FORM VI MATHEMATICS

Time allowed: 3 hours (5 minutes reading time) **Exam date:** 6th August 2003

Instructions:

All questions may be attempted.
All questions are of equal value.
Part marks are shown in boxes in the right margin.
All necessary working must be shown.
Marks may not be awarded for careless or badly arranged work.
Approved calculators and templates may be used.
A list of standard integrals is provided at the end of the examination paper.

Collection:

The writing booklets will be collected in one bundle.
Start each question in a new writing booklet.
If you use a second booklet for a question, place it inside the first. Don't staple.
Write your candidate number on each booklet.

Checklist:

SGS Writing Booklets required — 10 per boy.
Candidature 123 boys.

QUESTION ONE (Start a new writing booklet)

Marks

(a) (i) Evaluate $-2-(-5)$. **1**

(ii) Evaluate $(x^3 - x^2 + 2)$ when $x = -2$. **1**

(b) (i) Solve $(2x+3)(x-4) = 0$. **2**

(ii) Solve $2x = -7(500 - x)$. **2**

(c) Factorise completely $3m^2 - 12$. **1**

(d) (i) Write down the exact value of $\sin \dfrac{\pi}{4}$. **1**

(ii) Solve $\tan x = 1$, for $0^\circ \le x \le 360^\circ$. **2**

(e) Differentiate with respect to x:

(i) $y = 5x^2$ **1**

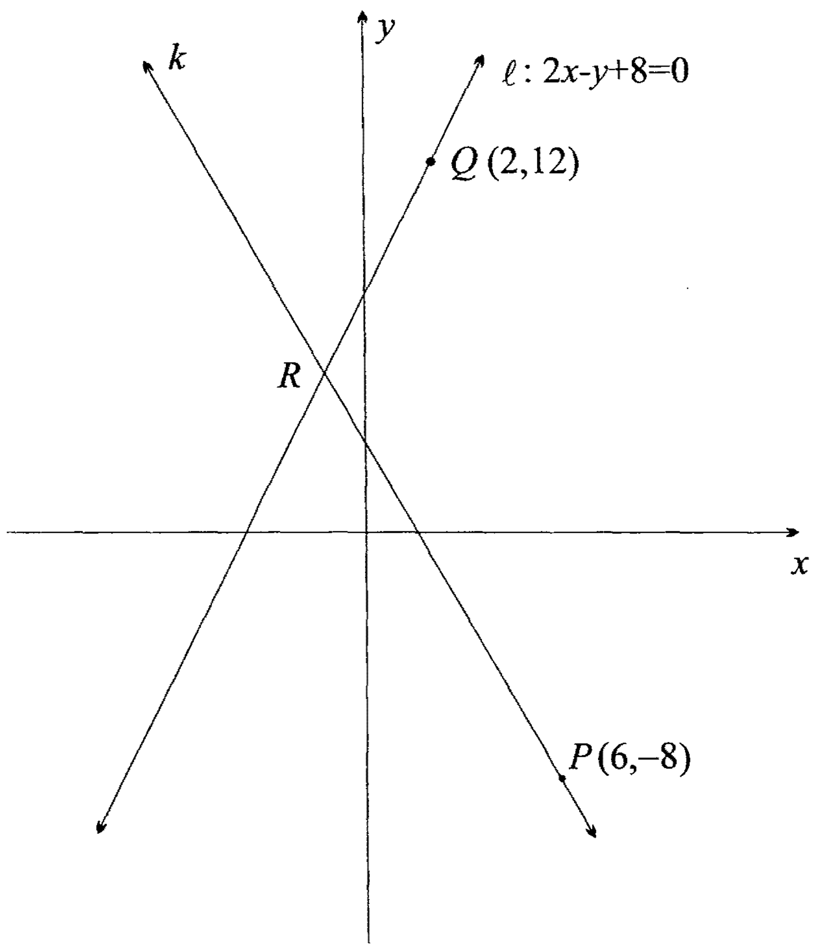
(ii) $y = \sin 2x$ **1**

QUESTION TWO (Start a new writing booklet)

Marks

(a) Find a primitive of $\dfrac{2x}{x^2+1}$. $\boxed{2}$

(b)



The diagram above shows the line ℓ: $2x - y + 8 = 0$ and the point $Q(2, 12)$ on it.
The line k has gradient -2 and passes through the point $P(6, -8)$.
The lines k and ℓ intersect at R.

(i) Show that the equation of the line k is $2x + y - 4 = 0$. $\boxed{1}$

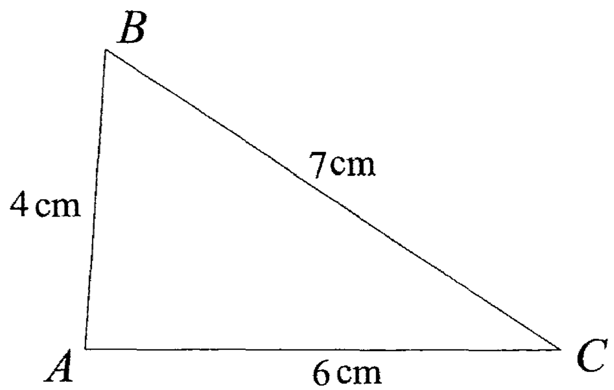
(ii) Show that the coordinates of R are $(-1, 6)$. $\boxed{1}$

(iii) Show that the distance QR is $3\sqrt{5}$. $\boxed{1}$

(iv) Find the perpendicular distance from P to the line ℓ. Leave your answer in surd form. $\boxed{2}$

(v) Find the area of the triangle PQR. $\boxed{1}$

(c)



In the diagram above, ABC is a triangle in which $AB = 4\,\text{cm}$, $BC = 7\,\text{cm}$ and $CA = 6\,\text{cm}$.

(i) Use the cosine rule to show that $\cos C = \dfrac{23}{28}$. **[1]**

(ii) Write down the size of $\angle C$, correct to the nearest degree. **[1]**

(iii) Calculate the area of the triangle ABC. Give your answer correct to the nearest square centimetre. **[2]**

QUESTION THREE (Start a new writing booklet)

(a) Differentiate with respect to x:

Marks

(i) $y = \dfrac{\log_e x}{x}$ **3**

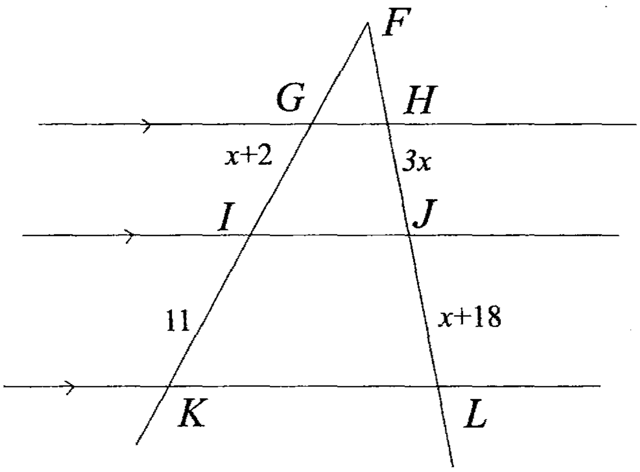
(ii) $y = e^x \cos x$ **2**

(b) In a certain arithmetic series, the first term is 13 and the sixth term is -7.

(i) Find the common difference. **1**

(ii) Find the value of the third term. **2**

(c)



In the diagram above, $GH \parallel IJ \parallel KL$. The lengths of the intervals GI, IK, HJ and JL are as shown.

(i) Give a reason why $\dfrac{x+2}{11} = \dfrac{3x}{x+18}$. **1**

(ii) Solve this equation to find x. **3**

QUESTION FOUR (Start a new writing booklet)

Marks

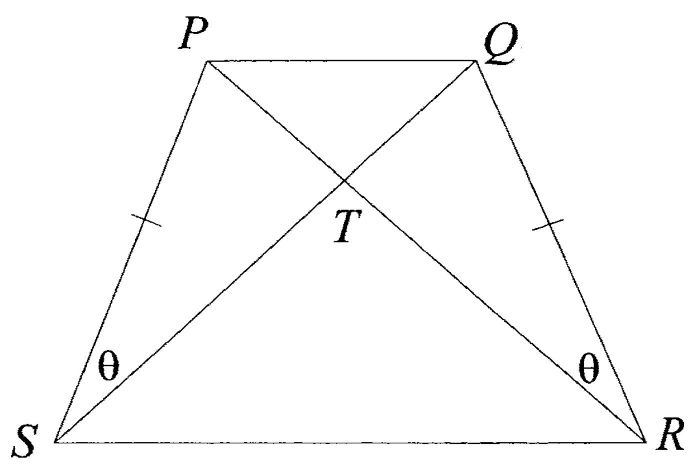
(a) Solve $\cot x = -\sqrt{3}$, for $0 \leq x \leq 2\pi$. **3**

(b) Find the exact value of:

(i) $\displaystyle\int_0^1 e^{3x}\,dx$ **2**

(ii) $\displaystyle\int_0^{\frac{\pi}{6}} \sin x\,dx$ **2**

(c)



In the diagram above, $PQRS$ is a quadrilateral, $\angle PSQ = \angle QRP = \theta$ and $PS = QR$. Copy this diagram into your answer booklet.

(i) Prove that $\triangle PTS$ is congruent to $\triangle QTR$. **3**

(ii) Give the reason why $TS = TR$. **1**

(iii) Prove that $\angle TSR = \angle TRS$. **1**

QUESTION FIVE (Start a new writing booklet)

Marks

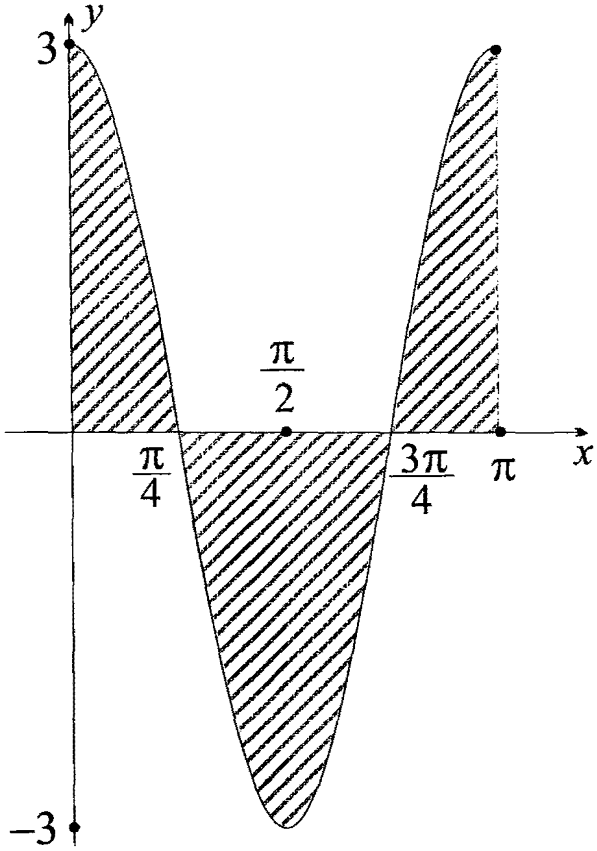
(a) Solve $\log_7 64 = 3\log_7 x$. **2**

(b) Thomas buys a new computer. He takes out a loan of \$3000 at a rate of 12% p.a. compounded monthly. He makes no repayments.

(i) Show that at the end of two years Thomas owes \$3809.20. **1**

(ii) How long after taking out the loan does he owe \$5000? Give your answer correct to the nearest month. **3**

(c)



The sketch above shows the curve $y = 3\cos 2x$, for $0 \le x \le \pi$. The region enclosed by the curve and the x-axis, from $x = 0$ to $x = \pi$, has been shaded.

(i) Write down the value of $\int_0^\pi 3\cos 2x\,dx$. **1**

(ii) Find the area of the shaded region. **3**

(iii) Copy the sketch above into your answer booklet and add the line $y = \frac{1}{2}x$ to your sketch. Use your sketch to find the number of solutions to the equation **2**

$$x = 6\cos 2x, \text{ for } 0 \le x \le \pi.$$

QUESTION SIX (Start a new writing booklet)

(a) The displacement x metres of a particle from the origin at time t seconds is given by the formula

$$x = t^3 - 21t^2.$$

Marks

(i) Find the velocity of the particle as a function of t. **1**

(ii) Find the acceleration of the particle as a function of t. **1**

(iii) Where is the particle when $t = 2$? **1**

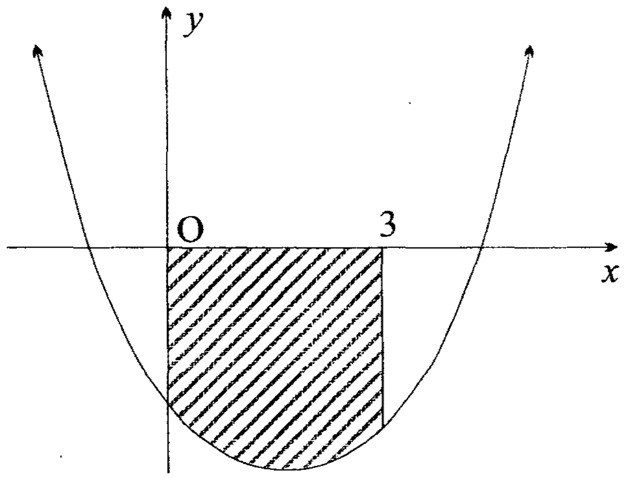
(iv) Find the times at which the particle is at the origin. **1**

(v) Find the times at which the particle is stationary. **1**

(vi) When is the acceleration of the particle zero and where is the particle then? **2**

(vii) When is the particle to the right of the origin? **2**

(b) **3**



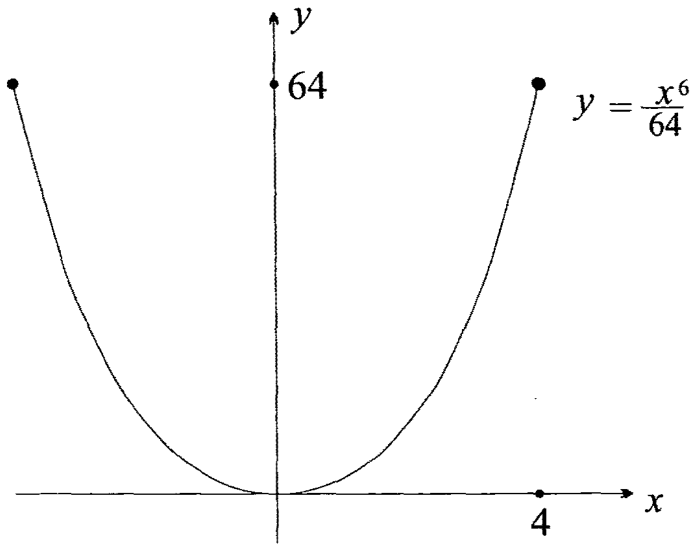
The sketch above shows the parabola $y = kx^2 - 6x - 8$, where k is a positive constant.

The shaded region has area 15 square units.

Find the value of k.

QUESTION SEVEN (Start a new writing booklet)

(a)



A bowl is formed by rotating the part of the curve $y = \dfrac{x^6}{64}$ between $x = 0$ and $x = 4$ about the y-axis.

Marks

(i) Show that $x^2 = 4y^{\frac{1}{3}}$. **1**

(ii) Find the volume of the bowl. **3**

(b) A jet engine uses fuel at a rate of R litres per minute.
The rate of fuel use t minutes after the engine starts operation is given by

$$R = 15 + \frac{10}{1+t}.$$

(i) What is R when $t = 0$? **1**

(ii) What is R when $t = 9$? **1**

(iii) What value does R approach as t becomes very large? **1**

(iv) Draw a sketch of R as a function of t. **3**

(v) Calculate the total amount of fuel burned during the first 9 minutes. Give your answer correct to the nearest litre. **2**

QUESTION EIGHT (Start a new writing booklet)

Marks

(a) Find the exact value of $\displaystyle\int_{\frac{1}{3}}^{\frac{1}{2}} \sec^2 \frac{\pi x}{2}\, dx$. $\boxed{2}$

(b) The sum S_n of the first n terms of a certain series is $2n + 3n^2$, for $n \geq 1$. Find an expression for the nth term T_n of this series. $\boxed{3}$

(c) The groundsman at School had trouble keeping pot plants alive because the boys kept kicking balls into them. At the beginning of the year 2000, all the plants were dead, so he replaced them with 256 new plants. He estimated that he would lose 25% of his plants each year as a result of the boys' boisterous behaviour, so he decided to buy P new plants at the beginning of each year in an effort to beautify the school environment. He bought his first lot of P plants at the beginning of 2001.

(i) Show that only 81 of the **original** 256 plants are left by the beginning of 2004. $\boxed{1}$

(ii) Show that by the beginning of 2004, before he buys his new plants, he has $81 + 3P(1 - 0{\cdot}75^3)$ plants alive in the grounds. $\boxed{2}$

(iii) How many plants should he buy each year to ensure that he has at least 200 plants left alive at the beginning of 2004? $\boxed{2}$

(d) A plague of locusts hit Gondor some time ago. $\boxed{2}$

The locust numbers increased for the first three years, and decreased thereafter.

The rate of change of the locust population declined for the first six years, but increased thereafter.

Draw a graph that represents this information. Remember to put time on the horizontal axis.

QUESTION NINE (Start a new writing booklet)

Marks

(a) The acceleration of a particle is given by $\ddot{x} = e^{-3t}$. The particle is initially stationary at the origin.

(i) Find the velocity function. **2**

(ii) Explain why the particle is stationary only once. **1**

(iii) Find the distance travelled during the first 3 seconds. **2**

(b) The population P of a small mining town is decreasing according to the equation $\dfrac{dP}{dt} = -kP$, where time t is measured in years and k is a positive constant.

In August 2000 it had a population of 3060. By August 2002, however, the population had halved.

(i) Show that $P = P_0 e^{-kt}$ is a solution of $\dfrac{dP}{dt} = -kP$. **1**

(ii) Write down the value of P_0. **1**

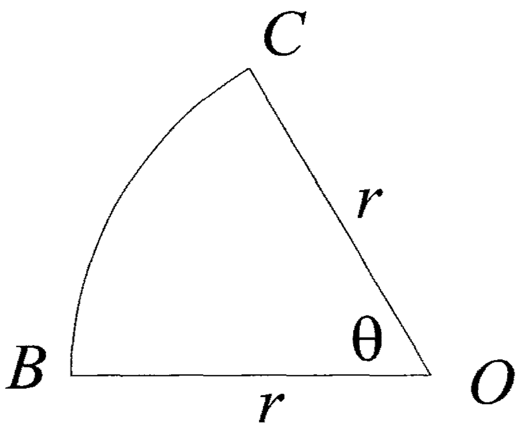
(iii) Show that the value of k is $\frac{1}{2}\log_e 2$ **2**

(iv) How many people are in the town in August 2003? **1**

(v) The mining company decrees that when there are fifty people left they must all leave and turn the lights out. When will this be? **2**

QUESTION TEN (Start a new writing booklet)

(a)



The diagram above shows a sector OBC of a circle with centre O and radius r cm. The arc BC subtends an angle θ radians at O.

Marks

(i) Show that the perimeter of the sector is $r(2+\theta)$. **1**

(ii) Given that the perimeter of the sector is 36 cm, show that its area is $A = \dfrac{648\,\theta}{(\theta+2)^2}$. **2**

(iii) Hence show that the maximum area of the sector is 81 square centimetres. **4**

(b) The point $P(s,t)$ lies on the parabola $y^2 = 4ax$ and the line $\ell x + my = 1$.

(i) Show that $\ell t^2 + 4amt - 4a = 0$. **2**

(ii) Show that if the line is a tangent to the parabola, then $am^2 + \ell = 0$. **2**

(iii) Show that the equation of the tangent at P is $y = amx + \dfrac{1}{m}$. **1**

MLS

SGS Trial 2003 2U

1. (a) (i) $-2-(-5)=3$

(ii) $-8-4+2=-10$

(b) (i) $(2x+3)(x-4)=0$

$x=-\frac{3}{2}$ or 4

(ii) $2x=-7(500-x)$

$2x=-3500+7x$

$5x=3500$

$x=700$

(c) $3m^2-12=3(m^2-4)$

$=3(m+2)(m-2)$

(d) (i) $\sin\frac{\pi}{4}=\frac{1}{\sqrt{2}}$

(ii) $\tan x=1$

$x=45^\circ\ 225^\circ$

(e) (i) $y=5x^2$

$\frac{dy}{dx}=10x$

(ii) $y=\sin 2x$

$\frac{dy}{dx}=2\cos 2x$

2 (a) $\int\frac{2x}{x^2+1}\,dx=\log_e(x^2+1)+c$

(give ✓ if they have $\log_e$(anything)

(don't worry about c)

(b) (i) $y+8=-2(x-6)$

$y+8=-2x+12$

$2x+y-4=0$

(ii)

Can solve eqns simultaneously (✓)

or Can show that $(-1,6)$ statisfies **both** eqns. (✓

$2x+y-4=0$ ①

$2x-y+8=0$ ②

add ①+② $4x+4=0$

$x=-1$

$y=2x+8$

$=6$

so $(-1,6)$ lies on both.

(iii) $QR=\sqrt{(2--1)^2+(12-6)^2}$

$=\sqrt{9+36}$

$=\sqrt{45}$

$=3\sqrt{5}$

(iv) dist $PL=\left|\frac{2\times 6+(-1)\times(-8)+8}{\sqrt{4+1}}\right|$

$=\frac{12+8+8}{\sqrt{5}}$

$=\frac{28}{\sqrt{5}}$

(v) Area of $\Delta PQR = \frac{1}{2} \times \text{base} \times \text{ht}$

$= \frac{1}{2} \times 3\sqrt{5} \times \frac{28}{\sqrt{5}}$

$= 3 \times 14$

$= 42 \ u^2$ ✓

(c) (i) $\cos C = \frac{a^2+b^2-c^2}{2ab}$

$= \frac{7^2+6^2-4^2}{2\times 7\times 6}$

$= \frac{49+36-16}{84}$

$= \frac{69}{84}$ ✓

$= \frac{23}{28}$

(ii) 35° ✓

(iii) Area $= \frac{1}{2} ab \sin C$ ✓

$= \frac{1}{2} \times 7 \times 6 \times \sin 35^\circ$ ✓

$= 12 \text{ cm}^2$

Q3. (a) (i) $y = \frac{\log_e x}{x}$

$\frac{dy}{dx} = \frac{x \cdot \frac{1}{x} - \log_e x}{x^2}$ ✓ ✓

$= \frac{1 - \log_e x}{x^2}$ ✓

(ii) $y = e^x \cos x$

$\frac{dy}{dx} = -\sin x \times e^x + e^x \cos x$ ✓✓

$= e^x(-\sin x + \cos x)$

(b) $a = 13$

$a + 5d = -7$

(i) $13 + 5d = -7$

$5d = -20$

$d = -4$ ✓

(ii) $T_3 = a + 2d$

$= 13 + 2(-4)$ ✓

$= 5$ ✓

(c) i) If two transversals cross 3 parallel lines, then the ratio of the intercepts on one transversal is the same as the ratio of the intercepts on the other transversals. ✓

(ii) $\frac{x+2}{11} = \frac{33x}{x+18}$ ✓

$(x+2)(x+18) = 33x$

$x^2 + 20x + 36 = 33x$

Q4 (a) $\cot x = -\sqrt{3}$

$\tan x = -\frac{1}{\sqrt{3}}$ ✓

	×
×	

related angle is $\frac{\pi}{6}$ ✓

$x = \frac{5\pi}{6}$ or $11\frac{\pi}{6}$ ✓

(b) (i) $\int_0^1 e^{3x}\,dx$

$= \left[\frac{1}{3}e^{3x}\right]_0^1$ ✓

$= \frac{1}{3}e^3 - \frac{1}{3}e^0$

$= \frac{1}{3}(e^3-1)$ or $\frac{1}{3}e^3 - \frac{1}{3}$ ✓

(ii) $\int_0^{\frac{\pi}{6}} \sin x\,dx$

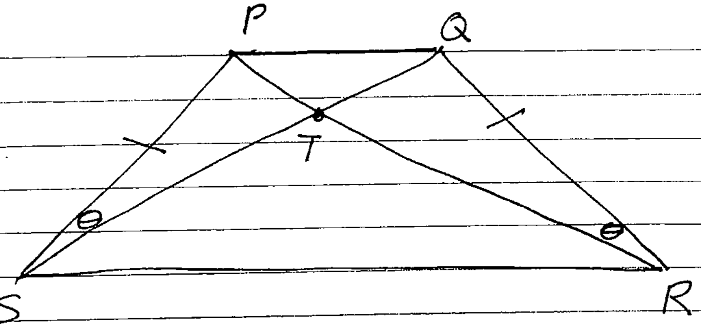
$= \left[-\cos x\right]_0^{\frac{\pi}{6}}$ ✓

$= -\cos\frac{\pi}{6} - -\cos 0$

$= -\frac{\sqrt{3}}{2} + 1$ ✓

$= 1 - \frac{\sqrt{3}}{2}$

c)



(i) In ΔPTS and ΔQTR

$\angle PSQ = \angle QRP$ (given)

$\angle PTS = \angle QTR$ (vertically opposite angles)

$PS = QR$ (given)

$\therefore \Delta PTS \equiv \Delta QTR$ (AAS) ✓✓✓

(ii) $TS = TR$ because the corresponding sides of congruent triangles are equal ✓

(iii) ΔTSR is isosceles ($TS = TR$)

The angles opposite equal sides are equa

so $\angle TSR = \angle TRS$ ✓

5.

(a) $\log_2 64 = 3\log_2 x$

$\log 64 = \log x^3$ ✓

$64 = x^3$

$x = 4$ ✓

(b) (i) 12% p.a $= 1\%$ per month.

Amount owed $= \$3000(1.01)^{24}$ ✓

$= \$3809.20$

(ii) find n if $5000 = 3000(1.01)^n$ ✓

$1.01^n = \frac{5}{3}$

$n \log 1.01 = \log \frac{5}{3}$ ✓

$n = \frac{\log 5/3}{\log 1.01}$

$\doteq 51$ months. ✓

(c) (i) 0 ✓

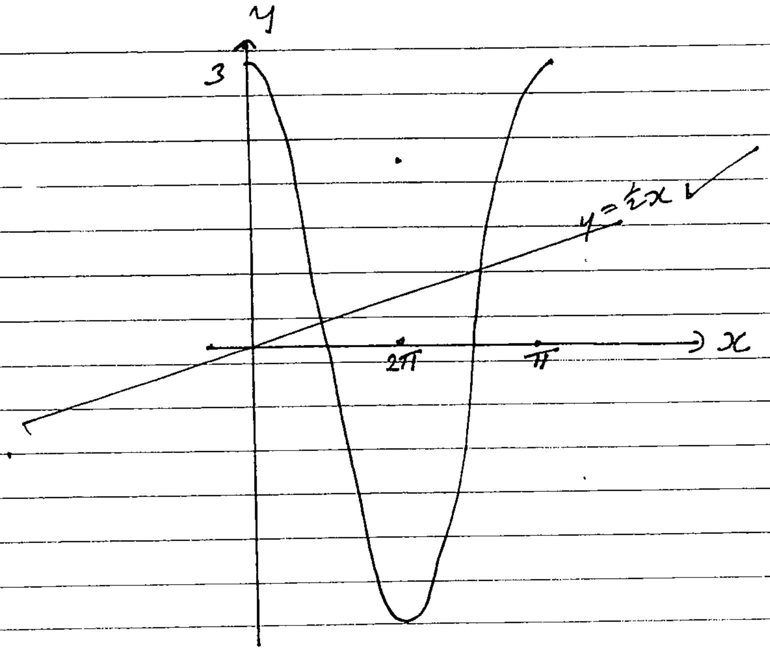
(ii) area $= 4\int_0^{\frac{\pi}{4}} 3\cos 2x \, dx$ ✓ (or equivalent integral)

$= 12\left[\frac{1}{2}\sin 2x\right]_0^{\frac{\pi}{4}}$ ✓

$= 6\left[\sin\frac{\pi}{2} - \sin 0\right]$

$= 6 \quad u^2$ ✓

∴ (iii) over.



number of solutions is 2. ✓

Q6.

(a) $x = t^3 - 21t^2$

(i) $v = \dot{x} = 3t^2 - 42t$

(ii) $a = \ddot{x} = 6t - 42$

(iii) $t=2$ $x = 2^3 - 21 \times 2^2$
$= -76$ m

76 m to the left of the origin

(iv) find t when $x = 0$

$t^3 - 21t^2 = 0$

$t^2(t-21) = 0$

$t = 0$ or 21 seconds

(v) stationary when $v = 0$.

$3t^2 - 42t = 0$

$3t(t-14) = 0$

$t = 0$ or 14 seconds

(vi) $6t - 42 = 0$

$t = 7$ seconds when $a = 0$

$t = 7$, $x = 7^3 - 21 \times 49$
$= -686$ m

(vii) $t^3 - 21t^2 > 0$

$t^2(t-21) > 0$

$t - 21 > 0$ since $t^2 > 0$

$t > 21$

It is to the right when t is greater than 21 seconds

(b) $\int_0^3 (kx^2 - 6x - 8)\,dx = -15$

area is below axis

$\left[\frac{k}{3}x^3 - 3x^2 - 8x\right]_0^3 = -15$

$(9k - 27 - 24) - (0) = -15$

$9k = -15 + 51$

$9k = 36$

$k = 4$

12.

(a) (i) $y = \frac{x^6}{64}$

$64y = x^6$

$x^2 = \sqrt[3]{64y}$

$= 4y^{\frac{1}{3}}$

(ii) $V = \pi\int_0^{64} x^2\,dy$

$= \pi\int_0^{64} 4y^{\frac{1}{3}}\,dy$

$= 4\pi\left[\frac{3y^{4/3}}{4}\right]_0^{64}$

$= 3\pi\left[y^{4/3}\right]_0^{64}$

$= 3\pi \times 256$

$= 768\pi \ u^3$

(b) (i) $t=0, \quad R = 15 + \frac{10}{1}$

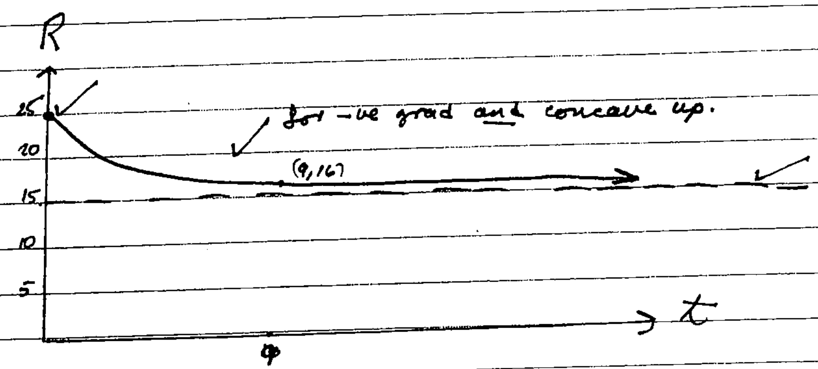
$= 25$ l/min

(ii) $t=9, \quad R = 15 + \frac{10}{10}$

$= 16$ l/min

(iii) as $t \to \infty$, $\frac{10}{1+t} \to 0$ and $R \to 15$

iv)



v) Fuel burned $= \int_0^9 \left(15 + \frac{10}{1+t}\right) dt$ (or equiv integral).

$= \left[15t + 10\log_e(1+t)\right]_0^9$

$= (135 + 10\log 10) - (0 + 10\log 1)$.

$= 135 + 23.025$

$\doteqdot 158$ l

8.

(a) $\displaystyle\int_{\frac{1}{3}}^{\frac{1}{2}} \sec^2 \frac{\pi x}{2}\, dx$

$= \frac{2}{\pi}\left[\tan \frac{\pi x}{2}\right]_{\frac{1}{3}}^{\frac{1}{2}}$

$= \frac{2}{\pi}\left(\tan\frac{\pi}{4} - \tan\frac{\pi}{6}\right)$

$= \frac{2}{\pi}\left(1 - \frac{1}{\sqrt{3}}\right)$

(b) $S_n = 2n + 3n^2$

$S_{n-1} = 2(n-1) + 3(n-1)^2$

$= 2n - 2 + 3n^2 - 6n + 3$

$= 3n^2 - 4n + 1$

$T_n = S_n - S_{n-1}$

$= 2n + 3n^2 - (3n^2 - 4n + 1)$

$= 2n + 3n^2 - 3n^2 + 4n - 1$

$= 6n - 1$

(c) (i) Number left $= 256(0.75)^4$

$= 81$

(ii) 1st lot of P plants becomes $P(0.75)^3$ Jan 2001

2nd lot of P plants becomes $P(0.75)^2$ Jan 2002

3rd lot of P plants become $P(0.75)^1$ Jan 2003

So, number of plants $= 81 + P(0.75^3 + 0.75^2 + 0.75)$

$= 81 + P$ (sum of GP, $a = 0.75$, $r = 0.75$, $n = 3$)

$= 81 + P\left(\dfrac{0.75(1 - 0.75^3)}{1 - 0.75}\right)$

$= 81 + P\left(\dfrac{\cancel{0.75}^{\,3}(1 - 0.75^3)}{\cancel{0.25}}\right)$

$= 81 + 3P(1 - 0.75^3)$

(iii) $81 + 3P(1 - 0.75^3) \geq 200$

$3P(1 - 0.75^3) \geq 119$

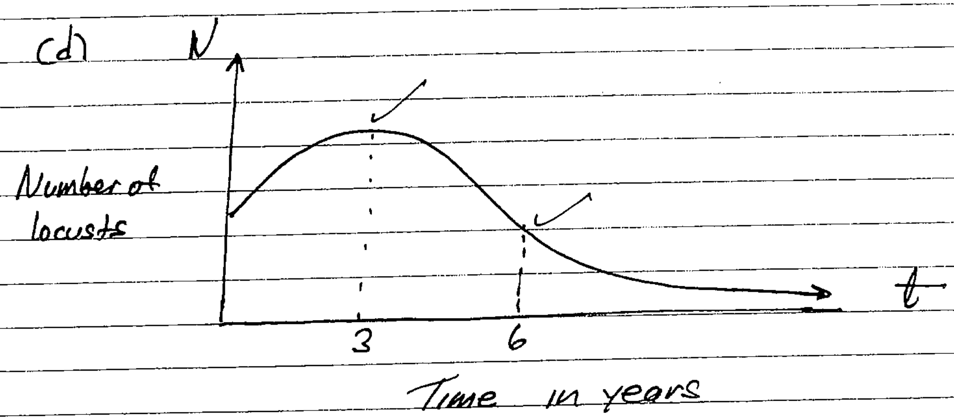
$P \geq \dfrac{119}{3(1 - 0.75^3)}$

$P \geq \dfrac{119}{1.73437}$

$P \geq 68.6$

so P ~~is~~ 69.

(d)



9.

(a) $\ddot{x} = e^{-3t}$

(i) $v = \dot{x} = \int e^{-3t}\,dt$

$\dot{x} = -\frac{1}{3}e^{-3t} + c$ ✓

$t=0,\quad 0 = -\frac{1}{3}e^{0} + c$

so $c = \frac{1}{3}$

and $v = -\frac{1}{3}e^{-3t} + \frac{1}{3}$ ✓

(ii) It is stationary when $-\frac{1}{3}e^{-3t} + \frac{1}{3} = 0$

so solve $\frac{1}{3}e^{-3t} = \frac{1}{3}$

$e^{-3t} = 1$ ✓

This has only one solution, $t=0$

(iii) Distance $= \int_0^3 \left(-\frac{1}{3}e^{-3t} + \frac{1}{3}\right) dt$ ✓

$= \left[\frac{1}{9}e^{-3t} + \frac{1}{3}t\right]_0^3$

$= \left(\frac{1}{9}e^{-9} + 1\right) - \left(\frac{1}{9}e^{0} + 0\right)$

$= \frac{1}{9}e^{-9} + \frac{8}{9}$ ✓

(b) $\frac{dP}{dt} = -kP$

(i) $P = P_0e^{-kt}$

$\frac{dP}{dt} = -kP_0e^{-kt}$ ✓

$= -kP$

(ii) when $t=0$, $P = 3060$ ✓

so $P_0 = 3060$

(iii) $t=2,\quad 1530 = 3060e^{-2k}$ ✓

$e^{-2k} = \frac{1}{2}$

$-2k = \log_e \frac{1}{2}$

$k = -\frac{1}{2}\log_e \frac{1}{2}$ ✓

$= -\frac{1}{2}\log_e 2^{-1}$

$= \frac{1}{2}\log 2$

(iv) $t=3,\quad P = 3060e^{-3/2\log 2}$

$= 3060 \times 2^{-3/2}$ ✓

$= 1082$

(v) $50 = 3060\, e^{(-\frac{1}{2}\ln 2)t}$

$\frac{5}{306} = e^{(-\frac{1}{2}\ln 2)t}$

$\log_e \frac{5}{306} = (-\frac{1}{2}\ln 2)\, t$

$t = \dfrac{\log \frac{5}{306}}{-\frac{1}{2}\log 2}$

$= 11.8709$ years.

So, light out ~~during~~ 11.8 years after August 2000 ie during 2012, during May 2012.

Q 10.

(a) (i) $P = 2r + r\theta$

$= r(2+\theta)$

(ii) $36 = r(2+\theta)$

so $r = \dfrac{36}{2+\theta}$

$A = \frac{1}{2} r^2 \theta$

$= \frac{1}{2}\left(\dfrac{36}{2+\theta}\right)^2 \theta$

$= \dfrac{648\,\theta}{(2+\theta)^2}$

(iii) $\dfrac{dA}{d\theta} = \dfrac{(2+\theta)^2\, 648 - 648\,\theta \times 2 \times (2+\theta)}{(2+\theta)^4}$

$= \dfrac{(2+\theta)648 - 1296\,\theta}{(2+\theta)^3}$

$= \dfrac{1296 - 648\,\theta}{(2+\theta)^3}$

at min/max, $\dfrac{dA}{d\theta} = 0$

solve $1296 - 648\,\theta = 0$

$\theta = \dfrac{1296}{648}$

$= 2$

Check for max

θ	1	2	3
$\frac{dA}{d\theta}$	$\frac{3\times648 - 2\times648}{27}$	0	$\frac{5\times648 - 6\times648}{5^3}$
	+ve	—	−ve

So we have maximum area at $\theta = 2$.

$$A = \frac{648\times2}{4^2}$$
$$= 81 \text{ cm}^2$$

(b)

(i) (s,t) satisfies both equations

so $t^2 = 4as$ ①

and $ls + mt = 1$ ②

from ① $s = \frac{t^2}{4a}$

from ② $s = \frac{1-mt}{l}$

so $\frac{t^2}{4a} = \frac{1-mt}{l}$

$4a - 4amt = lt^2$

$lt^2 + 4amt - 4a = 0$

say something about 1 value, or imply

(ii) If the line is a tangent then there is only 1 value of t that satisfies $lt^2 + 4amt - 4a = 0$

so we want $\Delta = 0$

$(4am)^2 + 4(4a)(l) = 0$

$16a^2m^2 + 16al = 0$

$a^2m^2 + al = 0$

$am + l = 0$ we can divide by a since $a \neq 0$ because $y^2 = 4ax$ is a parabola

(iii) For the line to be a tangent we want $l = -am^2$

so we have $-am^2x + my = 1$

$my = 1 + am^2x$

$y = amx + \frac{1}{m}$