

Name: .....

Maths Class: .....

**Year 12**  
**Mathematics**  
**Trial HSC**  
**August 2018**

*Time allowed: 180 minutes (plus 5 minutes reading time)*

**General Instructions:**

- Marks for each question are indicated on the question.
- Approved calculators may be used
- All necessary working should be shown
- Full marks may not be awarded for careless work or illegible writing
- **Begin each question on a new page**
- Write using black or blue pen
- All answers are to be in the writing booklet provided
- A reference sheet is provided at the rear of this Question Booklet, and may be removed at any time.

Section 1 Multiple Choice

Questions 1-10

10 Marks

Section II Questions 11-16

90 Marks

Total = 100 marks

## SECTION I

10 marks Attempt Questions 1–10

Allow about 15 minutes for this section Use the multiple-choice answer sheet for Questions 1–10.

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- The mass of 1 atom of oxygen is  $2.7 \times 10^{-23}$  grams. What is the mass of  $8 \times 10^{27}$  atoms of oxygen?

(A) 21600

(B) 2160000

(C)  $2.16 \times 10^5$

(D)  $2.16 \times 10^{51}$
- Which one of the following statements is true for the equation  $7x^2 - 5x + 2 = 0$ ?

(A) No real roots

(B) One real root

(C) Two real distinct roots

(D) Three real roots
- If  $y = e^{x^2}$  then  $\frac{dy}{dx} =$

(A)  $x^2 e^{x^2}$

(B)  $2x e^{x^2}$

(C)  $2e^{x^2}$

(D)  $2x^2 e^{x^2}$

4. The domain of the function  $f(x) = \frac{1}{\sqrt{4x^2 - 1}}$  is:

(A)  $-\frac{1}{2} < x < \frac{1}{2}$

(B)  $x < -\frac{1}{2}$  and  $x > \frac{1}{2}$

(C)  $x \leq -\frac{1}{2}$  and  $x \geq \frac{1}{2}$

(D)  $-\frac{1}{2} \leq x < \frac{1}{2}$

5. The solutions to the equation  $2\sin x + \sqrt{3} = 0$  for  $0 \leq x \leq 2\pi$  are:

(A)  $x = \frac{\pi}{3}$  or  $x = \frac{2\pi}{3}$

(B)  $x = \frac{\pi}{3}$  or  $x = \frac{4\pi}{3}$

(C)  $x = \frac{2\pi}{3}$  or  $x = \frac{4\pi}{3}$

(D)  $x = \frac{4\pi}{3}$  or  $x = \frac{5\pi}{3}$

6.  $\int_0^{\sqrt{2}} \frac{3x}{x^2+1} dx =$

(A)  $\frac{1}{2} \ln 3$

(B)  $\frac{2}{3} \ln 3$

(C)  $\frac{3}{2} \ln 3$

(D)  $3 \ln 3$

7.  $\frac{d}{dx} \ln\left(\frac{2x+1}{3x+2}\right) =$

(A)  $\frac{2}{3}$

(B)  $\ln\left(\frac{2}{3}\right)$

(C)  $\frac{1}{(2x+1)(3x+2)}$

(D) 1

8. If  $\tan x = \frac{-1}{k}$  and  $0 \leq x \leq \pi$  then  $\sec x =$

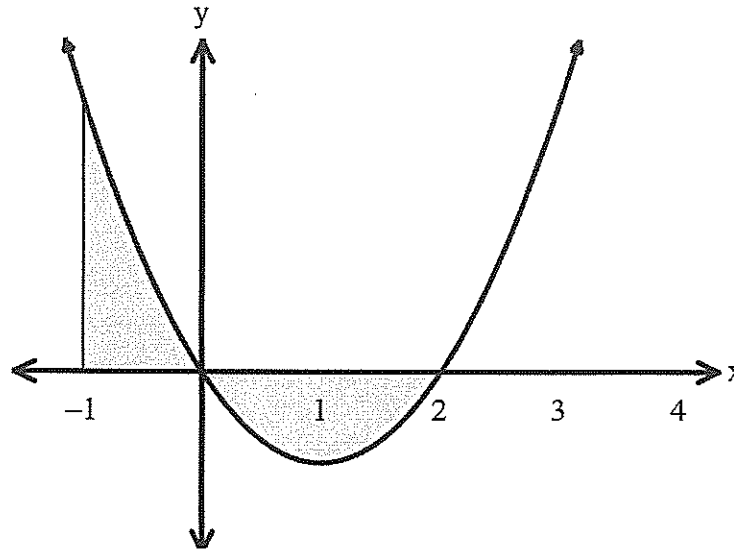
A.  $\frac{\sqrt{1+k^2}}{k}$

B.  $\frac{-\sqrt{1+k^2}}{k}$

C.  $\frac{k}{\sqrt{1+k^2}}$

D.  $-\frac{k}{\sqrt{1+k^2}}$

9. The area of the region shaded on the graph  $y = 3x(x - 2)$  below is:



- A. 0 square units  
B.  $\frac{8}{3}$  square units  
C. 8 square units  
D. 18 square units
10. Which of the following is TRUE?

- A.  $\int_0^1 e^{-x} dx < \int_1^2 e^{-x} dx$   
B.  $\int_0^1 e^x dx < \int_{-1}^0 e^x dx$   
C.  $\int_0^1 e^{-x} dx > \int_1^2 e^{-x} dx$   
D.  $\int_0^1 e^x dx > \int_1^2 e^x dx$

## SECTION II

90 marks

Attempt Questions 11–16

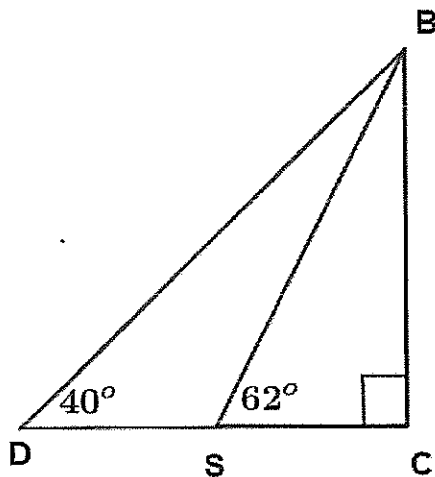
Allow about 2 hours and 45 minutes for this section.

Answer each question in your writing booklet.

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### QUESTION 11: (15 Marks)

- (a) Solve  $|1 - 3x| > 7$  2
- (b) Find  $a$  if  $\sqrt{48} + \sqrt{a} = 7\sqrt{3}$  2
- (c) Fully factorise  $x^3 - 125$  1
- (d) 3



From a position (S) out to sea, a sailor spies a bird perched at point (B) atop a vertical cliff, which he knows is 150m high.

He works out the angle of elevation from his position to the bird is  $62^\circ$

He rows out to sea a little further to a point D, and retakes the angle of elevation to the bird, which he now finds is only  $40^\circ$ .

How much further out to sea is he now than when he took the first reading?

(Give your answer to the nearest metre)

- (e) Solve the equation  $\cos^2 x + \cos x = 0$  for  $0 \leq x \leq 2\pi$  3
- (f) The roots of the quadratic equation  $2x^2 - 4x - 5 = 0$  are  $\alpha$  and  $\beta$ .  
Find the value of:
- i)  $\alpha + \beta$  1
- ii)  $\alpha \beta$  1
- iii)  $\alpha^2 + \beta^2$  1
- iv)  $\frac{1}{\alpha} + \frac{1}{\beta}$  1

**End of Question 11**

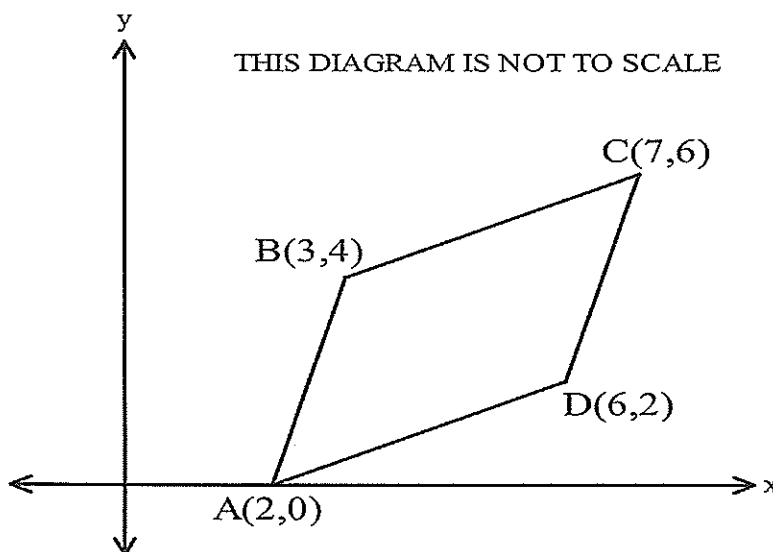
**QUESTION 12: (15 Marks)**

**Start a new page**

- (a) Find the equation of the tangent to the curve  $y = \frac{1}{x^2}$  at the point where  $x = -2$ . 3

Give your answer in general form.

(b)



- (i) Find the equation of the line BC 1
- (ii) Find the length of the perpendicular from D to BC 2
- (iii) Find the length of BC 1
- (iv) Find the area of the parallelogram ABCD 1

(c) For a certain base,  $a$ , you are given that

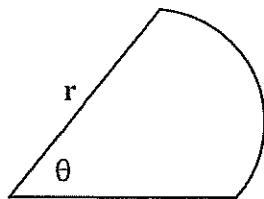
$$\log_a 5 = x \quad \text{and} \quad \log_a 2 = y$$

Find an expression for  $\log_a 12.5$

**2**



- (d) The area of the sector shown below is  $18\text{cm}^2$



- (i) Find an expression for  $r$  in terms of  $\theta$ . 1
- (ii) Show that the perimeter  $P$ , is given by  $P = \frac{6(2 + \theta)}{\sqrt{\theta}}$  1
- (iii) Find the minimum perimeter and the value of  $\theta$  for which that minimum exists. 3

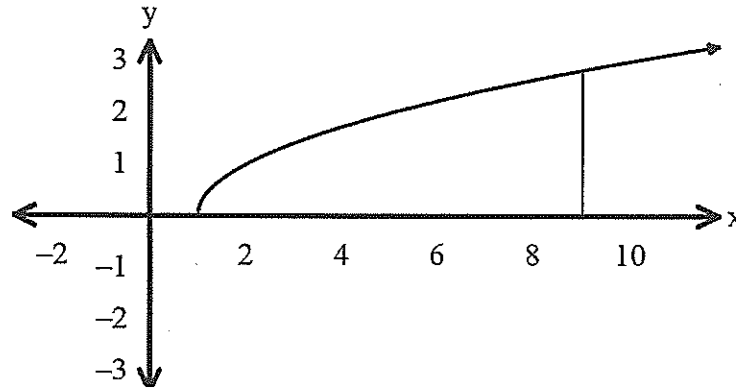
**End of Question 12**

### QUESTION 13 (15 Marks)

Start a new page

- (a) The following diagram shows the curve  $y = \sqrt{x - 1}$

3



Find the volume if the shaded area between the curve, the x-axis and the line  $x = 9$ , is revolved about the x-axis. (Give your answer correct to 2 decimal places)

- (b) Prove that  $\frac{\sin(\theta + 90^\circ)}{1 - \sin^2\theta} = \sec\theta$

3

- (c) Given the curve  $y = x^3 - 3x^2 - 9x + 1$

(i) Find  $\frac{dy}{dx}$  and  $\frac{d^2y}{dx^2}$

1

(ii) Find all stationary points and their nature

4

(iii) Find the point of inflexion

1

(iv) Sketch the graph using all of the information found above  
(Do NOT attempt to find the x-intercepts)

3

End of Question 13

**QUESTION 14: (15 Marks)**

**Start a new page**

(a) Find:

(i)  $\frac{d}{dx}(3\sin^2 x)$  2

(ii)  $\int \frac{3e^{2x}}{e^{2x} + 1} dx$  2

(iii)  $\int_0^{\frac{\pi}{2}} 2\cos \frac{x}{2} dx$  (give your answer in simplest rational form) 2

(b) (i) Explain why the series  $1 + \cos^2 x + \cos^4 x + \dots$  has a limiting sum, if  $0 < x < \frac{\pi}{2}$ . 1

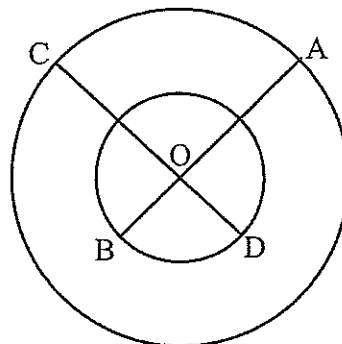
(ii) Show that this limiting sum is  $\operatorname{cosec}^2 x$ . 2

(c) The sum of the first and 2<sup>nd</sup> terms of a geometric series is 27, while the sum of the 4<sup>th</sup> and 5<sup>th</sup> terms is -125.

(i) Find the common ratio 2

(ii) Find the first 2 terms 1

(d) Two concentric circles have centre O. Two straight lines AB and CD, pass from one circumference to the other, through O, as shown.



Showing all steps and reasons, prove that  $AD = BC$  3

**End of Question 14**

**QUESTION 15: (15 Marks)**

Start a new page

- (a) A large balloon is being deflated and its rate of deflation is given by the formula

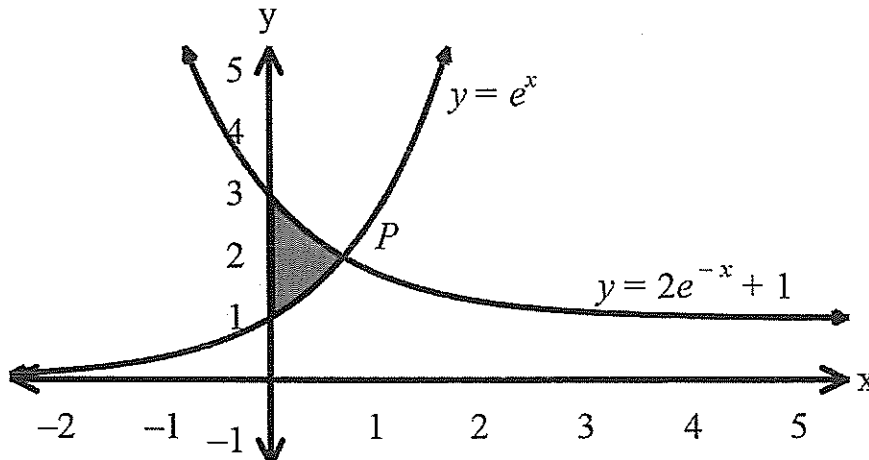
$$V = V_0 e^{-kt}$$

Where  $V$  is the volume at any particular time.

Initially, there is  $100m^3$  of air in the balloon, and after 20 seconds it has reduced to half of what it began.

- (i) Find the values of  $V_0$  and  $k$  (to 3 decimal places) 3
- (ii) How long will it take to deflate to a volume of  $5m^3$  ? (to the nearest second) 2
- (iii) Find the volume of the balloon after 1 minute (to the nearest  $m^3$ ) 1

- (b) The curves  $y = e^x$  and  $y = 2e^{-x} + 1$  are shown below.



- (i) By substitution, or otherwise, show that the  $x$ -value of  $P$  is  $x = \ln 2$  2
- (ii) Find the value of  $\int_0^{\ln 2} (2e^{-x} + 1) dx$  2
- (iii) Hence show that the shaded area above is  $\ln 2$  1

- (c) A person invests \$800 at the beginning of each year in a superannuation fund. Compound interest is paid at 10% per annum on the investment. The first \$800 is to be invested at the beginning of 2016 and the last is to be invested at the beginning of 2045.

Calculate to the nearest dollar:

- (i) The amount to which the 2016 investment will have grown by the beginning of 2046. 1
- (ii) The amount to which the total investment will have grown by the beginning of 2046. 3

**End of Question 15**

**QUESTION 16: (15 Marks)**

Start a new page

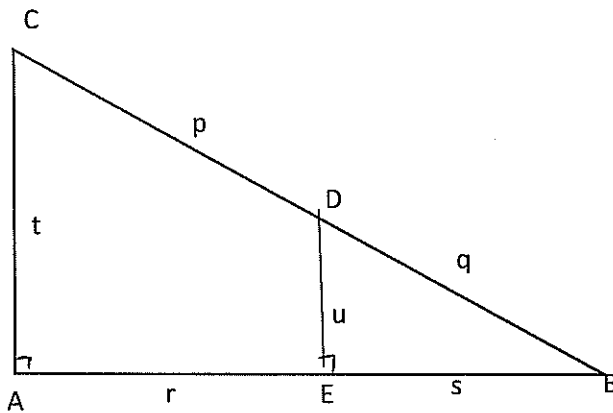
- (a) The table below represents the function  $f(x) = \sin^2 \frac{x}{2}$

$x$	0	$\frac{\pi}{2}$	$\pi$	$\frac{3\pi}{2}$	$2\pi$
$\sin^2 \frac{x}{2}$	0	0.79 0.5			

(i) Complete the table of values (Give each answer to 2 dec. places) 1

(ii) Using Simpson's Rule with 5 function values, find an approximation for  $\int_0^{2\pi} \sin^2 \frac{x}{2} dx$  to 2 decimal places. 2

- (b) In the following diagram,  $\angle CAB = \angle DEB = 90^\circ$   
 $CB = p$ ,  $DB = q$ ,  $CA = t$ ,  $DE = u$ ,  $AE = r$  and  $EB = s$

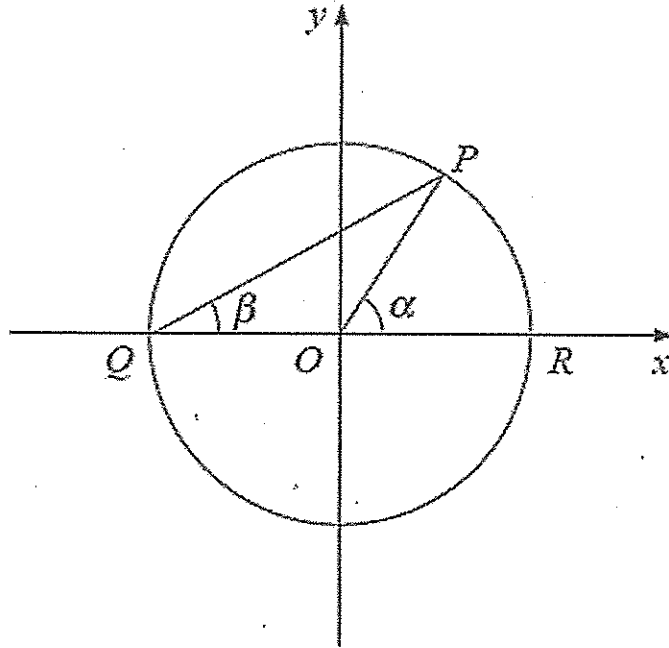


(i) Prove that  $\triangle ACB$  is similar to  $\triangle EDB$  2

(ii) Explain why  $\frac{t}{u} = \frac{p+q}{q}$  1

(iii) Deduce that  $\frac{t}{u} > \frac{r+s}{q}$  1

- (c) In the diagram, Q is the point  $(-1,0)$ , R is the point  $(1,0)$ , and P is another point on the circle with centre O and radius 1. Let  $\angle POR = \alpha$  and  $\angle PQR = \beta$ .  
Let  $\tan \beta = m$ .



- (i) Explain why  $\alpha = 2\beta$ . 1
- (ii) Find an expression for the equation of the line PQ in terms of  $m$ . 1
- (iii) Show that  $x$  coordinates of P and Q are solutions of the equation 1  

$$(1+m^2)x^2 + 2m^2x + m^2 - 1 = 0$$
- (iv) Find the coordinates of P in terms of  $m$ . 3
- (v) Deduce that  $\tan 2\beta = \frac{2 \tan \beta}{1 - \tan^2 \beta}$ . 2

**End of examination**

2018 TRIAL HSC - 2 UNIT - SOLUTIONS.

and MARKING SCHEME

ANSWER

MULTIPLE CHOICE

1.  $2.16 \times 10^5$

1. C

2.  $\Delta = 25 - 56$

$< 0 \Rightarrow$  no real roots.

2. A

3.  $y' = 2xe^{x^2}$

3. B

4.  $4x^2 - 1 > 0$

$\therefore x^2 > \frac{1}{4} \Rightarrow x < -\frac{1}{2}$  OR  $x > \frac{1}{2}$

4. B

5.  $\sin x = -\frac{\sqrt{3}}{2}$

$\therefore x = 240^\circ, 300^\circ$   
 $= \frac{4\pi}{3}$  OR  $\frac{5\pi}{3}$

5. D

6.  $\frac{3}{2} \int_0^{\sqrt{2}} \frac{2x}{x^2+1} dx = \frac{3}{2} \ln(x^2+1) \Big|_0^{\sqrt{2}}$

$= \frac{3}{2} [\ln(3) - \ln(1)]$

$= \frac{3}{2} \ln 3$

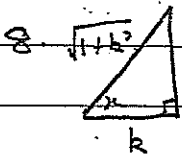
6. C

7.  $\frac{db}{dx} = \frac{2}{2x+1} - \frac{3}{3x+2}$

$= \frac{(2x+2) - (3x-3)}{(2x+1)(3x+2)}$

$= \frac{1}{(2x+1)(3x+2)}$

7. C



$\sec \theta = -\frac{\sqrt{1+k^2}}{b}$

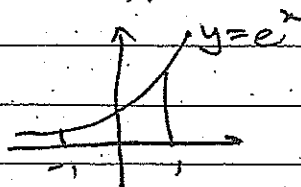
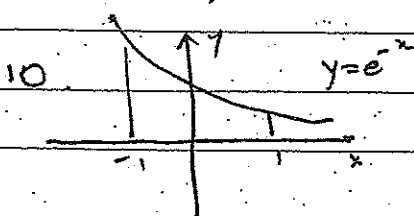
8. B

9.  $A_1 = \int_{-1}^0 (3x^2 - 6x) dx$       $A_2 = \left| \int_0^2 (3x^2 - 6x) dx \right|$

$= 4x^2$

$= 4x^2$

9. C



10. C



## SECTION 2:

### QUESTION 11:

(a)  $1 - 3x < 7$  or  $1 - 3x < -7$   
 $x < -2$  or  $x > \frac{8}{3}$

(b)  $4\sqrt{3} + \sqrt{a} = 7\sqrt{3}$   
 $\sqrt{a} = 3\sqrt{3}$   
 $a = 27$

(c)  $(x-5)(x^2+5x+25)$

(d)  $SC = \frac{150}{\tan 62^\circ}$   
 $\approx 79.76 \text{ m}$   
 $DC = \frac{150}{\tan 40^\circ}$   
 $\approx 178.76$

Difference  $\approx 99 \text{ m}$ .

(e)  $\cos x (\cos x + 1) = 0$   
 $\cos x = 0$  or  $\cos x = -1$   
 $x = \frac{\pi}{2}$  or  $\frac{3\pi}{2}$  or  $\pi$

(f) (i)  $\alpha + \beta = \frac{7}{2} = 2$

(ii)  $\alpha\beta = -\frac{5}{2}$

(iii)  $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$   
 $= 4 + 5$   
 $= 9$

(iv)  $\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha + \beta}{\alpha\beta}$   
 $= \frac{2}{-\frac{5}{2}}$   
 $= -\frac{4}{5}$

### QUESTION 12:

(a)  $\frac{dy}{dx} = -\frac{2}{x^3}$

At  $x = -2$   $m_T = \frac{1}{4}$ ,  $y = \frac{1}{4}$

Equation is:

$$y - \frac{1}{4} = \frac{1}{4}(x + 2)$$

$$4y - 1 = x + 2$$

$$x - 4y + 3 = 0$$

(b) (i)  $m_{BC} = \frac{1}{2}$

Equation is:  $y - 4 = \frac{1}{2}(x - 3)$

$$x - 2y + 5 = 0$$

(ii)  $P = \frac{|6 - 4 + 5|}{\sqrt{5}}$

$$= \frac{7}{\sqrt{5}}$$

(iii) length  $BC = \sqrt{2^2 + 4^2}$   
 $= \sqrt{20}$  or  $2\sqrt{5}$

(iv) Area  $= 2\sqrt{5} \times \frac{7}{\sqrt{5}}$   
 $= 14 \text{ u}^2$

(c)  $\log_a 2.5 = \log_a \left(\frac{25}{10}\right)$   
 $= 2 \log_a 5 - \log_a 2$   
 $= 2x - y$

(d) (i)  $A = \frac{1}{2} r^2 \theta = 18$   
 $r = \sqrt{\frac{36}{\theta}} = \frac{6}{\sqrt{\theta}}$

(ii)  $P = 2r + r\theta$   
 $= 12/\sqrt{\theta} + 6/\sqrt{\theta} \theta$   
 $= \frac{6}{\sqrt{\theta}}(2 + \theta)$

(iii)  $\frac{dP}{d\theta} = \frac{d}{d\theta} \left[ 12\theta^{-\frac{1}{2}} + 6\theta^{\frac{1}{2}} \right]$   
 $= -6\theta^{-\frac{3}{2}} + 3\theta^{-\frac{1}{2}}$   
 $= 3\theta^{-\frac{3}{2}} [-2 + \theta]$   
 $\frac{d^2y}{dx^2} = 9\theta^{-\frac{5}{2}} = \frac{9}{2}\theta^{-\frac{5}{2}}$

Q12 CONT...

At min T.P.,  $\frac{dP}{d\theta} = 0$

$\therefore \theta = 2$

$P = \frac{24}{\sqrt{2}}$   
 $= 12\sqrt{2}$

$P'' > 0$   
 $\Rightarrow$  MIN.

$\therefore$  MIN T.P. of  $12\sqrt{2}$  cm when  
 $\theta = 2^\circ$

$\therefore$  MIN at  $(3, -26)$

and

MAX at  $(-1, 6)$

(iii) At I.P.  
 $\frac{d^2y}{dx^2} = 0$

$\therefore \begin{cases} x = 1 \\ y = -10 \end{cases}$

QUESTION 13:

(a) VOL =  $\pi \int_1^9 y^2 dx$   
 $= \pi \int_1^9 (x-1) dx$   
 $= \pi \left[ \frac{1}{2}x^2 - x \right]_1^9$   
 $= \begin{cases} 32\pi \mu^2 \\ 100.48 \mu^2 \end{cases}$

(b)  $\frac{\sin(\theta + 90^\circ)}{1 - \sin\theta} = \frac{\cos\theta}{\cos^2\theta}$   
 $= \frac{1}{\cos\theta}$   
 $= \sec\theta$

(c) (i)  $\frac{dy}{dx} = 3x^2 - 6x - 9$   
 $\frac{d^2y}{dx^2} = 6x - 6$

(ii) At S.P.'s  $\frac{dy}{dx} = 0$

$\therefore 3(x-3)(x+1) = 0$

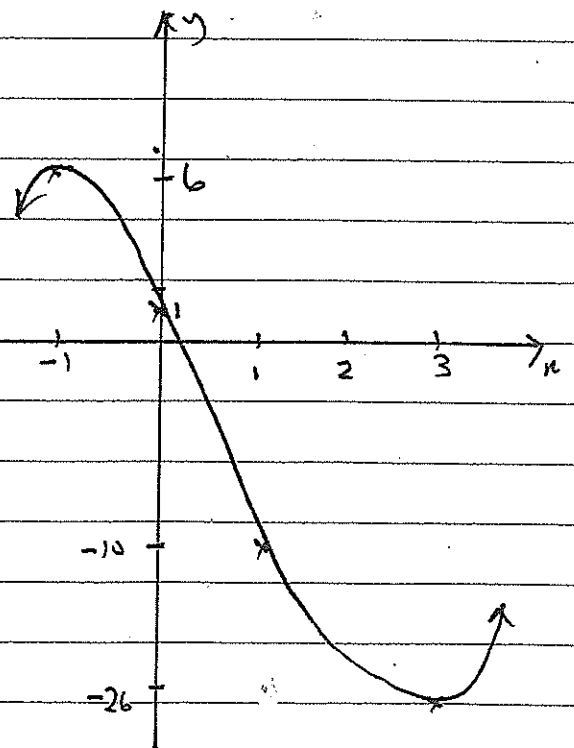
$\therefore \begin{cases} x = 3 \\ y = -26 \\ y'' > 0 \end{cases}$  OR  $\begin{cases} x = -1 \\ y = 6 \\ y'' < 0 \end{cases}$

x	0	1	2
y''	-6	0	6
	-ve	0	+ve

$\therefore \frac{d^2y}{dx^2}$  changes sign

$\therefore$  oblique I.P. at  $(1, -10)$

(iv)



QUESTION 14:

6(a) (i)  $6 \sin x \cos x = 3 \sin 2x$

(ii)  $\frac{3}{2} \ln(e^{2x} + 1) + k$

(iii)  $\left[ 4 \sin^2 \frac{x}{2} \right]_0^{\pi/2} = 4 \sin^2 \frac{\pi}{4}$   
 $= 4 \cdot \frac{1}{2}$   
 $= 2\sqrt{2}$

(b) (i)  $r = \cos^2 x$

Since  $0 \leq \cos^2 x \leq 1 \forall x$ then  $0 < \cos^2 x < 1$  for  $0 < x < \frac{\pi}{2}$ 

(ii)  $S_{\infty} = \frac{1}{1 - \cos^2 x}$   
 $= \frac{1}{\sin^2 x}$   
 $= \sec^2 x$

(c)  $a + ar = 27$

$\Rightarrow a(1+r) = 27 \quad (1)$

$\therefore ar^3 + ar^4 = -125$

$ar^3(1+r) = -125 \quad (2)$

By division (2)  $\div$  (1)

$r^3 = \frac{-125}{27}$

$r = -\frac{5}{3}$

Into (1)  $a(1 - \frac{5}{3}) = 27$

$a = \frac{27 \cdot 3}{-2}$

$= -\frac{81}{2}$

$T_1 = -\frac{81}{2}$

$T_2 = \frac{135}{2}$

(d) In  $\triangle BOC$  and  $\triangle DOA$ 

$BO = DO$  (equal radii)

$OC = OA$  (equal radii)

$\angle BOC = \angle DOA$  (vertically opposite angles)

 $\therefore \triangle BOC \cong \triangle DOA$  (SAS) $\therefore BC = AD$  [corresponding sides in congruent triangles]QUESTION 15:

(a) (i)  $V_0 = 100$

At  $t = 20$ ,  $V = 50$

$\therefore 0.5 = e^{-20k}$

$k = 0.035$

(ii) At  $V = 5$

$5 = 100 e^{-0.035t}$

$\therefore -0.035t = \ln(0.05)$

$\therefore t \approx 86 \text{ sec}$

(iii) At  $t = 60$

$V = 100 e^{-0.035 \times 60}$

$\approx 23 \text{ m}^3$

(b) (i)  $\begin{cases} y = e^x \text{ at } x = \ln 2 \\ y = 2 \end{cases}$

$y = 2$

$y = 2e^{-x} + 1 \text{ at } x = \ln 2$

$y = 2 \cdot \frac{1}{2} + 1$

$= 2$

 $\therefore x = \ln 2$  satisfies both lines

OR (SECOND METHOD)

$2e^{-x} + 1 = e^x$

$\therefore 2 + e^x = e^{2x}$

$(e^x + 1)(e^x - 2) = 0$

$e^x = -1$  or  $e^x = 2$

NO SOLUTION  $x = \ln 2$ 

(b) (ii)  $\int_0^{\ln 2} (2e^{-x} + 1) dx$

$= \left[ -2e^{-x} + x \right]_0^{\ln 2}$

$= -1 + \ln 2 + 2$

$= 1 + \ln 2$

QUEST 15 conts..

(b) (iii) 
$$\text{Area} = \int_0^{\ln 2} (2e^{-x} + 1) dx - \int_0^{\ln 2} e^{-x} dx$$

$$= 1 + \ln 2 - (2 - 1)$$

$$= \ln 2$$

(c) (i)  $800(1.1)^{30} \approx 13,959$

(ii) 
$$\text{TOTAL} = 800 \left[ (1.1)^0 + \dots + (1.1)^{30} \right]$$

$$= \frac{800(1.1)(1.1^{30} - 1)}{0.1}$$

$$\approx \$144,755$$

QUESTION 16:

(a)

$n$	0	$\frac{\pi}{2}$	$\pi$	$\frac{3\pi}{2}$	$2\pi$
$\sin^2 \frac{n}{2}$	0	0.5	1	0.5	0

(ii)  $A_1 \approx \frac{1}{3} \cdot \frac{\pi}{2} \cdot [0 + 1 + 4 \times 0.5]$   
 $A_2 \approx \frac{1}{3} \cdot \frac{\pi}{2} [1 + 0 + 4 \times 0.5]$   
 $\therefore \text{Area} \approx \frac{\pi}{6} (6)$   
 $\approx \pi \approx 3.14$

(b) (i) In  $\triangle ACB$  and  $\triangle EDB$   
 $\angle CAB = \angle DEB = 90^\circ$  (given)  
 $\therefore \angle KBA$  is common  
 $\therefore \triangle ACB \parallel \triangle EPB$  (equiangular)

(ii) Corresponding sides in similar triangles are in ratio  
 $\therefore \frac{t}{u} = \frac{p+q}{q}$

(iii) The hypotenuse is the largest side in any right angled triangle  
 $\therefore p+q > r+s$   
 $\therefore \frac{p+q}{q} > \frac{r+s}{q}$   
 $\therefore \frac{t}{u} > \frac{r+s}{q}$

(c) (i)  $\triangle OQP$  is isosceles and  $\angle POR$  is the external angle  
 $\therefore \alpha = 2\beta$

(ii) Since  $m = \tan \beta$  is the slope of  $QP$ ,  
 equation of  $QP$  is:  $y = mx + m$

(iii) By substituting into  $x^2 + y^2 = 1$   
 $x^2 + (mx+m)^2 = 1$   
 $\therefore x^2(1+m^2) + 2mx + (m^2-1) = 0$

(iv) Solving the q.e.,  

$$x = \frac{-2m \pm \sqrt{4m^2 - 4(1+m^2)(m^2-1)}}{2(1+m^2)}$$

leading to:  
 $x_p = \frac{1-m^2}{1+m^2}$  OR  $x = -1$   
 $\uparrow P$   $\quad \quad \quad \uparrow Q$

$\therefore y_p = mx_p + m$   
 $= \frac{2m}{1+m^2}$

(v)  $\tan \alpha$  is the slope of  $OP$   
 $\therefore m_{OP} = \frac{2m}{1+m^2} \div \frac{1-m^2}{1+m^2}$   
 $= \frac{2m}{1-m^2}$

AND  $\tan \alpha = \tan 2\beta$  (part (i))  
 $\therefore \tan 2\beta = \frac{2m}{1-m^2}$   
 $= \frac{2 \tan \beta}{1 - \tan^2 \beta}$

## Markers Comments – Year 12 – 2018 – Mathematics 2 Unit – HSC Trial

Q 11	<p>(a) A number of students used a whole page to answer this <b>simple</b> inequality. Some students floundered in the negative case. Revise preliminary inequalities and consult solutions.</p> <p>(b) A not insignificant number of students insisted on reinventing Pythagoras:  <math>\sqrt{c} = \sqrt{a} + \sqrt{b}</math> <b>DOES NOT SIMPLIFY TO</b> <math>c = a + b</math> (except in the trivial case of a or b being 0)</p> <p>The key to the question was spotting <math>\sqrt{3}</math> as the common factor, which would allow you to simplify and combine the surd terms to then square and arrive at solution.</p> <p>(e) Students who succeeded in this question factorised as a quadratic to quickly arrive at the solution: <math>\cos x = 0, \cos x = -1</math>. Students need to be more careful with boundary conditions and check their trig identities, using a calculator if necessary.</p>
Q 12	<p>a) Learn general form! <math>a &gt; 0</math>, no fractions, <math>ax + by + c = 0</math>. Some also made errors when differentiating to find the gradient of the tangent. Be careful with negative indices.</p> <p>b) ii) Needed the equation of the line in general form correctly before substituting into the perpendicular distance formula. Wrong general form led to incorrect values for a, b, c</p> <p>iv) Wrong formula. Area of parallelogram is NOT <math>\frac{1}{2}bh</math></p> <p>c) Incorrect use of log laws</p> <p>d) i) <math>r &gt; 0</math>. Read the context of the question, r is a radius so it must be positive only.</p> <p>ii) Careful when differentiating. Many made errors when using the quotient rule. You must test the minimum value to verify that it is a minimum! Remember to also substitute <math>\theta</math> into P for the minimum value of P.</p>
Q 13	<p>A) limit should have been from 1 to 9 not 0 to 9</p> <p>C) Must test for inflexion points, otherwise you only have a POSSIBLE inflexion point and for the sketch you HAD TO SHOW Y-intercept and make sure that the RELATIVE position of points are clear.</p>
Q 14	<p>a) i) A number of students did not use the chain rule correctly.</p> <p>b) i) A number of students did not notice that <math>\cos^2 x</math> is between 0 and 1.</p> <p>c) A number of students thought the series was an arithmetic series.</p> <p>d) Some geometry reasons were not appropriately written.</p>
Q 15	<p>a) i) State the value of <math>V_0</math> clearly.</p> <p>b) i) Much easier to substitute <math>x = \ln 2</math> into the curves but some students forgot to substitute it into <b>both</b> curves. Many factorising errors made when students tried to solve <math>2e^{-x} + 1 = e^x</math>.</p>

	<p>ii &amp; iii) Do not leave out steps of working. Especially for part iii which is a SHOW question worth 1 mark.</p> <p>c)i) Many used the wrong value of <math>n</math>.</p> <p>ii) Wrong value of <math>n</math> used. Those who tried to memorise and use a formula to answer the question without showing the development of the pattern and did not write the GP sum obtained the wrong answer.</p>
Q 16	<p>a) The Simpson's and trapezoidal rule questions should be an easy 3 marks. Some students are still losing marks through poor calculator work and incorrect application of the formula.</p> <p>b i) when students see the word <b>deduce</b>, they need to justify their assertions. In this case stating the fact that 'the hypotenuses is always the longest side in a triangle' was required and led to a quick and simple answer. Some students successfully used Pythagoras, which required more work.</p> <p>c ii) Far too many students laboured with this question. The answer in gradient point form is easily arrived in one line as both the gradient <math>m</math> and a point on the line <math>(-1,0)</math> are supplied in the question. Students need to carefully read and interpret the information they have been given.</p> <p>As a general note on trigonometry, students need to review their year 10 trigonometry and look for simple answers first.</p> <p>c iii) students found this question difficult. The required insight was the fact that points P and Q are the intersection points of line PQ and the unit circle which led to a substitution of the answer from c ii) into the equation of the unit circle which led to the answer with some straightforward algebra. Students who had not answered part (ii) could not arrive at the correct answer. Many students much time unsuccessfully attempting to work backwards, often substituting the <math>x</math> coordinate of Q to show that it satisfied the equation, however, this approach was futile as the same cannot be done for P whose <math>x</math> coordinate is clearly unknown at this point. Marks are generally not awarded for information from a later part of the question being used to answer an earlier part which leads to circular arguments.</p> <p>c iv) Students found the question hard. Students need to read and interpret questions carefully. All this question was asking was for students to solve the quadratic equation given in part iii) and recognise that the 2 solutions were points P and Q which was again explicitly stated in part iii). For full marks students needed to use the correct result from part ii).</p> <p>c v) Students found this question very hard. Many students wasted ink on a pointless regurgitation and circular manipulation the double angle formula from the reference sheet. The word <b>deduce</b> in a multi-part question is usually a strong signal to students that the previous parts of the question have been leading to the answer and need to be used. Students needed the correct answer from iv) for full marks.</p>