Student's Name:

Student Number:

Teacher's Name:

ABBOTSLEIGH

2016

HIGHER SCHOOL CERTIFICATE

Assessment 4

Mathematics Extension 1

General Instructions

- Reading time – 5 minutes
- Working time – 2 hours
- Write using black pen.
- **Board-approved** calculators may be used.
- A reference sheet is provided.
- All necessary working should be shown in every question.
- Make sure your HSC candidate Number is on the front cover of each booklet.
- Start a new booklet for Each Question.
- Answer the Multiple Choice questions on the answer sheet provided.
- If you do not attempt a whole question, you must still hand in the Writing Booklet, with the words **'NOT ATTEMPTED'** written clearly on the front cover.

Total marks – 70

- Attempt Sections 1 and 2.
- All questions are of equal value.

Section I Pages 3 - 7

10 marks

- Attempt Questions 1–10.
- Allow about 15 minutes for this section.

Section II Pages 8 - 12

60 marks

- Attempt Questions 11– 14.
- Allow about 1 hr and 45 minutes for this section.

Outcomes to be assessed:

Mathematics

Preliminary :

A student

P1 demonstrates confidence in using mathematics to obtain realistic solutions to problems.
P2 provides reasoning to support conclusions which are appropriate to the context.
P3 performs routine arithmetic and algebraic manipulation involving surds, simple rational expressions and trigonometric identities.
P4 chooses and applies appropriate arithmetic, algebraic, graphical, trigonometric and geometric techniques.
P5 understands the concept of a function and the relationship between a function and its graph.
P6 relates the derivative of a function to the slope of its graph.
P7 determines the derivative of a function through routine application of the rules of differentiation.
P8 understands and uses the language and notation of calculus.

HSC :

A student

H1 seeks to apply mathematical techniques to problems in a wide range of practical contexts.
H2 constructs arguments to prove and justify results.
H3 manipulates algebraic expressions involving logarithmic and exponential functions.
H4 expresses practical problems in mathematical terms based on simple given models.
H5 applies appropriate techniques from the study of calculus, geometry, probability, trigonometry and series to solve problems.
H6 uses the derivative to determine the features of the graph of a function.
H7 uses the features of a graph to deduce information about the derivative.
H8 uses techniques of integration to calculate areas and volumes.
H9 communicates using mathematical language, notation, diagrams and graphs.

Mathematics Extension 1

Preliminary:

A student

PE1 appreciates the role of mathematics in the solution of practical problems.
PE2 uses multi-step deductive reasoning in a variety of contexts.
PE3 solves problems involving permutations and combinations, inequalities, polynomials, circle geometry and parametric representations.
PE4 uses the parametric representation together with differentiation to identify geometric properties of parabolas.
PE5 determines derivatives which require the application of more than one rule of differentiation.
PE6 makes comprehensive use of mathematical language, diagrams and notation for communicating in a wide variety of situations.

HSC :

A student

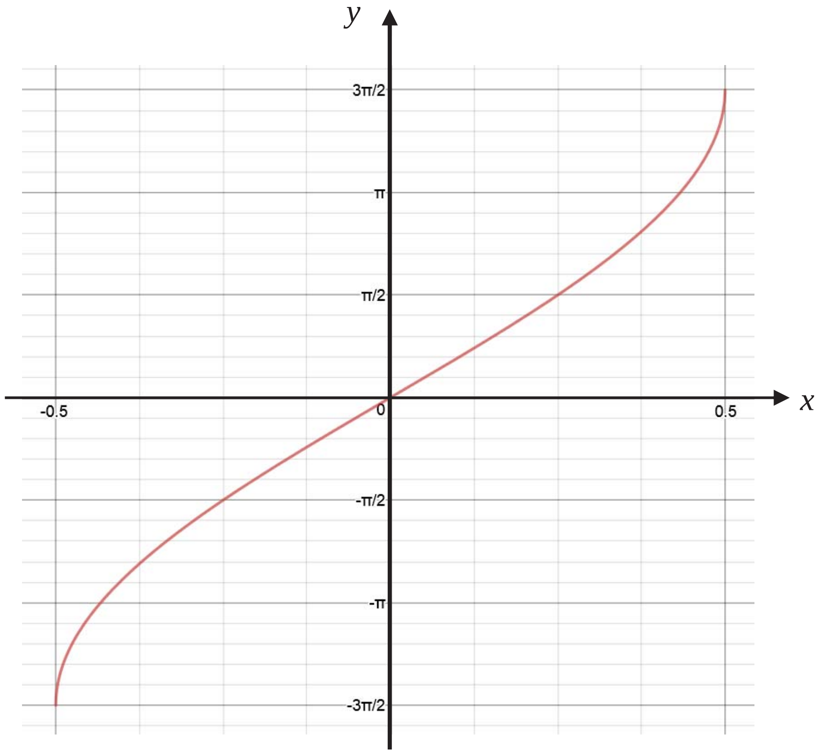
HE1 appreciates interrelationships between ideas drawn from different areas of mathematics.
HE2 uses inductive reasoning in the construction of proofs.
HE3 uses a variety of strategies to investigate mathematical models of situations involving projectiles.
HE4 uses the relationship between functions, inverse functions and their derivatives.
HE5 applies the chain rule to problems.
HE6 determines integrals by reduction to a standard form through a given substitution.
HE7 evaluates mathematical solutions to problems and communicates them in an appropriate form.

SECTION I

10 marks
Attempt Questions 1 – 10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1-10

1.



Which of the following is the equation of this graph?

(A) $y = \frac{1}{3}\sin^{-1}\frac{x}{2}$

(B) $y = \frac{1}{3}\sin^{-1}2x$

(C) $y = 3\sin^{-1}\frac{x}{2}$

(D) $y = 3\sin^{-1}2x$

2. The exact value of $\tan\frac{\pi}{12}$ is:

(A) $\frac{1}{2\sqrt{3}}$

(B) $2-\sqrt{3}$

(C) $2+\sqrt{3}$

(D) $\left(\sqrt{3}-1\right)^2$

3. $\displaystyle\int_0^{\frac{\pi}{8}} \sin^2 2x \; dx =$

(A) $\dfrac{\pi}{16} - \dfrac{1}{8}$

(B) $\dfrac{\pi}{8} - \dfrac{1}{4}$

(C) $\dfrac{\pi}{8} - \dfrac{1}{\sqrt{2}}$

(D) $\dfrac{\pi}{8} - \dfrac{1}{2\sqrt{2}}$

4. Which of the following is an equation of a curve that intersects at right angles every curve of the family $y = \dfrac{1}{x} + k$ (where k is a constant)?

(A) $y = x^2$

(B) $y = -x^2$

(C) $y = -\dfrac{1}{3}x^3$

(D) $y = \dfrac{1}{3}x^3$

5. What is the value of $\displaystyle\lim_{h \to 0} \frac{8\left(\frac{1}{2} + h\right)^8 - 8\left(\frac{1}{2}\right)^8}{h}$?

(A) 0

(B) $\dfrac{1}{2}$

(C) 1

(D) The limit does not exist.

6. Emma Jane made an error proving that $3^{2n}-1$ is divisible by 8 (for $n \geq 1$), using mathematical induction. Part of her proof is shown below.

Step 2: Assume the result true for $n = k$

$3^{2k}-1=8P$ where P is an integer.

Hence $3^{2k}=8P+1$

Step 3: To prove the result is true for $n = k+1$

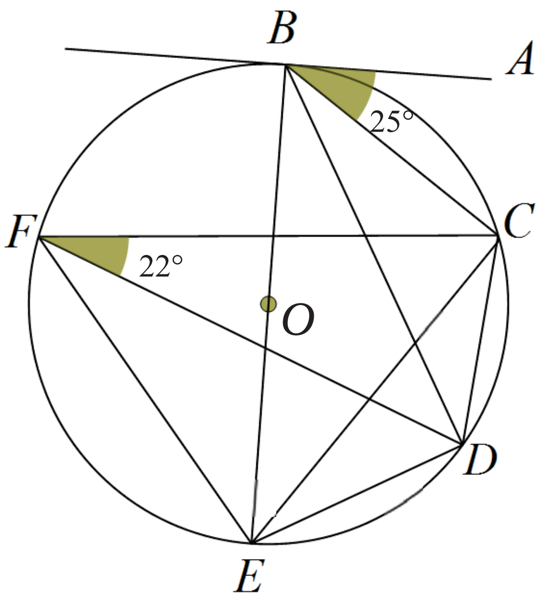
$RTP:$	$3^{2(k+1)}-1=8Q$ where Q is an integer.	Line 1
LHS	$= 3^{2k+2}-1$	Line 2
	$= 3^{2k}\times 3^2-1$	
	$= (8P+1)\times 3^2-1$ (*using the assumption*)	Line 3
	$= 72P+1-1$	Line 4
	$= 72\text{P}$	
	$= 8(9\text{P})$	
	$= 8Q$	
	$= RHS$	

In which line did Emma Jane make an error?

(A) Line 1 (B) Line 2 (C) Line 3 (D) Line 4

7. A circle with centre O has a tangent AB, diameter BE, $\angle ABC = 25°$ and $\angle CFD = 22°$.

NOT TO SCALE

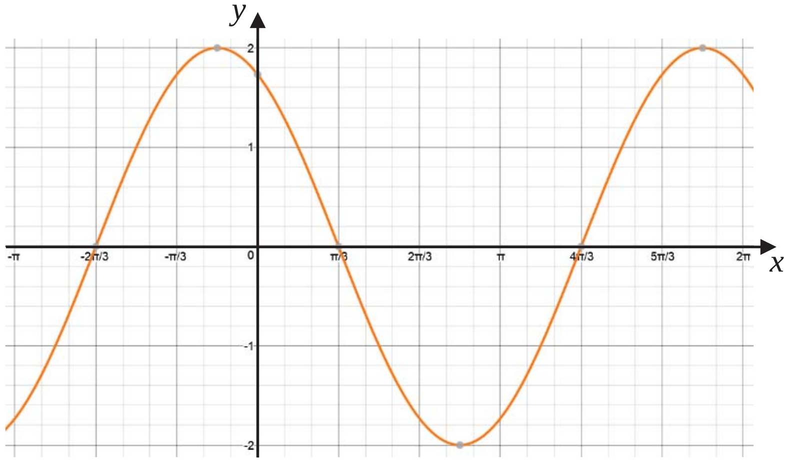


What is the size of $\angle EBD$?

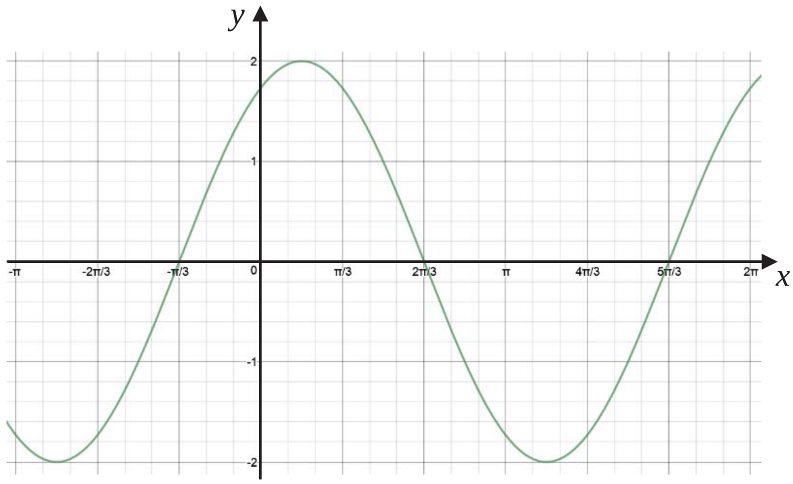
(A) 22° (B) 25° (C) 43° (D) 47°

8. The graph of $y = \sqrt{3}\sin x + \cos x$ is:

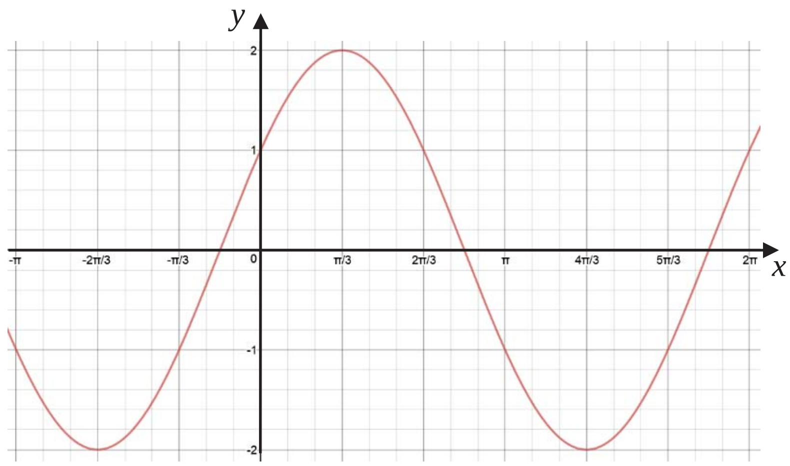
(A)



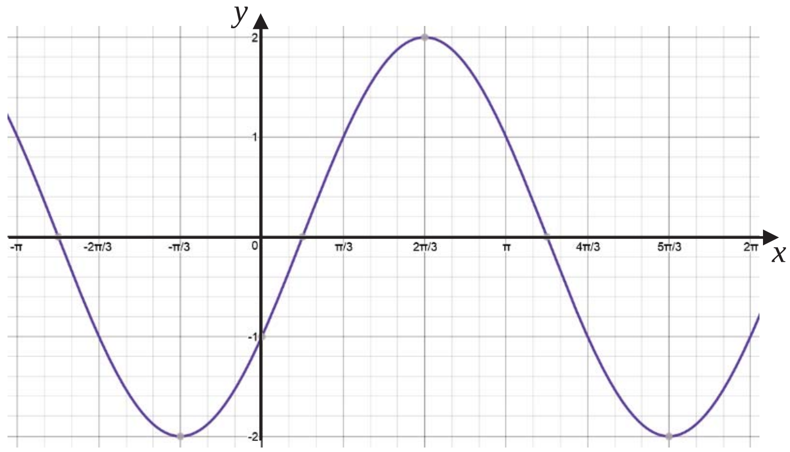
(B)



(C)



(D)



9. The radius of a circle is decreasing at a constant rate of $0.1\ \text{cm/s}$. In terms of the circumference, C, what is the rate of change of the area of the circle in cm^2/s?

(A) $-0.1C$

(B) $\dfrac{-0.1C}{2\pi}$

(C) $0.1^2 C$

(D) $0.1^2 \pi C$

10. Which of the following statements is **FALSE** ?

(A) $\cos^{-1}(-\theta) = -\cos^{-1}\theta$

(B) $\sin^{-1}(-\theta) = -\sin^{-1}\theta$

(C) $\tan^{-1}(-\theta) = -\tan^{-1}\theta$

(D) $\cos^{-1}(-\theta) = \pi - \cos^{-1}\theta$

End of Section I

Section II

60 marks

Attempt Questions 11-14
Allow about 1 hour and 45 minutes for this section

Answer each question in a SEPARATE writing booklet. Extra writing booklets are available.

In Questions 11-14, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) Use a SEPARATE writing booklet.

(a) Evaluate $\sum_{n=1}^{4} n\,!$ **1**

(b) Find $\int \frac{e^{3x}}{1+e^{3x}}\,dx$. **1**

(c) Solve $\frac{x}{x-3} \geq 3$. **3**

(d) Solve $\sin 2\theta = \cos\theta$ for $0 \leq \theta \leq \pi$. **2**

(e) The function $f(x) = x^2 - e^{(x-1)} + 1$ has only one real root.

(i) Show that the root lies between $x = 3$ and $x = 4$. **1**

(ii) Find a better approximation for the root using $x_1 = 3.5$ with one application of Newton's method. **2**

(f) The cubic polynomial $P(x) = ax^3 + bx^2 - 6x + 8$ has a factor of $(x-1)$ and a remainder of -24 when divided by $(x+2)$.

(i) Show that $a = 3$ and $b = -5$. **2**

(ii) Hence without using calculus, sketch *P(x)*, showing all axes intercepts. **3**

End of Question 11

Question 12 (15 marks) Use a SEPARATE writing booklet.

(a) Find the term independent of x in the expansion of $\left(5x^2+\dfrac{1}{x}\right)^{12}$. **3**

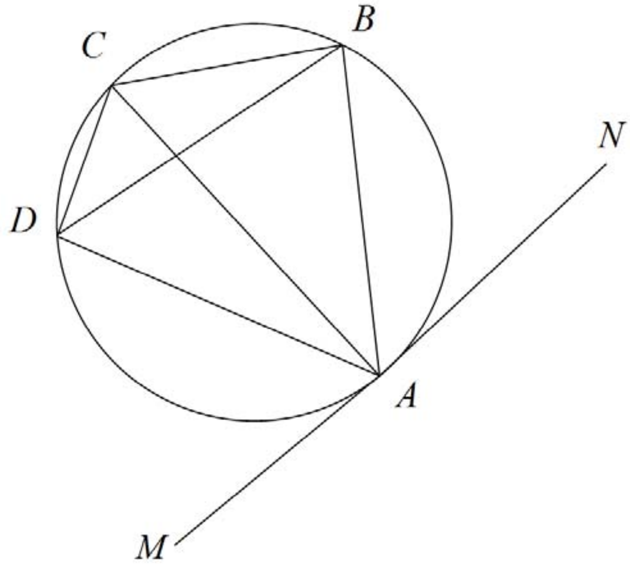
(b) $ABCD$ is a cyclic quadrilateral. MAN is the tangent at A to the circle through A, B, C and D. CA bisects $\angle DCB$.

Copy or trace the diagram into your writing booklet.

(i) Explain why $\angle BAN = \angle ACB$. **1**

(ii) Hence, or otherwise show that $MAN \parallel DB$. **3**

NOT TO SCALE



(c) Use the substitution $u = 1 - x^2$ to evaluate the definite integral **3**

$$\int_0^{\frac{\sqrt{3}}{2}} x\sqrt{1-x^2}\ dx.$$

(d) Let the cubic polynomial, $P(x) = x^3 - 3x^2 - 4x + 12$ have roots α, β and γ.

(i) Find the value of $\alpha + \beta + \gamma$. **1**

(ii) Find the value of $\dfrac{1}{\alpha}+\dfrac{1}{\beta}+\dfrac{1}{\gamma}$. **2**

(iii) Given that two of its roots have a sum of zero, find the values of α, β and γ. **2**

End of Question 12

Question 13 (15 marks) Use a SEPARATE writing booklet.

(a) Find $\displaystyle\int \frac{dx}{\sqrt{16-x^2}}$. **1**

(b) A particle is moving in a straight line. At time t seconds it has displacement x metres from a fixed point O on the line. Its velocity v ms^{-1}, is given by $v = \dfrac{2}{3\sqrt{x}}$.

Initially the particle is 1 metre to the right of O.

(i) Show that the acceleration, a ms^{-2}, of the particle is $\dfrac{-2}{9x^2}$. **2**

(ii) Show that $x = (t+1)^{\frac{2}{3}}$. **2**

(iii) Describe the motion of the particle. **2**

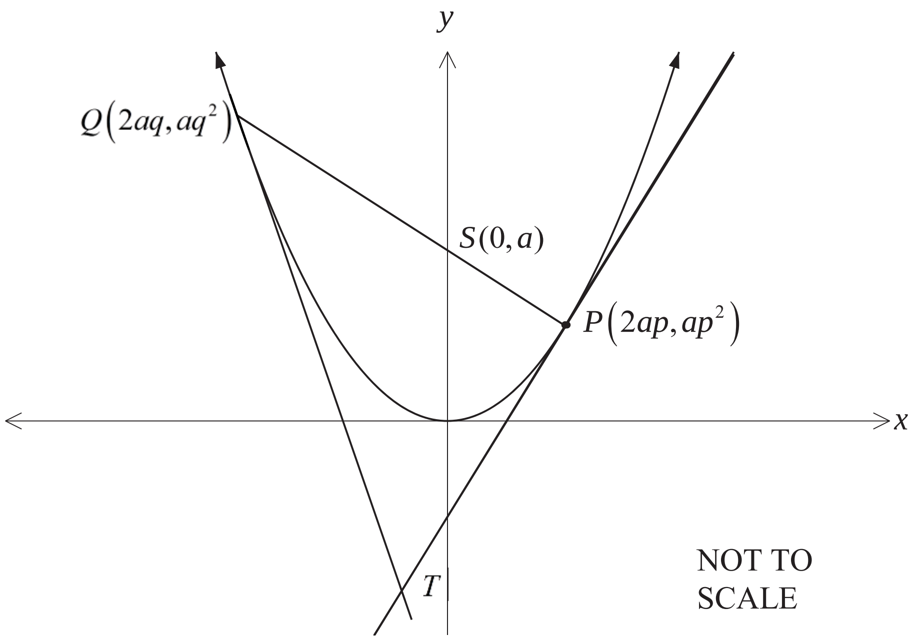
(c) Six letter words are formed from the letters of the word *CYCLIC*.

(i) Show that 120 six letter words can be formed. **1**

(ii) How many ways can a six letter word occur if there are no *C*'s together? **2**

Question 13 continues on page 11

(d) In the diagram below a focal chord PQ intersects the parabola $x^2 = 4ay$ at points $P(2ap, ap^2)$ and $Q(2aq, aq^2)$. The tangents to the parabola at points P and Q intersect at T.



(i) By considering the gradients of PQ and PS, show that $pq = -1$. **2**

(ii) Show that the acute angle between the focal chord PQ and the tangent PT to the parabola at P is given by $\tan^{-1}|q|$. **3**

End of Question 13

Question 14 (15 marks) Use a SEPARATE writing booklet.

(a) Alysha hits a golf ball off the ground with velocity V at an angle of projection θ to the horizontal. The equations of motion are as follows and **do not need to be proven**:

$\ddot{x} = 0$	$\ddot{y} = -g$
$\dot{x} = V\cos\theta$	$\dot{y} = V\sin\theta - gt$
$x = Vt\cos\theta$	$y = Vt\sin\theta - \frac{1}{2}gt^2$

(i) Show that Alysha's golf ball reaches a maximum height of $\dfrac{V^2\sin^2\theta}{2g}$. **2**

(ii) Kristine hits a second golf ball projected from the same horizontal plane. **4**

It has velocity $V \times \sqrt{\dfrac{5}{2}}$ and is projected at an angle $\dfrac{\theta}{2}$ to the horizontal.

What angles should Alysha and Kristine project their golf balls if they are to reach the same maximum height?

(b) By applying the binomial theorem to $(1+x)^{n-1}$ and integrating, show that: **3**

$$\frac{7}{1}\binom{n-1}{0} + \frac{7^2}{2}\binom{n-1}{1} + \frac{7^3}{3}\binom{n-1}{2} + \dots + \frac{7^n}{n}\binom{n-1}{n-1} = \frac{2^{3n}-1}{n}.$$

(c) (i) Prove that the graph of $y = \log_e x$ is concave down for all $x > 0$. **2**

(ii) Suppose $0 < a < b$ and consider the points $A(a, \log_e a)$ and $B(b, \log_e b)$ on the graph of $y = \log_e x$. Find the coordinates of the point P that divides the line segment AB internally in the ratio 2:1. **2**

(iii) Hence prove: $\dfrac{1}{3}\log_e a + \dfrac{2}{3}\log_e b < \log_e\left(\dfrac{1}{3}a + \dfrac{2}{3}b\right)$. **2**

End of paper

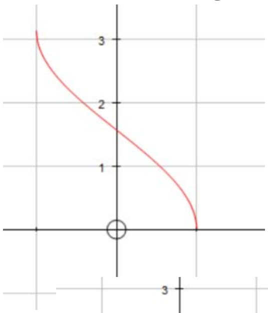
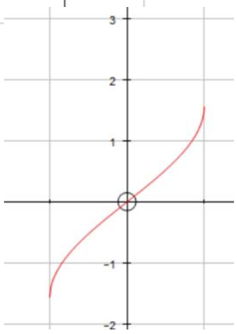
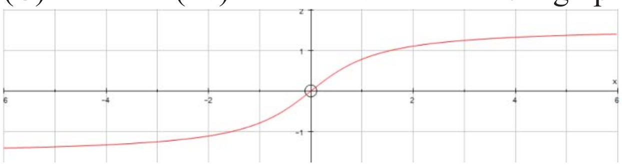
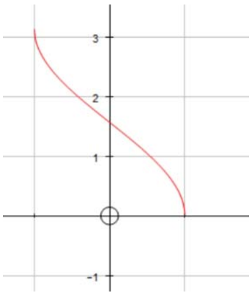
This paper is dedicated to Isaac Newton and the 350^{th} anniversary of several of his greatest achievements including the invention of calculus and groundbreaking work in optics.

These innovations alone would have made the year 1666 famous in the annals of science. But it was also in this year that the twenty- four-year old first began to conceive his greatest idea: the concept of gravity. By 1666 Newton had early versions of his three laws of motion and when applying them he calculated the force of attraction that held planets in their orbits, and the Moon in its orbit around Earth, as varying inversely with the square of their distance from the sun.

Extension 1 Mathematics Task 4 2016 Solutions:

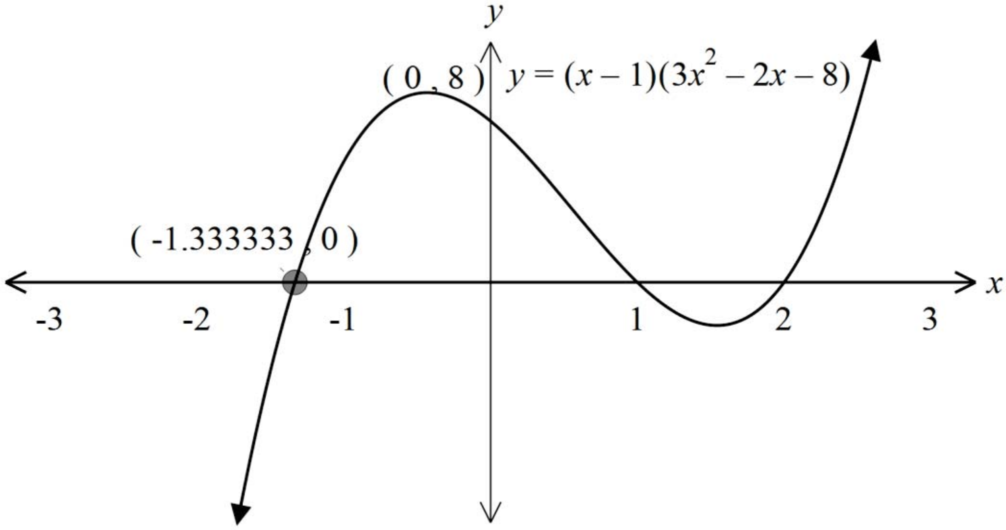
Question	Working	Solution
1	When $x = 0.5,\ y = \frac{3\pi}{2}$ Try D, $y = 3\sin^{-1}2x = 3\sin^{-1}(2\times\frac{1}{2}) = \frac{3\pi}{2}$ $\therefore D$	**1**
2	Substituting values into calculator or $\frac{\pi}{6}\times\frac{1}{2} = \frac{\pi}{12}$ Let $t = \tan\frac{\pi}{12}$ $\tan\theta = \frac{2t}{1-t^2}$ $\tan\frac{\pi}{6} = \frac{2t}{1-t^2}$ $\frac{1}{\sqrt{3}} = \frac{2t}{1-t^2}$ $1-t^2 = 2\sqrt{3}t$ $t^2 + 2\sqrt{3}t - 1 = 0$ $t = \frac{-2\sqrt{3}\pm\sqrt{(2\sqrt{3})^2+4}}{2} = \frac{-2\sqrt{3}\pm 4}{2} = -\sqrt{3}\pm 2$ but $t>0$ $\therefore t = -\sqrt{3}+2 = 2-\sqrt{3}$ $\therefore B$ Alternatively $\frac{\pi}{12} = \frac{4\pi}{12} - \frac{3\pi}{12} = \frac{\pi}{3} - \frac{\pi}{4}$ $\tan\frac{\pi}{12} = \tan\left(\frac{\pi}{3} - \frac{\pi}{4}\right)$	**1**
3	$\int_0^{\frac{\pi}{8}} \sin^2 2x\,dx = \int_0^{\frac{\pi}{8}} \frac{1}{2}(1-\cos 4x)\,dx$ $= \frac{1}{2}\left[x - \frac{1}{4}\sin 4x\right]_0^{\frac{\pi}{8}}$ $= \frac{1}{2}\left[(\frac{\pi}{8} - \frac{1}{4}\sin\frac{\pi}{2}) - 0\right]$ $= \frac{\pi}{16} - \frac{1}{8}$ $\therefore A$	**1**
4	$y = \frac{1}{x} + k \quad = x^{-1} + k$ $\frac{dy}{dx} = -\frac{1}{x^2}$ $y = \frac{1}{3}x^3$ $\frac{dy}{dx} = x^2$ $\therefore D$	**1**

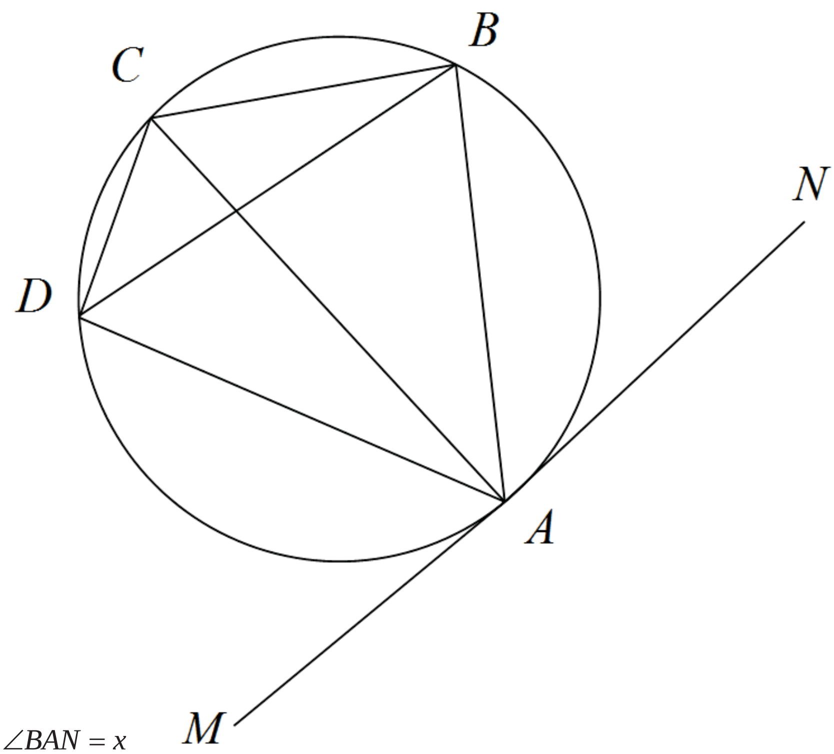
5	Using binomial expansions or by using calculator as $h \to 0$ $\frac{1}{2}$ $\therefore B$ Alternatively recognise as differentiation from first principles $8\left[\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}\right]$ where $f(x)=x^8$ at $x=\frac{1}{2}$ $8\left[\lim_{h\to 0}\frac{f(\frac{1}{2}+h)-f(\frac{1}{2})}{h}\right]$ $8\times 8x^7$ at $x=\frac{1}{2}$ $64\times\left(\frac{1}{2}\right)^7=\frac{1}{2}$	**1**
6	Line 4 $\therefore D$	**1**
7	$43°$ $\therefore C$	**1**
8	$y=\sqrt{3}\sin x+\cos x\equiv R\cos(x-\alpha)$ $R\cos x\cos\alpha+R\sin x\sin\alpha$ $r\sin\alpha=\sqrt{3}$ $r\cos\alpha=1$ $R=2,\tan\alpha=\sqrt{3},\alpha=\frac{\pi}{3}$ $2\cos(x-\frac{\pi}{3})$ Shift to right $\frac{\pi}{3}$ $\therefore$ C	**1**
9	$\frac{dr}{dt}=-0.1cm/s$ $A=\pi r^2$ $\frac{dA}{dr}=2\pi r$ $\frac{dA}{dt}=\frac{dA}{dr}\times\frac{dr}{dt}=2\pi r\times -0.1=-0.1C$ $\therefore A$	**1**

10	By considering odd/evenness of sketches of inverse trig functions show that only A is FALSE $\therefore A$ (A) $\cos^{-1}(-\theta) = -\cos^{-1}\theta$  from graph, FALSE (B) $\sin^{-1}(-\theta) = -\sin^{-1}\theta$  from graph, TRUE (C) $\tan^{-1}(-\theta) = -\tan^{-1}\theta$ from graph, TRUE  (D) $\cos^{-1}(-\theta) = \pi - \cos^{-1}\theta$ from graph, TRUE  .	**1**

Question	Working	Solution
11(a)	$\sum_{n=1}^{4} n! = 1 + 2! + 3! + 4!$ $\boxed{\checkmark}$ Answer $= 1 + 2 + 6 + 24 = 33$	**1**
11(b)	$\int \frac{e^{3x}}{1+e^{3x}}\,dx$ $= \frac{1}{3}\int \frac{3e^{3x}}{1+e^{3x}}\,dx$ $= \frac{1}{3}\ln\left\|1+e^{3x}\right\| + C$ $\boxed{\checkmark}$ $(no\ deduction\ \ for\ +C)$	**1**
11(c)	$\frac{x}{x-3} \ge 3$ $\times(x-3)^2$ to both sides $x \ne 3$ setting up critical points $x(x-3) \ge 3(x-3)^2$ $3(x-3)^2 - x(x-3) \le 0$ $\boxed{\checkmark}$ $(x-3)(3(x-3)-x) \le 0$ $(x-3)(3x-9-x) \le 0$ $(x-3)(2x-9) \le 0$ $x = \frac{9}{2}$ or 3 $\boxed{\checkmark}$ $3 < x \le \frac{9}{2}$ $\boxed{\checkmark}$ correct signs noting that $x \ne 3$	**3**
11(d)	$\sin 2\theta = \cos\theta$ for $0 \le \theta \le \pi$. $2\sin\theta\cos\theta - \cos\theta = 0$ $\boxed{\checkmark}$ double angle $\cos\theta(2\sin\theta - 1) = 0$ $\cos\theta = 0$ or $2\sin\theta - 1 = 0$ $\theta = \frac{\pi}{2}$ $2\sin\theta = 1$ $\sin\theta = \frac{1}{2}$ $\theta = \frac{\pi}{6}, \frac{5\pi}{6}$ 1 mark for all correct solutions in domain $\boxed{\checkmark}$	**2**
11(e)(i)	$f(x) = x^2 - e^{(x-1)} + 1$ $\left.\begin{array}{l} f(3) = 3^2 - e^{(3-1)} + 1 = 2.6109 \\ f(4) = 4^2 - e^{(4-1)} + 1 = -3.0855 \end{array}\right\}$ no marks for just substitutions Sketch or explain that since $f(x)$ is made up of continuous functions and $f(3) > 0$, $f(4) < 0$ there is a root in-between. $\boxed{\checkmark}$ explanation	**1**

11(e)(ii)	$f'(x)=2x-e^{(x-1)}$ $f(3.5)=3.5^2-e^{(3.5-1)}+1=1.0675$ $f'(3.5)=2(3.5)-e^{(3.5-1)}=-5.182$ ✓ for both substitutions CNE $x_2=x_1-\dfrac{f(x_1)}{f'(x_1)}$ $x_2=3.5-\dfrac{f(3.5)}{f'(3.5)}=3.5-\dfrac{1.0675}{-5.182}=3.7$ ✓ for substitution CNE	**2**
11(f)(i)	$P(x)=ax^3+bx^2-6x+8$ $(x-1)$ is a factor $\therefore P(1)=0$ or $P(-2)=-24$ ✓ $P(1)=a+b-6+8=0$ $a+b=-2$ ......(1) $P(-2)=a(-2)^3+b(-2)^2-6(-2)+8=-24$ $-8a+4b+12+8=-24$ $-8a+4b=-44$ $-2a+b=-11$ $2a-b=11$ ......(2) $(1)+(2)$ $3a=9$ $a=3$ $3+b=-2$ $b=-5$ ✓ correct progress to given solution	**2**

11(f)(ii)	$P(x)=(x-1)(3x^2-2x-8)$ $$\begin{array}{r} 3x^2-2x-8 \\ x-1\overline{)3x^3-5x^2-6x+8} \\ \underline{3x^3-3x^2} \\ -2x^2-6x \\ \underline{-2x^2+2x} \\ -8x+8 \\ \underline{-8x+8} \\ 0 \end{array}$$ $=(x-1)(x-2)(3x+4)$ ✓  ✓ roots ✓ shape and $y-$intercept	**3**
12(a)(i)	$\left(5x^2+\frac{1}{x}\right)^{12}$ General term $\binom{n}{k}a^{n-k}b^k$ $\binom{12}{k}(5x^2)^{12-k}(x^{-1})^k$ ✓ $=\binom{12}{k}5^{12-k}x^{24-2k}x^{-k}=\binom{12}{k}5^{12-k}x^{24-3k}$ Constant term $x^0=1$ $24-3k=0$ $3k=24$ $k=8$ ✓ $\therefore$ Term independent of x is : $\binom{12}{8}5^4=\frac{12!}{8!4!}5^4=309375$ ✓ *ISE*	**3**

12(b)(i)	 Let $\angle BAN = x$ $\angle BAN = \angle ACB = x$ (angle between tangent and chord equals angle in alternate segment) ☑	**1**
12(b)(ii)	$\angle DCA = \angle ACB = x$ (Given CA bisects angle $\angle DCB$) ☑ $\angle DCA = \angle DBA = x$ (angles in same segment) ☑ $\therefore \angle DBA = \angle BAN = x$ $\therefore MAN \parallel DB$ (alternate angles equal) ☑	**3**

12(c)	$\int_0^{\frac{\sqrt{3}}{2}} x\sqrt{1-x^2}\,dx$ $=-\frac{1}{2}\int_0^{\frac{\sqrt{3}}{2}} -2x\sqrt{1-x^2}\,dx$ $=-\frac{1}{2}\int_1^{\frac{1}{4}} \sqrt{u}\,du$ $=\frac{1}{2}\int_{\frac{1}{4}}^{1} \sqrt{u}\,du$ $\frac{1}{2}\left[\frac{u^{\frac{3}{2}}}{\frac{3}{2}}\right]_{\frac{1}{4}}^{1}$ ☑ (correct integration) $=\frac{1}{2}\times\frac{2}{3}\left[1-\frac{1}{4}^{\frac{3}{2}}\right]$ $=\frac{1}{3}\left[1-\frac{1}{8}\right]$ $=\frac{7}{24}$ ☑ $\left.\begin{array}{l} u=1-x^2 \\ du=-2x\,dx \\ \text{When } x=0,\ u=1, \\ x=\frac{\sqrt{3}}{2},\ u=1-\left(\frac{\sqrt{3}}{2}\right)^2=\frac{1}{4} \end{array}\right\}$ ☑ Note: Question asked for u substitution hence 2 marks if used reverse chain rule	**3**
12(d)(i)	$P(x)=x^3-3x^2-4x+12$ $\alpha+\beta+\gamma=3$ ☑	**1**
12(d)(ii)	$\frac{1}{\alpha}+\frac{1}{\beta}+\frac{1}{\gamma}=\frac{\alpha\beta+\alpha\gamma+\beta\gamma}{\alpha\beta\gamma}$ ☑ $\alpha\beta\gamma=-12$ $=\frac{-4}{-12}$ ☑ correct substitution $=\frac{1}{3}$	**2**
12(d)(iii)	Let $\alpha+\beta=0$ $\alpha=-\beta$ $\alpha+\beta+\gamma=3$ $\therefore \gamma=3$ ☑ $\alpha\beta\gamma=-12$ $-3\beta^2=-12$ or $\alpha\beta+\alpha\gamma+\beta\gamma=-4$ $-\beta^2-3\beta+3\beta=-4$ $\beta^2=4$ $\beta=\pm 2$ $\therefore$ Roots are 2,-2 and 3 ☑	**2**

13(a)	$\int \frac{dx}{\sqrt{16-x^2}}$ $= \sin^{-1}\frac{x}{4} + C$ ✓ constants and function	**1**
13(b)(i)	$v = \frac{2}{3\sqrt{x}} = \frac{2}{3}x^{-\frac{1}{2}}$ $v^2 = \frac{4}{9x}$ $\frac{1}{2}v^2 = \frac{2}{9x} = \frac{2}{9}x^{-1}$ ✓ $a = \frac{d\left(\frac{1}{2}v^2\right)}{dx} = -\frac{2}{9}x^{-2} = -\frac{2}{9x^2}$ ✓ showing derivative process	**2**
13(b)(ii)	$a = \frac{d^2x}{dt^2} = -\frac{2}{9x^2}$ $v = \frac{dx}{dt} = \frac{2}{3\sqrt{x}}$ $\frac{dt}{dx} = \frac{3\sqrt{x}}{2}$ $t = \int \frac{3\sqrt{x}}{2}\,dx = \frac{3}{2}\frac{x^{\frac{3}{2}}}{\left(\frac{3}{2}\right)} + C = x^{\frac{3}{2}} + C$ ✓ When $t = 0,\ x = 1$ $0 = 1 + C \qquad \therefore C = -1$ $\therefore t = x^{\frac{3}{2}} - 1$ ✓ $x^{\frac{3}{2}} = t + 1$ $x = (t+1)^{\frac{2}{3}}$	**2**
13(b)(iii)	Starting at 1m to the right of the origin, with a velocity of $\frac{2}{3}$ m/s , ✓ the particle ***always*** moves further to the right but is constantly slowing. Or mention that $v \neq 0$ or particle never stops ✓	**2**

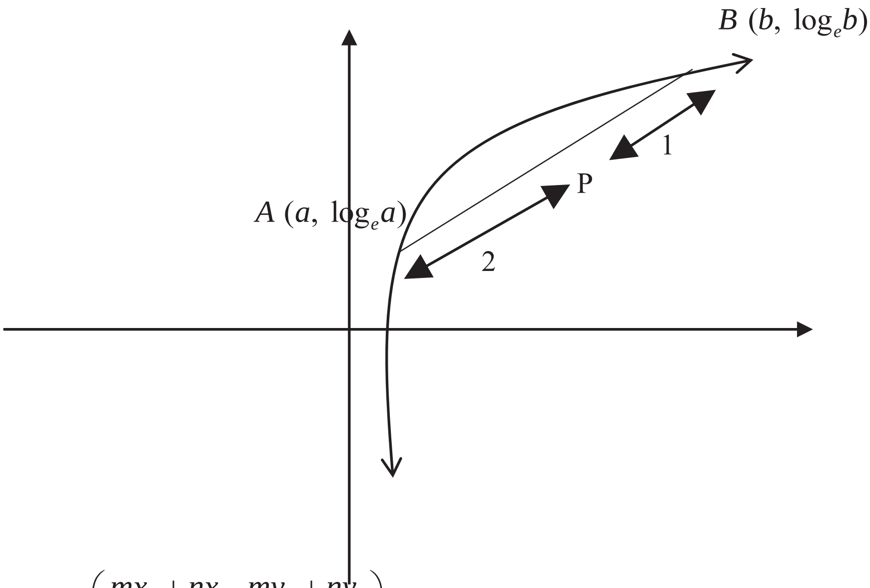
13(c)(i)	Number of words $= \dfrac{6!}{3!}$ $\boxed{\checkmark}$ $=120$	**1**
13(c)(ii)	No C's together $\underline{C}\ \_\ \underline{C}\ \_\ \underline{C}\ \_$ $\underline{C}\ \_\ \underline{C}\ \_\ \_\ \underline{C}$ $\underline{C}\ \_\ \_\ \underline{C}\ \_\ \underline{C}$ $\_\ \underline{C}\ \_\ \underline{C}\ \_\ \underline{C}$ #ways to arrange Cs $\times$ #ways to arrange Y,L,I $4\times 3! = 24$ $\boxed{\checkmark}$ $\boxed{\checkmark}$ OR _ Y _ L _ I _ possible positions of Cs - ${}^{4}_{3}C$ ways of positioning Cs $\boxed{\checkmark}$ Then $3!$ ways of arranging Y, L and I. Total no of arrangements $= {}^{4}_{3}C \times 3! = 24$ $\boxed{\checkmark}$	**2**
13(d)(i)	$m_{PQ} = \dfrac{ap^2 - aq^2}{2ap - 2aq}$ $= \dfrac{a(p-q)(p+q)}{2a(p-q)}$ $= \dfrac{(p+q)}{2}$ $\boxed{\checkmark}$ or equivalent statement of m_{PS} $m_{PS} = \dfrac{ap^2 - a}{2ap - 0}$ $= \dfrac{a(p^2-1)}{2ap}$ $= \dfrac{(p^2-1)}{2p}$ PQ and PS lie on the same line so $m_{PQ} = m_{PS}$ $\left.\begin{array}{l}\dfrac{(p+q)}{2} = \dfrac{(p^2-1)}{2p} \\ p^2 + pq = p^2 - 1\end{array}\right\}$ $\boxed{\checkmark}$ $\therefore pq = -1$	**2**

13(d)(ii)	Gradient of tangent at P is p. Acute angle between lines PQ and PT $\tan\theta = \left\lvert \dfrac{\dfrac{p+q}{2} - p}{1 + \dfrac{p+q}{2}.p} \right\rvert$ ☑ $= \left\lvert \dfrac{p+q-2p}{2+p^2+pq} \right\rvert$ $= \left\lvert \dfrac{q-p}{p^2+pq+2} \right\rvert$ But $pq = -1$ $= \left\lvert \dfrac{q-p}{p^2-1+2} \right\rvert$ $= \left\lvert \dfrac{q-p}{p^2+1} \right\rvert$ ☑ Using $pq = -1$ and simplification Now use $p = -\dfrac{1}{q}$ $\tan\theta = \left\lvert \dfrac{q+\dfrac{1}{q}}{\dfrac{1}{q^2}+1} \right\rvert$ $= \left\lvert \dfrac{q^3+q}{1+q^2} \right\rvert$ $= \left\lvert \dfrac{q(q^2+1)}{1+q^2} \right\rvert = \lvert q \rvert$ $\therefore \angle QPT = \tan^{-1}\lvert q \rvert$ ☑ using $pq = -1$		**3**

Question	Working	Solution
14(a)(i)	Max height when $\dot{y}=0$ $V\sin\theta - gt = 0$ $tg = V\sin\theta$ $t = \dfrac{V\sin\theta}{g}$ ☑ Sub into y $\left.\begin{aligned} y &= Vt\sin\theta - \frac{1}{2}gt^2 \\ &= \frac{V^2\sin^2\theta}{g} - \frac{1}{2}g\left(\frac{V\sin\theta}{g}\right)^2 \\ &= \frac{V^2\sin^2\theta}{g} - \frac{1}{2}g\left(\frac{V^2\sin^2\theta}{g^2}\right) \\ &= \frac{V^2\sin^2\theta}{2g}\end{aligned}\right\}$ ☑ process and simplification	**2**
14(a)(ii)	Kristine $\ddot{y} = -g$ $\dot{y} = -gt + C$ When $t=0, \dot{y} = V\sqrt{\frac{5}{2}}\sin\frac{\theta}{2}$ $\therefore \dot{y} = -gt + V\sqrt{\frac{5}{2}}\sin\frac{\theta}{2}$ $y = -\frac{1}{2}gt^2 + Vt\sqrt{\frac{5}{2}}\sin\frac{\theta}{2} + C$ When $t=0, y=0 \therefore C=0$ Kristine's max height when $\dot{y}=0$ $tg = V\sqrt{\frac{5}{2}}\sin\frac{\theta}{2}$ $t = \dfrac{V\sqrt{\frac{5}{2}}\sin\frac{\theta}{2}}{g}$ Sub into y	**4**

	$y=-\frac{1}{2}g\left(\frac{V\sqrt{\frac{5}{2}}\sin\frac{\theta}{2}}{g}\right)^2+V\left(\frac{V\sqrt{\frac{5}{2}}\sin\frac{\theta}{2}}{g}\right)\sqrt{\frac{5}{2}}\sin\frac{\theta}{2}$ $=-\frac{1}{2}g\frac{V^2\frac{5}{2}\sin^2\frac{\theta}{2}}{g^2}+\frac{V^2\frac{5}{2}\sin^2\frac{\theta}{2}}{g}$ $(Kris)=\frac{V^2\frac{5}{2}\sin^2\frac{\theta}{2}}{2g}$ $=\frac{5V^2\sin^2\frac{\theta}{2}}{4g}$ ☑ or using max height from (i) For same Max height $\frac{V^2\frac{5}{2}\sin^2\frac{\theta}{2}}{2g}=\frac{V^2\sin^2\theta}{2g}$ ☑ $\left.\begin{array}{l}\frac{5}{2}\sin^2\frac{\theta}{2}=\sin^2\theta\\ \sin^2\frac{\theta}{2}=\frac{2}{5}\sin^2\theta\\ \sin\frac{\theta}{2}=\sqrt{\frac{2}{5}}\sin\theta \quad \text{as } \theta \text{ and } \frac{\theta}{2} \text{ acute}\\ \sin\frac{\theta}{2}=\sqrt{\frac{2}{5}}2\sin\frac{\theta}{2}\cos\frac{\theta}{2}\\ \sqrt{\frac{2}{5}}2\sin\frac{\theta}{2}\cos\frac{\theta}{2}-\sin\frac{\theta}{2}=0\\ \sin\frac{\theta}{2}(\sqrt{\frac{2}{5}}2\cos\frac{\theta}{2}-1)=0\\ \sin\frac{\theta}{2}=0\\ \text{but } \theta=0\\ \text{no angle of projection}\end{array}\right\}$ ☑ $\left.\begin{array}{l}\sqrt{\frac{2}{5}}2\cos\frac{\theta}{2}=1\\ \cos\frac{\theta}{2}=\frac{1}{2}\sqrt{\frac{5}{2}}\\ \frac{\theta}{2}=\cos^{-1}\left(\frac{1}{2}\sqrt{\frac{5}{2}}\right)\\ \theta=75°31',\frac{\theta}{2}=37°46'\end{array}\right\}$ ☑	

Question	Working	Solution
14(b)(ii)	$\int (1+x)^{n-1}dx$ $=\dfrac{(1+x)^{n-1+1}}{n}+C$ $=\dfrac{(1+x)^{n}}{n}+C$ $(1+x)^{n-1}=\sum_{k=0}^{n-1}\binom{n-1}{k}x^k$ $=1+(n-1)x+\dfrac{(n-1)n}{2}x^2+\binom{n-1}{3}x^3+\ldots\ldots\binom{n-1}{n-1}x^{n-1}$ ☑ binomial expansion $\int 1+(n-1)x+\dfrac{(n-1)n}{2}x^2+\binom{n-1}{3}x^3+\ldots\ldots\binom{n-1}{n-1}x^{n-1}dx$ $=x+\dfrac{(n-1)}{2}x^2+\binom{n-1}{2}\dfrac{x^3}{3}+\binom{n-1}{3}\dfrac{x^4}{4}+\ldots\ldots\binom{n-1}{n-1}\dfrac{x^n}{n}+C=\dfrac{(1+x)^n}{n}$ ☑ integration of both sides When x=0, C=$\dfrac{1}{n}$ When $x=7$ $\dfrac{7}{1}\binom{n-1}{0}+\dfrac{7^2}{2}\binom{n-1}{1}+\dfrac{7^3}{3}\binom{n-1}{2}+\ldots+\dfrac{7^n}{n}\binom{n-1}{n-1}+\dfrac{1}{n}=\dfrac{(8)^n}{n}$ $\therefore \dfrac{7}{1}\binom{n-1}{0}+\dfrac{7^2}{2}\binom{n-1}{1}+\dfrac{7^3}{3}\binom{n-1}{2}+\ldots+\dfrac{7^n}{n}\binom{n-1}{n-1}=\dfrac{(8)^n}{n}-\dfrac{1}{n}$ $=\dfrac{2^{3n}-1}{n}$ ☑ ☑ sub x = 7 & simplifying	3

Question	Working	Solution
14(c)(i)	$y = \log_e x$ $\frac{dy}{dx} = \frac{1}{x} = x^{-1}$ $\frac{d^2y}{dx^2} = -1x^{-2} = -\frac{1}{x^2}$ $x^2 \geq 0$ so $\frac{d^2y}{dx^2} = \frac{1}{x^2} \leq 0$ and $x \neq 0$ $\therefore$ Concave down for all $x > 0$ ☑ derivative ☑ explanation	**2**
14(c)(ii)	 Division in ratio $\left(\frac{mx_2+nx_1}{m+n}, \frac{my_2+ny_1}{m+n}\right)$ $=\left(\frac{2\times b+1\times a}{3}, \frac{2\times \log_e b+1\times \log_e a}{3}\right)$ ☑ $=\left(\frac{2b+a}{3}, \frac{2\log_e b+\log_e a}{3}\right)$ ☑	**2**
14(c)(iii)	Since $\log_e x$ is concave down for all $x>0$, P $\left(\frac{2b+a}{3}, \frac{2\log_e b+\log_e a}{3}\right)$ is below the curve $y=\log_e x$ when $x=\frac{2b+a}{3}$ ☑ Let Q be point on $y=\log_e x$ above P at $x=\frac{2b+a}{3}$, $y=\log_e\left(\frac{2b+a}{3}\right)$ $\therefore \frac{2\log_e b+\log_e a}{3} < \log_e\left(\frac{2b+a}{3}\right)$ $\frac{1}{3}\log_e a+\frac{2}{3}\log_e b < \log_e\left(\frac{1}{3}a+\frac{2}{3}b\right)$ ☑	**2**