Student Number

Sydney Girls High School
2020
TRIAL HIGHER SCHOOL CERTIFICATE

EXAMINATION

Mathematics Advanced

General Instructions

- Reading time – 10 minutes
- Working time – 3 hours
- Write using black pen
- Calculators approved by NESA may be used
- A reference sheet is provided at the back of this paper
- For questions in Section II, show relevant mathematical reasoning and/ or calculations

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Total Marks:

100

Section I – 10 marks (pages 3–6)

- Attempt Questions 1–10
- Allow about 15 minutes for this section

Section II – 90 marks (pages 9–34)

- Attempt Questions 11–36
- Allow about 2 hours and 45 minutes for this section

THIS IS A TRIAL PAPER ONLY

It does not necessarily reflect the format or the content of the 2020 HSC Examination Paper in this subject.

Question	M.C	11-18	19-25	26-32	33-36	
Total						

Section I

10 marks

Attempt Questions 1-10

Allow about 15 minutes for this section

Use the multiple-choice answer sheet for questions 1-10

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1 The following table list the values of a function $y = f(x)$ for 3 values of x.

x	1.2	1.4	1.6
y	3	3.8	4.8

By using the Trapezoidal rule and the table of values, the best estimation of $\int_{1.2}^{1.6} f(x)\,dx$ is:

A. 3.08

B. 1.54

C. 0.77

D. 7.7

2 The solution to $h^2 > 3h + 4$ is given by:

A. $-1 < h < 4$

B. $h > 4$

C. $h < -1$

D. $h > 4$ or $h < -1$

3 The correct expression for the integral $\int \sin\frac{x}{4}\,dx$ is :

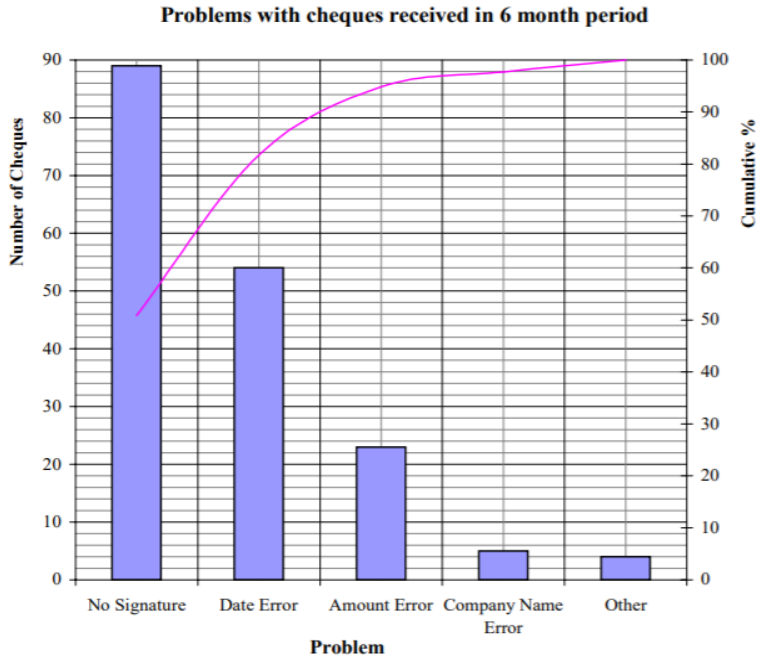
A. $-\cos\frac{x}{4} + c$

B. $4\cos\frac{x}{4} + c$

C. $-4\cos\frac{x}{4} + c$

D. $\frac{1}{4}\cos\frac{x}{4} + c$

4 The accounts department of a company records problems with cheques received from customers over a six-month period. The results are shown in the Pareto chart below.



Approximately what percentage of the problems are due to an Amount Error?

A. 23%
B. 5 %
C. 95%
D. 13%

5 The chance of a fisherman catching a fish of legal length is 3 in 5. If three fish are caught at random, what is the probability that exactly one is of legal length?

A. $\frac{36}{125}$

B. $\frac{12}{125}$

C. $\frac{7}{125}$

D. $\frac{9}{125}$

6 If $f(x) = b^x + b^{-x}$, then $\left(f(-x)\right)^2 =$

A. $f(2x) - 1$

B. $2 + f(2x)$

C. $f(2x) - 2$

D. $2 - f(2x)$

7 Let X and Y be independent events where $P(X) = 0.2$ and $P(Y) = 0.52$.
What is the value of $P(\overline{X} \cap \overline{Y})$?

A. 0.616
B. 0.72
C. 0.384
D. 0.28

8 If $y = -4\sin^3 3x$, then $\frac{dy}{dx} =$

A. $-12\cos^3 3x$
B. $-36\sin^2 3x \cdot \cos 3x$
C. $36\sin^3 3x \cdot \cos^2 3x$
D. $-12\sin^2 3x$

9 A set of data has a lower quartile of 10 and an upper quartile of 16. What is the maximum range for this set of data if there are no outliers?

A. 24
B. 22
C. 26
D. 20

10 The graph of $y=(x-2)^2(3+x)$ is dilated vertically by a factor of $\frac{1}{2}$ and horizontally by a factor of 3. It is then reflected across the y axis. The new equation is given by:

A. $y=-2(3x-2)^2(3+3x)$

B. $y=\frac{1}{2}(3x-2)^2(3+3x)$

C. $y=-\frac{1}{2}(\frac{x}{3}-2)^2(3+\frac{x}{3})$

D. $y=\frac{1}{2}(\frac{x}{3}+2)^2(3-\frac{x}{3})$

Mathematics Advanced

Section II

Answer Booklet

90 marks

Attempt Questions 11–36

Allow about 2 hours and 45 minutes for this section

Instructions Answer the questions in the spaces provided.

- Your responses should include relevant mathematical reasoning and/or calculations.
- Extra writing space is provided at the back of the question paper. If you use this space, clearly indicate which question you are answering.

Question 11 (2 marks)

Solve $|2x-7|=4$. 2

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Question 12 (3 marks)

On a golf course, the tee is a distance of 130 metres due west from the hole. On her first shot, Kelly hits the ball 100 metres but not at the correct angle. On her second shot she hits the ball 35 metres and gets it in the hole. On what bearing, a°, did she hit her first shot ? Give your answer correct to the nearest degree. 3

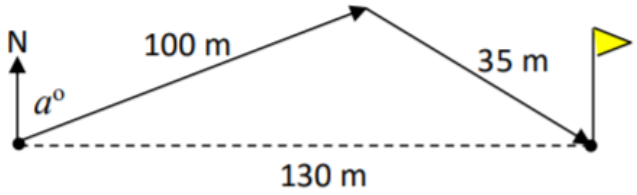

Diagram not to scale.

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Question 13 (3 marks)

The weight M, in kg, of an object on the moon is directly proportional to the weight E of the object on Earth. An astronaut who weighs 90 kg on Earth weighs 16.4 kg on the moon.

a) Find an equation for converting the weight of an object on Earth to its weight on the moon. 2

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b) An astronaut weighs 120 kg on Earth. How much will the astronaut weigh on the moon? 1

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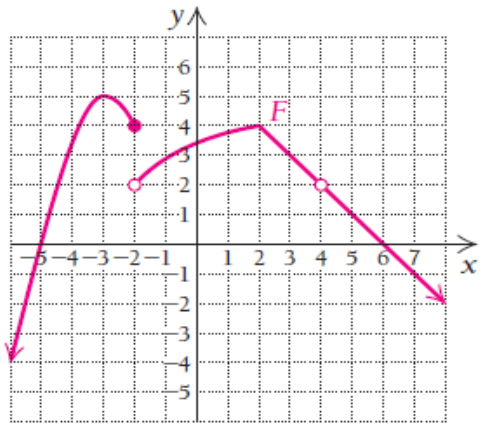
Question 14 (2 marks)

Consider the function shown below.



Write down the domain and range using interval notation. 2

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Question 15 (2 marks)

Given $\cos 2\theta = 0.5$ and $\frac{3\pi}{2} \le \theta \le 2\pi$, what is the value of $\sin\theta$? 2

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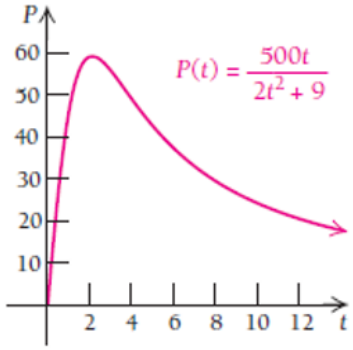
Question 16 (6 marks)

The population P, in thousands, of a small city is given by, $P(t) = \dfrac{500t}{2t^2 + 9}$, where t is the time in years.



a) Find the population after 10 months. 1

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b) Find the rate of change of the population with respect to time. 2

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c) Find the rate of change in the population at $t = 12$ years. 1

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d) Find the average rate of change in the population from $t = 10$ months to $t = 4$ years. 2

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Question 17 (2 marks)

Find the $x-$values for the points on the graph of $f(x) = -x^3 + 6x^2$ at which the normal has slope $\frac{-1}{9}$. 2

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Question 18 (1 marks)

Draw a graph that is continuous, but not differentiable at $x = 1$. 1

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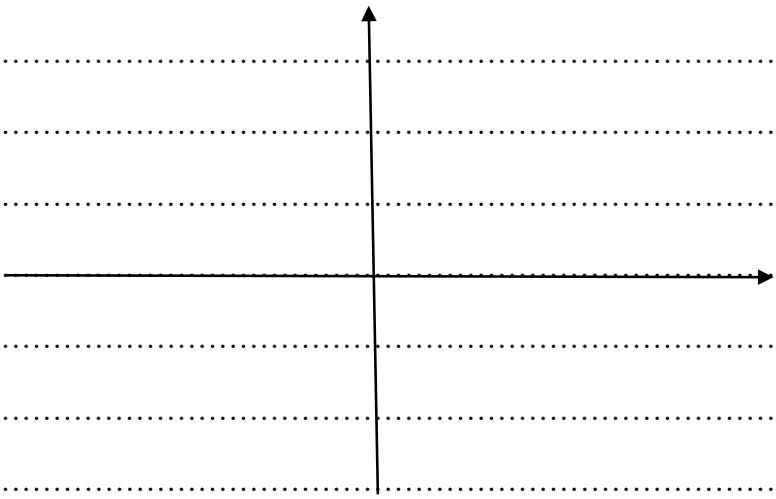

Question 19 (2 marks)

The data in the following table relates study time and test scores.

Study time in hours	Test grade in percent
7	83
8	85
9	88
10	91
11	93
12	95

Find a regression line for the data above, using accuracy to two decimal places. 2

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Question 20 (2 marks)

Find $\int 5x^2(x^3+1)^{10}\,dx$. 2

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Question 21 (4 marks)

Differentiate the following:

a) $f(x) = e^{-2x}(1+x^2)$ 2

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b) $f(x) = \ln\left(\frac{x^3+4}{6-x}\right)$ 2

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Question 22 (2 marks)

Show that $f(x) = -x^3 - 5x + 1$ is always decreasing. 2

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Question 23 (3 marks)

Find the area between the curve $y = x^2 - 3x$ and the $x-$axis over the interval $1 \le x \le 5$. 3

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Question 24 (4 marks)

The world population was approximately 6.04 billion at the beginning of 2000. It has been estimated that the population is growing exponentially at the rate of 0.016 per year.
The world population P is modelled by the equation $P = P_0 e^{kt}$, where k and P_0 are constants and t is the amount of time (in years) since the beginning of 2000.

a) Estimate the world population at the end of 2030. 2

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b) After how many years will the population be double the population in 2000? 2

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Question 25 (3 marks)

a) On the same axes sketch the graph of $y = |x|$ and $y = 1 - x^2$. 2

b) Shade in the region where both $y \geq |x|$ and $y \leq 1 - x^2$. 1

Question 26 (7 marks)

Consider the function $f(x) = 2x^3 - x^4$.

a) Find the coordinates of any stationary points on the curve $y = f(x)$. 2

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b) Determine their nature. 2

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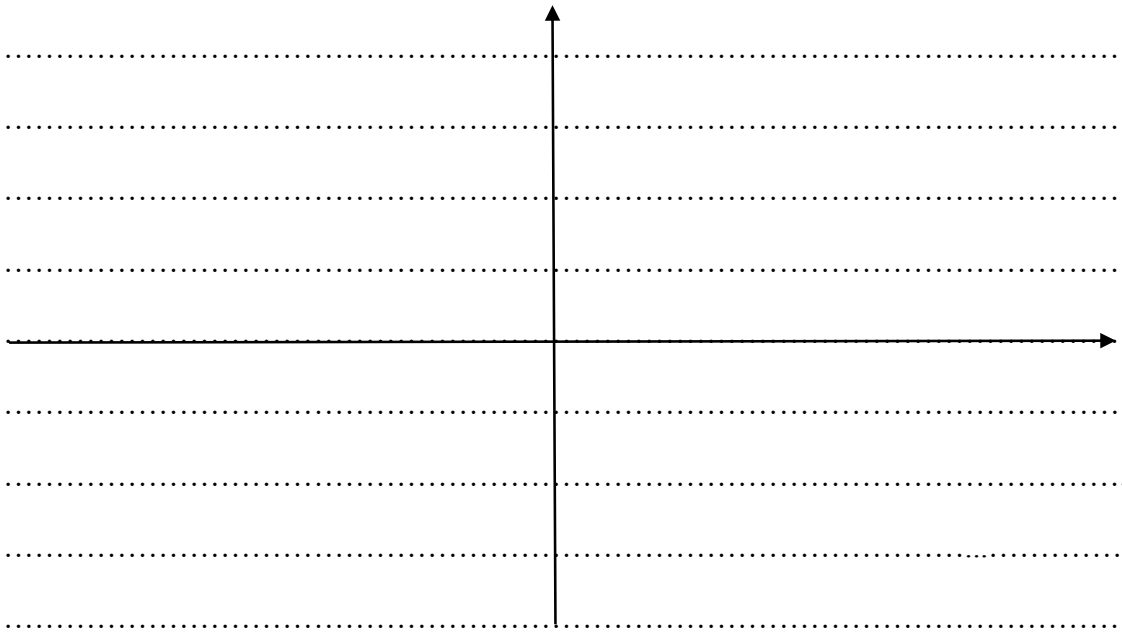
c) Find any points of inflection on the curve $y = f(x)$. 1

d) Sketch the function $y = f(x)$ for $x \in [-2,2]$. 2



Question 27 (4 marks)

Use the standard normal distribution table on the opposite page, or otherwise, to answer the questions below.

a) Find $P(-1.93 \le z \le 0.79)$. 2

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b) A large class takes an exam, and the students' scores are normally distributed. The mean score is $\mu = 72$ and the standard deviation is $\sigma = 4.5$. Students who score in the top 10% receive an A grade. What is the minimum score needed to get an A grade? 2

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Table : The standard normal distribution

The table below provides some values of the probabilities for the standard normal distribution.

$$\text{i.e. } \Phi(z) = P(Z \leq z) = \int_{-\infty}^{z} \phi(t)dt$$

z	+0.00	+0.01	+0.02	+0.03	+0.04	+0.05	+0.06	+0.07	+0.08	+0.09
0.0	0.50000	0.50399	0.50798	0.51197	0.51595	0.51994	0.52392	0.52790	0.53188	0.53586
0.1	0.53983	0.54380	0.54776	0.55172	0.55567	0.55962	0.56360	0.56749	0.57142	0.57535
0.2	0.57926	0.58317	0.58706	0.59095	0.59483	0.59871	0.60257	0.60642	0.61026	0.61409
0.3	0.61791	0.62172	0.62552	0.62930	0.63307	0.63683	0.64058	0.64431	0.64803	0.65173
0.4	0.65542	0.65910	0.66276	0.66640	0.67003	0.67364	0.67724	0.68082	0.68439	0.68793
0.5	0.69146	0.69497	0.69847	0.70194	0.70540	0.70884	0.71226	0.71566	0.71904	0.72240
0.6	0.72575	0.72907	0.73237	0.73565	0.73891	0.74215	0.74537	0.74857	0.75175	0.75490
0.7	0.75804	0.76115	0.76424	0.76730	0.77035	0.77337	0.77637	0.77935	0.78230	0.78524
0.8	0.78814	0.79103	0.79389	0.79673	0.79955	0.80234	0.80511	0.80785	0.81057	0.81327
0.9	0.81594	0.81859	0.82121	0.82381	0.82639	0.82894	0.83147	0.83398	0.83646	0.83891
1.0	0.84134	0.84375	0.84614	0.84849	0.85083	0.85314	0.85543	0.85769	0.85993	0.86214
1.1	0.86433	0.86650	0.86864	0.87076	0.87286	0.87493	0.87698	0.87900	0.88100	0.88298
1.2	0.88493	0.88686	0.88877	0.89065	0.89251	0.89435	0.89617	0.89796	0.89973	0.90147
1.3	0.90320	0.90490	0.90658	0.90824	0.90988	0.91149	0.91308	0.91466	0.91621	0.91774
1.4	0.91924	0.92073	0.92220	0.92364	0.92507	0.92647	0.92785	0.92922	0.93056	0.93189
1.5	0.93319	0.93448	0.93574	0.93699	0.93822	0.93943	0.94062	0.94179	0.94295	0.94408
1.6	0.94520	0.94630	0.94738	0.94845	0.94950	0.95053	0.95154	0.95254	0.95352	0.95449
1.7	0.95543	0.95637	0.95728	0.95818	0.95907	0.95994	0.96080	0.96164	0.96246	0.96327
1.8	0.96407	0.96485	0.96562	0.96638	0.96712	0.96784	0.96856	0.96926	0.96995	0.97062
1.9	0.97128	0.97193	0.97257	0.97320	0.97381	0.97441	0.97500	0.97558	0.97615	0.97670
2.0	0.97725	0.97778	0.97831	0.97882	0.97932	0.97982	0.98030	0.98077	0.98124	0.98169
2.1	0.98214	0.98257	0.98300	0.98341	0.98382	0.98422	0.98461	0.98500	0.98537	0.98574
2.2	0.98610	0.98645	0.98679	0.98713	0.98745	0.98778	0.98809	0.98840	0.98870	0.98899
2.3	0.98928	0.98956	0.98983	0.99010	0.99036	0.99061	0.99086	0.99111	0.99134	0.99158
2.4	0.99180	0.99202	0.99224	0.99245	0.99266	0.99286	0.99305	0.99324	0.99343	0.99361
2.5	0.99379	0.99396	0.99413	0.99430	0.99446	0.99461	0.99477	0.99492	0.99506	0.99520
2.6	0.99534	0.99547	0.99560	0.99573	0.99585	0.99598	0.99609	0.99621	0.99632	0.99643
2.7	0.99653	0.99664	0.99674	0.99683	0.99693	0.99702	0.99711	0.99720	0.99728	0.99736
2.8	0.99744	0.99752	0.99760	0.99767	0.99774	0.99781	0.99788	0.99795	0.99801	0.99807
2.9	0.99813	0.99819	0.99825	0.99831	0.99836	0.99841	0.99846	0.99851	0.99856	0.99861
3.0	0.99865	0.99869	0.99874	0.99878	0.99882	0.99886	0.99889	0.99893	0.99896	0.99900
3.1	0.99903	0.99906	0.99910	0.99913	0.99916	0.99918	0.99921	0.99924	0.99926	0.99929
3.2	0.99931	0.99934	0.99936	0.99938	0.99940	0.99942	0.99944	0.99946	0.99948	0.99950
3.3	0.99952	0.99953	0.99955	0.99957	0.99958	0.99960	0.99961	0.99962	0.99964	0.99965
3.4	0.99966	0.99968	0.99969	0.99970	0.99971	0.99972	0.99973	0.99974	0.99975	0.99976
3.5	0.99977	0.99978	0.99978	0.99979	0.99980	0.99981	0.99981	0.99982	0.99983	0.99983

Question 28 (5 marks)

Consider the functions $f(x) = \sqrt[3]{3-2x}$ and $g(x) = 1+3x^2$.

a) Find $f \circ g(x)$ 2

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b) Evaluate $g(f'(1))$ 3

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Question 29 (2 marks)

If $a = \log_{\frac{1}{x}} \frac{1}{N}$ such that $x > 0$ and $N > 0$, show that $a = \log_x N$. 2

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Question 30 (5 marks)

The probability density function $f(x) = \frac{1}{3}(x^2 + 1)$ is defined over the interval $[0,1]$.

Find the value of :

a) $E(X)$. 2

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b) the variance. 2

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c) the standard deviation, correct to 2 decimal places. 1

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Question 31 (5 marks)

Consider the function $y = \frac{8}{x^2 - 4}$.

a) Write down the domain. 1

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b) Find the horizontal and vertical asymptotes. 2

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c) Sketch the function, showing the main features. 2

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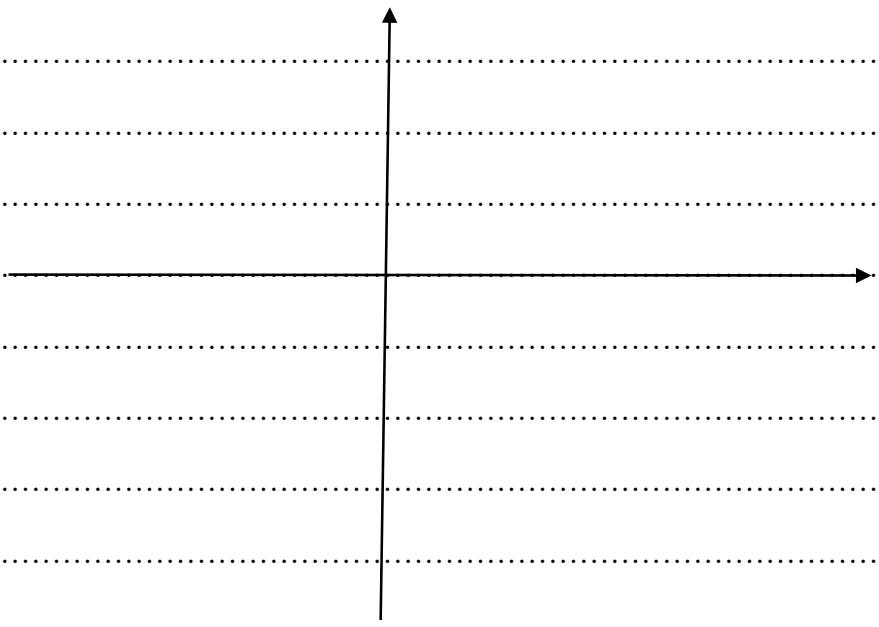

Question 32 (7 marks)

The velocity of a particle travelling along the x axis is given by $v = 2 - \frac{10}{t+2}\ ms^{-1}$, where t is the time in seconds .

a) When is the particle stationary? 1

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b) What happens to the velocity as $t \to \infty$? 1

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c) Find the acceleration when the particle is stationary. 2

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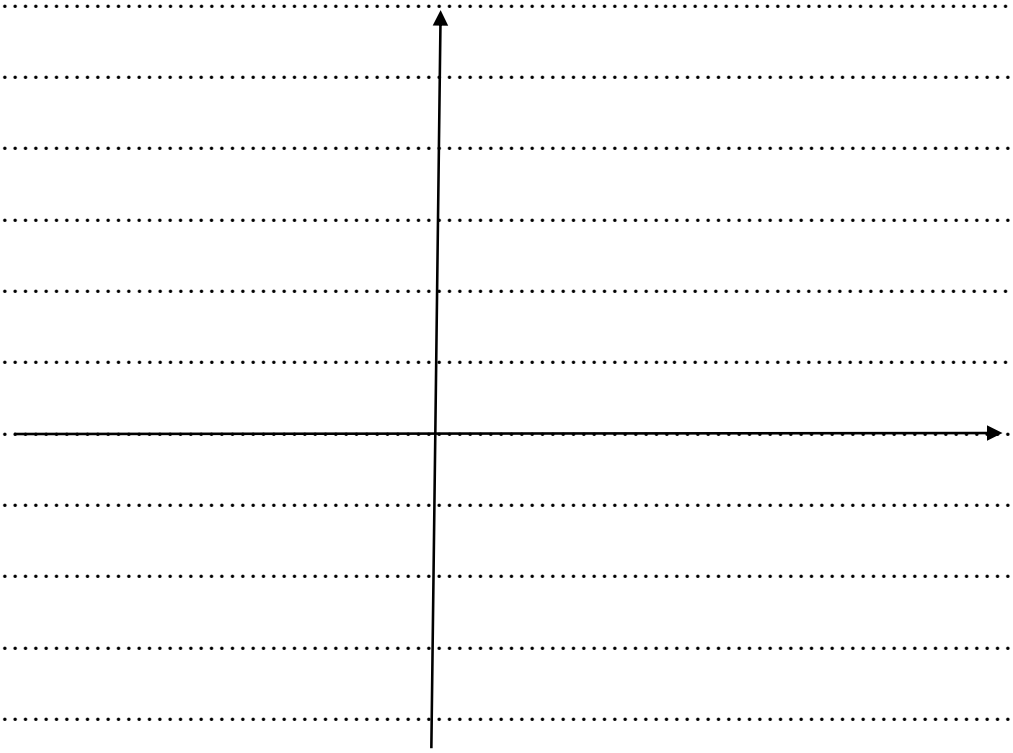
d) Find the distance travelled in the first 5 seconds, in exact form. 3

Question 33 (2 marks)

Evaluate $\int_0^7 f(x)\,dx$, where $f(x) = 3 - |x - 2|$. 3

Question 34 (4 marks)

a) Sketch $f(x) = -3\cos\left(2x + \frac{\pi}{3}\right) + 2$ for $-\pi \leq x \leq \pi$. 3



b) Hence, state the number of solutions to the equation $-3\cos\left(2x + \frac{\pi}{3}\right) + 2 = 0.5x + 1$ for $-\pi \leq x \leq \pi$. 1

Question 35 (6 marks)

A fence, 8 m high, is parallel to the wall of a building and 1 m from the building.

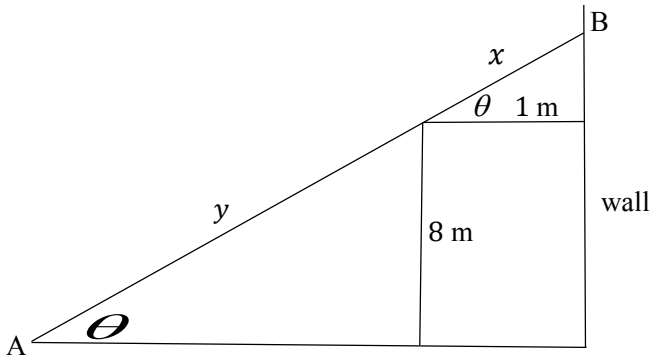


Diagram not to scale.

a) Write an expression for the length of AB in terms of θ. 2

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b) Find the shortest length of the plank AB that can go over the fence, from the level ground, in order to rest against the wall. 3

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Question 36 (2 marks)

Neville lives in a town where one third of the days are rainy days. If it is a rainy day, the chance of heavy traffic is $\frac{1}{2}$. If it is not a rainy day, the chance of heavy traffic is $\frac{1}{4}$. If it is a rainy day and there is heavy traffic, there is a 50% chance that Neville will arrive late to work. On the other hand , the probability of being late is reduced to $\frac{1}{8}$ if it is not a rainy day and there is no heavy traffic. In other situations, the probability of Neville being late to work is 0.35. On a random day, Neville arrives late to work. What is the probability that it is a rainy day? 2

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The End

Sydney Girls High School

Mathematics Faculty

Multiple Choice Answer Sheet
Trial HSC Mathematics

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample $2 + 4 = ?$ **(A)** 2 **(B)** 6 **(C)** 8 **(D)** 9

A ○ B ● C ○ D ○

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ● B ⊗ C ○ D ○

If you change your mind and have crossed out what you consider to be the correct answer, then indicate this by writing the word *correct* and drawing an arrow as follows:

correct

A ⊗ B ⊗ C ○ D ○

Student Number: Advanced.

Completely fill the response oval representing the most correct answer.

1. A ○ B● C○ D○
2. A ○ B○ C○ D●
3. A ○ B○ C● D○
4. A ○ B○ C○ D●
5. A ● B○ C○ D○
6. A ○ B● C○ D○
7. A ○ B○ C● D○
8. A ○ B● C○ D○
9. A ● B○ C○ D○
10. A ○ B○ C○ D●

Question 11 (2 marks)

Solve $|2x-7|=4$. 2

$2x-7=4$ or $2x-7=-4$

$2x=11$ $2x=3$

$x=\frac{11}{2}$ $x=3/2$

$x=5\frac{1}{2}$

Question 12 (3 marks)

On a golf course, the tee is a distance of 130 metres due west from the hole. On her first shot, Kelly hits the ball 100 metres but not at the correct angle. On her second shot she hits the ball 35 metres and gets it in the hole. On what bearing, a°, did she hit her first shot ? Give your answer correct to the nearest degree. 3

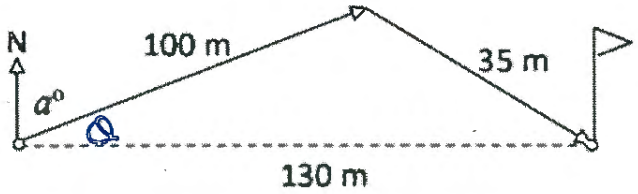


Diagram not to scale.

$\cos\theta = \frac{100^2+130^2-35^2}{2\times100\times130}$

$\theta \doteq 9^\circ 4'$

$\therefore a = 90^\circ - 9^\circ 4'$

$\doteq 81^\circ$ (nearest degree)

Question 13 (3 marks)

The weight M, in kg, of an object on the moon is directly proportional to the weight E of the object on Earth. An astronaut who weighs 90 kg on Earth weighs 16.4 kg on the moon.

a) Find an equation for converting the weight of an object on Earth to its weight on the moon. 2

$M \propto kE$

$M = kE$

$16.4 = k \times 90$

$k = \frac{16.4}{90}$

$k = \frac{41}{225}$

$\therefore M = \frac{41}{225} E$

b) An astronaut weighs 120 kg on Earth. How much will the astronaut weigh on the moon? 1

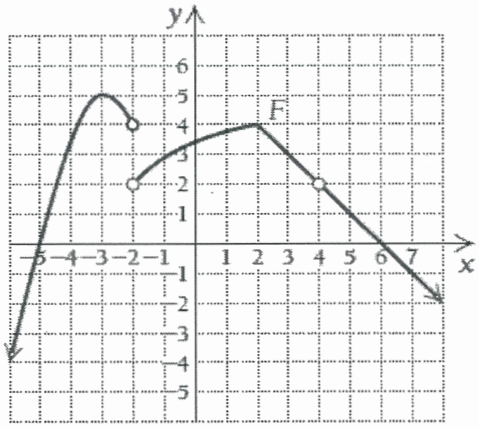
$M = \frac{41}{225} \times 120$

$= 21.87 kg \ (2 dp)$

∴ The astronaut will weigh 21.87kg on the moon.

Question 14 (2 marks)

Consider the function shown below.



Write down the domain and range using interval notation. 2

Domain $(-\infty, 4) \cup (4, +\infty)$

Range $(-\infty, 5]$

Note these were the only acceptable answers. You needed to use full interval notation. 1 mark for each correct answer.

Question 15 (2 marks)

Given $\cos 2\theta = 0.5$ and $\frac{3\pi}{2} \le \theta \le 2\pi$, what is the value of $\sin\theta$? 2

$\cos 2\theta = \frac{1}{2}$

$2\theta = \frac{\pi}{3}, \frac{5\pi}{3}, \frac{7\pi}{3}, \frac{11\pi}{3}$

$\theta = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

$\theta = \frac{11\pi}{6}$, $\sin\theta$ $\frac{3\pi}{2} \le \theta \le 2\pi$

$\sin\frac{11\pi}{6} = -\frac{1}{2}$

OR

$\cos 2\theta = 0.5$

$1 - 2\sin^2\theta = \frac{1}{2}$

$2\sin^2\theta = \frac{1}{2}$

$\sin^2\theta = \frac{1}{4}$

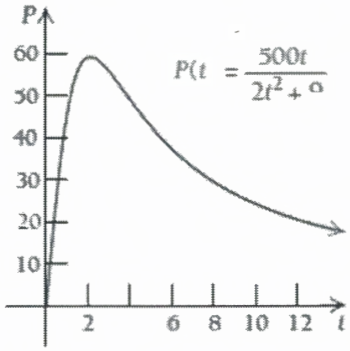
$\sin\theta = \pm\frac{1}{2}$

$\sin\theta = -\frac{1}{2}$

for the above domain.

Question 16 (6 marks)

The population P, in thousands, of a small city is given by, $P(t) = \dfrac{500t}{2t^2+9}$, where t is the time in years.



a) Find the population after 10 months. 1

Let $t = \frac{10}{12} = \frac{5}{6}$

$P(10 \text{ months}) = \dfrac{500 \times \frac{5}{6}}{2(5/6)^2 + 9}$

$\doteq 40.106$ (thousands)

$\doteq 40.12$ (thousands)

* Be accurate with your approximations.

b) Find the rate of change of the population with respect to time. 2

$P'(t) = \dfrac{(2t^2+9) \times 500 - (500t \times 4t)}{(2t^2+9)^2}$

$= \dfrac{1000t^2 + 4500 - 2000t^2}{(2t^2+9)^2}$

$= \dfrac{4500 - 1000t^2}{(2t^2+9)^2}$

c) Find the rate of change in the population at $t = 12$ years. 1

$$P'(12) = \frac{4500 - 1000(12)^2}{(2 \times 12^2 + 9)^2}$$

$$\doteq -1.58$$

d) Find the average rate of change in the population from $t = 10$ months to $t = 4$ years. 2

$$\text{Av rate change} = \frac{P(4) - P\left(\frac{5}{6}\right)}{4 - \frac{5}{6}}$$

$$\doteq \frac{48.78 - 40.12}{\frac{19}{6}}$$

$$\doteq 2.73 \text{ thousand.}$$

A lot of students had trouble with this question. $P(t)$ needed to be used rather than $P'(t)$. The denominator was the difference between the time periods.

Question 17 (2 marks)

Find the $x-$values for the points on the graph of $f(x)=-x^3+6x^2$ at which the normal has slope $\frac{-1}{9}$. 2

$f'(x) = -3x^2 + 12x$ (marks were not awarded for differentiation)

$m_N = -\frac{1}{9} \quad \therefore m_T = 9$ (1 mark for recognizing this)

Let $f'(x) = 9$

$-3x^2 + 12x = 9$

$3x^2 - 12x + 9 = 0$

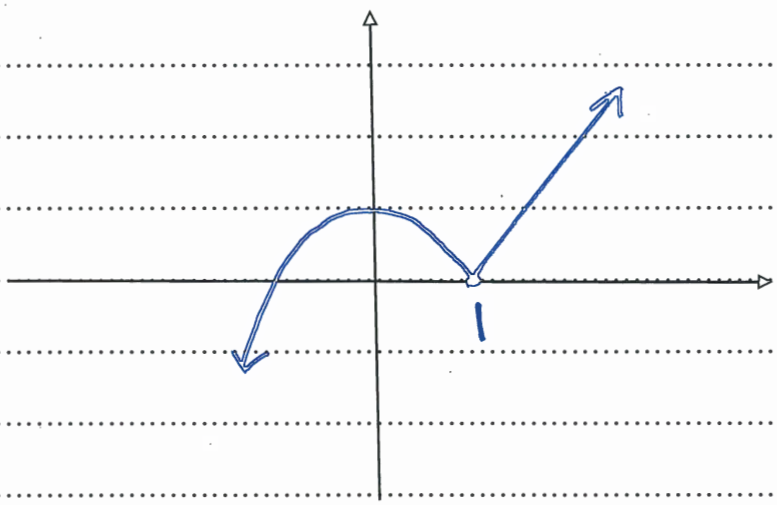
$x^2 - 4x + 3 = 0$

$(x-3)(x-1) = 0$ (1 mark for solving)

$\therefore x = 1, \; x = 3$ are the x-values for the points where the normal has a slope of $\frac{-1}{9}$

Question 18 (1 marks)

Draw a graph that is continuous, but not differentiable at $x=1$. 1



A large number of students drew graphs that were not continuous.
An infinite gradient at $x=1$ was accepted if continuous.

Question 19 (2 marks)

The data in the following table relates study time and test scores.

Study time in hours	Test grade in percent
7	83
8	85
9	88
10	91
11	93
12	95

Find a regression line for the data above, using accuracy to two decimal places. 2

$y = bx + a$

$a \doteq 65.55$ (2 d.p.)

$b \doteq 2.49$ (2 d.p.)

Regression line:

$\therefore \quad y = 2.49x + 65.55$

1 mark awarded if a and b were correct but incorrectly placed into the formula.

Question 20 (2 marks)

Find $\int 5x^2(x^3+1)^{10}\,dx$. 2

$\frac{d}{dx}(x^3+1)^{11} = 11(x^3+1)^{10} \times 3x^2$

$= 33x^2(x^3+1)^{10}$

$\therefore \int 5x^2(x^3+1)^{10}\,dx = \frac{5}{33}(x^3+1)^{11} + c$

Question 21 (4 marks)

Differentiate the following:

a) $f(x)=e^{-2x}(1+x^2)$ 2

$f'(x) = e^{-2x} \times 2x + (1+x^2) \times -2e^{-2x}$

$= 2e^{-2x}(x-(1+x^2))$

$= 2e^{-2x}(x-1-x^2)$

b) $f(x)=\ln\left(\frac{x^3+4}{6-x}\right)$ 2

$f(x) = \ln(x^3+4) - \ln(6-x)$

$f'(x) = \frac{3x^2}{x^3+4} - \frac{-1}{6-x}$

$= \frac{3x^2}{x^3+4} + \frac{1}{6-x}$

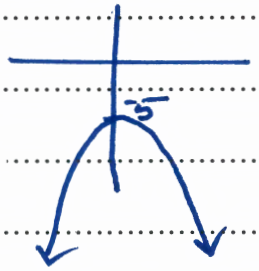
OR

$= \frac{2(9x^2-x^3+2)}{(x^3+4)(6-x)}$ if differentiated the long way.

Question 22 (2 marks)

Show that $f(x) = -x^3 - 5x + 1$ is always decreasing. 2

$f'(x) = -3x^2 - 5$



$f'(x) < 0$ for all values of x

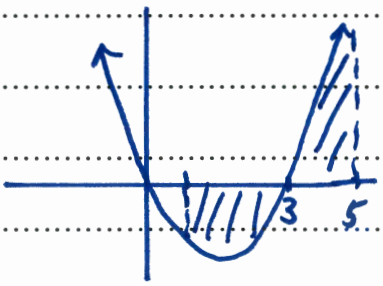
$\therefore f(x)$ is always decreasing.

Most students received 1 mark for correct differentiation. To receive the 2nd mark you needed to explain clearly why the $f(x)$ was always decreasing. A sketch is very helpful here.

Question 23 (3 marks)

Find the area between the curve $y = x^2 - 3x$ and the $x-$axis over the interval $1 \le x \le 5$. 3

A sketch is helpful to recognize you need to do the two regions seperately.



$$\text{Area} = \left| \int_1^3 (x^2 - 3x)\,dx \right| + \int_3^5 (x^2 - 3x)\,dx$$

$$= \left| \left[\frac{x^3}{3} - \frac{3x^2}{2} \right]_1^3 \right| + \left[\frac{x^3}{3} - \frac{3x^2}{2} \right]_3^5$$

$$= \left| \left(9 - \frac{27}{2}\right) - \left(\frac{1}{3} - \frac{3}{2}\right) \right| + \left(\frac{125}{3} - \frac{75}{2} - \left(9 - \frac{27}{2}\right) \right)$$

$$= \left| -\frac{10}{3} \right| + \frac{26}{3}$$

$$= \frac{36}{3}$$

$$= 12 \text{ units}^2$$

Roughly
- 1 mark for separation
- 1 mark for correct integration
- 1 mark for correct arithmetic

Question 24 (4 marks)

The world population was approximately 6.04 billion at the beginning of 2000. It has been estimated that the population is growing exponentially at the rate of 0.016 per year.
The world population P is modelled by the equation $P = P_0 e^{kt}$, where k and P_0 are constants and t is the amount of time (in years) since the beginning of 2000.

a) Estimate the world population at the end of 2030. 2

$P = 6.04e^{0.016t}$

let $t = 31$ (since end of 2030)

$P_{31} = 6.04e^{0.016 \times 31}$

$\doteq 9.9185$ billion

$\doteq 9.919$ billion (3 dp)

1 mark for recognising $t=31$, if not but used formula correctly 1 mark was awarded.

b) After how many years will the population be double the population in 2000? 2

$2P_0 = P_0 e^{0.016t}$

$2 = e^{0.016t}$

$\ln 2 = \ln e^{0.016t}$

$0.016t = \ln 2$

$t = \frac{\ln 2}{0.016}$

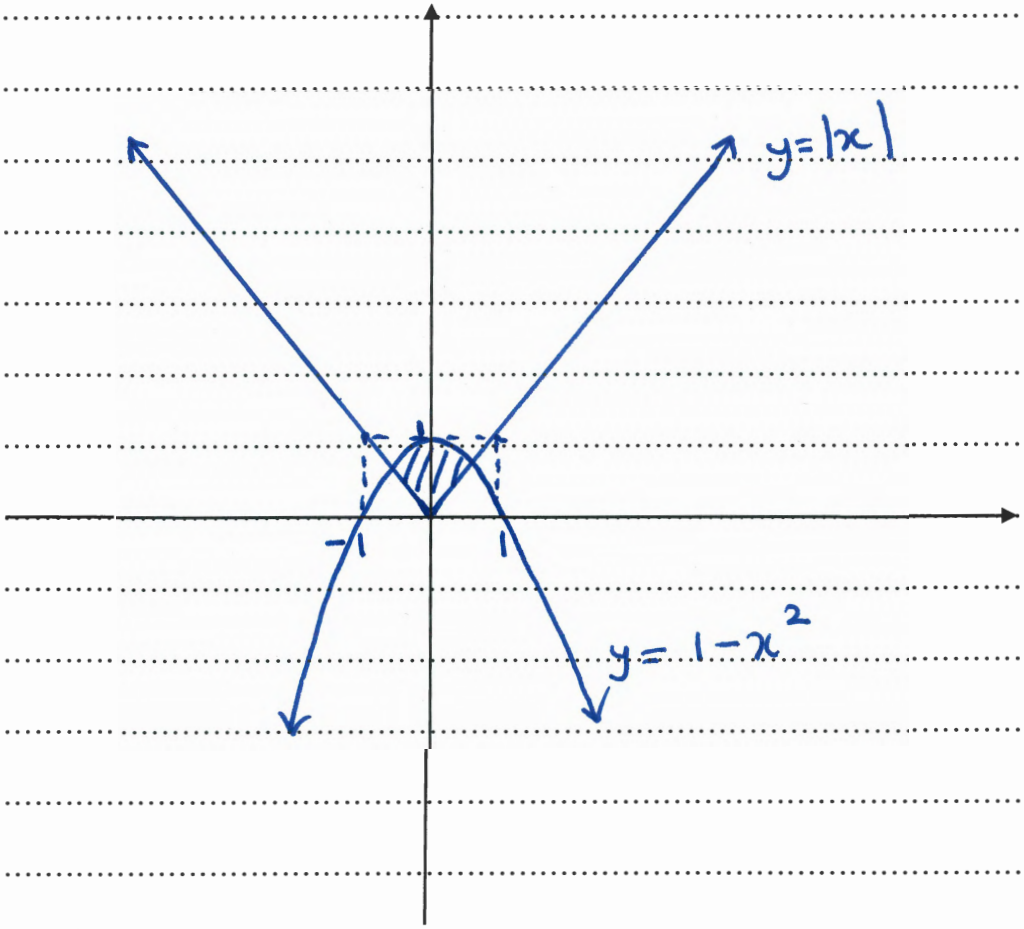
$t \doteq 43.32$ yrs.

If you devised your own k in (a) but used it correctly in (b) two marks were awarded. ie c.f.p.r. (correct from previous response)

Question 25 (3 marks)

a) On the same axes sketch the graph of $y = |x|$ and $y = 1 - x^2$. 2

b) Shade in the region where both $y \geq |x|$ and $y \leq 1 - x^2$. 1



Note: intercepts are required to be labelled.

Question 26 (7 marks)

Consider the function $f(x) = 2x^3 - x^4$.

a) Find the coordinates of any stationary points on the curve $y = f(x)$. 2

$f'(x) = 6x^2 - 4x^3 = 0$

$2x^2(3 - 2x) = 0$

$x = 0$ or $x = \frac{3}{2}$ ✓

$y = 0$ $y = \frac{27}{16}$

∴ Stationary points are $(0,0)$ and $\left(\frac{3}{2}, \frac{27}{16}\right)$ ✓

b) Determine their nature. 2

x	-1	0	1	$\frac{3}{2}$	2
$f'(x)$	10	0	2	0	-8
	/	—	/	—	\

✓

∴ $(0,0)$ is a HPOI ✓

$\left(\frac{3}{2}, \frac{27}{16}\right)$ is a maximum turning point

c) Find any points of inflection on the curve $y = f(x)$. 1

$f''(x) = 12x - 12x^2 = 0$

$12x(1-x) = 0$

$x = 0$ or $x = 1$

x	$\frac{1}{2}$	1	2
$f''(x)$	3	0	-24

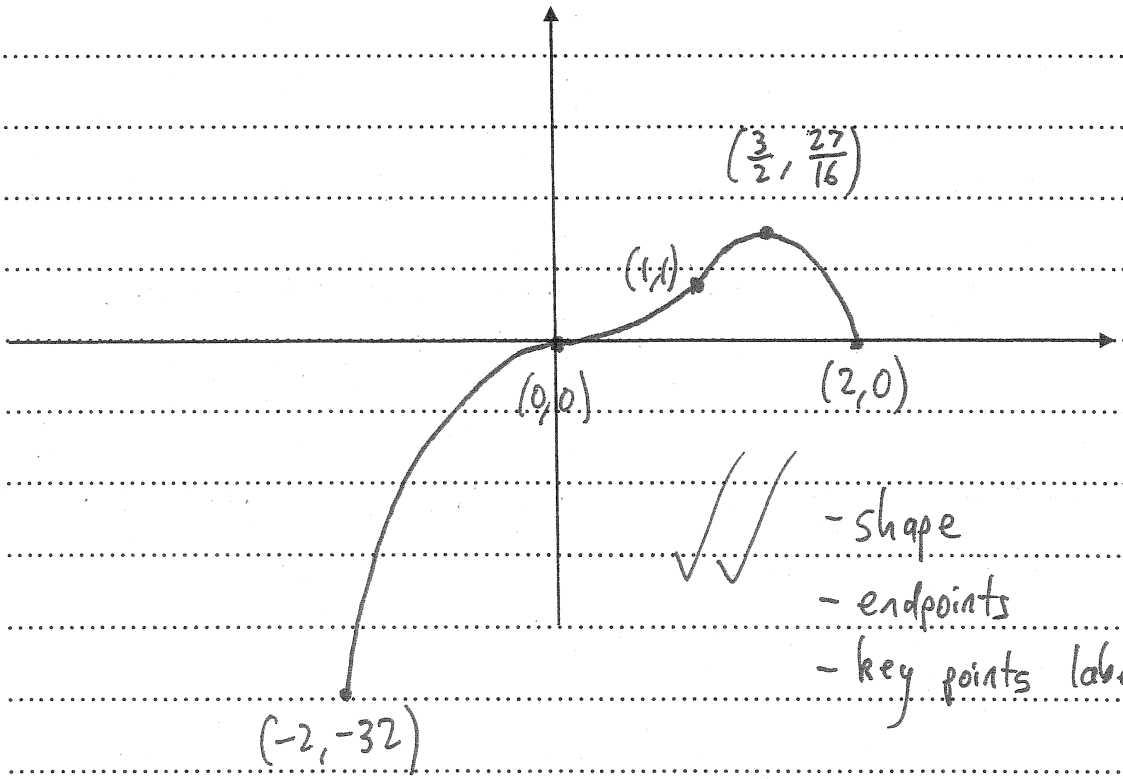
$\therefore$ concavity changes ✓

When $x = 1$, $y = 1$

$\therefore$ $(1,1)$ is a POI

& $(0,0)$ is a HPOI (from (b))

d) Sketch the function $y = f(x)$ for $x \in [-2,2]$. 2



✓✓ - shape
- endpoints
- key points labelled

Question 27 (4 marks)

Use the standard normal distribution table on the opposite page, or otherwise, to answer the questions below.

a) Find $P(-1.93 \leq z \leq 0.79)$. 2

$P(-1.93 \leq z \leq 0.79) = P(z \leq 0.79) - P(z \leq -1.93)$

$= P(z \leq 0.79) - P(z \geq 1.93)$

$= P(z \leq 0.79) - (1 - P(z \leq 1.93))$ ✓

$= 0.78524 - (1 - 0.97320)$

$= 0.75844$ ✓

b) A large class takes an exam, and the students' scores are normally distributed. The mean score is $\mu = 72$ and the standard deviation is $\sigma = 4.5$. Students who score in the top 10% receive an A grade. What is the minimum score needed to get an A grade? 2

An A-grade is greater than 90% of scores.

From the table, $P(z \leq 1.28) \doteq 0.9$ ✓

∴ Minimum score to receive an A-grade has a z-score of 1.28:

$z = \frac{x - \mu}{\sigma}$

$1.28 = \frac{x - 72}{4.5}$

$x = 72 + 4.5 \times 1.28$

$= 77.76$ ✓

Question 28 (5 marks)

Consider the functions $f(x)=\sqrt[3]{3-2x}$ and $g(x)=1+3x^2$.

a) Find $f\circ g(x)$ 2

$f\circ g(x) = f(g(x))$ ✓

$= f(1+3x^2)$

$= \sqrt[3]{3-2(1+3x^2)}$ ✓

$= \sqrt[3]{1-6x^2}$

b) Evaluate $g(f'(1))$ 3

$f(x) = (3-2x)^{\frac{1}{3}}$

$f'(x) = \frac{1}{3}(3-2x)^{-\frac{2}{3}} \times -2$

$= -\frac{2}{3}(3-2x)^{-\frac{2}{3}}$ ✓

$f'(1) = -\frac{2}{3}(3-2)^{-\frac{2}{3}}$

$= -\frac{2}{3}$ ✓

$g(f'(1)) = g(-\frac{2}{3})$

$= 1+3(-\frac{2}{3})^2$ ✓

$= 1+\frac{4}{3}$

$= \frac{7}{3}$

Question 29 (3 marks)

If $a = \log_{\frac{1}{x}} \frac{1}{N}$ such that $x > 0$ and $N > 0$, show that $a = \log_x N$. 2

Method 1:

$a = \log_{\frac{1}{x}} \frac{1}{N}$

$\left(\frac{1}{x}\right)^a = \frac{1}{N}$ (definition of log) ✓

$\frac{1}{x^a} = \frac{1}{N}$

$x^a = N$ ✓

$a = \log_x N$ (definition of log)

Method 2: $a = \log_{\frac{1}{x}} \frac{1}{N}$

$= \frac{\log_x \frac{1}{N}}{\log_x \frac{1}{x}}$ (change of base) ✓

$= \frac{\log_x (N^{-1})}{\log_x (x^{-1})}$

$= \frac{-\log_x N}{-\log_x x}$ (since $\log_a (b^c) = c \log_a b$)

$= \frac{\log_x N}{1}$ ✓ (since $\log_x x = 1$)

$= \log_x N$

Question 30 (5 marks)

The probability density function $f(x)=\frac{1}{3}(x^2+1)$ is defined over the interval $[0,1]$.

Find the value of :

a) $E(X)$. 2

$E(X) = \int_0^1 x\, f(x)\, dx$ ✓

$= \frac{1}{3}\int_0^1 x\left(x^2+1\right) dx$

$= \frac{1}{3}\int_0^1 \left(x^3+x\right) dx$

$= \frac{1}{3}\left[\frac{x^4}{4}+\frac{x^2}{2}\right]_0^1$

$= \frac{1}{3}\left[\frac{1}{4}+\frac{1}{2}\right]$ ✓

$= \frac{1}{4}$

b) the variance. 2

$Var(X) = \int_0^1 x^2 f(x)\, dx - \left(E(X)\right)^2$ ✓

$= \frac{1}{3}\int_0^1 \left(x^4+x^2\right) dx - \left(\frac{1}{4}\right)^2$

$= \frac{1}{3}\left[\frac{x^5}{5}+\frac{x^3}{3}\right]_0^1 - \frac{1}{16}$

$= \frac{1}{3}\left[\frac{1}{5}+\frac{1}{3}\right] - \frac{1}{16}$ ✓

$= \frac{83}{720}$

c) the standard deviation, correct to 2 decimal places. 1

$\sigma = \sqrt{\frac{83}{720}} \doteq 0.34 \ (2 \text{ d.p.})$ ✓

Question 31 (5 marks)

Consider the function $y = \frac{8}{x^2 - 4}$.

a) Write down the domain. 1

Domain: all real x, $x \neq \pm 2$ ✓

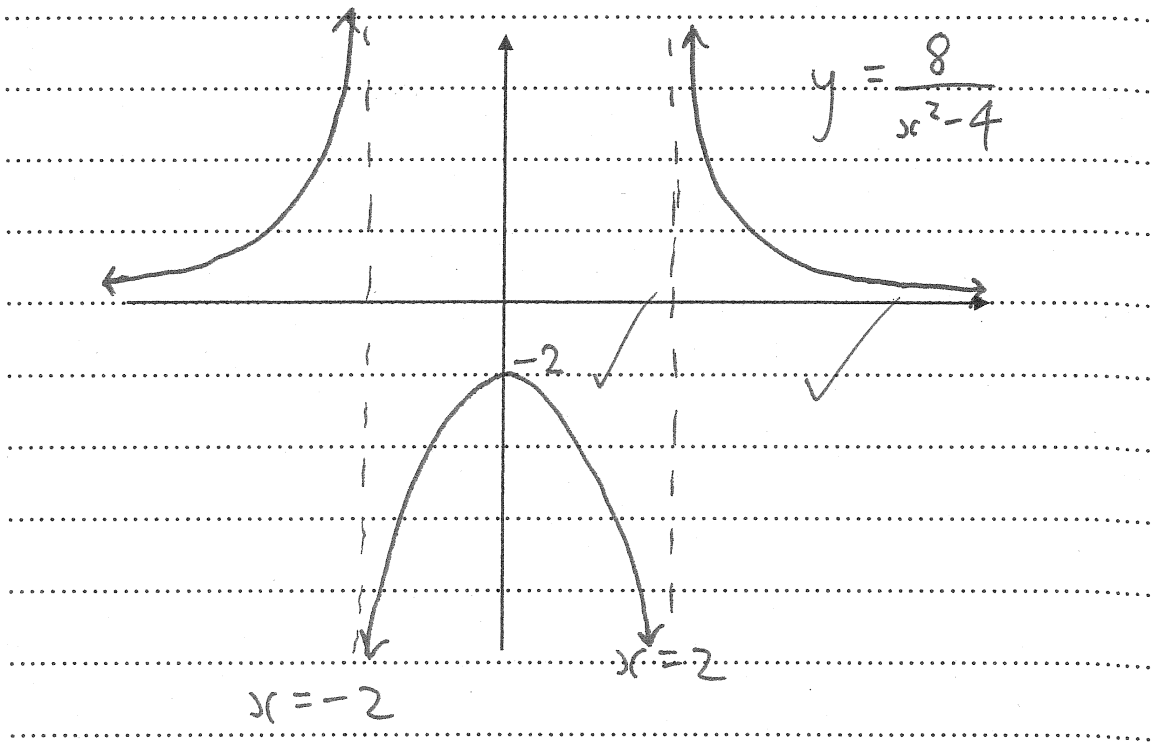
b) Find the horizontal and vertical asymptotes. 2

Vertical asymptotes: $x = \pm 2$ ✓

Horizontal asymptotes: $\lim_{x \to \pm\infty} \frac{8}{x^2 - 4} = 0$

$\therefore y = 0$ ✓

c) Sketch the function, showing the main features. 2



Question 32 (7 marks)

The velocity of a particle travelling along the x axis is given by $v = 2 - \frac{10}{t+2}\ ms^{-1}$, where t is the time in seconds .

a) When is the particle stationary? 1

$v = 0$

$2 - \frac{10}{t+2} = 0$

$\frac{10}{t+2} = 2$

$10 = 2t + 4$

$2t = 6 \Rightarrow t = 3$ ✓

b) What happens to the velocity as $t \to \infty$? 1

As $t \to \infty$, $v \to 2 - 0$

$= 2$ ✓

c) Find the acceleration when the particle is stationary. 2

$a = \frac{dv}{dt}$ ✓ $\qquad v = 2 - 10(t+2)^{-1}$

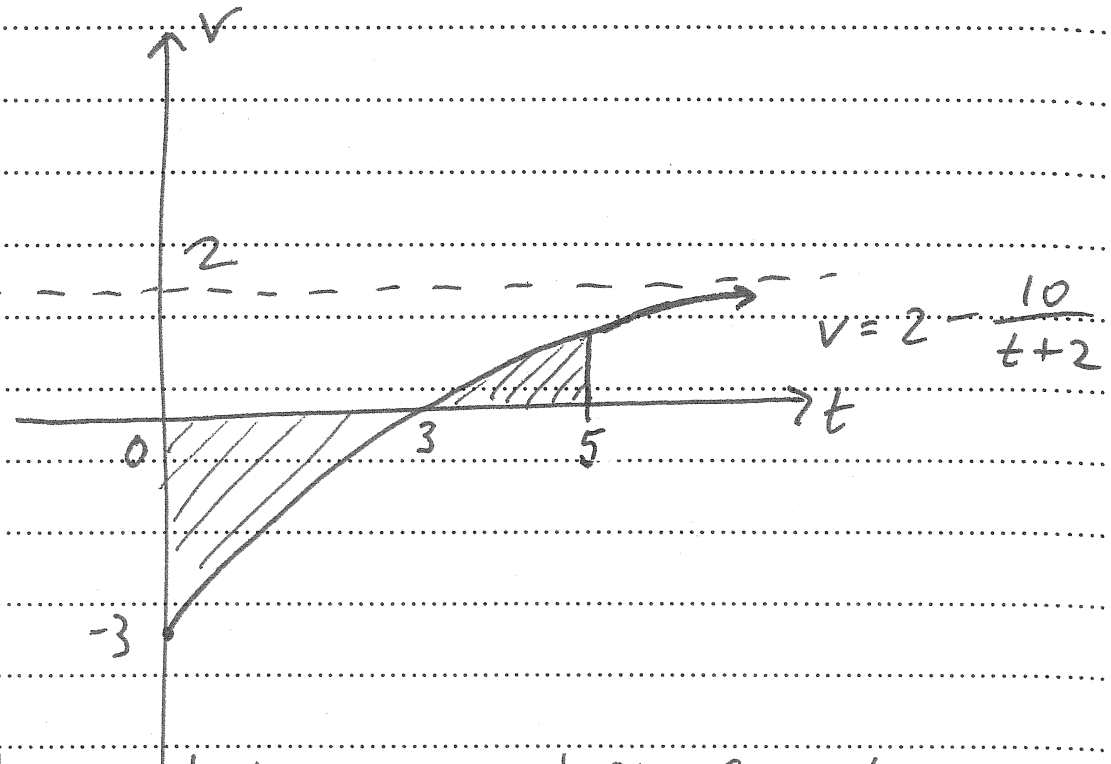
$= -10 \times -1(t+2)^{-2}$

$= 10(t+2)^{-2}$

When $t = 3$, $\quad a = 10(3+2)^{-2}$ ✓

$= 0.4\ ms^{-2}$

d) Find the distance travelled in the first 5 seconds, in exact form. 3



The particle moves left from $t=0$ until $t=3$. Then the particle turns around and moves right from $t=3$ until $t=5$. Since displacement is the integral of velocity, the distance travelled in the first 5 seconds is:

$$\left|\int_0^3 \left(2-\frac{10}{t+2}\right)dt\right| + \int_3^5 \left(2-\frac{10}{t+2}\right)dt \quad \checkmark$$

$$= \left|\left[2t-10\ln(t+2)\right]_0^3\right| + \left[2t-10\ln(t+2)\right]_3^5 \quad \checkmark$$

$$= \left|6-10\ln 5-(0-10\ln 2)\right| + 10-10\ln 7-(6-10\ln 5) \quad \checkmark$$

$$= -6+10\ln 5-10\ln 2+4-10\ln 7+10\ln 5$$

$$= 20\ln 5-10\ln 2-10\ln 7-2$$

$$= 10\ln\left(\frac{25}{14}\right)-2 \quad \text{(by the log laws)}$$

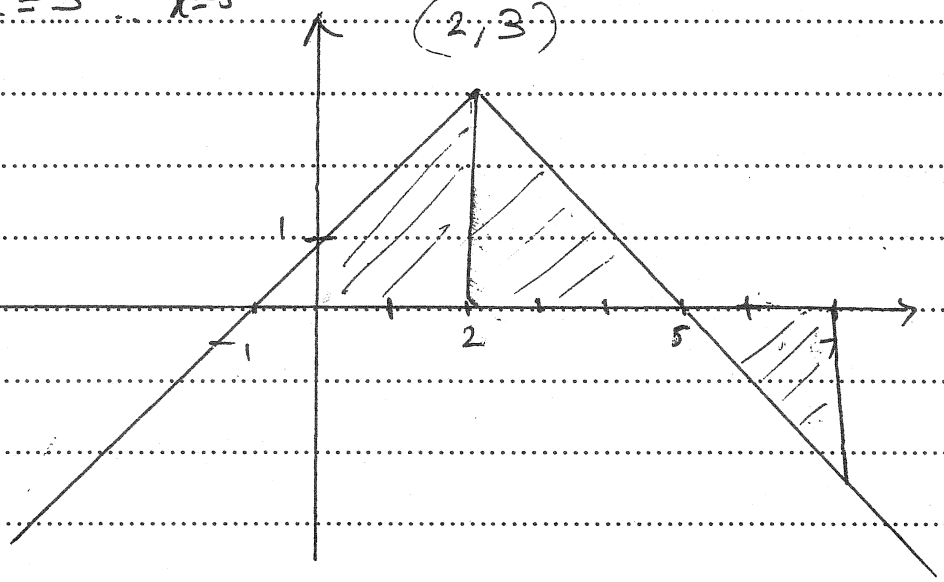
Question 33 (2 marks)

Evaluate $\int_0^7 f(x)\,dx$, where $f(x)=3-|x-2|$. 3

$|x-2| = 3$ $\quad x-2=-3 \therefore x=-1$

$x-2=3 \therefore x=5$

or



$$\int_0^7 -|x-2|+3\,dx = \frac{1}{2}\times 2(1+3)+\frac{1}{2}\times 3\times 3-\frac{1}{2}\times 2\times 2$$

$$= 4+\frac{9}{2}-\frac{4}{2}$$

$$= 6\frac{1}{2}$$

Many students didn't use this method & couldn't get the correct answer

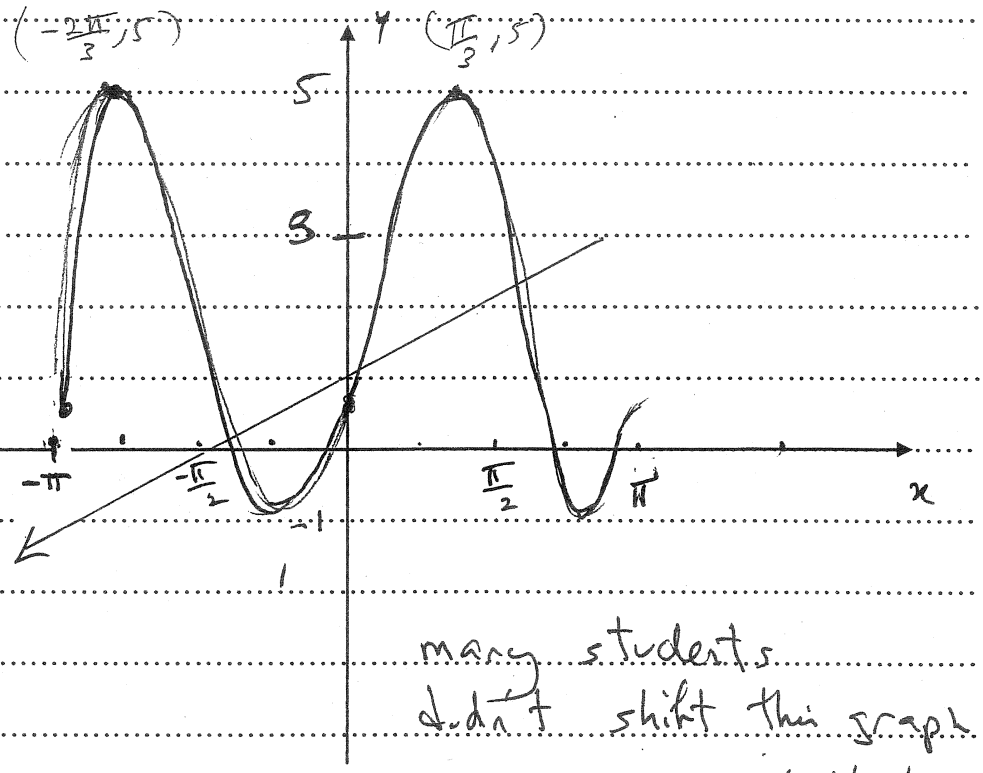
Question 34 (4 marks)

a) Sketch $f(x) = -3\cos\left(2x + \frac{\pi}{3}\right) + 2$ for $-\pi \le x \le \pi$. 3

$a = 3$

$P = \frac{2\pi}{2}$

$= \pi$



many students didn't shift this graph correctly ... didn't have the correct max pts

b) Hence, state the number of solutions to the equation $-3\cos\left(2x + \frac{\pi}{3}\right) + 2 = 0.5x + 1$ for $-\pi \le x \le \pi$. 1

3 solutions

students had to draw $y = 0.5x + 1$ to get the answe

many didn't

Question 35 (6 marks)

A fence, 8 m high, is parallel to the wall of a building and 1 m from the building.

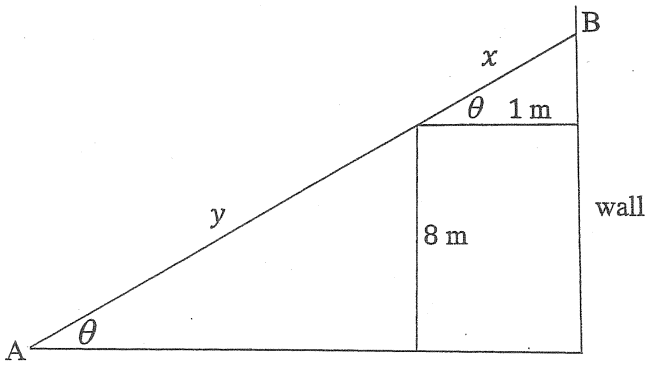


Diagram not to scale.

a) Write an expression for the length of AB in terms of θ. 2

$AB = \frac{8}{\sin\theta} + \frac{1}{\cos\theta}$

b) Find the shortest length of the plank AB that can go over the fence, from the level ground, in order to rest against the wall. 3

$AB = 8(\sin\theta)^{-1} + (\cos\theta)^{-1}$

$\frac{dAB}{d\theta} = -8(\sin\theta)^{-2} \cdot \cos\theta - (\cos\theta)^{-2} \cdot -\sin\theta$

$= \frac{-8\cos\theta}{\sin^2\theta} + \frac{\sin\theta}{\cos^2\theta}$

$= \frac{-8\cos^3\theta + \sin^3\theta}{\sin^2\theta\cos^2\theta}$

many student couldn't do The differentiation correctly

$\frac{dAB}{d\theta} = 0$

$-8\cos^3\theta + \sin^3\theta = 0$

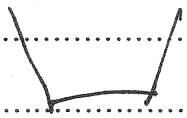
$\sin^3\theta = 8\cos^3\theta$

$\tan^3\theta = 8$

$\tan\theta = 2$

$\theta = 63^\circ$

θ	62°	63°	65°
y'	-1.6	0	0.93



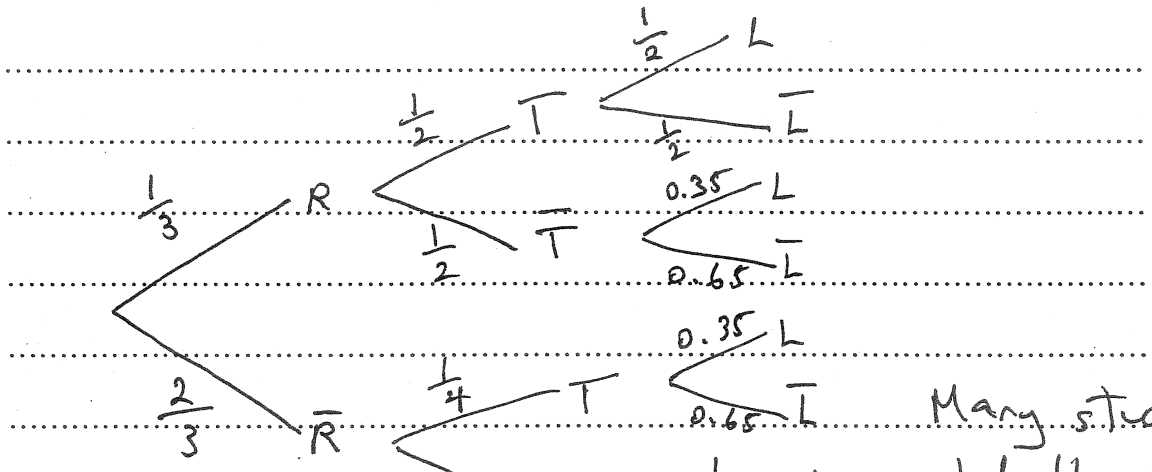
$AB = \frac{8}{\sin 63} + \frac{1}{\cos 63}$

$= 11.18\ m$

some students didn't test for minimum. They must test. I didn't deduct mark but 1 mark can be lost.

Question 36 (2 marks)

Neville lives in a town where one third of the days are rainy days. If it is a rainy day, the chance of heavy traffic is $\frac{1}{2}$. If it is not a rainy day, the chance of heavy traffic is $\frac{1}{4}$. If it is a rainy day and there is heavy traffic, there is a 50% chance that Neville will arrive late to work. On the other hand , the probability of being late is reduced to $\frac{1}{8}$ if it is not a rainy day and there is no heavy traffic. In other situations, the probability of Neville being late to work is 0.35. On a random day, Neville arrives late to work. What is the probability that it is a rainy day? 2



Many students did not use the conditional probability correctly

$$P(R/L) = \frac{P(L \cap R)}{P(L)}$$

$$= \frac{\frac{1}{3} \times \frac{1}{2} \times \frac{1}{2} + \frac{1}{3} \times \frac{1}{2} \times \frac{35}{100}}{\frac{1}{3} \times \frac{1}{2} \times \frac{1}{2} + \frac{1}{3} \times \frac{1}{2} \times \frac{35}{100} + \frac{2}{3} \times \frac{1}{4} \times \frac{35}{100} + \frac{2}{3} \times \frac{3}{4} \times}$$

$$= \frac{34}{63}$$

The End