

Name: .....

Teacher: .....

Year 12  
**Mathematics**  
**Trial HSC**

**August, 2017**

*Time allowed: 3 hours plus 5 minutes reading time*

***General Instructions:***

- Reading time – 5 minutes
- Working time – 3 hours
- Marks for each question are indicated on the question.
- Approved calculators may be used
- All necessary working should be shown
- Full marks may not be awarded for careless work or illegible writing
- **Begin each question on a new page**
- Write using black or blue pen
- All answers are to be in the writing booklet provided
- A reference sheet is provided

**Total marks – 100**

**Section I – 10 Marks**

- Attempt Question 1-10 on the sheet provided
- Allow about 15 minutes for this section

**Section II – 90 Marks**

- Attempt Questions 11-16
- Allow about 2 hours and 45 minutes for this section

## Section I

10 marks

Attempt Questions 1–10

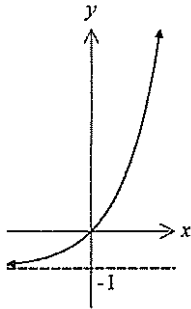
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1–10.

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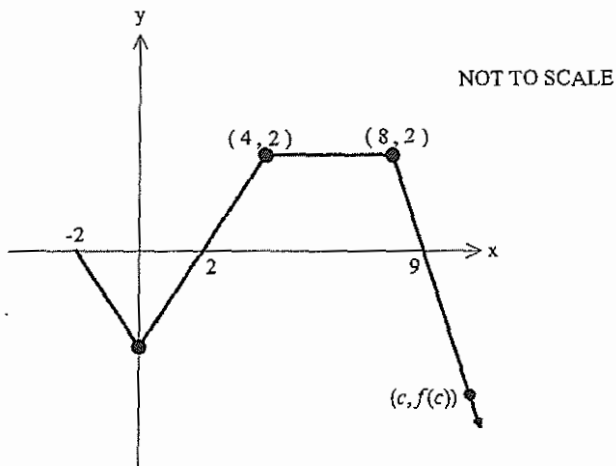
1. The solution to  $x^2 - 4 < -3$  is:
- (A)  $-2 < x < 2$
  - (B)  $x < -2, x > 2$
  - (C)  $-1 < x < 1$
  - (D)  $x < -1, x > 1$
2. An infinite geometric series has first term 4 and a limiting sum of 6.  
What is the common ratio?
- (A)  $\frac{1}{6}$
  - (B)  $\frac{1}{5}$
  - (C)  $\frac{1}{4}$
  - (D)  $\frac{1}{3}$
3. What is a possible primitive function for  $2x^{-4} + 5x$ ?
- (A)  $-\frac{2}{3x^3} + \frac{5x^2}{2} + 12$
  - (B)  $-\frac{1}{6x^3} + \frac{5x^2}{2}$
  - (C)  $\frac{2}{3x^3} + \frac{5x^2}{2} + 12$
  - (D)  $\frac{1}{6x^3} + \frac{5x^2}{2}$

4. The quadratic equation  $x^2 + 5x - 4 = 0$  has roots  $\alpha$  and  $\beta$ .  
What is the value of  $2\alpha^2\beta + 2\alpha\beta^2$ ?
- (A)  $-20$   
(B)  $40$   
(C)  $-40$   
(D)  $20$
5. What are the solutions of  $\tan 2\theta = 1$  for  $0 \leq \theta \leq 360^\circ$ ?
- (A)  $\theta = 45^\circ, 225^\circ$   
(B)  $\theta = 45^\circ, 225^\circ, 405^\circ, 585^\circ$   
(C)  $\theta = 22\frac{1}{2}^\circ, 112\frac{1}{2}^\circ$   
(D)  $\theta = 22\frac{1}{2}^\circ, 112\frac{1}{2}^\circ, 202\frac{1}{2}^\circ, 292\frac{1}{2}^\circ$
6. What is a possible equation for the following graph?



- (A)  $y = e^{x-1}$   
(B)  $y = e^x + 1$   
(C)  $y = e^x - 1$   
(D)  $y = e^{x+1}$

7. Consider the graph below:



For what value of  $C$  would  $\int_{-2}^C f(x) dx = -2$  be true?

- (A) 10
  - (B) 11
  - (C) 12
  - (D) 13
8. What is the value of  $\sum_{n=1}^5 n(n-1)$  ?

- (A) 50
- (B) 40
- (C) 30
- (D) 20

9. For what values of  $x$  is the curve  $f(x) = 2x^3 + x^2$  both concave down and decreasing?

(A)  $-\frac{1}{6} < x < 0$

(B)  $-3 < x < 0$

(C)  $-3 < x < -\frac{2}{12}$

(D)  $-\frac{1}{3} < x < -\frac{1}{6}$

10. A parabola has a focus  $(0,6)$  and directrix of  $y = 2$ .

What is the equation of the parabola?

(A)  $x^2 = -8(y - 4)$

(B)  $x^2 = -16(y - 5)$

(C)  $x^2 = 8(y - 4)$

(D)  $x^2 = 16(y - 5)$

## Section II

Total marks – 90

Attempt Questions 11–16

Allow about 2 hours 45 minutes for this section.

Begin each question on a NEW page.

In Questions 11–16, your responses should include relevant mathematical reasoning and/or calculations.

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Question 11 (15 marks) Begin a NEW page.

- a) Evaluate  $\frac{7.4^2 - e^2}{\sqrt{12 - \sqrt{2}}}$  to 4 significant figures. 2
- b) Rationalise the denominator of  $\frac{5\sqrt{2}}{2\sqrt{2} - 3}$  2
- c) Fully factorise  $x^6 - 27$ . 2
- d) Solve the equation  $|5 - x| = 3x$ . 2
- e) If  $\sin\theta = \frac{7}{10}$  and  $\tan\theta < 0$ , find the exact value of  $\sec\theta$ . 2
- f) Simplify  $\lim_{x \rightarrow \infty} \frac{2x^3 - 4x^2 + 7}{x^3 + 3x + 1}$  2
- g) Find the equation of the normal to the curve  $y = 4e^{2(x-1)}$  at  $x = 1$ . 3

End of Question 11

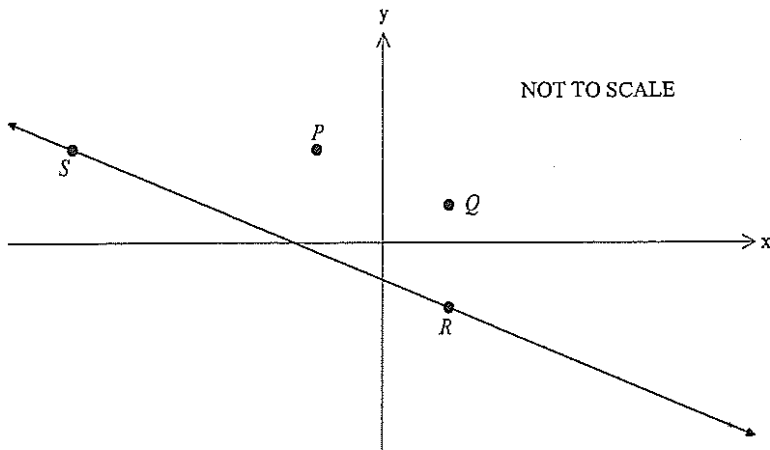
Question 12 (15 marks) Begin a NEW page.

- a) Differentiate the following with respect to  $x$
- i.  $(3x^2 + 4)^5$  2
  - ii.  $x^2 \tan x$  2
  - iii.  $\frac{\sin x}{e^{-x}}$  2
- b) Find the area under the curve  $y = |2x - 1|$  bounded by  $x = -4$  and  $x = 2$  2
- c) Sketch the curve  $y = 4 \sin(2x) + 1$  between  $-\pi \leq x \leq \pi$  3  
showing all important features (You DO NOT need to find  $x$ -intercepts).  
(Make your graph at least a third of a page)
- d) A function  $y = f(x)$  has  $\frac{dy}{dx} = 3x - 4$  and passes through  $(1, 4)$ . Find  $f(x)$ . 2
- e) Shade the region represented by the intersection of  $x^2 + (y - 3)^2 \leq 4$  2  
and  $x + y > 3$ .

End of Question 12

Question 13 (15 marks) Begin a NEW page.

- a) The points  $P(-3,5)$  and  $Q(3,2)$  are shown on the number plane below.



The equation of the line passing points S and R is  $y = -\frac{1}{2}x - 2$

- i. Find the gradient of  $PQ$ . Explain why PQRS is a trapezium. 2
- ii. Find the length of  $PQ$  in exact form. 2
- iii. Given that line  $QR$  is parallel to the  $y$ -axis, state the coordinates of R. 1
- iv. Find the perpendicular distance from  $P$  to the line  $RS$ . 2
- v. If the length of  $RS$  is  $\sqrt{95}$  units find the area of PQRS correct to 2 decimal places 2

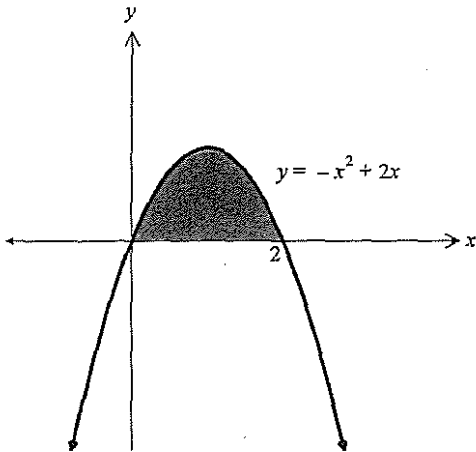
Question 13 continues on page 9



Question 13 (continued)

- b) The graph of  $y = -x^2 + 2x$  is shown below.

3



Find the volume of the solid of revolution formed when the shaded region is rotated about the  $x$ -axis.

- c) Given the function  $f(x) = 3^{\cos x}$

- i. Copy and complete the table for  $y = f(x)$  in your exam booklet.  
(Round your answers to 3 decimals places)

1

$x$	0	1	2	3	4
$y$	3.000				

- ii. Apply the Trapezoidal rule with 4 subintervals to find an approximation of

2

$$\int_0^4 3^{\cos x} dx$$

correct to 2 decimal places.

End of Question 13

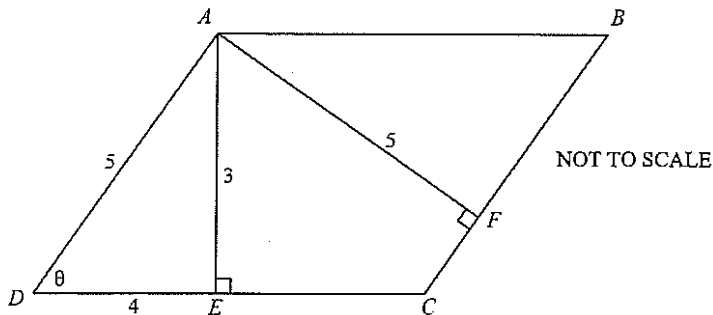
**Question 14** (15 marks) Begin a NEW page.

a) Consider the function

$$f(x) = \frac{x^2 + 15}{5x}$$

- i. Show that the function is odd. 1
- ii. Show that there is no value of  $x$  for which  $f(x) = 0$ . 1
- iii. State the vertical asymptote of  $y = f(x)$ . 1
- iv. Find the stationary points and determine their nature. 3
- v. Sketch the graph of  $y = f(x)$  showing all important features. 2

b) In the diagram below,  $ABCD$  is a parallelogram.



Copy the diagram into your booklet

- i. Prove that if  $\angle ADE = \theta$ , then  $\angle EAF = \theta$  (give reasons). 2
  - ii. Hence, using the cosine rule, find the exact length of  $EF$  2
- c) Find the value of  $m$  for which the equation  $(m - 4)x^2 - 6x + 7 = 0$  has one root twice the other. 3

**End of Question 14**

Question 15 (15 marks) Begin a NEW page.

a) Given the equation of the parabola  $4y - 20 = x^2 + 12x + 36$ :

- i. Find the coordinates of the vertex.
- ii. Find coordinates of the focus.
- iii. Find the equation of the directrix.

2

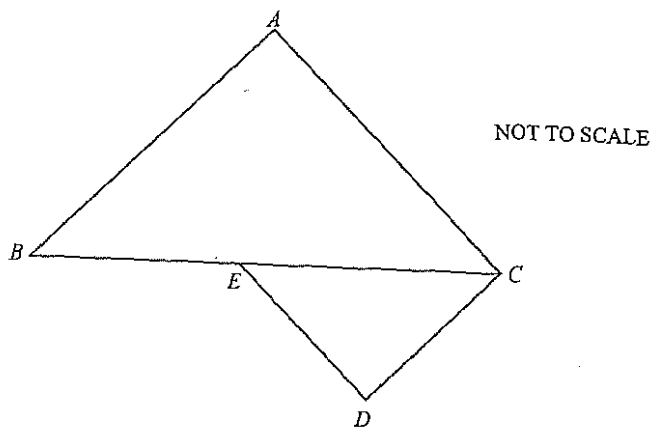
1

1

b) Find  $\int (10x - 4)^5 dx$ .

1

c) In the diagram  $CD \parallel AB$  and  $DE \parallel CA$ .  $AC = 15\text{cm}$ ,  $AB = 18\text{cm}$ ,  $CD = 8\text{cm}$  and  $BE = 12\text{cm}$ .



Copy the diagram into your booklet adding in all given information.

- i. Prove  $\triangle ABC \parallel\parallel \triangle DCE$
- ii. Hence find the length of  $BC$ .

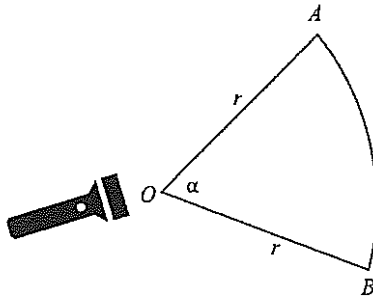
2

2

Question 15 continues on page 12

Question 15 (continued)

- d) The diagram below shows a sector  $OAB$  of a circle with centre  $O$  and a radius  $r$  cm created by the light of a torch.



- i. Show that the perimeter of the light sector  $OAB$  is  $r(2 + \alpha)$  1
- ii. Given that the perimeter of the light sector  $OAB$  is 6m, show that the area illuminated is given by: 2
- $$A = \frac{18\alpha}{(\alpha + 2)^2}$$
- iii. Hence show that the maximum illuminated area is  $2.25\text{m}^2$ . 3

End of Question 15

Question 16 (15 marks) Begin a NEW page.

a) Show that

3

$$\frac{\cos A}{1 + \sin A} + \frac{1 + \sin A}{\cos A} = \frac{2}{\cos A}$$

b) The 4<sup>th</sup> term of an arithmetic sequence is 18 and the sum of the first 10 terms is 195. Find the first term.

3

c) Mr Steve has a travel fund of \$55 000. The account accrues interest at 5.4% p.a. compounded monthly. He withdraws \$1 500 per month, after interest is paid, to pay for his travel adventures.

i. Show that the amount left at the end of the 2<sup>nd</sup> month is given by

2

$$A_2 = 55000 \times 1.0045^2 - 1500(1.0045 + 1)$$

ii. If  $A_n$  is the amount left after  $n$  months, show that:

2

$$A_n = 55000(1.0045)^n - 1500 \left[ \frac{1.0045^n - 1}{0.0045} \right]$$

iii. Hence find the number of months Mr Steve can travel before his funds run out.

2

iv. If after 12 months Mr Steve decides to travel overseas, and increases his withdrawals to \$3 000 per month, how many more months can he now afford to travel.

3

End of Paper

# Solutions

## MULTIPLE CHOICE ANSWER SHEET

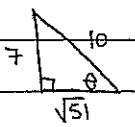


### Mathematics 2 unit Trial HSC August 2017

Completely fill the response oval representing the most correct answer.

Do not remove this sheet from the answer booklet.

1. A  B  C  D
2. A  B  C  D
3. A  B  C  D
4. A  B  C  D
5. A  B  C  D
6. A  B  C  D
7. A  B  C  D
8. A  B  C  D
9. A  B  C  D
10. A  B  C  D

Q11	test: $ 5 - \frac{5}{4}  = \frac{15}{4}$
a) 14.5596....	$3(\frac{5}{4}) = \frac{15}{4}$
$\approx 14.56$ (4 sig figs)	$\therefore \text{LHS} = \text{RHS}$
	$\therefore x = \frac{5}{4}$ only
b) $\frac{5\sqrt{2}}{2\sqrt{2}-3} \times \frac{2\sqrt{2}+3}{2\sqrt{2}+3}$	e) 
$= \frac{10 \times 2 + 15\sqrt{2}}{8-9}$	$\tan \theta < 0, \sin \theta > 0$ $\therefore$ quadrant 2.
$= \frac{20 + 15\sqrt{2}}{-1}$	$\sec \theta = -\frac{10}{\sqrt{51}}$ or $-\frac{10\sqrt{51}}{51}$
$= -20 - 15\sqrt{2}$	f. $\lim_{x \rightarrow \infty} \frac{2x^3 - 4x^2 + 7}{x^3 + 3x + 1} = 2$
c) $x^6 - 27$	g. $\frac{dy}{dx} = 8e^{2(x-1)}$
$= (x^2)^3 - 3^3$	at $x=1$
$= (x^2 - 3)(x^4 + 3x^2 + 9)$	$m_T = 8$ ; $m_N = -\frac{1}{8}$
d) $ 5-x  = 3x$	Using $m = -\frac{1}{8}$ and $(1, 4)$
$5-x = -3x$	eqn of normal
$2x = -5$	$y - 4 = -\frac{1}{8}(x-1)$
$x = -\frac{5}{2}$	$y - 4 = -\frac{x}{8} + \frac{1}{8}$
test: $ 5 + \frac{5}{2}  = \frac{15}{2}$	$y = -\frac{x}{8} + \frac{33}{8}$
$3(\frac{5}{2}) = -\frac{15}{2}$	$x + 8y - 33 = 0$
$\therefore \text{LHS} \neq \text{RHS}$	
$5-x = 3x$	
$5 = 4x$	
$x = \frac{5}{4}$	

Q12.

a) i.  $\frac{d}{dx} (3x^2+4)^5 = 5 \times 6x (3x^2+4)^4$   
 $= 30x (3x^2+4)^4$

ii.  $\frac{d}{dx} x^2 \tan x \Rightarrow$

$u = x^2 \quad v = \tan x$

$u' = 2x \quad v' = \sec^2 x$

$\frac{d}{dx} x^2 \tan x = 2x \tan x + x^2 \sec^2 x$

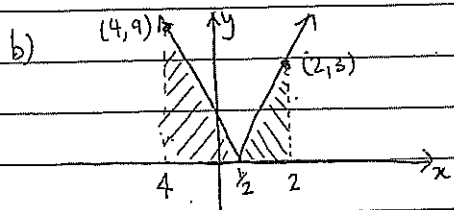
iii.  $\frac{d}{dx} \frac{\sin x}{e^{-x}}$

$u = \sin x \quad v = e^{-x}$

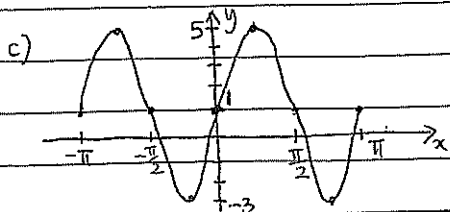
$u' = \cos x \quad v' = -e^{-x}$

$\frac{d}{dx} \frac{\sin x}{e^{-x}} = \frac{\cos x e^{-x} + e^{-x} \sin x}{(e^{-x})^2}$

$\frac{\cos x + \sin x}{e^{-x}}$



$\int_{-4}^2 |2x-1| dx = \frac{1}{2} \times 9 \times 9 + \frac{1}{2} \times 3 \times 3$   
 $= 22 \frac{1}{2} u^2$



d)  $\frac{dy}{dx} = 3x-4$

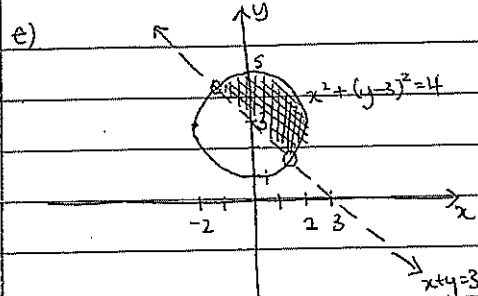
$y = \int 3x-4 dx$   
 $= \frac{3}{2}x^2 - 4x + C$

sub (1,4)

$4 = \frac{3}{2} - 4 + C$

$C = \frac{13}{2}$

$\therefore y = \frac{3}{2}x^2 - 4x + \frac{13}{2}$



Q13.

a) i.  $M_{PQ} = \frac{5-2}{-3-3}$   
 $= \frac{3}{-6}$   
 $= -\frac{1}{2} = m_{RS}$

$\therefore PQRS$  is a trapezium as  $PQ \parallel RS$ .

ii.  $d_{PR} = \sqrt{(-3-3)^2 + (5-2)^2}$   
 $= \sqrt{36+9}$   
 $= \sqrt{45}$   
 $= 3\sqrt{5}$

iii.  $R(3, -\frac{7}{2})$

iv.  $d_{PR} = \frac{|1(-3) + 2(5) + 4|}{\sqrt{1^2 + 2^2}}$   
 $= \frac{|11|}{\sqrt{5}}$   
 $= \frac{11\sqrt{5}}{5}$

v.  $A = \frac{11\sqrt{5}}{5} \times \frac{1}{2} (\sqrt{45} + 3\sqrt{5})$   
 $= \frac{11\sqrt{5}}{10} (\sqrt{45} + 3\sqrt{5})$   
 $= 40.473 \dots$   
 $\approx 40.47 u^2$  (2 d.p.)

b)  $V = \pi \int_0^2 (-x^2+2x)^2 dx$   
 $= \pi \int_0^2 x^4 - 4x^3 + 4x^2 dx$   
 $= \pi \left[ \frac{x^5}{5} - x^4 + \frac{4}{3}x^3 \right]_0^2$   
 $= \pi \left( \frac{16}{15} \right)$   
 $= \frac{16\pi}{15} u^3$   
 $\approx 3.3510 u^3$  (4 d.p.)

c) i. 

$x$	0	1	2	3	4
$y$	3.000	1.810	0.633	0.337	0.488

ii.  $\int_0^4 3^{\cos x} dx \approx$   
 $\approx \frac{1}{2} (3+0.488+2(1.810+0.633+0.337))$   
 $\approx 4.524$   
 $\approx 4.52$  (2 d.p.)

Q14.

a) i.  $f(-x) = \frac{(-x)^2 + 15}{5(-x)}$   
 $= \frac{x^2 + 15}{-5}$   
 $= -\frac{x^2 + 15}{5}$   
 $= -f(x)$

$\therefore$  odd function.

ii. let  $f(x) = 0$   
 $0 = \frac{x^2 + 15}{5x}$   
 $0 = x^2 + 15$   
 $x^2 = -15$   
 no solution  
 $\therefore$  no x-value,  $f(x) \neq 0$ .

iii.  $x = 0$

iv.  $y = \frac{x^2 + 15}{5x}$   
 $= \frac{x}{5} + \frac{3}{x}$   
 $\frac{dy}{dx} = \frac{1}{5} - \frac{3}{x^2}$   
 at  $\frac{dy}{dx} = 0$ , stationary pt.  
 $0 = \frac{1}{5} - \frac{3}{x^2}$   
 $\frac{3}{x^2} = \frac{1}{5}$   
 $15 = x^2$   
 $x = \pm\sqrt{15}$

test:

x	-4	$-\sqrt{15}$	-3	3	$\sqrt{15}$	4
y'	$\frac{1}{80}$	0	$-\frac{2}{15}$	$-\frac{2}{15}$	0	$\frac{1}{80}$

so:  $(-\sqrt{15}, -\frac{6\sqrt{15}}{5})$  maximum

$(\sqrt{15}, \frac{6\sqrt{15}}{5})$  minimum

v.

b) i.  $\angle ECF = 180^\circ - \theta^\circ$   
 (co-interior angles,  $AD \parallel BC$ )  
 $180^\circ - \theta^\circ + 90^\circ + 90^\circ + \angle EAF = 360^\circ$   
 (angle sum of quadrilateral)  
 $\angle EAF = \theta^\circ$

ii.  $EF^2 = 3^2 + 5^2 - 2(3)(5)\cos\theta$   
 $\cos\theta = \frac{4}{5}$  (from  $\triangle ADE$ )  
 $EF^2 = 9 + 25 - 30 \times (\frac{4}{5})$   
 $= 10$   
 $EF = \sqrt{10}$

c) Roots are  $\alpha$  and  $2\alpha$   
 so  $\alpha + 2\alpha = -\frac{b}{a}$   
 $3\alpha = \frac{6}{m-4}$   
 $\alpha = \frac{2}{m-4}$   
 $\alpha \times 2\alpha = \frac{c}{a}$

$2\alpha^2 = \frac{7}{m-4}$   
 sub  $\alpha = \frac{2}{m-4}$   
 $2\left(\frac{2}{m-4}\right)^2 = \frac{7}{m-4}$   
 $\frac{8}{(m-4)^2} = \frac{7}{m-4}$

for quadratic,  $m-4 \neq 0$  so divide off.  
 $\frac{8}{m-4} = 7$   
 $8 = 7m - 28$   
 $7m = 36$   
 $m = \frac{36}{7}$  or  $5\frac{1}{7}$



Q15	ii. $6 = r(2+\theta)$ $r = \frac{6}{2+\theta}$
a) $(x+6)^2 = 4(y-5)$	Area of sector = $\frac{1}{2}r^2\theta$ $= \frac{1}{2} \times \left(\frac{6}{2+\theta}\right)^2 \times \theta$ $= \frac{18\theta}{(2+\theta)^2}$
i. $V(-6, -5)$	
ii. $S(-6, -4)$	iii. $A = \frac{18\theta}{(2+\theta)^2} > \theta \neq -2$
iii. $y = -6$	$u = 18\theta \quad v = (2+\theta)^2$ $u' = 18 \quad v' = 2(2+\theta)$
b) $\frac{(10x-4)^6}{60} + C$	$\frac{dA}{d\theta} = \frac{18(2+\theta)^2 - 36\theta(2+\theta)}{(2+\theta)^4}$ $= \frac{18(2+\theta) - 36\theta}{(2+\theta)^3}$
c) i. In $\triangle ABC$ and $\triangle DCE$	For max; $\frac{dA}{d\theta} = 0$ $0 = \frac{18(2+\theta) - 36\theta}{(2+\theta)^3}$ $0 = 36 + 18\theta - 36\theta$ $0 = 36 - 18\theta$ $18\theta = 36$ $\theta = 2$
1. $\angle ACB = \angle DEC$	test
(Alternate angles, $AC \parallel ED$ )	$\theta \parallel 1 \quad 2 \quad 3$
2. $\angle ABC = \angle ECD$	$A' \parallel \frac{2}{3} \quad 0 \quad -\frac{18}{125}$
(Alternate angles, $AB \parallel CD$ )	$\therefore \max$
$\therefore \triangle ABC \parallel \triangle DCE$ (equiangular)	Max Area = $\frac{18 \times 2}{(2+2)^2}$ $= 2.25 \text{ m}^2$
ii. $\frac{AB}{DC} = \frac{BC}{CE} = \frac{AC}{DE}$	
(matching sides in ratio, similar triangles)	
$\frac{18}{8} = \frac{12+CE}{CE}$	
$18CE = 96 + 8CE$	
$10CE = 96$	
$CE = 9.6$	
$BE = 9.6 + 12$	
$= 21.6 \text{ cm}$	
d) i. $P = r + r + r\theta$ ( $l = r\theta$ )	
$= r(2+\theta)$	

Q16.	$= 55000 \times 1.0045^2 - 1500 \times 1.0045 - 1500$ $= 55000 \times 1.0045^2 - 1500(1.0045 + 1)$
a) LHS = $\frac{\cos A}{1 + \sin A} + \frac{1 + \sin A}{\cos A}$	ii. $A_3 = 55000 \times 1.0045^3 - 1500(1.0045^2 + 1.0045 + 1)$ $A_n = 55000 \times 1.0045^n - 1500(1 + 1.0045 + \dots + 1.0045^{n-1})$ $= 55000 \times 1.0045^n - 1500 \left[ \frac{1(1.0045^n - 1)}{1.0045 - 1} \right]$ $= 55000 \times 1.0045^n - 1500 \left[ \frac{1.0045^n - 1}{0.0045} \right]$
$= \frac{\cos^2 A + (1 + \sin A)^2}{\cos A(1 + \sin A)}$	$= 55000 \times 1.0045^n - 1500 \left[ \frac{1.0045^n - 1}{0.0045} \right]$
$= \frac{\cos^2 A + 1 + 2\sin A + \sin^2 A}{\cos A(1 + \sin A)}$	iii. $0 = 55000 \times 1.0045^n - 1500 \left( \frac{1.0045^n - 1}{0.0045} \right)$ $1500 \left( \frac{1.0045^n - 1}{0.0045} \right) = 55000 \times 1.0045^n$ $1.0045^n - 1 = \frac{55000 \times 0.0045}{1500} \times 1.0045^n$
$= \frac{\cos^2 A + \sin^2 A + 1 + 2\sin A}{\cos A(1 + \sin A)}$	$1.0045^n - 1 = \frac{33}{200} (1.0045^n) = 1$
$= \frac{2 + 2\sin A}{\cos A(1 + \sin A)}$	$\frac{167}{200} (1.0045^n) = 1$
$= \frac{2(1 + \sin A)}{\cos A(1 + \sin A)}$	$1.0045^n = \frac{200}{167}$
$= \frac{2}{\cos A}$	$\ln \frac{200}{167} = n \ln 1.0045$ $n = 40.16 \dots$ $\approx 40 \text{ months}$
$= \text{RHS.}$	iv. $A_{12} = 55000 \times 1.0045^{12} - 1500 \left( \frac{1.0045^{12} - 1}{0.0045} \right)$ $= 39592.37071 \dots$
b) $T_4 = 18$	
$18 = a + 3d$ ①	
$S_{10} = 195$	
$195 = 5(2a + 9d)$ ②	
$3 \times ① : 54 = 3a + 9d$	
$② : 39 = 2a + 9d$	
$15 = a + 0$	
$\therefore a = 15$	
c) i. $A_1 = 55000 \times 1.0045 - 1500$	$1^{\text{st}}$ month after 12 (i.e. 13 <sup>th</sup> )
$A_2 = A_1 \times 1.0045 - 1500$	$A_1 = A_{12} \times 1.0045 - 3000$
$= (55000 \times 1.0045 - 1500) \times 1.0045 - 1500$	$A_2 = (A_{12} \times 1.0045 - 3000) \times 1.0045 - 3000$

$$= A_{12} \times 1.0045^2 - 3000(1 + 1.0045)$$

$$A_3 = A_{12} \times 1.0045^3 - 3000(1 + 1.0045 + 1.0045^2)$$

$$A_n = A_{12} \times 1.0045^n - 3000(1 + 1.0045 + \dots + 1.0045^{n-1})$$

$$= A_{12} \times 1.0045^n - 3000 \left[ \frac{1.0045^n - 1}{0.0045} \right]$$

$$\text{Let } A_n = 0$$

$$0 = A_{12} \times 1.0045^n - 3000 \left[ \frac{1.0045^n - 1}{0.0045} \right]$$

$$3000 \left[ \frac{1.0045^n - 1}{0.0045} \right] = A_{12} \times 1.0045^n$$

$$1.0045^n - 1 = \frac{A_{12} \times 0.0045}{3000} \times 1.0045^n$$

$$1.0045^n - \left( \frac{A_{12} \times 0.0045}{3000} \right) \times 1.0045^n = 1$$

$$1.0045^n \left( 1 - \frac{A_{12} \times 0.0045}{3000} \right) = 1$$

$$1.0045^n = 1 \div \left( 1 - \frac{A_{12} \times 0.0045}{3000} \right)$$

$$\dots = 1.068138245 \dots$$

$$n = \frac{\ln 1.068138245 \dots}{\ln 1.0045}$$

$$= 13.6361 \dots$$

$$\doteq 13$$

$\therefore$  He can travel 13 more months.

(can travel 25 months altogether)