Sydney
Technical
High School

Name: ....

Maths Class: ....

Year 12
Mathematics
Trial HSC
August 2019

Time allowed: 180 minutes (plus 5 minutes reading time)

General Instructions:

- Marks for each question are indicated on the question.
- Approved calculators may be used
- All necessary working should be shown
- Full marks may not be awarded for careless work or illegible writing
- ***Begin each question on a new page***
- Write using black or blue pen
- All answers are to be in the writing booklet provided
- A reference sheet is provided

Section 1	Multiple Choice Questions 1-10 10 Marks
Section II	Questions 11-16 90 Marks

Total = 100 marks

SECTION 1

10 marks
Attempt Questions 1-10
Allow about 15 minutes for this section
Use the multiple-choice answer sheet for Questions 1-10.

1. What is the value of $8^{-2.1}$ correct to two decimal places?

   (A) 0.013

   (B) 0.01

   (C) -0.01

   (D) -12.55

2. What are the co-ordinates of the x-intercept of the line $y = -\frac{1}{3}x - 2$?

   (A) (-6, 0)

   (B) (7, 0)

   (C) (0, -2)

   (D) (0, -6)

3. For what values of x is the curve $f(x) = 3x^3 + x^2 - 1$ concave down?

   (A) $x < -\frac{1}{9}$

   (B) $x > -\frac{1}{9}$

   (C) $x > -9$

   (D) $x < -9$

4. What are the solutions of $\sqrt{3}\tan x = -1$ for $0 \leq x \leq 2\pi$?

(A) $\frac{2\pi}{3}$ and $\frac{4\pi}{3}$

(B) $\frac{2\pi}{3}$ and $\frac{5\pi}{3}$

(C) $\frac{5\pi}{6}$ and $\frac{7\pi}{6}$

(D) $\frac{5\pi}{6}$ and $\frac{11\pi}{6}$

5. Find the equation of the line with a y intercept of -5 and parallel to the line joining (1,3) to the origin.

(A) $y = -3x + 5$

(B) $y = -3x - 5$

(C) $y = 3x + 5$

(D) $y = 3x - 5$

6. The quadratic equation $x^2 + 3x - 1 = 0$ has roots a and b.

What is the value of $-2ab + (a + b)$?

(A) 1

(B) -1

(C) 2

(D) -2

7. What is the correct expression for $\int \sin\left(\frac{x}{5}\right) dx$?

(A) $-\frac{1}{5}\cos\left(\frac{x}{5}\right) + C$

(B) $5\cos\left(\frac{x}{5}\right) + C$

(C) $-5\cos\left(\frac{x}{5}\right) + C$

(D) $-5\cos 5x + C$

8. What is the derivative of $(1 - \ln x)^{-2}$?

(A) $\dfrac{-2(1 - \ln x)^3}{x}$

(B) $\dfrac{2(1 - \ln x)^3}{x}$

(C) $\dfrac{2}{x(1 - \ln x)^3}$

(D) $-\dfrac{2}{x(1 - \ln x)^3}$

9. What is the value of $\sum_{r=1}^{15}(2r + 1)$?

(A) 109

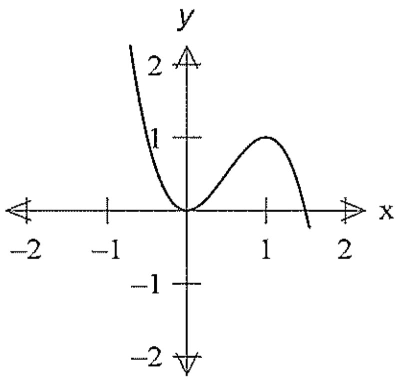
(B) 255

(C) 225

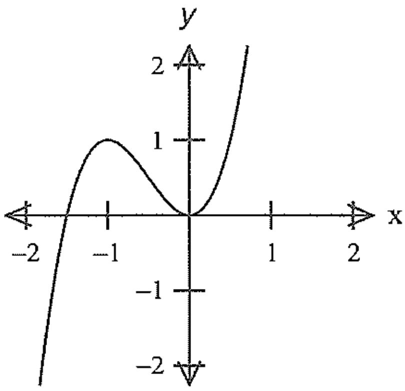
(D) 235

10. Which of the following is the graph of $y = 2x^3 - 3x^2$?

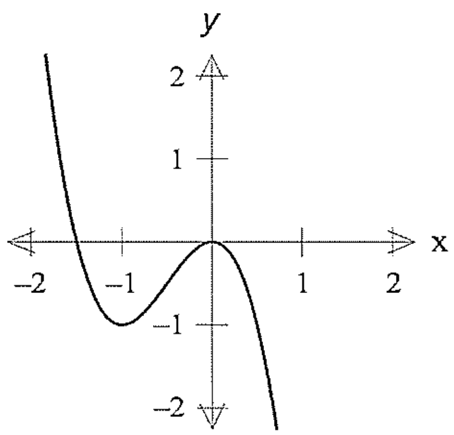
(A)



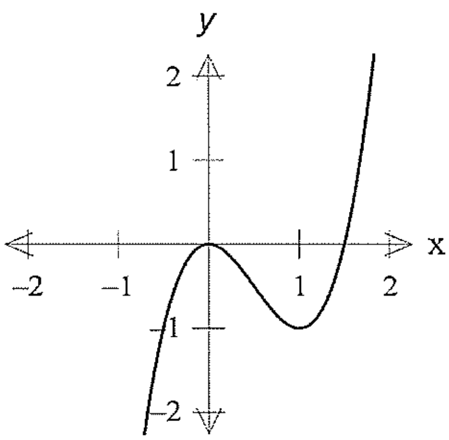
(B)



(C)



(D)



END OF SECTION 1

SECTION II

90 marks
Attempt Questions 11-16
Allow about 2 hours and 45 minutes for this section
In Questions 11-16, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) Start a new page **Marks**

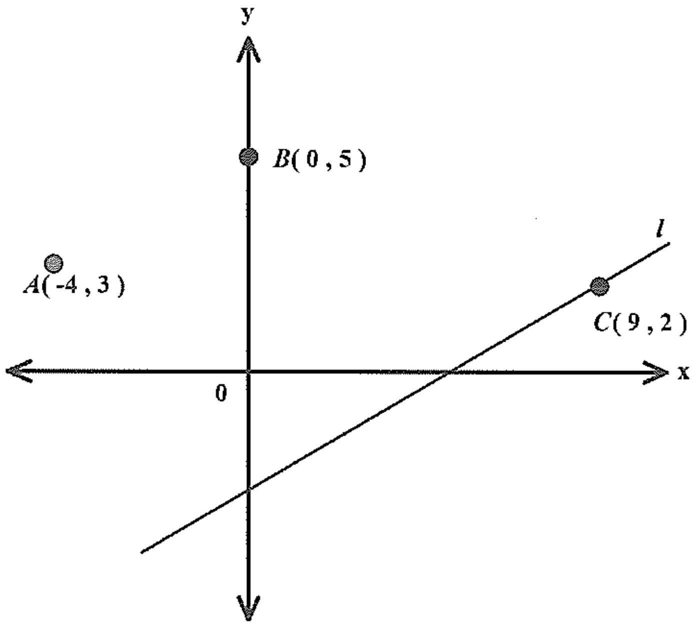
(a) Rationalise the denominator of $\dfrac{3}{\sqrt{5}-1}$. 1

(b) Determine $\lim\limits_{x \to 3} \dfrac{2x^2 - 18}{x - 3}$. 2

(c) Factorise $4x^2 - 14x + 6$. 2

(d) Solve $|3x - 1| < 2$. 2

(e) The diagram shows the points A (-4, 3), B (0, 5) and C (9, 2), and the line l which passes through C and is parallel to AB.



Copy the diagram into your answer booklet.

(i) Find the length AB. 1

(ii) Show that the equation of the line l is $x - 2y - 5 = 0$. 2

(iii) Find the coordinates of point D, where the line l meets the x-axis. 1

(iv) Prove that ABCD is a parallelogram. 2

(f) Differentiate with respect to x

(i) $\sqrt[3]{x^2}$ 1

(ii) $\tan(2x + 1)$ 1

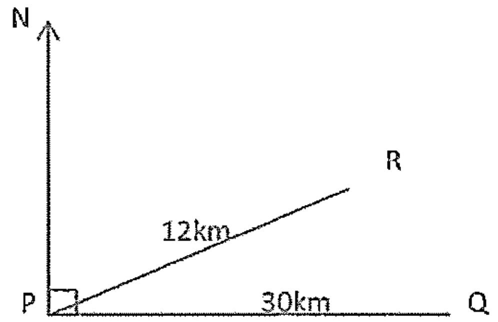
Question 12 (15 marks) **Start a new page**

(a) Find the equation of the tangent to the curve $y = e^{2x} + 1$ where $x = 3$. 2

(b) Find the coordinates of the focus of the parabola $x^2 = 16(y - 2)$. 2

(c) State the period and amplitude of $y = 3\sin 2x$. 2

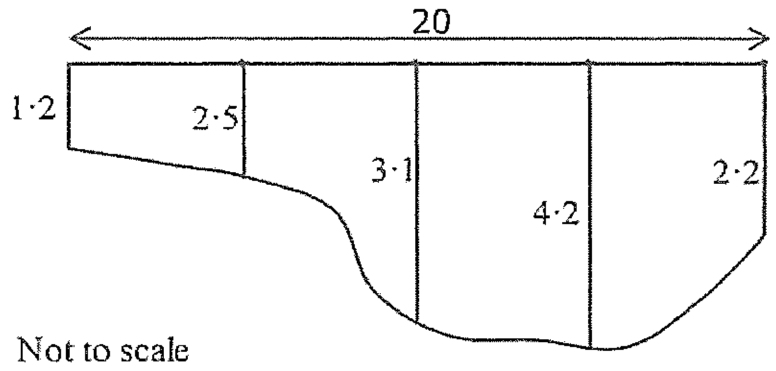
(d) The diagram below shows a point P which is 30km due west of point Q.
The point R is 12km from P and has a bearing of 070° from P.



(i) Find the distance of R from Q (to 1 decimal place). 1
(ii) Find the bearing of R from Q (to the nearest degree). 2

(e) Find $\displaystyle\int \frac{4x}{x^2 + 6}\, dx$. 2

(f) At a certain location, a river is 20 metres wide. At this location the depth of the river in metres has been measured at 5 metre intervals. The cross section of the river is shown below.



(i) Use Simpson's rule with the five depth measurements to calculate the approximate area of the cross-section. 2

(ii) The river flows at 0.4 metres per second. Calculate the approximate volume of water flowing through the cross-section in 10 seconds. 2

Question 13 (15 marks) Start a new page

(a) (i) On the same set of axes, sketch the graphs of $y = x^2 - 4$ and $y = x + 1$ 2

(ii) On the graph above, shade the region defined by $y \geq x^2 - 4$ and $y \leq x + 1$ 1

(b) (i) Show that $\frac{d}{dx}(x\ln x - x) = \ln x$. 2

(ii) Hence evaluate $\int_1^e \ln x \, dx$. 2

(c) Consider the curve given by $y = x^3 - 3x + 2$

(i) State the y-intercept. 1

(ii) Find the coordinates of the stationary points and determine their nature. 3

(iii) Find the coordinates of any points of inflexion. 2

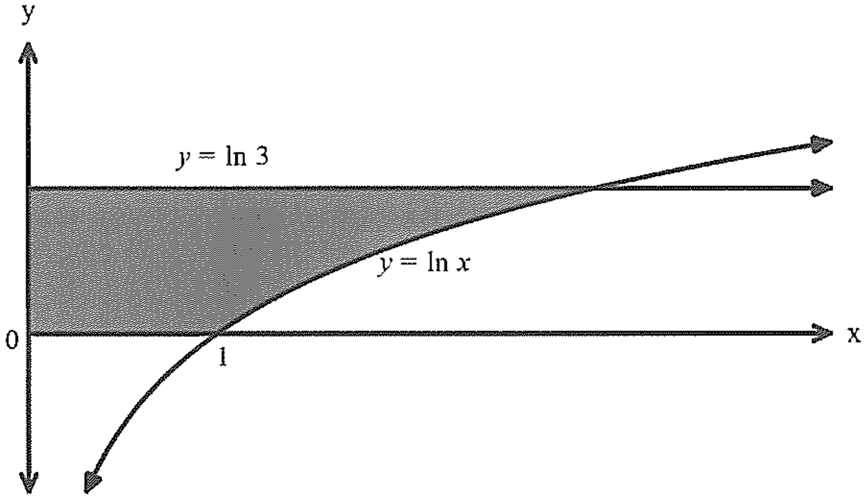
(iv) Sketch the curve, showing all of the above information. 2

Question 14 (15 marks) **Start a new page**

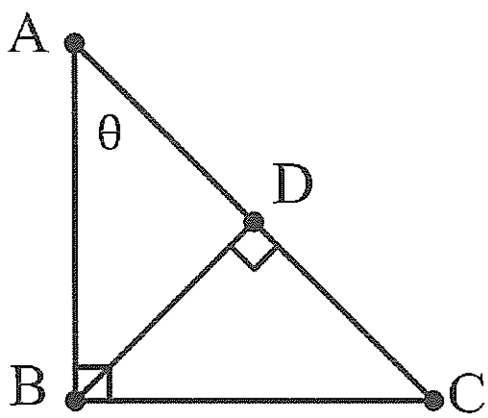
(a) Solve the equation $\ln(x-2) - \ln(x+2) = 1$.
Give your answer correct to four decimal places. **2**

(b) Given $f'(x) = e^{1-x}$ and $f(1) = 3$, find $f(x)$. **2**

(c) The shaded region below is rotated about the y-axis. Find the exact volume of this solid. **3**



(d) A triangle is right-angled at B. D is the point on AC such that BD is perpendicular to AC.



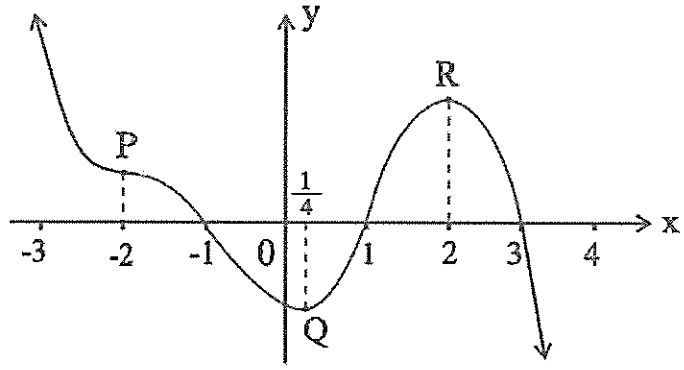
(i) Given that $6AD + BC = 5AC$, show that $6\cos\theta + \tan\theta = 5\sec\theta$. 2

(ii) Deduce that $6\sin^2\theta - \sin\theta - 1 = 0$ 3

(iii) Find θ 3

Question 15 (15 marks) **Start a new page**

(a) The diagram below shows the graph of a function $y = f(x)$. The graph has a horizontal point of inflexion at P, a minimum turning point at Q and a maximum turning point at R.



(i) Sketch the graph $y = f'(x)$ 2

(ii) For which values of x is the derivative $f'(x)$ positive? 1

(iii) For which values of x is the derivative $f'(x)$ negative? 1

(b) For what values of k is the quadratic $(k+2)x^2 + 4\sqrt{3}\,x + 5 - k$ positive definite? 3

(c) A prize fund of \$200 000 is established. Interest is earned at a rate of 6% p.a. compounded monthly. At the end of each year a prize of \$20 000 is awarded. Let A_n dollars denote the amount remaining in the prize fund at the end of the n^{th} year just after the award of the prize for that year.

(i) Show that $A_3 = 200\,000 \times 1.005^{36} - 20\,000\left(1.005^{24} + 1.005^{12} + 1\right)$. 2

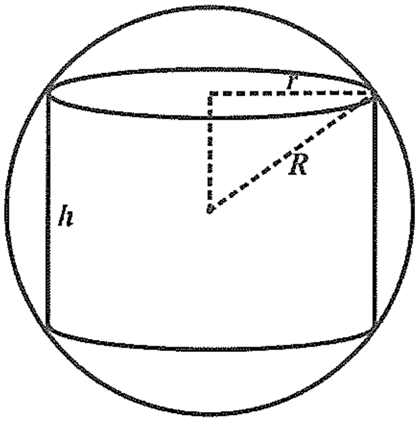
(ii) Write down an expression for A_n and hence find the amount remaining in the prize fund just after the 10^{th} award of the prize. 3

(d) Evaluate $\displaystyle\int_{\ln 2}^{\ln 4} \frac{e^x + 1}{e^x + x}\,dx$, expressing the answer in the form $\ln a$ for some constant a. 3

Question 16 (15 marks) **Start a new page**

(a) The perimeter of a sector is 50cm. If the angle at the centre is 2 radians, find the radius of the circle. 2

(b)

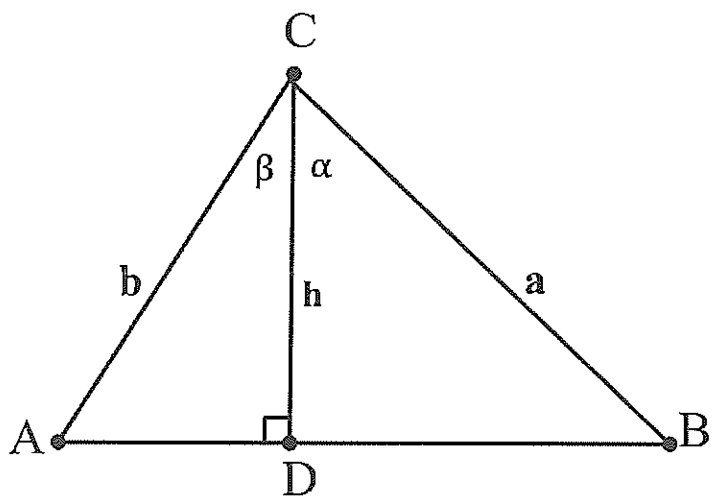


In the diagram, a cylinder of radius r and height h is inscribed in a sphere of radius R.

(i) Show that the volume V of the cylinder is given by $V^2 = 4\pi^2 r^4(R^2 - r^2)$. 2

(ii) Find in terms of R the maximum volume of the cylinder. 3

(c)



The diagram above show a triangle ABC, and CD is perpendicular to AB. It is given that BC $= a$, AC $= b$, $\angle ACD = \beta$ and $\angle BCD = \alpha$.

(i) By using triangles ACD and BCD, show that $h = b\cos\beta = a\cos\alpha$. 1

(ii) Show that the area of triangle ACD is equal to $\frac{1}{2}ab\sin\beta\cos\alpha$ 1

(iii) Find another expression for the area of triangle BCD in terms of a, b, α and β. 1

(iv) Show that the area of triangle ABC is equal to $\frac{1}{2}ab\sin(\alpha + \beta)$ 2

(v) Hence, but not otherwise, deduce that $\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$ 3

END OF EXAMINATION

Q1 0.01 B

Q2 When $y=0$,

$$0 = -\frac{1}{3}x - 2$$

$$2 = -\frac{1}{3}x$$

$$x = -6$$

A

Q3 Concave down when $f''(x) < 0$

$$f'(x) = 9x^2 + 2x$$

$$f''(x) = 18x + 2$$

$$18x + 2 < 0$$

$$18x < -2$$

$$x < -\frac{1}{9}$$

A

Q4 $\sqrt{3}\tan x = -1$

$$\tan x = -\frac{1}{\sqrt{3}}$$

S	A
T	C

$$x = 150^\circ, 330^\circ$$

$$x = \frac{5\pi}{6}, \frac{11\pi}{6}$$

D

Q5 $y = mx + b$

$b = -5$

$m = \dfrac{3-0}{1-0} = 3$

$y = 3x - 5$

D

Q6 $-2ab + (a+b)$

$= -2(-1) + (-3)$

$= 2 - 3$

$= -1$

B

Q7 $\int \sin\left(\frac{x}{5}\right) dx$

$= -5\cos\left(\frac{x}{5}\right) + C$

C

Q8

$$-2(1-\ln x)^{-3}\left(\frac{-1}{x}\right)$$

$$=\frac{2}{x(1-\ln x)^3}$$

C

Q9 $\sum_{r=1}^{15}(2r+1)$

$$=3+5+7+\dots+31$$

$$\frac{3+31}{2}\times 15=255$$

B

Q10

$$y=2x^3-3x^2$$

$$y=x^2(2x-3)$$

D

Q11 (a)

$$\frac{3}{\sqrt{5}-1} \times \frac{\sqrt{5}+1}{\sqrt{5}+1}$$

$$= \frac{3\sqrt{5}+3}{5-1}$$

$$= \frac{3\sqrt{5}+3}{4}$$

Q11 (b) $\lim\limits_{x \to 3} \frac{2(x^2-9)}{x-3}$

$$= \lim_{x \to 3} \frac{2(x+3)(x-3)}{x-3}$$

$$= \lim_{x \to 3} (2(x+3))$$

$$= 6 \times 2$$

$$= 12$$

Q11 (c) $4x^2 - 14x + 6$

$$= 2(2x^2 - 7x + 3)$$

$$= 2(2x-1)(x-3)$$

Q11(d) $-2 < 3x - 1 < 2$

$$-1 < 3x < 3$$

$$-\frac{1}{3} < x < 1$$

Q11(e)(i)

$AB = \sqrt{(5-3)^2 + (0+4)^2}$

$= 2\sqrt{5}$ units

Q11(e)(ii)

$m_{AB} = \dfrac{5-3}{0+4} = \dfrac{1}{2}$

$y - 2 = \frac{1}{2}(x-9)$

$2y - 4 = x - 9$

$x - 2y - 5 = 0$

Q11(e)(iii)

At point D, $y=0$

$x - 2(0) - 5 = 0$

$x = 5$

$D(5, 0)$

Q11(e)(iv)

$AB \parallel CD$ (given)

$CD = \sqrt{(9-5)^2 + (2-0)^2} = \sqrt{20} = AB$

In ABCD, one pair of sides are equal and parallel so ABCD is a parallelogram.

Q 11 (f)(i)

$$\sqrt[3]{x^2} = x^{\frac{3}{2}}$$

$$\frac{d}{dx}\left(x^{\frac{3}{2}}\right)$$

$$= \frac{3}{2} x^{-\frac{1}{3}}$$

Q 11 (f)(ii)

$$\frac{d}{dx}\left(\tan(2x+1)\right)$$

$$= 2\sec^2(2x+1)$$

Q12(a)

$$y' = 2e^{2x}$$

$$y'\big|_{x=3} = 2e^{6}$$

$$\frac{y-(e^{6}+1)}{x-3} = 2e^{6}$$

$$y - e^{6} - 1 = 2e^{6}x - 6e^{6}$$

$$y = 2e^{6}x - 5e^{6} + 1$$

Q12(b) $4a = 16$

$a = 4$

Vertex : $(0, 2)$

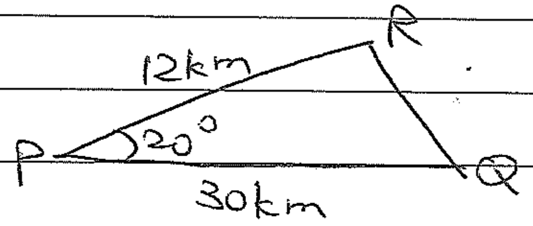
Focus : $(0, 6)$

Q12(c) $y = 3\sin 2x$

Amplitude : 3

Period $= \frac{2\pi}{2} = \pi$

Q12(d)(i)



In $\triangle PQR$,

$$RQ^2 = 12^2 + 30^2 - 2(12)(30)\cos 20^\circ$$

$$RQ^2 = 367.42$$

$RQ = 19.2$ km (1 decimal place)

Q12(d)(ii)

In $\triangle PQR$,

$$\frac{\sin \angle RQP}{12} = \frac{\sin 20^\circ}{19.2}$$

$$\sin \angle RQP = \frac{12 \sin 20^\circ}{19.2}$$

$$\angle RQP = 12.34^\circ$$

Bearing of R from Q

$= 12.34^\circ + 270^\circ$

$= 282.34^\circ T$

$\approx 282^\circ T$ (to the nearest degree)

Q12(e) $\int \frac{4x}{x^2+6}\,dx$

$$= 2\int \frac{2x}{x^2+6}\,dx$$

$$= 2\ln(x^2+6) + C$$

Q 12 (f) (i)

$$\text{Area} = \frac{5}{3}\left[1.2 + 2.2 + 4(2.5 + 4.2) + 2(3.1)\right]$$

$$= 60\frac{2}{3}\ m^2$$

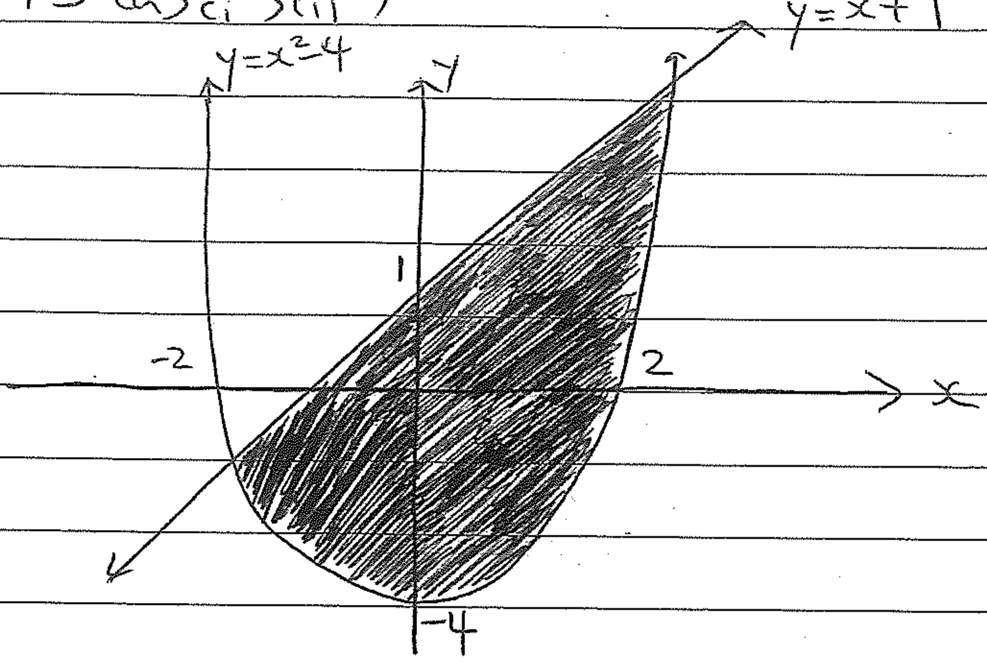
Q 12 (f) (ii)

Volume of water flowing through the cross-section in 10 seconds

$$= 60\tfrac{2}{3} \times 10 \times 0.4$$

$$= 242\frac{2}{3}\ m^3$$

Q13 (a)(i)(ii)



Q13(b)(i)

$$\frac{d}{dx}(x\ln x - x)$$

$$= \ln x + \frac{x}{x} - 1$$

$$= \ln x$$

Q13(b)(ii) $\int_1^e \ln x \, dx$

$$= \left[x\ln x - x \right]_1^e$$

$$= (e-e) - (0-1)$$

$$= 1$$

Q 13 (c)(i)

y - intercept is 2

Q 13(c)(ii)

For any stationary points, $\frac{dy}{dx}=0$

$$3x^2-3=0$$

$$x=\pm 1$$

At $x=1$, $y=0$.

$$\frac{d^2y}{dx^2}=6x$$

$$\left.\frac{d^2y}{dx^2}\right|_{(1,0)}=6>0$$

$\therefore$ $(1,0)$ is a minimum point

At $x=-1$, $y=4$, $\left.\frac{d^2y}{dx^2}\right|_{(-1,4)}=-6<0$

$\therefore$ $(-1,4)$ is a maximum point.

Q 13(c)(iii) For any points of inflexion,

$\frac{d^2y}{dx^2}=0$ and concavity changes

$$\frac{d^2y}{dx^2}=0$$

$$6x=0$$

$$x=0$$

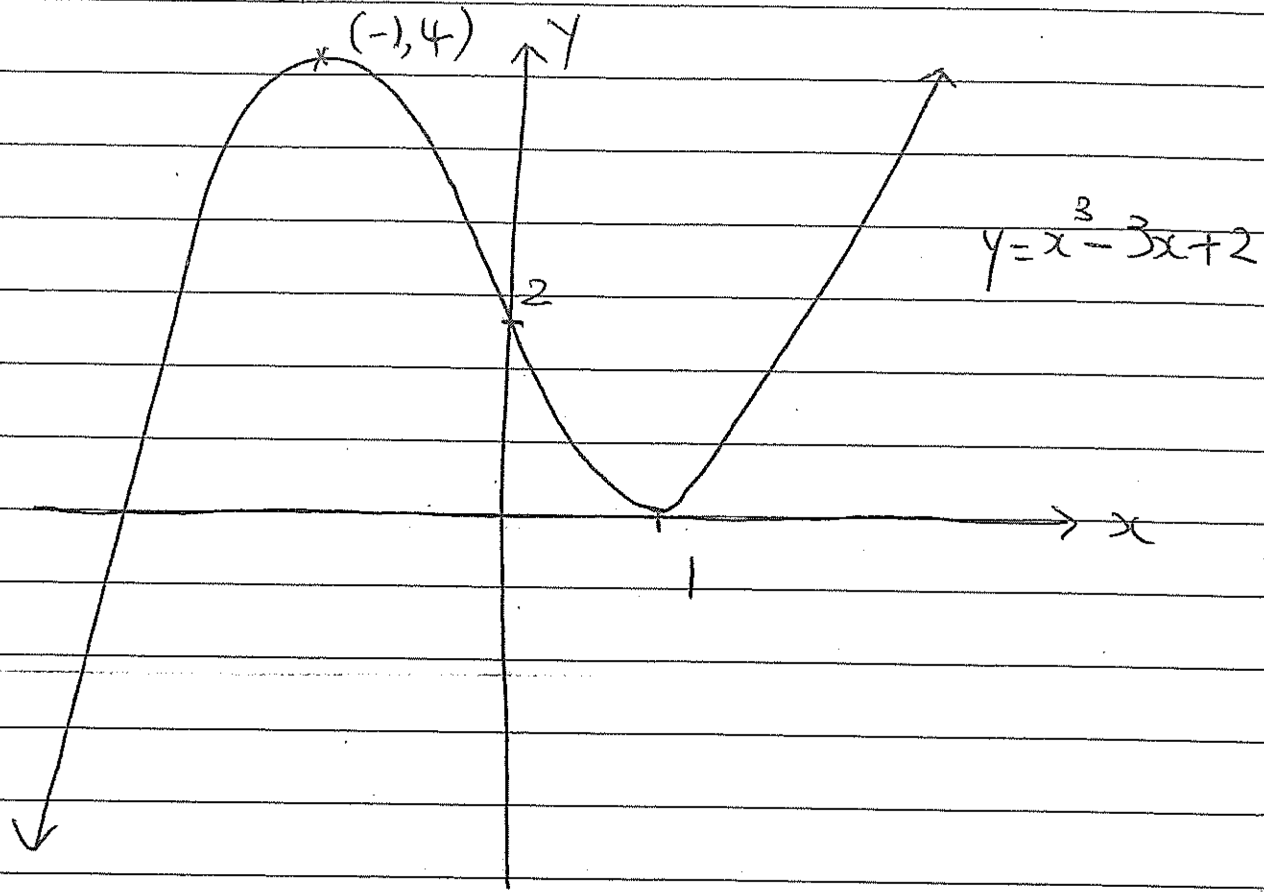
When $x=0$, $y=2$.

x	-1	0	1
$\frac{d^2y}{dx^2}$	-6	0	6

$\therefore$ Concavity changes at $x=0$

$(0,2)$ is a point of inflexion

Q 13 (c)(iv)



Q 14(a) $\ln(x-2) - \ln(x+2) = 1$

$$\ln\left(\frac{x-2}{x+2}\right) = 1$$

$$\frac{x-2}{x+2} = e$$

$$x - 2 = ex + 2e$$

$$x - ex = 2e + 2$$

$$x = \frac{2e+2}{1-e}$$

$$x = -4.3279$$

But substitute $x = -4.3279$ into

$$\ln(x-2) - \ln(x+2) = 1$$

$LHS = \ln(-6.3279) - \ln(-2.3279)$

ln(any negative number) doesn't exist.

So $x = -4.3279$ is not a valid solution

Q 14(b) $f'(x) = e^{1-x}$

$$f(x) = -e^{1-x} + C$$

At $x = 1$, $y = 3$

$$3 = -e^{1-1} + C$$

$$3 = -1 + C$$

$$C = 4$$

$$f(x) = -e^{1-x} + 4$$

Q14 (c) $y = \ln x$, $e^y = x$

$$V = \pi \int_0^{\ln 3} x^2 \, dy$$

$$= \pi \int_0^{\ln 3} e^{2y} \, dy$$

$$= \pi \left[\frac{e^{2y}}{2} \right]_0^{\ln 3}$$

$$= 4\pi \text{ units}$$

Q14 (d) (i) $6AD + BC = 5AC$

$$\cos\theta = \frac{AD}{AB}$$

$$AB \cos\theta = AD$$

$$\tan\theta = \frac{BC}{AB} \qquad \cos\theta = \frac{AB}{AC}$$

$$BC = AB\tan\theta \qquad AC = \frac{AB}{\cos\theta}$$

Substitute $BC = AB\tan\theta$ and $AC = \frac{AB}{\cos\theta}$ into $6AD + BC = 5AC$

$$6\,AB\cos\theta + AB\tan\theta = \frac{5AB}{\cos\theta}$$

$$6\cos\theta + \tan\theta = 5\sec\theta$$

Q 14 (d) (ii)

$$6\cos\theta + \tan\theta = 5\sec\theta$$

$$6\cos\theta + \tan\theta = \frac{5}{\cos\theta}$$

$$\cancel{6\cos^2\theta + \frac{\sin\theta}{\cos\theta} \times \cos\theta = 5}$$

$$6\cos^2\theta + \sin\theta = 5$$

$$6\cos^2\theta + \sin\theta - 5 = 0$$

$$6(1-\sin^2\theta) + \sin\theta - 5 = 0$$

$$6 - 6\sin^2\theta + \sin\theta = 5$$

$$6\sin^2\theta - \sin\theta - 1 = 0$$

Q 14(d)(iii)

$$6\sin^2\theta - \sin\theta - 1 = 0$$

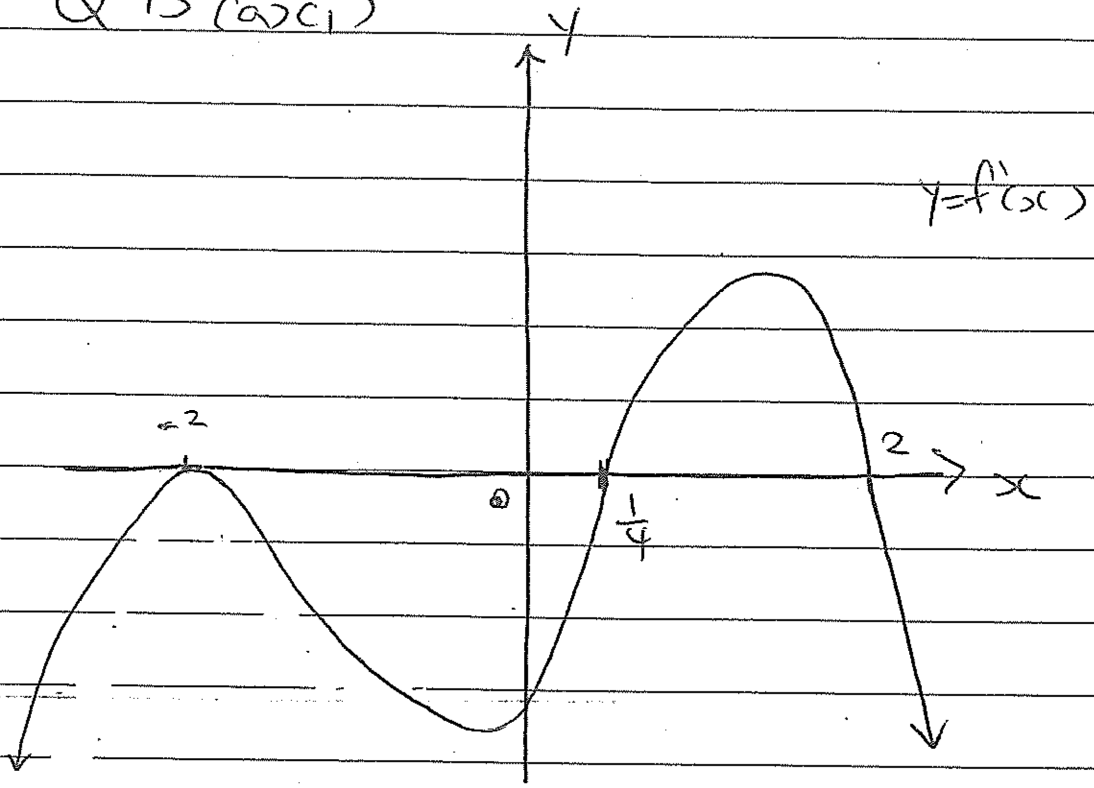
$$(2\sin\theta - 1)(3\sin\theta + 1) = 0$$

$$\sin\theta = \frac{1}{2} \quad \text{or} \quad \sin\theta = -\frac{1}{3}$$

$$\theta = 30^\circ \quad \text{or} \quad \theta = 199^\circ 28' \text{ (rejected)}$$

$$\text{So } \theta = 30^\circ$$

Q 15 (a)(i)



Q 15 (a)(ii) $\frac{1}{4} < x < 2$

Q 15 (a)(iii) $x < \frac{1}{4}$ and $x > 2$

Q 15 (b) For a positive definite,

$(k+2)x^2 + 4\sqrt{3}x + 5 - k$

$a > 0$ and $\Delta < 0$

$k+2 > 0$ $\qquad$ $b^2 - 4ac < 0$

$k > -2$ $\qquad$ $(4\sqrt{3})^2 + 4(k+2)(5-k) < 0$

$48 - 4(5k - k^2 + 10 - 2k) < 0$

$48 - 4k^2 - 12k - 40 < 0$

$4k^2 - 12k + 8 < 0$

$(k^2 - 3k + 2) < 0$

$(k-2)(k-1) < 0$

$1 < k < 2$

$\therefore$ $1 < k < 2$

Q15 (c)(i) $r = 0.06 \div 12 = 0.005$ per month

$A_1 = 200000 \times 1.005^{12} - 20000$

$A_2 = A_1 \times 1.005^{12} - 20000$

$= (200000 \times 1.005^{12} - 20000)1.005^{12} - 20000$

$= 200000 \times 1.005^{24} - 20000 \times 1.005^{12} - 20000$

$= 200000 \times 1.005^{24} - 20000\ (1.005^{12} - 1)$

$A_3 = A_2 \times (1.005^{12}) - 20000$

$= \left(200000 \times 1.005^{24} - 20000(1.005^{12} - 1)\right) \times 1.005^{12} - 20000$

$= 200000 \times 1.005^{36} - 20000 \times 1.005^{12}(1.005^{12} - 1) - 20000$

$= 200000 \times 1.005^{36} - 20000\ (1.005^{24} + 1.005^{12} + 1)$

Q15 (c)(ii)

$A_n = 200000(1.005^{12})^n - 20000\left[(1.005^{12})^{n-1} + (1.005^{12})^{n-2} + \dots + 1.005^{12} + 1\right]$

$= 200000(1.005^{12})^n - 20000\left[\dfrac{(1.005^{12})^n - 1}{(1.005)^{12} - 1}\right]$

$A_{10} = 200000(1.005^{12})^{10} - 20000\left[\dfrac{(1.005^{12})^{10} - 1}{(1.005)^{12} - 1}\right]$

$= 200000(1.005^{120}) - 20000 \times 13.28511355$

$= \$\ 98177.07584$

Q 15 (d) $\displaystyle\int_{\ln 2}^{\ln 4} \frac{e^x+1}{e^x+x}\,dx$

$= \left[\ln(e^x+x)\right]_{\ln 2}^{\ln 4}$

$= (\ln(e^{\ln 4}+\ln 4)) - (\ln(e^{\ln 2}+\ln 2))$

$= \ln(4+\ln 4) - \ln(2+\ln 2)$

$= \ln\left(\frac{4+\ln 4}{2+\ln 2}\right)$

$= \ln\left(\frac{4+\ln(2^2)}{2+\ln 2}\right)$

$= \ln\left(\frac{4+2\ln 2}{2+\ln 2}\right)$

$= \ln\left(\frac{2(2+\ln 2)}{2+\ln 2}\right)$

$= \ln 2$

Q 16(a) $P = 50\text{cm}, \theta = 2, r = ?$

Perimeter of a sector $= 2r + r\theta$

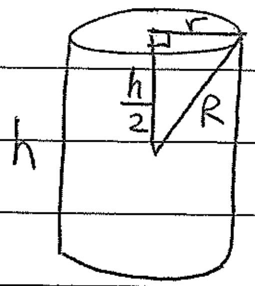
$2r + r\theta = 50$

$2r + 2r = 50$

$4r = 50$

$r = 12.5\text{cm}$

Q16(b)(i) $V = \pi r^2 h$



$\left(\frac{h}{2}\right)^2 = R^2 - r^2$

$\frac{h^2}{4} = R^2 - r^2$

$h^2 = 4(R^2 - r^2)$

$V^2 = \pi^2 r^4 h^2$

$V^2 = \pi^2 r^4 (4(R^2 - r^2))$

$V^2 = 4\pi^2 r^4 (R^2 - r^2)$

Q16 (b) (ii)

$V^2 = 4\pi^2 r^4 (R^2 - r^2) = 4\pi^2 R^2 r^4 - 4\pi^2 r^6$

$\frac{d(V^2)}{dr} = 16\pi^2 R^2 r^3 - 24\pi^2 r^5$

$= 24\pi^2 \left(\frac{2}{3} R^2 r^3 - r^5\right)$

Stationary points occur when $\frac{d(V^2)}{dr} = 0$

$24\pi^2 \left(\frac{2}{3} R^2 r^3 - r^5\right) = 0$

$\frac{2}{3} R^2 r^3 = r^5$

$\frac{2}{3} R^2 = r^2$

$\frac{d^2(V^2)}{dr^2} = 48\pi^2 R^2 r^2 - 120\pi^2 r^4$

$= 120\pi^2 r^2 \left(\frac{2}{5} R^2 - r^2\right)$

$\left.\frac{d(V^2)}{dr^2}\right|_{r^2 = \frac{2}{3}R^2} = 120\pi^2 \left(\frac{2}{3} R^2\right)\left(\frac{2}{5} R^2 - \frac{2}{3} R^2\right)$

$= 120\pi^2 \left(\frac{2}{3} R^2\right)\left(-\frac{4}{15} R^2\right)$

which is negative.

So $r^2 = \frac{2}{3} R^2$ gives the maximum V^2.

Hence $r^2 = \frac{2}{3} R^2$ gives the maximum V, volume of the cylinder.

$V^2 = 4\pi^2 r^4 (R^2 - r^2)$

$V = 2\pi r^2 (R^2 - r^2)^{\frac{1}{2}}$

Sub. $r^2 = \frac{2}{3}R^2$ into $V = 2\pi r^2 (R^2 - r^2)^{\frac{1}{2}}$

$$V = 2\pi r^2 (R^2 - r^2)^{\frac{1}{2}}$$
$$= 2\pi \left(\frac{2}{3}R^2\right)\left(R^2 - \left(\frac{2}{3}R^2\right)\right)^{\frac{1}{2}}$$
$$= \frac{4}{3}\pi R^2 \left(R^2 - \frac{2}{3}R^2\right)^{\frac{1}{2}}$$
$$= \frac{4}{3}\pi R^2 \left(\frac{R^2}{3}\right)^{\frac{1}{2}}$$
$$= \frac{4}{3}\pi R^2 \left(\frac{R}{\sqrt{3}}\right)$$
$$= \frac{4}{3\sqrt{3}}\pi R^3$$

So the maximum volume of the cylinder
$$= \frac{4}{3\sqrt{3}}\pi R^3$$

Q 16(c)(i)

In $\triangle ADC$,
$\cos\beta = \frac{h}{b}$
$h = b\cos\beta$

In $\triangle BCD$,
$\cos\alpha = \frac{h}{a}$
$h = a\cos\alpha$

$\therefore\ b\cos\beta = a\cos\alpha$

Q16(c)(ii) Area of $\triangle ACD = \frac{1}{2}(AD)(h)$
$$= \frac{1}{2}(b\sin\beta)(a\cos\alpha)$$
$$= \frac{1}{2}ab\sin\beta\cos\alpha$$

Q16(c)(iii)

$$\text{Area}_{BCD} = \frac{1}{2}(BD)(CD)$$
$$= \frac{1}{2}(a\sin\alpha)(b\cos\beta)$$
$$= \frac{1}{2}ab\sin\alpha\cos\beta$$

Q16(C)(iv)

Area of $\triangle ABC$

$$= \frac{1}{2}(AC)(BC)(\sin\angle ACB)$$
$$= \frac{1}{2}ba\sin(\alpha+\beta)$$
$$= \frac{1}{2}ab\sin(\alpha+\beta)$$

Q16(c)(v)

$$\text{Area}_{ABC} = \text{Area}_{ACD} + \text{Area}_{BCD}$$

$$\frac{1}{2}ab\sin(\alpha+\beta) = \frac{1}{2}ab\sin\beta\cos\alpha + \frac{1}{2}ab\sin\alpha\cos\beta$$

Divide both sides by $\frac{1}{2}ab$,

$$\sin(\alpha+\beta) = \sin\beta\cos\alpha + \sin\alpha\cos\beta$$

TRIAL HSC 2019 2 UNIT – Markers comments

Question 11:

a) Some students multiplied the numerator by $3\sqrt{5}-1$, perhaps thinking of the "difference" of two squares.

c) This question asked for factorising, not solving an equation. The common factor of 2 needs to be taken out, not just the two binomial factors. Moreover, this 2 does not disappear (you can only divide by 2 if the expression is equal to 0).

d) The answer needed to be a combined inequality $-\frac{1}{3} < x < 1$. Separating the inequalities implies that, for example, $x = -2$ is a valid solution.

e) Several students only substituted the point $(9, 2)$. This is not enough, as it only demonstrates that it is a possible line (in particular, you need to establish that the gradient is correct). Successful responses used $y - y_1 = m(x - x_1)$

Students should also be aware of the appropriate tests to establish that a quadrilateral is a parallelogram. One pair of parallel sides is not sufficient (this could be a trapezium). Successful students used one of:

- Opposite sides are equal
- Opposite sides are parallel
- One pair of opposite equal sides
- Diagonals bisect each other (i.e. midpoints are equal)

Using properties of a parallelogram to prove that a shape is, in fact, a parallelogram, was not accepted. In general students should know to avoid using a result to prove itself.

f) Students using the chain rule were very rarely successful, and should instead know to do basic simplifications before differentiating: $\sqrt[3]{x^2} = x^{\frac{2}{3}}$.

Question 12:

e) Some students did not realise that the question is related to $\int \frac{f'(x)}{f(x)}\, dx = \ln(f(x)) + C$.

f) i) Some students did not use the correct Simpson's rule formula.

f) ii) Some students did simple arithmetic errors such as 0.4 x 10 = 40.

Question 13:

a) Take more care with graphs-use a ruler for axes-label intercepts- use a solid line for graphs.

b) Setting out a "show that" must be clear and all working shown.

c) iii. Change in concavity must be verified! iv. Some sketches were dreadful! Take more pride!

Question 14:

a) Students did not know the log law $lna - lnb = \ln(\frac{a}{b})$.

c) The shaded region is rotated about the y-axis but some students did not use dy.

d) i) ii) Some students did not show appropriate reasoning when doing 'show questions'.

d) iii) Some students did not know that Θ cannot be greater than 30° given that Θ is in a right-angled triangle.

Question 15:

a) (i) Many did not know that point of inflexion meant the curve is stationary at $x = -2$. Negative slope means the graph is completely below the x-axis on derivative sketch.

(ii) & (iii) Many incorrectly looked at the slope of the derivative rather than where the curve is above or below the x-axis. Positive values means where the derivative curve is above the x-axis and negative values means where the derivative curve is below the x-axis.

b) Most students incorrectly thought positive definite means $\Delta > 0$. It means the graph has no x-intercepts so the discriminant must be less than 0. You must also show that $a > 0$.

c)(i) Show means do NOT skip steps. Write all working for how you came up with r and progress from A_1 through to A_3.

(ii) Many had the wrong value for r. Carefully look at the difference between terms in your expression for A_n.

d) Many students were able to get to an answer of $\frac{4+ln4}{2+ln2}$ but did not factorise and use log laws to simplify it further. Practice log law manipulations and look for common factors to cancel terms. If necessary, you could even check the value of $\frac{4+ln4}{2+ln2}$ in your calculator to come up with a value of 2.

Question 16:

a) The equation $l = r\theta$ assumes that θ is in radians. When given an angle of 2 radians, students only need to substitute $\theta = 2$ rather than incorrect unit conversions.

b) i) "Show that" questions require clear communication. The use of Pythagoras ($\left(\frac{h}{2}\right)^2 + r^2 = R^2$) to substitute h^2 should be obvious to an marker.

ii) Students differentiating V rather than V^2 made life difficult for themselves, although a few were successful. Students frequently forgot to substitute their point or test that their point is a maximum. This is absolutely always needed, even if it's the only solution to $\frac{dV^2}{dr} = 0$ (and in this case, it was not).

c) The aim of this question was to prove an identity $\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$. This is in fact an Extension 1 result, but several Extension 1 students attempted to use this result to prove itself. This is inappropriate and resulted in lost marks. Moreover, "show that" questions in general should never start with what is to be proven.

Students also need to know that even if they are unable to complete earlier parts of a question, they can still use "shown" results.