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ASCHAM SCHOOL

MATHEMATICS EXTENSION 1

TRIAL EXAMINATION 2003

Time allowed: 2 hours plus 5 minutes reading time

All questions may be attempted

All questions are of equal value.

All necessary working should be shown in every question.

Approved calculators may be used.

Standard integrals are printed at the end of the exam paper.

Start each question in a new booklet.

QUESTION 1

a) Find $\frac{d}{dx}\sin^{-1}2x$ (1)

b) Find $\int\frac{5}{2+3x^2}dx$ (2)

c) Solve for x: $\frac{2}{x}\geq x-1$ (2)

d) Find the acute angle between the lines y = -x and $\sqrt{3}y=x$ (2)

e) Find the exact value of $\cos(\sin^{-1}(-\frac{1}{4}))$ (2)

f) Use the substitution u = 1 + x to find $\int_1^2\frac{1-x}{(1+x)^3}dx$ (3)

QUESTION 2

a) Find $\int\sin^2 2x\,dx$ (2)

b) Find the co-ordinates of the point P which divides AB externally in the ratio 2:3 where A is (1,-4) and B(6,9). (2)

c) Solve $|3x-2|>x+1$ (3)

d) α and β are acute angles such that $\cos\alpha=\frac{3}{5}$ and $\sin\beta=\frac{1}{\sqrt{5}}$.

Without finding the size of either angle, show that $\alpha=2\beta$. (2)

e) The roots of the equation $x^3-6x^2+3x+k=0$ are consecutive terms of an Arithmetic Sequence. Find k. (3)

14
2
1
8

QUESTION 3

a) A spherical bubble is expanding so that its volume is increasing at the constant rate of $12mm^3$ per second. What is the rate of increase of the radius when the surface area is $500mm^2$? (4)

b) Find θ if $\sin\theta - \cos\theta = 1$ (4)

c) Prove by mathematical induction that

$$1\times 2^0 + 2\times 2^1 + 3\times 2^2 + \ldots\ldots + n\times 2^{n-1} = 1 + (n-1)2^n \text{ for all integers } n \geq 1$$ (4)

QUESTION 4

a) A particle moves on a line so that its distance from the origin at time t is x and its velocity is v.

i) Prove $\dfrac{d^2x}{dt^2} = \dfrac{d}{dx}\left(\dfrac{1}{2}v^2\right)$ (2)

ii) If $\dfrac{d^2x}{dt^2} = n^2(3-x)$ where n is a constant

and if the particle is released from rest at x = 0,

show $\dfrac{1}{2}v^2 - n^2\left(3x - \dfrac{1}{2}x^2\right) = 0$ (2)

iii) Hence show that the particle never moves outside a certain interval. (2)

b) i) Draw a large sketch of $y = \dfrac{x+4}{x(x+8)}$, showing all essential features. (4)

ii) Find the area bounded by the curve $y = \dfrac{x+4}{x(x+8)}$ and the x axis between x = 1 and x = 2. (2)

$m = \left(\dfrac{1-7}{2}, \dfrac{8+4}{2}\right)$

$= \left(-\dfrac{6}{2}, \dfrac{12}{2}\right)$

$= (-3, +6)$

$y =$

$y = 4x - 6$ — ①

$3x + 2y = -1$ — ②

$3x + 2(4x - 6) = -1$

$3x + 8x - 12 = -1$

$11x - 12 = -1$

QUESTION 5

a) For what values of x will $1-\tan^2 x+\tan^4 x-\tan^6 x+...$ have a limiting sum for $0 \leq x \leq 2\pi$? (4)

b) A particle is oscillating in simple harmonic motion such that its displacement x metres from a given origin O satisfies the equation

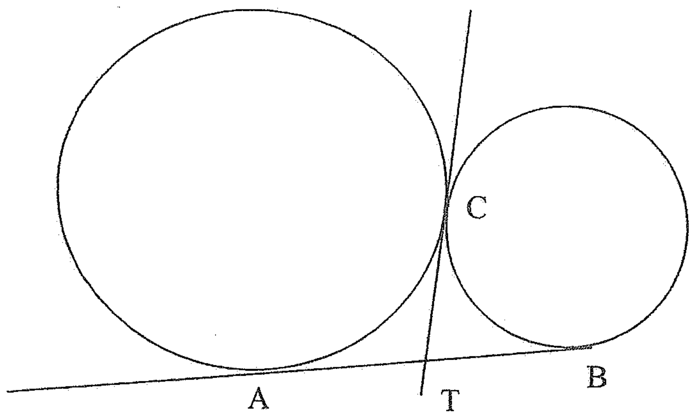
$$\frac{d^2x}{dt^2} = -4x \quad \text{where t is the time in seconds.}$$

i) Show that $x = a\cos(2t+\beta)$ is a possible equation of motion for the particle when a and β are constants. (1)

ii) The particle is observed at time t = 0 to have a velocity of 2m/s and a displacement from the origin of 4m. Find the amplitude of oscillation. (3)

iii) Determine the maximum velocity of the particle. (1)

b)



Two circles touch externally at C. The circles are touched by the common tangent at A and B respectively. The common tangent at C meets AB in T.

Show that ∠ACB = 90°. (3)

QUESTION 6

a) A particle is fired from a point O on the floor of a horizontal tunnel of height 15 metres at a speed of 30m/s and at an angle θ above the horizontal where $0 < \theta < \frac{\pi}{2}$.

Assume that the horizontal displacement x metres and the vertical displacement y metres of the particle from O at time t seconds after firing are given by

$x = 30t\cos\theta$ and $y = 30t\sin\theta - 5t^2$.

Find the maximum horizontal range of the particle along the tunnel. (5)

b) A function is defined as $f(x) = 1 - \cos\frac{x}{2}$ where $0 \le x \le a$

i) Find the largest value of a for which the inverse function $f^{-1}(x)$ exists. (2)

ii) Find $f^{-1}(x)$ (1)

iii) Sketch the graph of y = $f^{-1}(x)$ (2)

iv) Find the area enclosed between the curve y = $f^{-1}(x)$, the x axis and x = 2. (2)

QUESTION 7

a) The following question appears in a textbook:

Use Newton's method to find an approximation to the solution of f(x)= -5. Take x_0 = ____ *as a first approximation.*

Christiana writes down:

$$x_1 = 1.8 - \frac{-0.10}{-5.91}$$

i) What is x_0? (1)

ii) It is known that f(x) = asinx +bx, where a and b are integers.

Write down 2 equations involving a and b. (3)

Do not solve the equations.

b) The straight line y = mx + b meets the parabola $x^2 = 4ay$ at the points P($2ap, ap^2$) and Q($2aq, aq^2$).

i) Find the equation of the chord PQ and hence or otherwise show that $pq = \frac{-b}{a}$ (2)

ii) Prove that $p^2 + q^2 = 4m^2 + \frac{2b}{a}$ (1)

iii) Given that the equation of the normal to the parabola at P is

$x + py = 2ap + ap^3$ and that N, the point of intersection of the

normals at P and Q, has co-ordinates

$$\left[-apq(p+q), a(2+p^2+pq+q^2)\right],$$

express these co-ordinates in terms of a, m and b. (2)

iv) Suppose that the chord PQ is free to move while maintaining a fixed gradient. Find the locus of N and show that this locus is a straight line.
Verify that this line is a normal to the parabola. (3)

END OF EXAM

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FORM 6 MATHEMATICS EXTENSION 1 TRIAL EXAM 2003

QUESTION 1

a) $\frac{d}{dx}\sin^{-1}2x = \frac{2}{\sqrt{1-4x^2}}$ (1)

b) $\int \frac{5}{2+3x^2}\,dx = \int \frac{5}{3\left(\frac{2}{3}+x^2\right)}\,dx$ ✓

$= \frac{5}{3}\cdot\frac{\sqrt{3}}{\sqrt{2}}\tan^{-1}\frac{\sqrt{3}}{\sqrt{2}}x + c$

$= \frac{5}{\sqrt{6}}\tan^{-1}\sqrt{\frac{3}{2}}\,x + c$ ✓ (2)

$\left(= \frac{5\sqrt{6}}{6}\tan^{-1}\frac{\sqrt{6}}{2}x + c\right)$

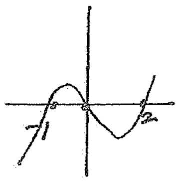
c) $\frac{2}{x} \geq x-1$

$2x \geq x^3 - x^2$

$x^3 - x^2 - 2x \leq 0$

$x(x^2-x-2) \leq 0$

$x(x-2)(x+1) \leq 0$



$x \leq -1$ ✓, or $0 < x \leq 2$ ✓ (2)

d) $y = -x \quad m_1 = -1$

$y = \frac{x}{\sqrt{3}} \quad m_2 = \frac{1}{\sqrt{3}}$

$\tan\alpha = \left|\frac{-1-\frac{1}{\sqrt{3}}}{1+(-1)\left(\frac{1}{\sqrt{3}}\right)}\right|$ ✓

$= \frac{-\sqrt{3}-1}{\sqrt{3}} \div \frac{\sqrt{3}-1}{\sqrt{3}}$

$= \left|\frac{-\sqrt{3}-1}{\sqrt{3}-1}\right|$ (2)

$\alpha = 75^\circ$ ✓

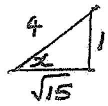
∴ acute angle between the 2 given lines is 75°.

e) $\cos\left(\sin^{-1}\left(-\frac{1}{4}\right)\right) = \cos\left(-\sin^{-1}\left(\frac{1}{4}\right)\right)$

let $\sin^{-1}\left(\frac{1}{4}\right) = x$

$\sin x = \frac{1}{4}, \quad -\frac{\pi}{2} < x < \frac{\pi}{2}$

$\therefore \cos\left(\sin^{-1}\left(-\frac{1}{4}\right)\right)$ ✓



$= \cos[-x]$

$= \cos x$ (2)

$= \frac{\sqrt{15}}{4}$ ✓

f) $\int_1^2 \frac{1-x}{(1+x)^3}\,dx$

let $u = 1+x$
$du = dx$
$x=1 \quad u=2$
$x=2 \quad u=3$
$x = u-1$

$= \int_2^3 \frac{1-(u-1)}{u^3}\,du$

$= \int_2^3 \frac{2-u}{u^3}\,du$ ✓

$= \int_2^3 \frac{2}{u^3} - \frac{1}{u^2}\,du$

$= \left[\frac{2u^{-2}}{-2} - \frac{u^{-1}}{-1}\right]_2^3$

$= \left[-\frac{1}{u^2} + \frac{1}{u}\right]_2^3$ ✓ (3)

$= -\frac{1}{9} + \frac{1}{3} - \left(-\frac{1}{4} + \frac{1}{2}\right)$

$= -\frac{1}{36}$ ✓

QUESTION 2

a) $\int \sin^2 2x\,dx$

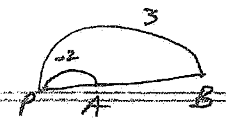
$\cos 4x = 1 - 2\sin^2 2x$
$2\sin^2 2x = 1 - \cos 4x$

$= \frac{1}{2}\int(1-\cos 4x)\,dx$ ✓

$= \frac{1}{2}\left[x - \frac{1}{4}\sin 4x\right] + c$

$= \frac{1}{2}x - \frac{1}{8}\sin 4x + c$ (2)

✓



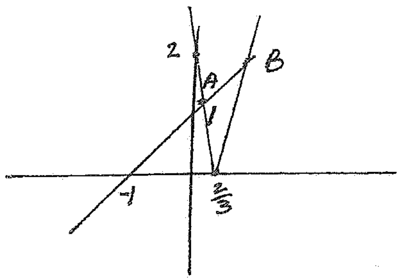
b) $\overset{k}{-2} : \overset{l}{3}$

$$P = \left(\frac{kx_2 + lx_1}{k+l}, \frac{ky_2 + lx_1}{k+l}\right)$$

$$= \left(\frac{-2\times 6 + 3\times 1}{1}, \frac{-2\times 9 + 3\times -4}{1}\right)$$

$$= (-9, -30)$$

c) $|3x-2| > x+1$



✓

At A: $-3x+2 = x+1$

$-4x = -1$

$x = \frac{1}{4}$

(3)

At B: $3x-2 = x+1$ ✓

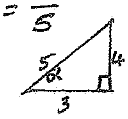
$2x = 3$

$x = \frac{3}{2}$

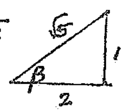
$\therefore |3x-2| > x+1$ for

$x < \frac{1}{4}$ or $x > 1\frac{1}{2}$ ✓

d) $\cos\alpha = \frac{3}{5}$



$\sin\beta = \frac{1}{\sqrt{5}}$



$\sin 2\beta = 2\sin\beta\cos\beta$

$= 2\times\frac{1}{\sqrt{5}}\times\frac{2}{\sqrt{5}}$

$= \frac{4}{5}$ ✓

$\sin\alpha = \frac{4}{5}$ ✓

(2)

$\therefore \sin 2\beta = \sin\alpha$

$\therefore 2\beta = \alpha$

e) $x^3 - 6x^2 + 3x + k = 0$ ✓

let roots be $\alpha - d, \alpha, \alpha + d$

$3\alpha = 6$ ✓

$\alpha = 2$

Sub $\alpha = 2$ in eqn for x:

(3)

$8 - 24 + 6 + k = 0$ ✓

$k = 10$

OR

$\alpha(\alpha - d) + \alpha(\alpha + d) + \alpha^2 - d^2 = 3$

$\alpha^2 - \alpha d + \alpha^2 + \alpha d + \alpha^2 - d^2 = 3$

$3\alpha^2 - d^2 = 3$

$12 - d^2 = 3$

$d = \pm 3$

$(\alpha - d)\alpha(\alpha + d) = -k$

$(2-3)2(2+3) = -k$

$-10 = -k$

$k = 10$.

QUESTION 3

a) $\frac{dV}{dt} = 12\ mm^3/s$

$V = \frac{4}{3}\pi r^3$

$\frac{dV}{dr} = 4\pi r^2$ ✓

$4\pi r^2 = 500$

$\frac{dr}{dt} = \frac{dr}{dV}\cdot\frac{dV}{dt}$ ✓

$= \frac{1}{4\pi r^2}\times 12$ ✓

$= \frac{12}{500}$ ✓

(4)

$= \frac{3}{125}$

$\therefore$ radius is increasing at rate of $\frac{3}{125}$ mm/sec.

b) $\sin\theta - \cos\theta = 1$

Let $\tan\frac{\theta}{2} = t$

$$\frac{2t}{1+t^2} - \frac{1-t^2}{1+t^2} = 1 \quad ✓$$

$$2t - 1 + t^2 = 1 + t^2$$

$$2t = 2$$

$$t = 1$$

$$\therefore \tan\frac{\theta}{2} = 1$$

$$\frac{\theta}{2} = 45°, 225° \ldots\ldots$$

$$\theta = 90°, 450 \ldots\ldots \quad ✓$$

Test $\theta = 180°$:

$$\text{lhs} = 0 - (-1) \quad ✓$$

$$= 1 = \text{rhs}$$

(4)

$$\therefore \theta = 90° + 360n \quad \text{or} \quad 180° + 360n$$

or $\theta = \frac{\pi}{2} + 2\pi n$ or $\pi + 2\pi n$, $n \in J$ ✓

OR Using subsidiary angle method:

$$\sin\theta - \cos\theta = \sqrt{2}\sin(\theta - \alpha)$$

$$\cos\alpha = \frac{1}{\sqrt{2}} \quad \alpha = \frac{\pi}{4}$$

$$\sqrt{2}\sin\left(\theta - \frac{\pi}{4}\right) = 1$$

$$\sin\left(\theta - \frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$$

$$\theta - \frac{\pi}{4} = \frac{\pi}{4}, \frac{3\pi}{4} \ldots\ldots$$

$$\theta = \frac{\pi}{2}, \pi \ldots\ldots$$

$$\theta = \frac{\pi}{2} + 2\pi n \quad \text{or} \quad \pi + 2\pi n$$

OR square both sides of eqn.

c) Prove

$$1\times2^0 + 2\times2^1 + 3\times2^2 + \ldots + n\times2^{n-1}$$
$$= 1 + (n-1)2^n \quad \text{for } n \geq 1$$

Step 1 Prove True for $n=1$

lhs $= 1\times2^0 = 1$ rhs $= 1 + (1-1)2^1 = 1$ ✓

$\therefore$ True for $n=1$

Step 2 Assume true for $n=k$ ✓

$$\therefore 1\times2^0 + 2\times2^1 + \ldots k\times2^{k-1} = 1 + (k-1)2^k$$

Step 3 Prove true for $n = k+1$ if true for $n=k$

ie. prove $1\times2^0 + \ldots k\times2^{k-1} + (k+1)2^k = 1 + k2^{k+1}$

Proof:
$$\text{lhs} = 1 + (k-1)2^k + (k+1)2^k$$
$$= 1 + k.2^k - 2^k + k.2^k + 2^k$$
$$= 1 + 2k.2^k$$
$$= 1 + k.2^{k+1} \quad ✓$$
$$= \text{rhs}$$

$\therefore$ True for $n=k+1$ if true for $n=k$

Step 4 Conclusion

Statement true for $n=1$ & true for $n=k+1$ if true for $n=k$. $\therefore$ true for $n = 2, 3, 4 \ldots\ldots$

ie. for all integers ≥ 1

$\therefore$ Proved by induction ✓

(4)

QUESTION 4

a) i) $\frac{d}{dx}\left(\frac{1}{2}v^2\right) = \frac{d}{dv}\left(\frac{1}{2}v^2\right)\frac{dv}{dx}$

$= v\frac{dv}{dx}$

$= \frac{dx}{dt}\cdot\frac{dv}{dx}$

$= \frac{dv}{dt}$

$= \frac{d^2x}{dt^2}$ (2)

ii) $\frac{d^2x}{dt^2} = n^2(3-x)$

$\frac{d}{dx}\left(\frac{1}{2}v^2\right) = n^2(3-x)$

$\frac{1}{2}v^2 = n^2\left(3x - \frac{x^2}{2}\right) + c$ ✓

$\left.\begin{array}{l}x=0\\ v=0\end{array}\right]$ $0 = n^2(0-0) + c$

$c = 0$ ✓

$\therefore \frac{1}{2}v^2 = n^2\left(3x - \frac{1}{2}x^2\right)$ (2)

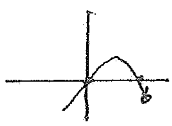
$\therefore \frac{1}{2}v^2 - n^2\left(3x - \frac{1}{2}x^2\right) = 0$

iii) $\frac{1}{2}v^2 \geq 0$ for all x.

$\therefore 3x - \frac{1}{2}x^2 \geq 0$ ✓

$6x - x^2 \geq 0$

$x(6-x) \geq 0$



$0 \leq x \leq 6$

$\therefore$ particle never moves ✓
outside the interval
$0 \leq x \leq 6$. (2)

b) i) $y = \frac{x+4}{x(x+8)}$ ✓

Vert asymptotes: $x=0,\ x=-8$

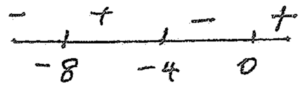
x intercept : $x=-4$ ✓

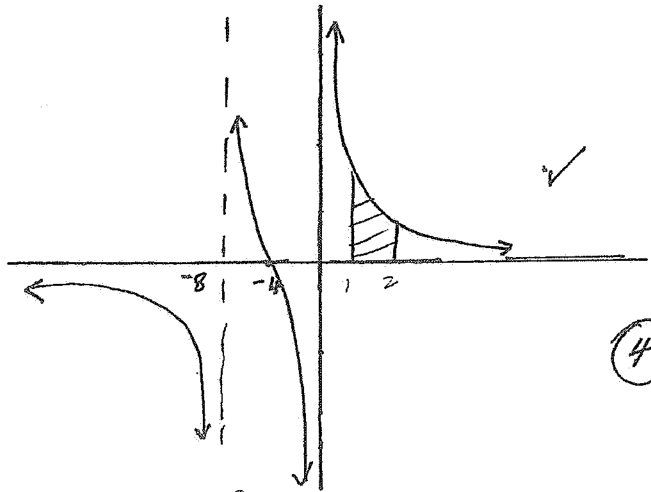
Horiz asymptote: $y: \lim_{x\to\infty} \frac{x+4}{x^2+8x}$

$= \lim_{x\to\infty} \frac{\frac{1}{x}+\frac{4}{x^2}}{1+8/x}$

$= 0.$ ✓

Sign Diag:





(4)

ii) Area $= \int_1^2 \frac{x+4}{x^2+8x}\,dx$

$= \frac{1}{2}\left[\log(x^2+8x)\right]_1^2$ ✓

$= \frac{1}{2}(\log 20 - \log 9)$

$= \frac{1}{2}\log\frac{20}{9}\ u^2$ ✓ (2)

QUESTION 5

a) $1 - \tan^2 x + \tan^4 x - \ldots$

lim sum exists if $|r| < 1$ ✓

ie. $-1 < -\tan^2 x < 1$

$1 > \tan^2 x > -1$

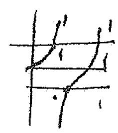
$-1 < \tan^2 x < 1$

But $\tan^2 x$ always +ve

ie $-\tan^2 x$ always > -1

So solve $\tan^2 x < 1$ ✓

ie. $-1 < \tan x < 1$



$0 < x < \frac{\pi}{4}$ or $\frac{3\pi}{4} < x < \frac{5\pi}{4}$

or $\frac{7\pi}{4} < x < 2\pi$ ✓✓ (4)

b) $\frac{d^2x}{dt^2} = -4x$

i) $x = a\cos(2t+\beta)$

$v = -2a\sin(2t+\beta)$

$\ddot{x} = -4a\cos(2t+\beta)$

$= -4x$ ①

ii) $t=0$ $v=2$ $x=4$.

$4 = a\cos\beta$ — ① ✓

$2 = -2a\sin\beta$ ✓

$1 = -a\sin\beta$ — ②

$4^2+1^2 = a^2\cos^2\beta + a^2\sin^2\beta$

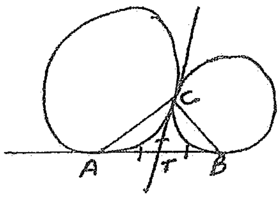
$17 = a^2$

$a = \sqrt{17}$ ✓ ③

iii) $v = -2\sqrt{17}\sin(2t+\beta)$

$\therefore$ max $v = 2\sqrt{17}$ m/sec. ①

b)



✓

AT = TC (tangents from ext pt
CT = TB (to circle are equal)

Let $\angle TAC = x$

$\therefore \angle TCA = x$ (base ∠s isos △ TAC)

Let $\angle TCB = y$ ✓

$\therefore \angle TBC = y$ (base ∠s isos △TCB)

$2x + 2y = 180°$ (∠sum △ ABC) ✓

$\therefore x + y = 90°$

$\therefore \angle ACB = 90°$

③

(−2 marks if use 45°)

QUESTION 6

a) $x = 30t\cos\theta$ (i) $y = 30t\sin\theta - 5t^2$ (ii)

$\dot{y} = 30\sin\theta - 10t$

Max range when $y=0$

$30t\sin\theta - 5t^2 = 0$

$t(30\sin\theta - 5t) = 0$

$t = 6\sin\theta$ — (iii) ✓

$y = 15$ when $\dot{y} = 0$

$30\sin\theta - 10t = 0$

$t = 3\sin\theta$ ✓

Sub in (ii)

$15 = 30(3\sin\theta)\sin\theta - 5(9\sin^2\theta)$

$15 = 90\sin^2\theta - 45\sin^2\theta$

$15 = 45\sin^2\theta$

$\sin^2\theta = \frac{1}{3}$

$\sin\theta = \pm\frac{1}{\sqrt{3}}$ ✓

But $0 < \theta < \frac{\pi}{2}$, $\therefore \sin\theta = \frac{1}{\sqrt{3}}$ (iv)

Sub iii) & iv) in i)

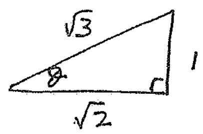
$x = 30(6\sin\theta)\cos\theta$ ✓

$= 180 \times \frac{1}{\sqrt{3}} \times \frac{\sqrt{2}}{\sqrt{3}}$

$= 60\sqrt{2}$

✓

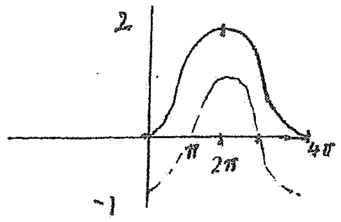
$\therefore$ max horiz range is $60\sqrt{2}$ metres.



⑤

b) $f(x) = 1 - \cos\frac{x}{2}, \; 0 \le x \le a$

i)



Period $= \frac{2\pi}{\frac{1}{2}}$

$= 4\pi$ ✓

Largest a is 2π ✓ ②

ie. $f^{-1}(x)$ exists for $0 \le x \le 2\pi$

ii) $y = 1 - \cos\frac{x}{2}$

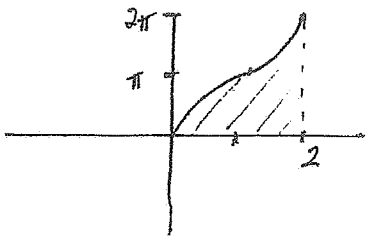
For inv: $x = 1 - \cos\frac{y}{2}$

$\cos\frac{y}{2} = 1 - x$

$\frac{y}{2} = \cos^{-1}(1-x)$

$y = 2\cos^{-1}(1-x)$ ✓ ①

iii)



②

R: $0 \le y \le 2\pi$

D: $-1 \le 1 - x \le 1$

$-2 \le -x \le 0$

$2 \ge x \ge 0$

$x = 2 \quad y = 2\cos^{-1}(-1)$

$= 2\pi$

iv) Reqd area $= \frac{1}{2}$ Area of rect

$= \frac{1}{2}\, 2 \times 2\pi$

$= 2\pi \; u^2$

②

OR →

$$\text{Area} = 4\pi - \int_0^{2\pi} \left(1 - \cos\tfrac{y}{2}\right) dy$$

$= 4\pi - \left[y - 2\sin\frac{y}{2}\right]_0^{2\pi}$

$= 4\pi - [2\pi - 0 - (0 - 0)]$

$= 2\pi u^2$.

QUESTION 7

a) $x_1 = x_0 - \frac{P(x)}{P'(x)}$ ✓ ①

i) $x_0 = 1{\cdot}8$

ii) $P(x) = f(x) + 5$

$= a\sin x + bx + 5$ ✓

$P(1{\cdot}8) = -0{\cdot}10$

$\therefore a\sin 1{\cdot}8 + 1{\cdot}8b + 5 = -0{\cdot}10$

or ✓

$a\sin 1{\cdot}8 + 1{\cdot}8b + 5{\cdot}10 = 0$

$P'(1{\cdot}8) = -5{\cdot}91$

$P'(x) = a\cos x + b$

$\therefore a\cos 1{\cdot}8 + b = -5{\cdot}91$

✓

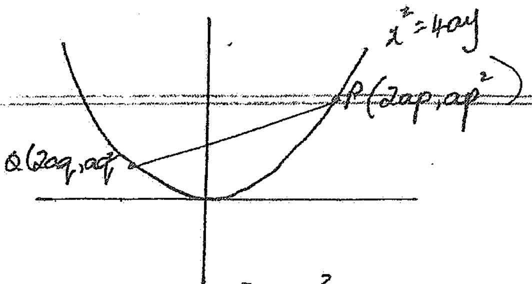
③

−1 mark if

$a\sin 1{\cdot}8 + b1{\cdot}8 = -0{\cdot}1$

$a\cos 1{\cdot}8 + b = -5{\cdot}91$

b)



i) For PQ: $m = \dfrac{ap^2-aq^2}{2ap-2aq}$

$= \dfrac{a(p-q)(p+q)}{2a(p-q)}$

$= \dfrac{p+q}{2}$

$y - ap^2 = \dfrac{p+q}{2}(x-2ap)$

$2y - 2ap^2 = (p+q)x - 2ap^2 - 2apq$

$2y = (p+q)x - 2apq$

$y = \left(\dfrac{p+q}{2}\right)x - apq$ ✓

This can be written as

$y = mx + b$

$\therefore -apq = b$

$pq = -b/a$ —— (1) ✓ 2

ii) $\dfrac{p+q}{2} = m$ —— (ii) ✓

$\dfrac{p^2+2pq+q^2}{4} = m^2$

$p^2+q^2+2pq = 4m^2$ 2

$p^2+q^2+2(-b/a) = 4m^2$ ✓

$p^2+q^2 = 4m^2 + \dfrac{2b}{a}$

iii) $N\left[-apq(p+q),\ a(2+p^2+pq+q^2)\right]$

$x = -apq(p+q)$

$x = -a \times \dfrac{-b}{a}(p+q)$ ✓

$x = b(p+q)$ $\therefore x = b \times 2m$

$= 2bm$

from (ii)

$y = a(2+p^2+pq+q^2)$

$= a\left(2 + 4m^2 + \dfrac{2b}{a} - \dfrac{b}{a}\right)$

$= a\left(2 + 4m^2 + \dfrac{b}{a}\right)$

$= 2a + 4am^2 + b$. ✓ 2

v) Locus of N:

$x = 2bm$ $y = 2a + 4am^2 + b$

$b = \dfrac{x}{2m} \rightarrow y = 2a + 4am^2 + \dfrac{x}{2m}$

$2my = 4am + 8am^3 + x$

$x - 2my = -4am - 8am^3$

$x + (-2m)y = 2a(-2m) + a(-2m)^3$ ✓

Compare this with equation of normal to parabola at P

$x + py = 2ap + ap^3$

p has been replaced by $-2m$. ✓

$\therefore$ locus of N is a straight line which is a normal to the parabola at point $(2a \times -2m,\ a(-2m)^2)$

ie. at $(-4am,\ 4am^2)$.

2

$7x - 3y = -41$

$5x - 4y = -33 \quad (2)$

$-3y = -41 - 7x$

$3y = 41 + 7x$

$y = \dfrac{41 + 7x}{3} \quad -(1)$

sub (1) → (2)

$5x - 4\left(\dfrac{41 + 7x}{3}\right) = -33$

$5x - \dfrac{(164 + 28x)}{3} = -33$

$15x - 164 - 28x = -99$

$-13x = 65$

$x = -5$

$y = \dfrac{41 + 7(-5)}{3}$

$= 2.$