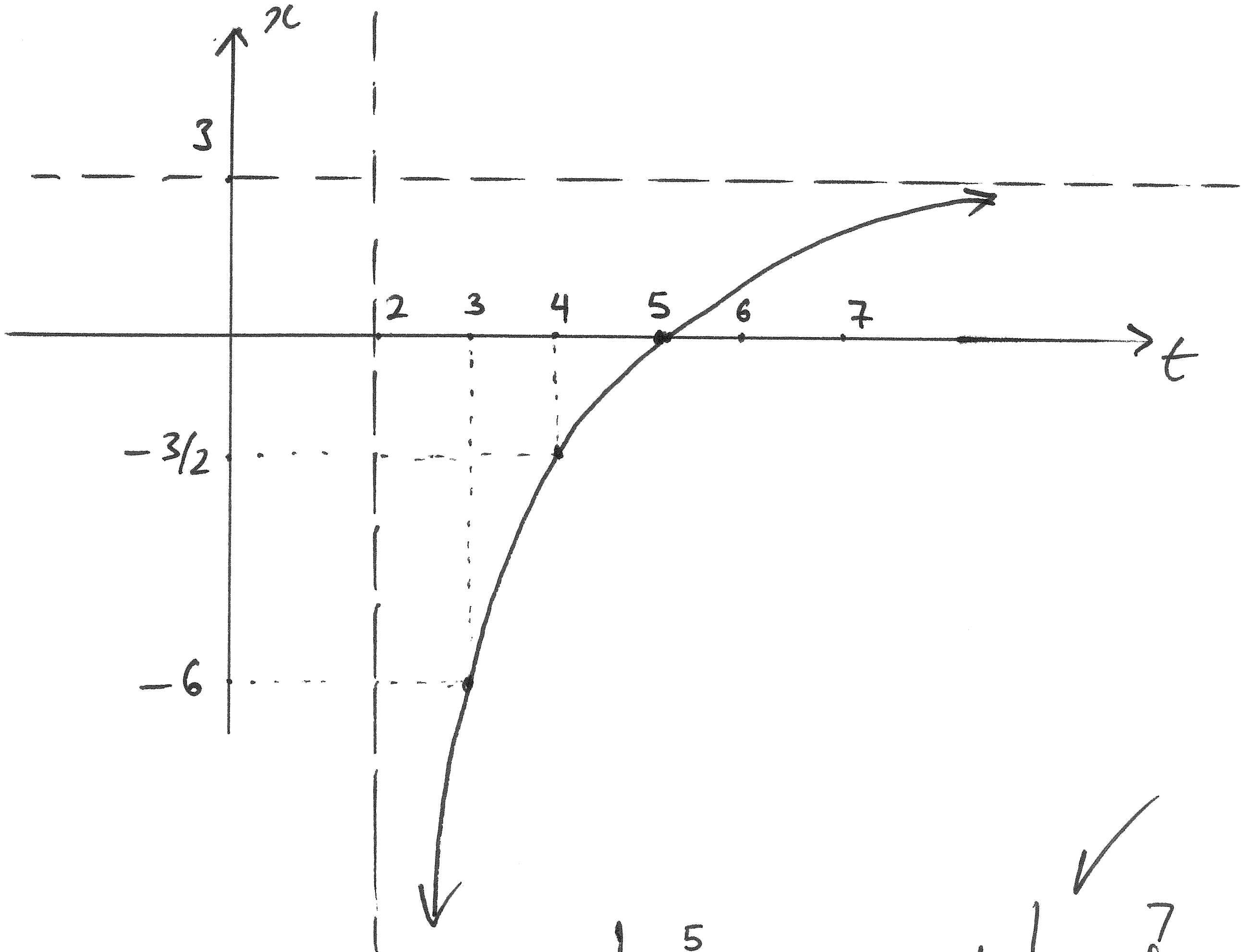
$\therefore$ The particle is moving to the Left.

ii) $\dot{x} = \dfrac{3t - 15}{t-2}$

$\dot{x} \geq 0 \quad \therefore \quad 3t - 15 \geq 0$

$t \geq 5$ sec

OR $t = 5$ sec



$$\text{Distance travelled} = \left| \int_3^5 \left(3 - \frac{9}{t-2}\right) dt \right| + \int_3^7 \left(3 - \frac{9}{t-2}\right) dt$$

$$d = \left| \Big[3t - 9\ln(t-2) \Big]_3^5 \right| + \Big[3t - 9\ln(t-2) \Big]_5^7$$

$$d = \left| 15 - 9\ln 3 - 9 \right| + 21 - 9\ln 5 - 15 + 9\ln 3$$

$$d = 5.29 \text{ (m)}$$

$\boxed{16}$

c) Let the numbers be x and $\frac{1}{x}$

Sum: $S = x + \frac{1}{x} = \frac{x^2+1}{x}$

$$\frac{dS}{dx} = \frac{2x(x) - (x^2+1)}{x^2} = \frac{x^2-1}{x^2} \checkmark$$

$$\frac{dS}{dx} = 0 \quad \therefore \quad x^2 - 1 = 0$$
$$x = 1 \quad \text{as } x > 0$$

$$\frac{d^2S}{dx^2} = \frac{2x(x^2) - 2x(x^2-1)}{x^4} = \frac{2}{x^3} \checkmark$$

$S''(1) = \frac{2}{1} > 0$ Thus the Sum is minimum when $x = 1$ OR $\checkmark$

$S = 1 + \frac{1}{1} = 2$

OR $S \geq 2$ OR $x + \frac{1}{x} \geq 2$

* To get full mark, students must apply the skill of calculus to prove not by algebraically.