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CANDIDATE NUMBER

2020 Trial Examination

Form VI Mathematics Advanced

Wednesday 12th August 2020

General Instructions

- Reading time — 10 minutes
- Working time — 3 hours
- Attempt all questions.
- Write using black pen.
- Calculators approved by NESA may be used.
- A loose reference sheet is provided separate to this paper.
- Remove the central staple: you should have this cover booklet with Section I and 4 booklets for Section II.

Total Marks: 100

Section I (10 marks) Questions 1–10

- This section is multiple-choice. Each question is worth 1 mark.
- Record your answers on the provided answer sheet.

Section II (90 marks) Questions 11–32

- Relevant mathematical reasoning and calculations are required.
- Answer the questions in this paper in the spaces provided.
- This section is divided in four parts. Extra writing paper is provided at the end of each part.

Collection

- Write your candidate number on each booklet and on the multiple choice sheet.
- Place everything inside this cover booklet.

Checklist

- Reference sheet
- Multiple-choice answer sheet
- Candidature: 92 pupils

Writer: RDP/BDD

Section I

Questions in this section are multiple-choice.

Choose a single best answer for each question and record it on the provided answer sheet.

1. Which of the following correctly expresses y as the subject of the formula $2x - 5y + 3 = 0$?

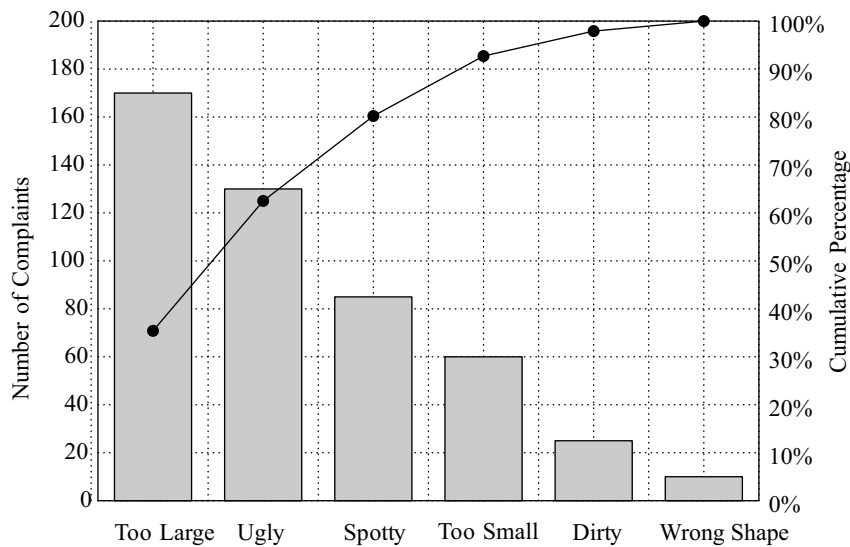
(A) $y = 2x + 3$

(B) $y = \frac{2}{5}x + 3$

(C) $y = \frac{2}{5}x + \frac{3}{5}$

(D) $y = \frac{2}{5}x - \frac{3}{5}$

2. The number of complaints for the return of a particular item is shown in the Pareto chart below.



What percentage of the total number of complaints do the three largest complaints account for?

(A) 60%

(B) 70%

(C) 80%

(D) 90%

3. A particle is moving with velocity $v = t^2 - 10t + 21$, $t > 0$. When is the particle stationary?

- (A) When $t = 3$.
- (B) When $t = 5$.
- (C) When $t = 3$ or $t = 7$.
- (D) When $t = 5$ or $t = 7$.

4. A pupil graphs the following relations. Which relation is many-to-one?

- (A) $x^2 + y^2 = 4$
- (B) $y = \sqrt{4 - x^2}$
- (C) $y = x^3 + 3$
- (D) $y = 2x - 4$

5. What amount must be invested now at 5% per annum, compounded monthly, so that in four years time it will have grown to \$50 000?

- (A) \$38 772
- (B) \$39 176
- (C) \$40 954
- (D) \$41 135

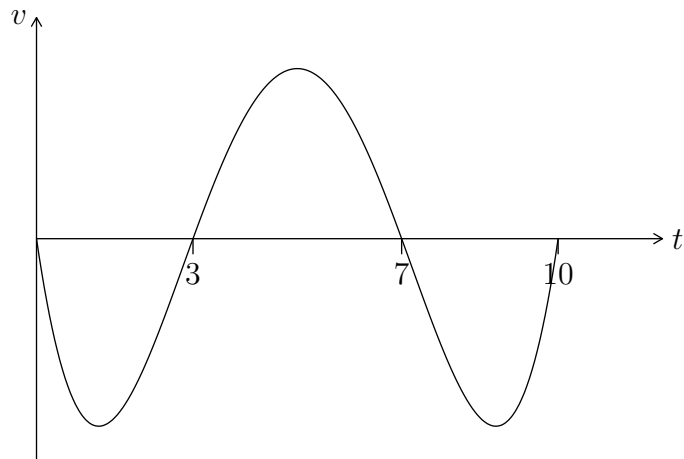
6. The graph $y = x^2 + 4x + 7$ is reflected in the y -axis, followed by a translation 2 units to the right. What is the equation of the new graph?

- (A) $y = (-x - 2)^2 + 4(-x - 2) + 7$
- (B) $y = (-x + 2)^2 + 4(-x + 2) + 7$
- (C) $-y = (x + 2)^2 + 4(x + 2) + 7$
- (D) $-y = (x - 2)^2 + 4(x - 2) + 7$

7. The function $y = f(x)$ has second derivative $y'' = 2(x - 1)^2(x - 3)$. A pupil is asked to find the x -coordinate(s) of any point(s) of inflection. What should his final answer be?
- (A) $x = 1$
 - (B) $x = 2$
 - (C) $x = 3$
 - (D) $x = 1$ or $x = 3$
8. What is the domain of the function $y = \frac{1}{\sqrt{7 - x}}$?
- (A) $(-\infty, 7)$
 - (B) $(-\infty, 7]$
 - (C) $(7, \infty)$
 - (D) $[7, \infty)$
9. Which one of the following is NOT true about the function $f(x) = |6 - 2x|$?
- (A) $f(x) \geq 0$ for all x
 - (B) The graph of f is continuous everywhere
 - (C) There is a local minimum at $x = 3$
 - (D) The graph of f is differentiable everywhere

The paper continues on the next page

10. A particle moves along a straight line. Its velocity v at time t is shown in the graph below.



For what value of t is the displacement of the particle a maximum?

- (A) 1
- (B) 3
- (C) 5
- (D) 7

End of Section I

The paper continues in the next section

QUESTION ELEVEN (2 marks)

Marks

Find $\int \frac{2x + 3}{x^2} dx$.

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QUESTION TWELVE (2 marks)

Marks

Solve $|2x - 1| > 5$.

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QUESTION THIRTEEN (3 marks)

Marks

Find the equation of the tangent to $y = \ln(2x - 5)$ at $x = 3$.

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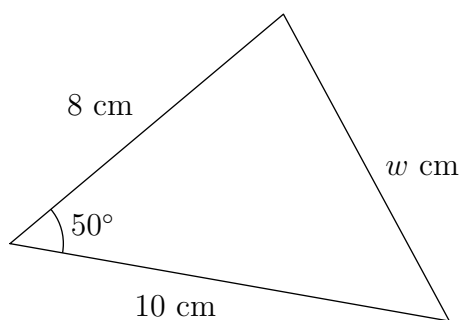
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QUESTION FOURTEEN (2 marks)

Marks

The diagram shows a triangle with sides of length w cm, 8 cm and 10 cm along with an angle of 50° .

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Use the cosine rule to calculate the value of w , correct to two significant figures.

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QUESTION FIFTEEN (3 marks)

Marks

Solve $\sqrt{3} \tan x = -1$ on the domain $0 \leq x \leq 3\pi$.

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QUESTION SIXTEEN (2 marks)

Marks

If $f'(x) = 6x^2 + 3$ and $f(2) = 8$, find $f(x)$.

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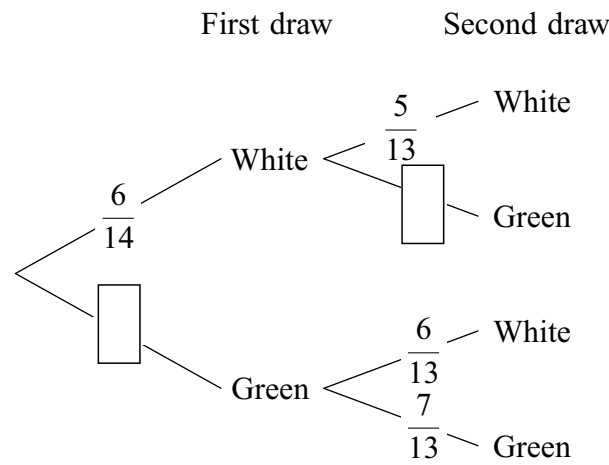
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QUESTION SEVENTEEN (3 marks)

Marks

A bag contains 8 green balls and 6 white balls. Two balls are selected at random without replacement. A partially completed tree diagram is shown below.

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Complete the tree diagram and calculate the probability of selecting two balls of different colours.

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QUESTION EIGHTEEN (3 marks)

Marks

On his 1st birthday, Timmy was given \$5 by his Aunty Ruth. On each subsequent birthday, Aunty Ruth gave Timmy \$2 more than the previous year.

- (a) How much did Aunty Ruth give Timmy on his 20th birthday?

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- (b) If Timmy saves the money every year, how old will he be when he has received over \$100 in total?

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QUESTION NINETEEN (3 marks)

Marks

The bank has provided Jocelyn with the following two brief annuity tables to help her make financial decisions.

Future value of an annuity with instalments of \$1

Periods n	<i>Interest rate per period</i>				
	1%	2%	3%	4%	5%
5	5.1010	5.2040	5.3091	5.4163	5.5256
10	10.4622	10.9497	11.4639	12.0061	12.5779
15	16.0969	17.2934	18.5989	20.0236	21.5786
20	22.0190	24.2974	26.8704	29.7781	33.0660

Present value of an annuity with instalments of \$1

Periods n	<i>Interest rate per period</i>				
	1%	2%	3%	4%	5%
5	4.8534	4.7135	4.5797	4.4518	4.3295
10	9.4713	8.9826	8.5302	8.1109	7.7217
15	13.8651	12.8493	11.9379	11.1184	10.3797
20	18.0456	16.3514	14.8775	13.5903	12.4622

Jocelyn pays \$5000 per annum into her superannuation scheme, which pays 4% per annum, compounded yearly.

- (a) What is the total value of her fund at the end of 20 years? Give your answer correct to the nearest dollar. 1

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- (b) Sammy wins the lottery and invests all \$80000 at 4% per annum for 20 years, compounded annually. How much would Jocelyn need to increase her annual repayments to match Sammy’s payout after 20 years? Give your answer correct to the nearest dollar. 2

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QUESTION TWENTY (6 marks)

Marks

(a) Solve the equation $x^2 - 6 = x$.

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(b) In the space below, sketch the graphs of $y = x^2 - 6$ and $y = x$. Label the coordinates of any x or y intercepts and any points of intersection of the graphs.

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(c) Find the values of x for which $x^2 - 6 > x$.

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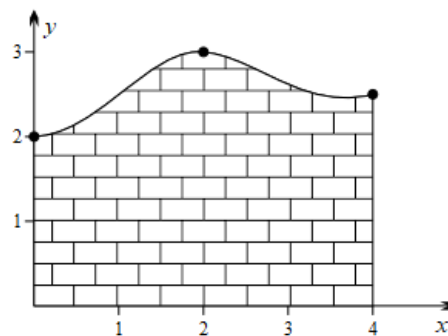
QUESTION TWENTY-ONE (2 marks)

Marks

The height y of a sculptured brick wall at 3 points is tabulated below. Both x and y are measured in metres. Use the trapezoidal rule to estimate the cross-sectional area of the wall.

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x	0	2	4
y	2	3	2.5



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QUESTION TWENTY-TWO (9 marks)

Marks

Consider the function $f(x) = \frac{x}{1+x^2}$.

(a) Determine algebraically whether the function is even, odd, or neither.

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(b) Show that $f'(x) = \frac{1-x^2}{(1+x^2)^2}$.

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(c) Determine the coordinates of the stationary points of $y = f(x)$ and verify their nature.

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(d) Sketch the graph of $y = f(x)$, showing the information found in parts (a) and (c). You need not show points of inflection, but any asymptotes should be clear. 3

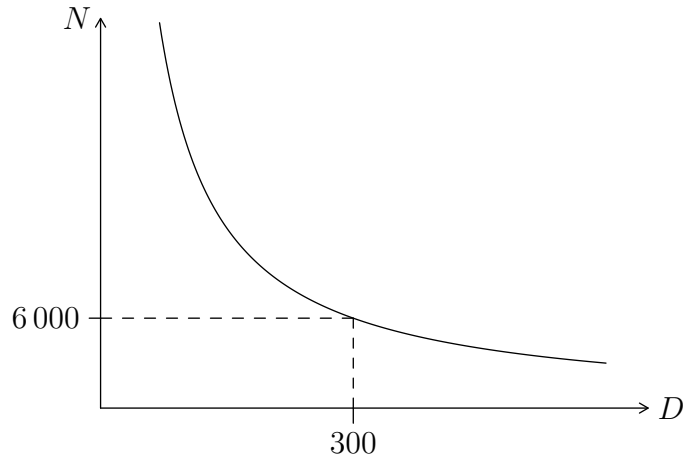
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QUESTION TWENTY-THREE (5 marks)

Marks

A concert is to be held. The number of people expected to attend N varies inversely with distance D metres from the concert to the city centre.

If the distance from the city centre is 300 m then 6000 people are expected to attend, as shown below.



- (a) Find an equation for the graph in terms of N and D in the form $N = \frac{k}{D}$, where k is a constant. 1

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- (b) The concert needs an attendance of at least 2000 in order to make a profit. Find the maximum distance the concert can be from the city centre and still make a profit. 1

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- (c) At a point on the graph, the number of people attending is decreasing at the rate of 5 people per 4 metres. Find the value of D for this point. 3

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QUESTION TWENTY-FOUR (6 marks)

Marks

A experimenter records the following data:

4 6 8 8 9 10 12 12 17 24

- (a) Use the standard criterion for outliers on your reference sheet to determine any outliers in the data. 3

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- (b) Find the mean and variance for the dataset. 2

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- (c) Write down a dataset with the same variance but a mean of 15. 1

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QUESTION TWENTY-FIVE (4 marks)

Marks

A probability distribution function $p(x)$ is tabulated below.

4

x	1	2	3	4
$p(x)$	$2m$	$1 - 8m$	$4m$	0.1

Find the value of the constant m and hence find the expected value $E(X)$.

Find m :.....

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Find $E(X)$:.....

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QUESTION TWENTY-SIX (5 marks)

Marks

A 20 kg block of a radioactive isotope is decaying, and the amount M kg remaining after t years is given by the equation

$$M = 20e^{-0.05t}$$

- (a) Find the mass of the isotope remaining after 2 years. Give your answer correct to 3 significant figures. 1

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- (b) How long until half the isotope has decayed? Give your answer correct to 3 significant figures. 2

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- (c) Show that M satisfies the equation $\frac{dM}{dt} = -0.05M$ and hence find the rate of decay when 5 kg of the isotope remains. Give your answer correct to 2 significant figures. 2

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QUESTION TWENTY-SEVEN (5 marks)

Marks

- (a) Sketch the graphs of $y = 10 \sin \frac{\pi}{6}x$ and $y = x$ for $0 \leq x \leq 12$ in the space provided below. Mark the point of intersection at $(5, 5)$. 2

- (b) Find a simplified exact expression for the area bounded by the two graphs for $0 \leq x \leq 5$. 3

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QUESTION TWENTY-EIGHT (4 marks)

Marks

Data from 200 pupils is tabulated below. The data gathered indicates which hand they write with and their favourite colour.

	Green	Red	Blue	Total
Left	17	8	21	46
Right	44	71	39	154
Total	61	79	60	200

A pupil is selected at random.

- (a) Find the probability that the pupil is right-handed.

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- (b) Find the probability that the pupil prefers green, given that they are left-handed.

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- (c) Are the events of being left-handed and preferring green independent? Justify your answer with a calculation.

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QUESTION TWENTY-NINE (6 marks)

Marks

Janice deposits \$5000 into her savings account every year on January 1. During the year, the account receives compound interest of 6% per annum, where interest is added to her account at the end of each year.

Let J_n be the value of Janice’s account immediately after her n th investment.

(a) Show that $J_2 = \$10\,300$.

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(b) Show that

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$$J_n = \frac{250\,000(1.06^n - 1)}{3}$$

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(c) Determine the minimum number of years until Janice’s account contains \$100 000.

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QUESTION THIRTY (5 marks)

Marks

Let $y = xe^{x^2-3x}$.

(a) Show that

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$$\frac{dy}{dx} = (2x^2 - 3x + 1)e^{x^2-3x}$$

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(b) For what values of x is y decreasing?

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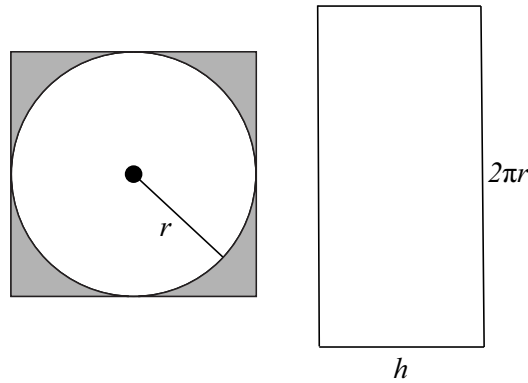
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QUESTION THIRTY-ONE (6 marks)

Marks

A company is manufacturing barrels with a constant volume specified by the customer. The barrels are open cylinders with a circular base but no top. Each barrel has height h metres, radius r metres and volume V cubic metres.

In the manufacturing process, the circular base and the material for the curved side are cut from flat sheets of metal, which are then folded to form the open cylinder. The metal used to construct a barrel is shown by the total area in the diagram below. After the circular base is removed, the remaining metal is wasted.



- (a) Show that the area of the sheet metal required to construct a barrel is given by

2

$$A = \frac{2V}{r} + 4r^2$$

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Section I

Questions in this section are multiple-choice.

Choose a single best answer for each question and record it on the provided answer sheet.

1. Which of the following correctly expresses y as the subject of the formula $2x - 5y + 3 = 0$?

(A) $y = 2x + 3$

(B) $y = \frac{2}{5}x + 3$

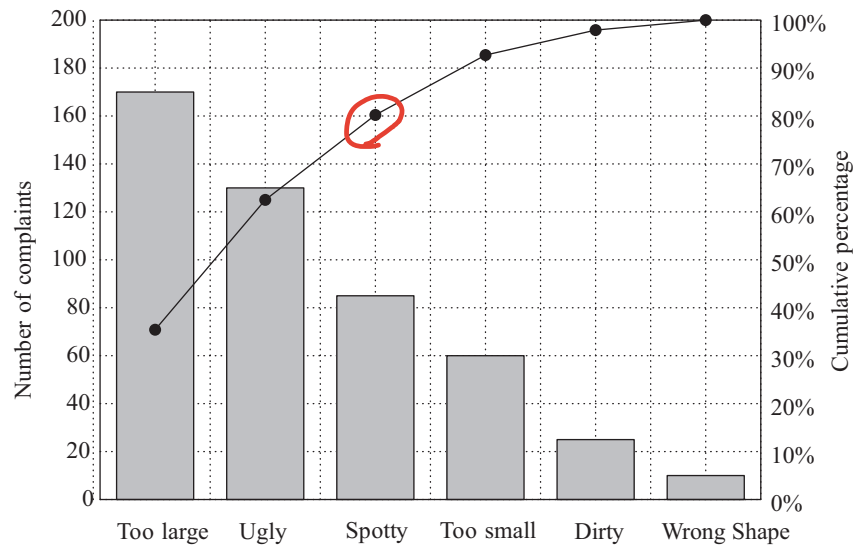
(C) $y = \frac{2}{5}x + \frac{3}{5}$

(D) $y = \frac{2}{5}x - \frac{3}{5}$

$$5y = 2x + 3$$

$$\left(\frac{\cdot}{5}\right) y = \frac{2}{5}x + \frac{3}{5}$$

2. The number of complaints for the return of a particular item is shown in the Pareto chart below.



What percentage of the total number of complaints do the three largest complaints account for?

(A) 60%

(B) 70%

(C) 80%

(D) 90%

3. A particle is moving with velocity $v = t^2 - 10t + 21$, $t > 0$. When is the particle stationary?

- (A) When $t = 3$.
 (B) When $t = 5$.
 (C) When $t = 3$ or $t = 7$.
 (D) When $t = 5$ or $t = 7$.

$$\begin{aligned} v &= 0 \\ t^2 - 10t + 21 &= 0 \\ (t - 7)(t - 3) &= 0 \\ t &= 3, 7 \end{aligned}$$

4. A pupil graphs the following relations. Which relation is many-to-one?

- (A) $x^2 + y^2 = 4$
 (B) $y = \sqrt{4 - x^2}$ ← upper semicircle
 (C) $y = x^3 + 3$
 (D) $y = 2x - 4$

5. What amount must be invested now at 5% per annum, compounded monthly, so that in four years time it will have grown to \$50 000?

- (A) \$38 772
 (B) \$39 176
 (C) \$40 954
 (D) \$41 135

$$\begin{aligned} P &= A(1+r)^n \\ A &= \frac{P}{(1+r)^n} \\ &= \frac{50000}{\left(1 + \frac{0.05}{12}\right)^{48}} \end{aligned}$$

6. The graph $y = x^2 + 4x + 7$ is reflected in the y -axis, followed by a translation 2 units to the right. What is the equation of the new graph?

- (A) $y = (-x - 2)^2 + 4(-x - 2) + 7$
 (B) $y = (-x + 2)^2 + 4(-x + 2) + 7$
 (C) $-y = (x + 2)^2 + 4(x + 2) + 7$
 (D) $-y = (x - 2)^2 + 4(x - 2) + 7$

$$\begin{aligned} y &= x^2 + 4x + 7 \\ &\downarrow \text{reflect in } y\text{-axis} \\ y &= (-x)^2 + 4(-x) + 7 \\ &= x^2 - 4x + 7 \\ &\downarrow \text{translate 2 right} \\ y &= (x - 2)^2 - 4(x - 2) + 7 \\ &= (-x + 2)^2 + 4(-x + 2) + 7 \end{aligned}$$

7. The function $y = f(x)$ has second derivative $y'' = 2(x-1)^2(x-3)$. A pupil is asked to find the x -coordinate(s) of any point(s) of inflection. What should his final answer be?

(A) $x = 1$

(B) $x = 2$

(C) $x = 3$

(D) $x = 1$ or $x = 3$

 y'' will change sign about $x=3$ y'' will not change sign about $x=1$

8. What is the domain of the function $y = \frac{1}{\sqrt{7-x}}$?

(A) $(-\infty, 7)$

(B) $(-\infty, 7]$

(C) $(7, \infty)$

(D) $[7, \infty)$

$7-x \neq 0$

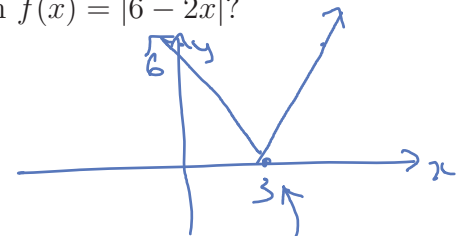
and $7-x > 0$

i.e. $7-x > 0$

$x < 7$

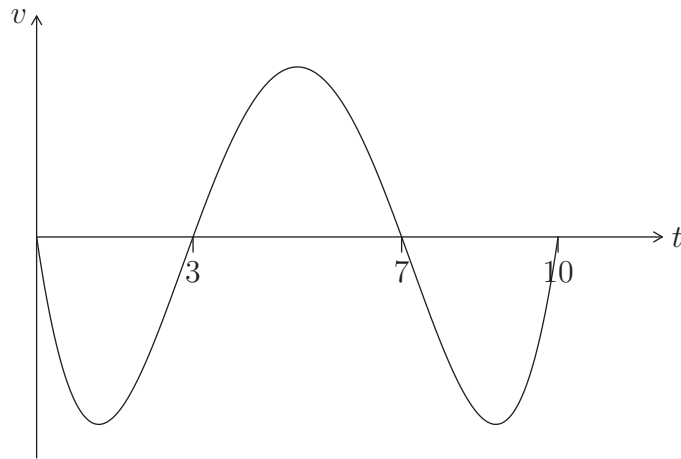
9. Which one of the following is NOT true about the function $f(x) = |6 - 2x|$?

(A) $f(x) \geq 0$ for all x

(B) The graph of f is continuous everywhere(C) There is a local minimum at $x = 3$ (D) The graph of f is differentiable everywhere"sharp" at $x=3$

The paper continues on the next page

10. A particle moves along a straight line. Its velocity v at time t is shown in the graph below.



For what value of t is the displacement of the particle a maximum?

- (A) 1
- (B) 3
- (C) 5
- (D) 7

End of Section I

The paper continues in the next section

QUESTION ELEVEN (2 marks)

Marks

Find $\int \frac{2x+3}{x^2} dx$.

2

$$\int \frac{2x+3}{x^2} dx = \int \left(\frac{2}{x} + \frac{3}{x^2} \right) dx \quad \checkmark$$

$$= \int (2x^{-1} + 3x^{-2}) dx$$

$$= 2 \ln x + \frac{3x^{-1}}{-1} + C$$

$$= 2 \ln x - \frac{3}{x} + C \quad \checkmark$$

QUESTION TWELVE (2 marks)

Marks

Solve $|2x - 1| > 5$.

2

$$2x - 1 > 5 \quad \text{or} \quad 2x - 1 < -5 \quad \checkmark$$

$$x > 3 \quad \text{or} \quad x < -2 \quad \checkmark$$

QUESTION THIRTEEN (3 marks)

Marks

Find the equation of the tangent to $y = \ln(2x - 5)$ at $x = 3$.

3

$$y' = \frac{2}{2x-5}$$

$$= \frac{2}{6-5} \quad \text{at } x=3$$

$$= 2$$

$$\text{at } x=3, y = \ln(1) = 0$$

$$y - y_1 = m(x - x_1)$$

$$y - 0 = 2(x - 3)$$

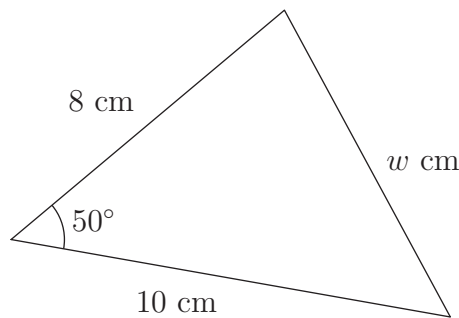
$$y = 2x - 6$$

QUESTION FOURTEEN (2 marks)

Marks

The diagram shows a triangle with sides of length w cm, 8 cm and 10 cm along with an angle of 50° .

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Use the cosine rule to calculate the value of w , correct to two significant figures.

$$w^2 = 8^2 + 10^2 - 2 \times 8 \times 10 \times \cos 50^\circ$$

$$= 64 + 100 - 160 \cos 50^\circ$$

$$= 7.8 \quad (2 \text{ sig. figs.})$$

QUESTION FIFTEEN (3 marks)

Marks

Solve $\sqrt{3} \tan x = -1$ on the domain $0 \leq x \leq 3\pi$.

3

$$\tan x = -\frac{1}{\sqrt{3}}$$

✓ related angle

$$x = \pi - \frac{\pi}{6} \text{ or } 2\pi - \frac{\pi}{6} \text{ or } 3\pi - \frac{\pi}{6}$$

$$= \frac{5\pi}{6}, \frac{11\pi}{6} \text{ or } \frac{17\pi}{6}$$

QUESTION SIXTEEN (2 marks)

Marks

(a) If $f'(x) = 6x^2 + 3$ and $f(2) = 8$, find $f(x)$.

2

$$f(x) = \frac{6x^3}{3} + 3x + C$$

$$8 = 2(2)^3 + 3(2) + C$$

$$8 = 16 + 6 + C$$

$$C = 8 - 16 - 6$$

$$= -14$$

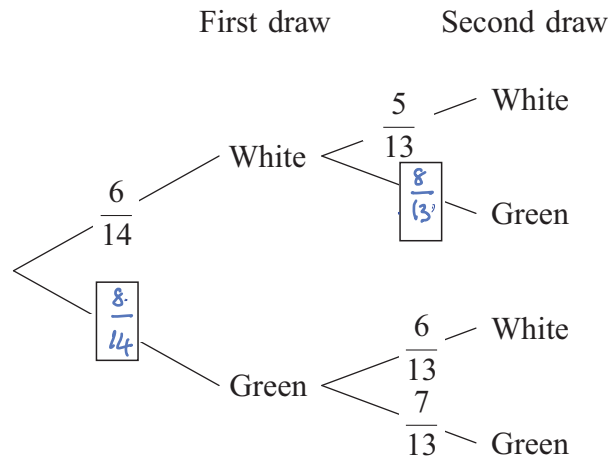
$$f(x) = 2x^3 + 3x - 14$$

QUESTION SEVENTEEN (3 marks)

Marks

A bag contains 8 green balls and 6 white balls. Two balls are selected at random without replacement. A partially completed tree diagram is shown below.

3



Complete the tree diagram and calculate the probability of selecting two balls of different colours.

$$P(2 \text{ diff. colours}) = P(WG \text{ or } GW)$$

$$= \frac{6}{14} \times \frac{8}{13} + \frac{8}{14} \times \frac{6}{13}$$

$$= \frac{48}{91}$$



The paper continues on the next page

QUESTION EIGHTEEN (3 marks)

Marks

On his 1st birthday, Timmy was given \$5 by his Aunty Ruth. On each subsequent birthday, Aunty Ruth gave Timmy \$2 more than the previous year.

- (a) How much did Aunty Ruth give Timmy on his 20th birthday?

1

$$\begin{aligned}
 T_n &= a + (n-1)d \\
 &= 5 + (20-1) \times 2 \\
 &= 5 + 38 \\
 &= 43
 \end{aligned}$$

Timmy was given \$43.

- (b) If Timmy saves the money every year, how old will he be when he has received over \$100 in total?

2

$$S_n = \frac{n}{2}(2a + (n-1)d) > 100$$

$$\frac{n}{2}(10 + (n-1)2) > 100$$

$$n(n+4) > 100$$

By trial & error, $n > 8$

He has received over \$100 after his 9th Birthday.

QUESTION NINETEEN (3 marks)

Marks

The bank has provided Jocelyn with the following two brief annuity tables to help her make financial decisions.

Future value of an annuity with instalments of \$1

Periods n	Interest rate per period				
	1%	2%	3%	4%	5%
5	5.1010	5.2040	5.3091	5.4163	5.5256
10	10.4622	10.9497	11.4639	12.0061	12.5779
15	16.0969	17.2934	18.5989	20.0236	21.5786
20	22.0190	24.2974	26.8704	29.7781	33.0660

Present value of an annuity with instalments of \$1

Periods n	Interest rate per period				
	1%	2%	3%	4%	5%
5	4.8534	4.7135	4.5797	4.4518	4.3295
10	9.4713	8.9826	8.5302	8.1109	7.7217
15	13.8651	12.8493	11.9379	11.1184	10.3797
20	18.0456	16.3514	14.8775	13.5903	12.4622

Jocelyn pays \$5000 per annum into her superannuation scheme, which pays 4% per annum, compounded yearly.

- (a) What is the total value of her fund at the end of 20 years? Give your answer correct to the nearest dollar. 1

$$5000 \times 29.7781 = \$148891$$

- (b) Sammy wins the lottery and invests all \$80000 at 4% per annum for 20 years, compounded annually. How much would Jocelyn need to increase her annual repayments to match Sammy's payout after 20 years? Give your answer correct to the nearest dollar. 2

Method 1: Using FV table

$$\text{Payment} \times 29.7781 = 80000 \times (1.04)^{20}$$

$$\text{Payment} = \$5887$$

Method 2 Using PV table

$$\text{Payment} = 80000 \div 13.5903$$

$$= \$5887$$

Jocelyn needs to pay \$877 more.

QUESTION TWENTY (6 marks)

Marks

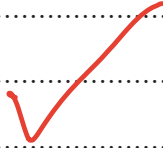
(a) Solve the equation $x^2 - 6 = x$.

1

$$x^2 - x - 6 = 0$$

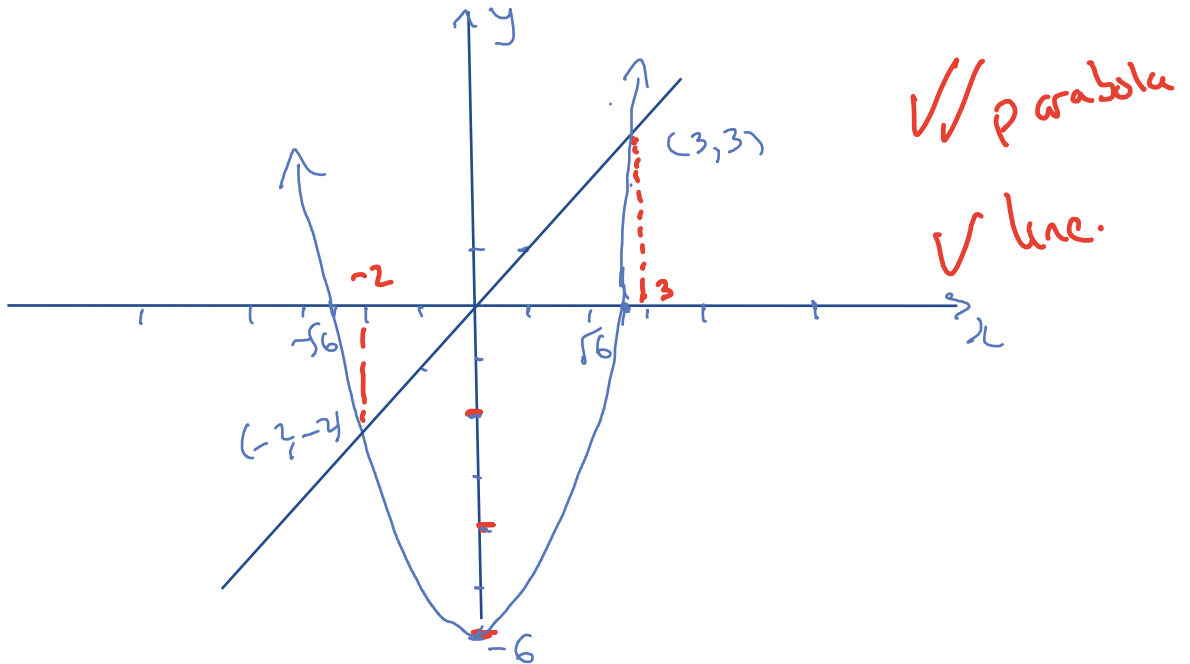
$$(x - 3)(x + 2) = 0$$

$$x = 3 \text{ or } -2$$



(b) In the space below, sketch the graphs of $y = x^2 - 6$ and $y = x$. Label the coordinates of any x or y intercepts and any points of intersection of the graphs.

3



(c) Find the values of x for which $x^2 - 6 > x$.

2

$$x < -2 \text{ or } x > 3$$



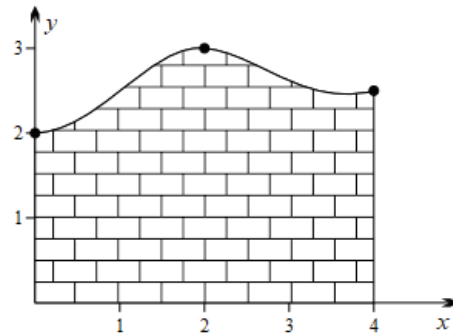
QUESTION TWENTY-ONE (2 marks)

Marks

The height y of a sculptured brick wall at 3 points is tabulated below. Both x and y are measured in metres. Use the trapezoidal rule to estimate the cross-sectional area of the wall.

2

x	0	2	4
y	2	3	2.5



$$\begin{aligned} \text{Area} &\doteq \frac{h}{2} (f(a_1) + 2f(a_2) + f(a_3)) \\ &= \frac{2}{2} (2 + 2 \times 3 + 2.5) \\ &= 10.5 \text{ m}^2 \end{aligned}$$

The paper continues on the next page

QUESTION TWENTY-TWO (9 marks)

Marks

Consider the function $f(x) = \frac{x}{1+x^2}$.

- (a) Determine algebraically whether the function is even, odd, or neither.
- 1

$$f(-x) = \frac{(-x)}{1+(-x)^2}$$

$$= \frac{-x}{1+x^2}$$

$$= -f(x)$$

$f(x)$ is odd. ✓

- (b) Show that
- $f'(x) = \frac{1-x^2}{(1+x^2)^2}$
- .
- 2

$$f'(x) = \frac{1 \cdot x(1+x^2) - x(2x)}{(1+x^2)^2}$$

$$= \frac{1+x^2-2x^2}{(1+x^2)^2}$$

$$= \frac{1-x^2}{(1+x^2)^2}$$

(quotient rule) ✓

- (c) Determine the coordinates of the stationary points of
- $y = f(x)$
- and verify their nature.
- 3

$$1-x^2 = 0$$

$$x = \pm 1 \quad \checkmark$$

$$\text{When } x=1, y = \frac{1}{2} \quad \checkmark$$

$$x=-1, y = -\frac{1}{2} \quad \checkmark$$

x	-2	-1	0	1	2
$f'(x)$	$-\frac{3}{25}$	0	1	0	$-\frac{3}{25}$

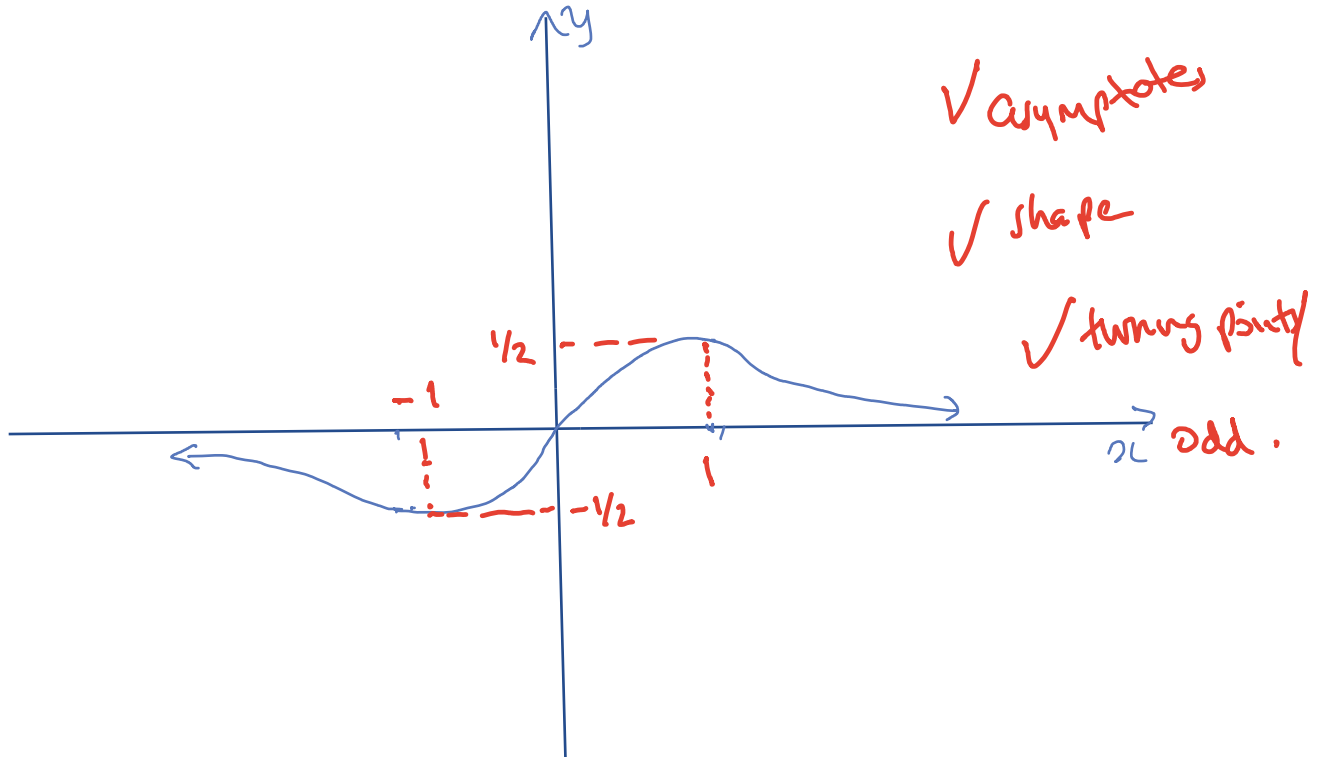
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$(1, \frac{1}{2})$ is a local minimum

$(-1, -\frac{1}{2})$ is a local maximum

Conclusion required

- (d) Sketch the graph of $y = f(x)$, showing the information found in parts (a) and (c). You need not show points of inflection, but any asymptotes should be clear. 3



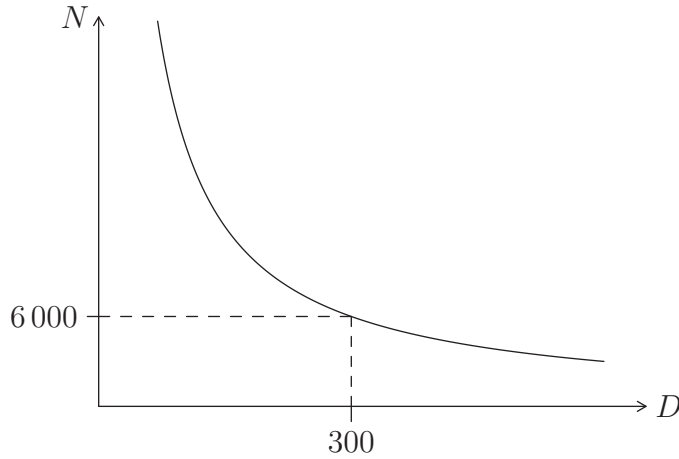
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QUESTION TWENTY-THREE (5 marks)

Marks

A concert is to be held. The number of people expected to attend N varies inversely with distance D metres from the concert to the city centre.

If the distance from the city centre is 300m then 6000 people are expected to attend, as shown below.



- (a) Find an equation for the graph in terms of N and D in the form $N = \frac{k}{D}$, where k is a real constant. 1

When $D=300, N=6000$

$$6000 = \frac{k}{300}$$

$$k = 18 \times 10^5$$

$$N = \frac{1800000}{D}$$

- (b) The concert needs an attendance of at least 2000 in order to make a profit. Find the maximum distance the concert can be from the city centre and still make a profit. 1

Need $\frac{1800000}{D} \geq 2000$

$$D \leq 900$$

- (c) At a point on the graph, the number of people attending is decreasing at the rate of 5 people per 4 metres. Find the value of D for this point. 3

$$N = \frac{k}{D}$$

$$N' = -\frac{k}{D^2}$$

$$D^2 = -\frac{k}{N'}$$

$$= -\frac{1800000}{-1.25}$$

$$= +1440000 \text{ m}$$

$$D = 1200 \text{ m}$$

$$= 1.2 \text{ km}$$

QUESTION TWENTY-FOUR (6 marks)

Marks

A experimenter records the following data:

4 6 (8) 8 9 | 10 12 (12) 17 24

- (a) Use the standard criterion for outliers on your reference sheet to determine any outliers in the data. 3

$$IQR = 12 - 8$$

$$= 4$$

Outlier if $> 12 + 1.5 \times 4 = 18$ ✓
 or if $< 8 - 1.5 \times 4 = 2$ ✓
 thus 24 is an outlier. ✓

- (b) Find the mean and variance for the dataset. 2

From a calculator, $\bar{x} = 11$ ✓

$$\sigma_n \doteq 5.5$$

$$\text{and variance} = \sigma_n^2$$

$$= 30.4$$

- (c) Write down a dataset with the same variance but mean 15. 1

Add 4 to all values

8 10 12 12 13 14 16 16 21 28 ✓

QUESTION TWENTY-FIVE (4 marks)

Marks

A probability distribution function $p(x)$ is tabulated below.

4

x	1	2	3	4
$p(x)$	$2m$	$1 - 8m$	$4m$	0.1

Find the value of the constant m and hence find the expected value $E(X)$.Find m : We need $p(x) \geq 0$ and $\sum p(x) = 1$

$$2m + 1 - 8m + 4m + 0.1 = 1$$

$$-2m + 1.1 = 1$$

$$2m = 0.1$$

$$m = 0.05$$

Find $E(X)$:

x	1	2	3	4
$p(x)$	0.1	0.6	0.2	0.1

$$E(X) = 1 \times 0.1 + 2 \times 0.6 + 3 \times 0.2 + 4 \times 0.1$$

$$= 0.1 + 1.2 + 0.6 + 0.4$$

$$= 2.3$$

The paper continues on the next page

QUESTION TWENTY-SIX (5 marks)

Marks

A 20 kg block of a radioactive isotope is decaying, and the amount M kg remaining after t years is given by the equation

$$M = 20e^{-0.05t}$$

- (a) Find the mass of the isotope remaining after 2 years. Give your answer correct to 3 significant figures. 1

$$M = 20e^{-0.05 \times 2}$$

$$= 18.1 \text{ kg}$$

- (b) How long will it be until half the isotope has decayed? Give your answer correct to 3 significant figures. 2

$$10 = 20e^{-0.05t}$$

$$-0.05t = \ln \frac{1}{2}$$

$$t = \frac{\ln 2}{0.05}$$

$$\approx 13.9 \text{ y}$$

After 13.9 years.

- (c) Show that M satisfies the equation $\frac{dM}{dt} = -0.05M$ and hence find the rate of decay when 5 kg of the isotope remains. Give your answer correct to 2 significant figures. 2

$$M = 20e^{-0.05t}$$

$$\frac{dM}{dt} = 20 \times -0.05e^{-0.05t}$$

$$= -0.05(20e^{-0.05t})$$

$$= -0.05M$$

when $M = 5 \text{ kg}$,

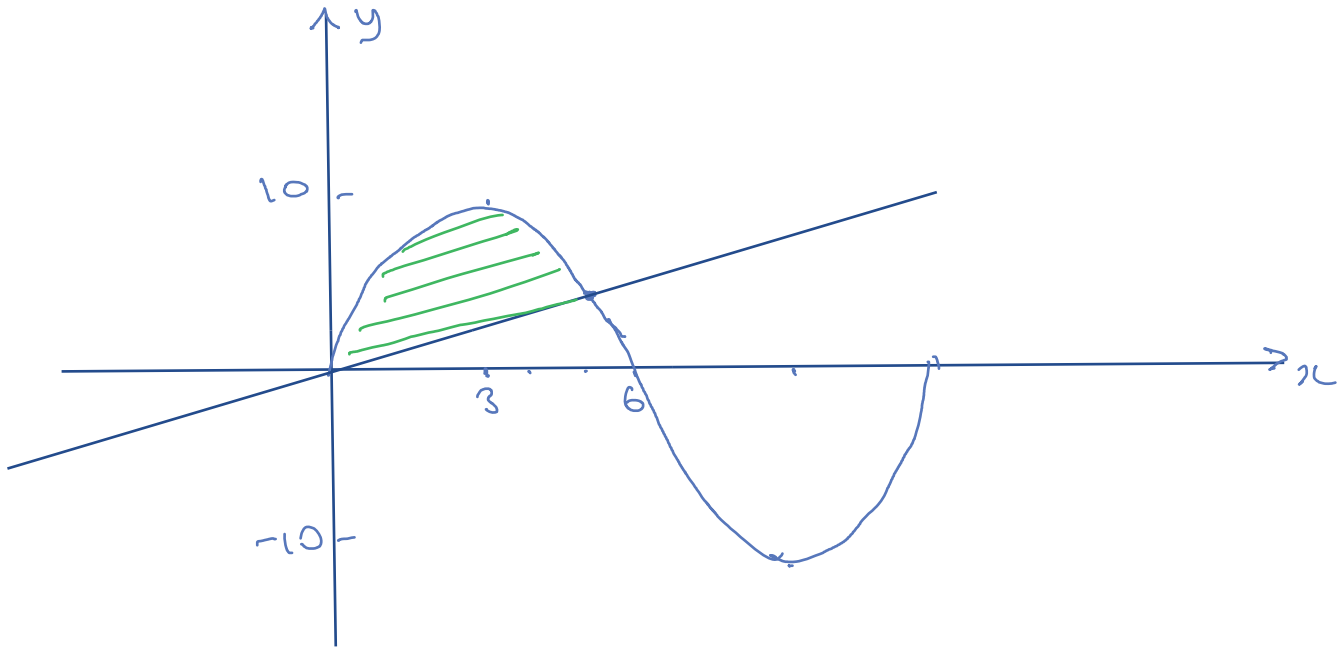
$$\frac{dM}{dt} = -0.05 \times 5$$

$$= -0.25 \text{ kg/y}$$

QUESTION TWENTY-SEVEN (5 marks)

Marks

- (a) Sketch the graphs of $y = 10 \sin \frac{\pi}{6}x$ and $y = x$ for $0 \leq x \leq 12$ in the space provided 2
 below. Mark the point of intersection at $(5, 5)$.



- (b) Find a simplified exact expression for the area bounded by the two graphs for $0 \leq x \leq 5$. 3

$$\int_0^5 \left(10 \sin \frac{\pi x}{6} - x \right) dx = \left[-10 \times \frac{6}{\pi} \cos \frac{\pi x}{6} - \frac{x^2}{2} \right]_0^5$$

$$= \left(-\frac{60}{\pi} \cos \frac{5\pi}{6} - \frac{25}{2} \right) - \left(-\frac{60}{\pi} - 0 \right)$$

$$= \frac{60\sqrt{3}}{\pi} - \frac{25}{2} + \frac{60}{\pi}$$

QUESTION TWENTY-EIGHT (4 marks)

Marks

Data from 200 pupils is tabulated below. The data gathered indicates which hand they write with and their favourite colour.

	Green	Red	Blue	Total
Left	17	8	21	46
Right	44	71	39	154
Total	61	79	60	200

A pupil is selected at random.

- (a) Find the probability that the pupil is right-handed. 1

$$\frac{154}{200} = \frac{77}{100} \text{ or } 0.77 \quad \checkmark$$

- (b) Find the probability that the pupil prefers green, given that they are left-handed. 1

$$P(\text{green} | \text{left}) = \frac{17}{46} \quad \checkmark$$

- (c) Are the events of being left-handed and preferring green independent? Justify your answer with a calculation. 2

$$P(\text{green}) = \frac{61}{200} \quad \checkmark \quad P(\text{green} | \text{left}) = \frac{17}{46}$$

Since these differ, the events are dependent. ✓

Alternative:

$$P(\text{green and left}) = \frac{17}{200} \quad P(\text{green}) \times P(\text{left}) = \frac{61}{200} \times \frac{46}{200}$$

Since these differ, the events are dependent

QUESTION TWENTY-NINE (6 marks)

Marks

Janice deposits \$5000 into her savings account every year on January 1. During the year, the account receives compound interest of 6% per annum, where interest is added to her account at the end of each year.

Let J_n be the value of Janice's account immediately after her n th investment.

(a) Show that $J_2 = \$10\,300$.

1

$$J_1 = 5000$$

$$J_2 = 5000 \times 1.06 + 5000$$

$$= 10300$$

(b) Show that

2

$$J_n = \frac{250\,000(1.06^n - 1)}{3}$$

$$J_2 = m \times 1.06 + m \quad \text{where } m = 5000 \text{ is the annual deposit}$$

$$J_3 = (m \times 1.06 + m) \times 1.06 + m$$

$$= m + 1.06m + 1.06^2m$$

!

$$J_n = m + 1.06m + 1.06^2m + \dots + 1.06^{n-1}m$$

$$= m \frac{(1.06^n - 1)}{1.06 - 1} \quad (\text{sum of a G.P., } n \text{ terms, } r = 1.06)$$

$$= \frac{250\,000}{3} (1.06^n - 1) \quad \text{using } m = 5000$$

Must follow from line above

(c) Determine the minimum number of years until Janice's account contains \$100 000.

3

Solve $J_n = 100\,000$, ie

$$100\,000 = \frac{25\,000}{3} (1.06^n - 1)$$

$$\frac{30}{25} = 1.06^n - 1$$

$$1.06^n = 1 + \frac{30}{25}$$

$$= 2.2$$

$$n = \log_{1.06} 2.2$$

$$= \frac{\log 2.2}{\log 1.06}$$

$$\approx 13.5$$

QUESTION THIRTY (5 marks)

Marks

on the payment of the 14th
installment at the start of the 14th year

Let $y = xe^{x^2-3x}$.

(a) Show that

2

$$\frac{dy}{dx} = (2x^2 - 3x + 1)e^{x^2-3x}$$

$$y' = 1 \times e^{x^2-3x} + x \times (2x-3) e^{x^2-3x}$$

$$= (1 + 2x^2 - 3x) e^{x^2-3x}$$

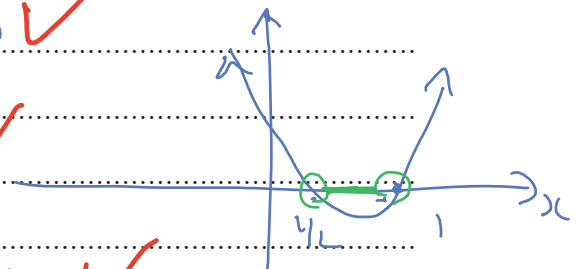
(b) For what values of x is y decreasing?

3

$$y' = 0 \text{ when } (2x^2 - 3x + 1) = 0$$

$$(2x-1)(x-1) = 0$$

$$x = \frac{1}{2}, 1$$



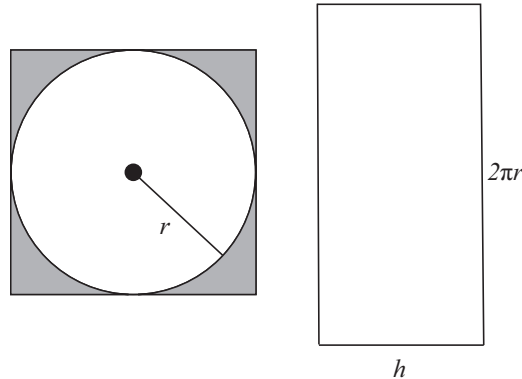
hence $y' < 0$ when $\frac{1}{2} < x < 1$

QUESTION THIRTY-ONE (6 marks)

Marks

A company is manufacturing barrels with a constant volume specified by the customer. The barrels are open cylinders with a circular base but no top. Each barrel has height h metres, radius r metres and volume V cubic metres.

In the manufacturing process, the circular base and the material for the curved side are cut from flat sheets of metal, which are then folded to form the open cylinder. The metal used to construct a barrel is shown by the total area in the diagram below. The shaded metal remaining after the circular base is removed is wasted.



- (a) Show that the area of the sheet metal required to construct a barrel is given by

2

$$A = \frac{2V}{r} + 4r^2$$

Area of metal required $A = (2r)^2 + 2\pi r \times h$ ✓
 but $V = \pi r^2 h$ $\therefore h = \frac{V}{\pi r^2}$

$$A = 4r^2 + 2\pi r \times \frac{V}{\pi r^2}$$
 ✓

$$= 4r^2 + \frac{2V}{r}$$

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- (b) Find the minimum area of metal A required to make a barrel. Your result will depend on V . 4

$$\frac{dA}{dr} = -\frac{2V}{r^2} + 8r$$

$$= 0 \text{ when } \frac{2V}{r^2} = 8r$$

$$r^3 = \frac{V}{4}$$

$$\frac{d^2A}{dr^2} = -2V \times -2r^{-3} + 8$$

$$= 8 + \frac{4V}{r^3}$$

$$= 8 + 4V \times \frac{4}{V}$$

$$= 24$$

> 0 \cup (concave up)

so we have a local minimum.

The minimum amount of metal required is

$$A = 4r^2 + \frac{2V}{r}$$

$$= \frac{4r^3 + 2V}{r}$$

$$= \frac{3V}{r}$$

$$= \frac{3V}{\sqrt[3]{\frac{V}{4}}}$$

$$= 3\sqrt[3]{4} \times 3V^{2/3}$$

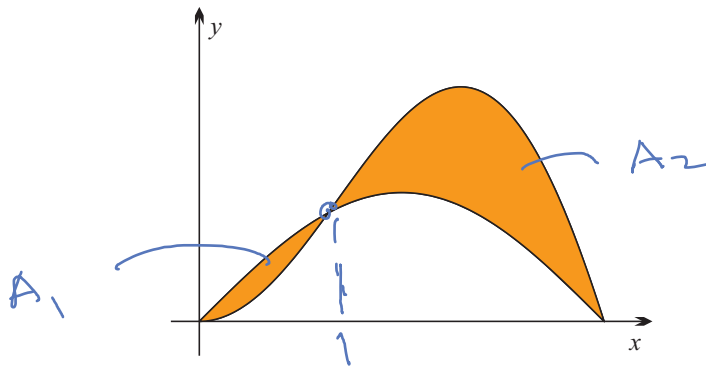
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QUESTION THIRTY-TWO (4 marks)

Marks

A company's new logo is the region bounded by $y = \sin x$ and $y = x \sin x$ for $0 \leq x \leq \pi$. 4

This region is shown shaded in the diagram below.



✓ split area
Correct integrals

Calculate the area of the logo, correct to 2 decimal places.

You may wish to assume the result

$$\frac{d}{dx}(x \cos x) = \cos x - x \sin x.$$

The hint tells us: $x \cos x = \int \cos x dx - \int x \sin x dx$
 $= \sin x - \int x \sin x dx$

hence $\int x \sin x dx = \sin x - x \cos x$ *

The curves intersect at $x = 1$ To calculate the area $0 < x < 1$ note that $\sin x > x \sin x$, so

$$A_1 = \int_0^1 (\sin x - x \sin x) dx$$

$$= \left[-\cos x - \sin x + x \cos x \right]_0^1 \quad \text{from } x$$

$$= (-\cos 1 - \sin 1 + 1 \cos 1) - (-\cos 0 - \sin 0 - 0 \cos 0)$$

$$= 1 - \sin 1$$

Similarly

$$A_2 = \int_1^\pi (x \sin x - \sin x) dx$$

$$= \left[\sin x - x \cos x + \cos x \right]_1^\pi$$

$$= (\sin \pi - \pi \cos \pi + \cos \pi) - (\sin 1 - \cos 1 + \cos 1)$$

$$= \pi - 1 - \sin 1$$

The total area of the logo is

$$A = A_1 + A_2$$

$$= | -\sin 1 | + \pi - | \sin 1 | - 1$$

$$= \pi - 2\sin 1$$

$$\doteq 1.46 \text{ units}^2$$