QUESTION ONE (Start a new answer booklet)

Marks

2 (a) Find the value of A correct to 2 significant figures given $B = 6{\cdot}4$ and $C = 3{\cdot}5$ and:

$$\frac{1}{A} = \frac{1}{B} + \frac{1}{C}$$

2 (b) Solve the equation $4(x - 1) = 3 - 2x$.

2 (c) Factorise fully $a - a^3$.

2 (d) Find the values of x for which $|x + 2| < 3$.

2 (e) Express the fraction $\dfrac{1}{\sqrt{5} - 1}$ with a rational denominator.

2 (f) When 16% sales tax is added on, the selling price of an item becomes \$290. What is the selling price to a customer who has a sales tax exemption?

QUESTION TWO (Start a new answer booklet)

A, B and C are the points $(-2, 2)$, $(-1, -5)$ and $(6, -6)$ respectively.

Marks

1 (a) Sketch the triangle ABC on a number plane.

1 (b) Show that the midpoint P of AC has coordinates $(2, -2)$.

1 (c) Find the gradient of BP.

2 (d) Show that BP is perpendicular to AC.

1 (e) Show that the line BP has equation $y = x - 4$.

2 (f) Find the coordinates of D if P is the midpoint of the interval BD.

1 (g) What kind of special quadrilateral is $ABCD$?

1 (h) Show that the diagonal AC has length $8\sqrt{2}$ units.

2 (i) Find the area of quadrilateral $ABCD$.

(Exam continues next page ...)

QUESTION THREE (Start a new answer booklet)

Marks

2 (a) Find the gradient of the tangent to the curve $y = 2x^3 + 4$ at the point $(-5, -246)$.

3 (b) Find $\dfrac{dy}{dx}$ given:

(i) $y = (3x + 7)^5$,

(ii) $y = \log_e(3x + 7)$,

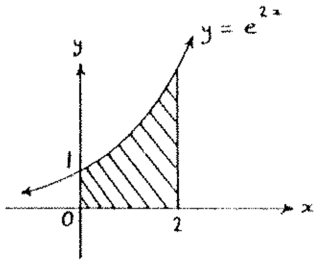
(iii) $y = \sin(3x + 7)$.

4 (c) (i) Find $\int (12x^2 + 7)\, dx$.

(ii) Find $\int \frac{1}{x^2}\, dx$.

(iii) Find $\int_0^{\frac{\pi}{3}} \sec^2 x\, dx$.

3 (d)



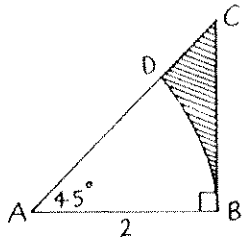
The diagram shows the region bounded by the curve $y = e^{2x}$, the x axis, the y axis and the line $x = 2$. Find the exact area of this region.

(Exam continues overleaf ...)

QUESTION FOUR (Start a new answer booklet)

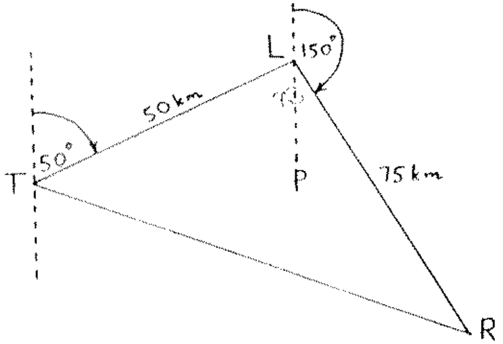
Marks

[4] (a)



In the diagram above, triangle ABC has a right-angle at B, angle $BAC = 45°$ and $AB = 2$ units. A circular arc, centre A and radius AB, cuts the side AC at D. Find the exact area of the shaded portion BCD.

[4] (b)



A motorist drives 50 km from town T to landmark L on a bearing of 050°. He then drives 75 km to resort R on a bearing of 150°.

(i) Explain why $\angle TLR = 80°$.

(ii) Use the cosine rule to find the distance between T and R to the nearest kilometre.

[4] (c) (i) Write down a formula in terms of n, a and d for the sum of the first n terms of an arithmetic series.

(ii) The weekly wages of the 100 employees in a certain company add up to \$85 000. If these wages are listed in ascending order from the most junior Office Clerk to the Managing Director, they form an arithmetic sequence with a common difference of \$10. What is the weekly wage of the most junior Office Clerk?

(Exam continues next page ...)

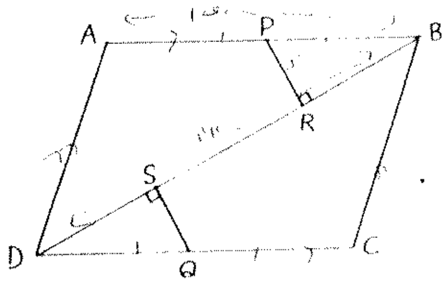
QUESTION FIVE (Start a new answer booklet)

Marks

[3] (a) (i) Solve $\log_3 x - \log_3(x-4) = 2$.

(ii) Solve $\sin x = \frac{1}{2}$, where $0 < x < 2\pi$.

[4] (b)



In the diagram above, $ABCD$ is a parallelogram and P, Q are the midpoints of the sides AB, DC respectively. The intervals PR and QS are drawn perpendicular to the diagonal DB.

(i) Prove that the triangles BPR and DQS are congruent.

(ii) If $AB = 10$ cm, $PR = 3$ cm and $BD = 14$ cm, find the length of SR.

[5] (c) The equation $(x-2)^2 = -6(y-\frac{3}{2})$ represents a parabola.

(i) Write down the coordinates of the vertex.

(ii) Find the focal length.

(iii) Find the x-intercepts of the parabola.

(iv) Sketch the graph of the parabola, and show the focus and directrix on your diagram.

(Exam continues overleaf ...)

QUESTION SIX (Start a new answer booklet)

The function $f(x)$ is defined by the rule $f(x) = e^{-\frac{1}{2}x^2}$.

Marks

[1] (a) State the domain of $f(x)$.

[2] (b) Show that $f(x)$ is an even function. About which line is an even function symmetrical?

[1] (c) Explain why $f'(x) = -x\,e^{-\frac{1}{2}x^2}$.

[2] (d) Find the stationary point of $y = f(x)$ and determine its nature.

[1] (e) Use the product rule to show that $f''(x) = (x^2 - 1)e^{-\frac{1}{2}x^2}$.

[2] (f) Find the two points at which $f''(x) = 0$ and show that they are both points of inflexion.

[1] (g) What value does $f(x)$ approach as x becomes large?

[2] (h) Use the information found in the parts above to sketch the graph of $y = f(x)$.

(Exam continues next page ...)

QUESTION SEVEN (Start a new answer booklet)

Marks

[5] (a) (i) Sketch the graph of the function $y = \cos 2x$ for $-\pi \leq x \leq \pi$.

(ii) On the same diagram, sketch the line $x + 2y = 1$.

(iii) Hence determine the number of solutions of the equation $2\cos 2x = 1 - x$.

(iv) Let the negative solution be $x = N$. Indicate N on your diagram.

[7] (b) James had a full drink bottle containing 500 ml of Gatorade. He drank from it so that the volume, V millilitres, of Gatorade in the bottle changed at a rate given by:

$$\frac{dV}{dt} = \left(\tfrac{2}{5}t - 20\right) \text{ millilitres per second.}$$

(i) Find a formula for V.

(ii) Show that it took James 50 seconds to drink the contents of the bottle.

(iii) How long, to the nearest second, did it take James to drink half the contents of the bottle?

(Exam continues overleaf ...)

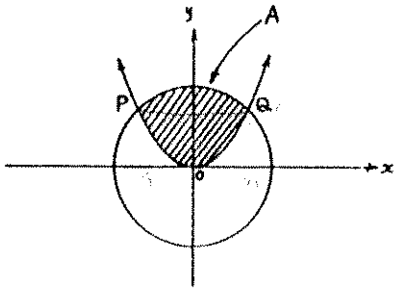
QUESTION EIGHT (Start a new answer booklet)

Marks

[3] (a) (i) Show that $\int_0^2 \frac{1}{1+x}\,dx = \ln 3$.

(ii) Use Simpson's rule with five function values to find an approximation to $\ln 3$.

[5] (b)



The region A is bounded by the curve $y = x^2$ and a minor arc of the circle $x^2 + y^2 = 12$.

(i) By solving the two equations simultaneously, show that points P and Q both have y-coordinate 3.

(ii) Prove that the volume generated when the region A is rotated about the y-axis is $\pi(16\sqrt{3} - \frac{45}{2})$ units^3. (CARE: The rotation is about the y-axis, not the x-axis.)

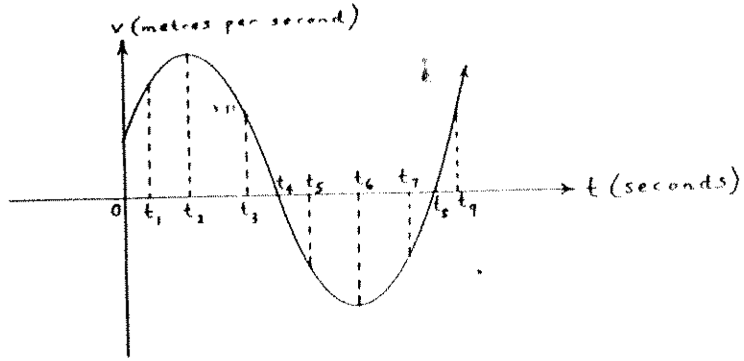
[4] (c) The population W of Williamtown is increasing exponentially according to the equation $W = W_0 e^{0.02t}$, while the population H of Hectorville is decreasing exponentially according to the equation $H = H_0 e^{-0.01t}$. If the current populations of Williamtown and Hectorville are 8000 and 12 000 respectively, how long, to the nearest year, will it be before their populations are the same?

(Exam continues next page ...)

QUESTION NINE (Start a new answer booklet)

Marks

[4] (a)



The velocity-time graph of a particle moving in a straight line is shown above.

(i) At which of the times from $t = 0$ up to $t = t_9$ is:

(α) the velocity zero,

(β) the acceleration zero?

(ii) At which time is the acceleration greatest in magnitude?

(iii) What is the physical significance of the area between the graph and the t axis from $t = 0$ to $t = t_4$?

[8] (b) Steve won the New Year Jackpot Lottery on 1st January 1970. The prize was 1 million dollars. He decided to deposit the entire amount into an account which paid interest at the rate of 8% per annum. The interest was calculated quarterly and compounded quarterly. Steve then made an annual withdrawal of \$50 000, starting on 1st January 1971.

(i) Write down an expression for the amount in Steve's account immediately after his first withdrawal.

(ii) Show that the amount in Steve's account immediately after his third withdrawal is given by the expression:

$$10^6 \times 1{\cdot}02^{12} - 50\,000(1 + 1{\cdot}02^4 + 1{\cdot}02^8).$$

(iii) How much, to the nearest dollar, was left in Steve's account immediately after his twentieth withdrawal?

(Exam continues overleaf ...)

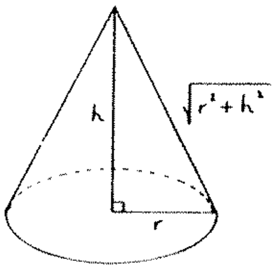
QUESTION TEN (Start a new answer booklet)

Marks

[4] (a) (i) Show that $5p^2 + 6p + 5 = 5\left(p + \frac{3}{5}\right)^2 + \frac{16}{5}$, for all p.

(ii) Hence or otherwise, show that the equation $(p-1)x^2 + (3p+1)x + (p+1) = 0$ has two real solutions for all real values of p except $p = 1$.

[8] (b)



A right circular cone has base radius r and height h. As r and h vary, its curved surface area $\pi r\sqrt{r^2+h^2}$ is kept constant. Let $\pi r\sqrt{r^2+h^2} = K$, where K is a constant.

(i) Show that $V^2 = \frac{1}{9}r^2(K^2 - \pi^2 r^4)$, where V is the volume of the cone. (Note that $V = \frac{1}{3}\pi r^2 h$.)

(ii) Let $Q = V^2$. Show that $\frac{dQ}{dr} = 0$ when $r^4 = \frac{K^2}{3\pi^2}$.

(iii) Show that the maximum value of Q (and hence the maximum value of V) occurs when $h = \sqrt{2}\,r$.

DS

The following list of standard integrals may be used:

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1},\ n \neq -1;\ x \neq 0, \text{ if } n < 0,$$

$$\int \frac{1}{x}\,dx = \ln|x|,\ x \neq 0,$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax},\ a \neq 0,$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax,\ a \neq 0,$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax,\ a \neq 0,$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax,\ a \neq 0,$$

$$\int \sec ax \tan ax\,dx = \frac{1}{a}\sec ax,\ a \neq 0,$$

$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a},\ a \neq 0,$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\frac{x}{a},\ a > 0,\ -a < x < a,$$

$$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \ln\left|x + \sqrt{x^2-a^2}\right|,\ |x| > |a|,$$

$$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \ln\left(x + \sqrt{x^2+a^2}\right).$$