ASCHAM SCHOOL

MATHEMATICS EXTENSION 1

TRIAL EXAMINATION 2005

Time allowed: 2 hours plus 5 minutes reading time

Total marks 84

- Attempt all questions.
- All questions are of equal value.
- Write using black or blue pen.
- Board-approved calculators may be used.
- A table of standard integrals is provided.
- All necessary working should be shown in every question.
- Do each of the 7 questions in a separate booklet.

QUESTION 1

a) Solve $\frac{3}{x-1} \geq 2$ **(3)**

b) Find the coordinates of a point P which divides the line AB internally in the ratio 5:2 if A is (-3,2) and B(5,-1). **(2)**

c) Find $\frac{d}{dx}\sin^{-1} x$ **(1)**

d) Find $\int \operatorname{cosec}^2 3x\, dx$ **(1)**

e) Find $\int \frac{\ln x}{2x}\, dx$ using the substitution u = ln x **(2)**

f) Draw the graph of $y = 2\cos^{-1}(1-x)$ **(3)**

QUESTION 2

a) Evaluate $\lim_{x\to 0} \frac{\sin\frac{x}{2}}{3x}$ **(1)**

b) Find $\int \sin x \cos^2 x\, dx$ **(1)**

c) Solve for x: $|2x-1| > x+3$ **(3)**

d) For what values of m does the line y = m(x+1) have no intersection with the parabola $y = 2x^2$ **(3)**

e) Solve $\sin 2x = \cos x$ **(4)**

QUESTION 3

a) i) Sketch the curve $y = 3\sin 2x$ for $0 \le x \le \frac{\pi}{2}$ **(1)**

ii) The area bounded by the curve and the x axis is rotated about the x axis to form a solid.
Find the volume of this solid in exact form. **(3)**

b) i) Express $3\cos x + 4\sin x$ in the form $R\cos(x-\alpha)$ **(2)**

ii) Hence or otherwise solve
$3\cos x + 4\sin x = 2$ for $0 \le x \le \pi$,
correct to 2 decimal places **(3)**

c) A vertical tower of height h metres stands on horizontal ground. From a point P on the ground due east of the tower the angle of elevation of the top of the tower is 45°. From a point Q on the ground south of the tower, the angle of elevation of the top of the tower is 30°.
If the distance PQ is 40m, find the height of the tower. **(3)**

QUESTION 4

a) The acceleration of a particle travelling in a straight line is given by $\frac{d^2x}{dt^2} = -4x$ where x measures displacement in metres from its initial position and t is in seconds. Initially the velocity of the particle is -6m/s when x = 0.

i) Find an expression for the speed of the particle in terms of x. **(3)**

ii) Find the maximum (positive) acceleration of the particle. **(2)**

iii) Find the time taken by the particle to reach this maximum acceleration from its starting point. **(1)**

b) At any time t the rate of cooling of the temperature T of a body when the surrounding temperature is S is given by the equation
$\frac{dT}{dt} = -k(T-S)$ for some constant k

i) Show that $T = S + Ae^{-kt}$ for some constant A satisfies the equation. **(1)**

ii) A metal rod has a temperature of 1390°C and cools to 1060°C in 10 minutes when the surrounding temperature is 30°C. find how much longer it will take the rod to cool to 110°C to the nearest minute. **(5)**

QUESTION 5

a) Prove by mathematical induction that for all positive integers n

$$1\times5+2\times6+3\times7+.....n(n+4)=\frac{1}{6}n(n+1)(2n+13)$$ **(5)**

b) P(x) is a monic polynomial of degree 3 which leaves a remainder of 12 when divided by $x+3$.
The roots of P(x) = 0 have a sum of 3 and a product of 72.

i) Show that $P(x)=x^3-3x^2-46x-72$ **(4)**

ii) Solve $P(x)>0$ **(3)**

QUESTION 6

a) $P(2ap,ap^2)$ and $Q(2aq,aq^2)$ lie on the parabola $x^2=4ay$

i) Show that the equation of PQ is $y=\frac{(p+q)x}{2}-apq$ **(1)**

ii) If PQ produced passes through (a,0) show that $p+q=2pq$ **(1)**

iii) Find the coordinates of M, the midpoint of PQ. **(1)**

iv) Find the cartesian equation of the locus of M as P and Q move on the parabola subject to the condition that PQ passes through (a,0) **(3)**

b) A 5 metre ladder rests with one end against a vertical wall and the other end on horizontal ground which is level with the base of the wall. The end in contact with the ground slips away from the wall at a constant rate of 0.1 m/s.
Find the rate at which the angle between the ladder and the ground is decreasing when the end of the ladder is 3m from the wall. **(3)**

c) Given the function $f(x)=3\sin^{-1}x+3\cos^{-1}x$

i) Find f ' (x) and hence state what can be deduced about the graph of $y=f(x)$. **(1)**

ii) Draw the graph of $y=f(x)$ **(2)**

QUESTION 7

a) A stone is thrown from the top of a vertical cliff over the water of a lake. The height of the cliff is 15 metres above the level of the water, the initial speed of the stone is 20 metres per second and the angle of projection is θ degrees above the horizontal.

Take the acceleration due to gravity to be 10 metres per second per second and taking the origin at the cliff top, let the horizontal and vertical displacements of the stone after t seconds be

$x = 20t\cos\theta$ and $y = -5t^2 + 20t\sin\theta$

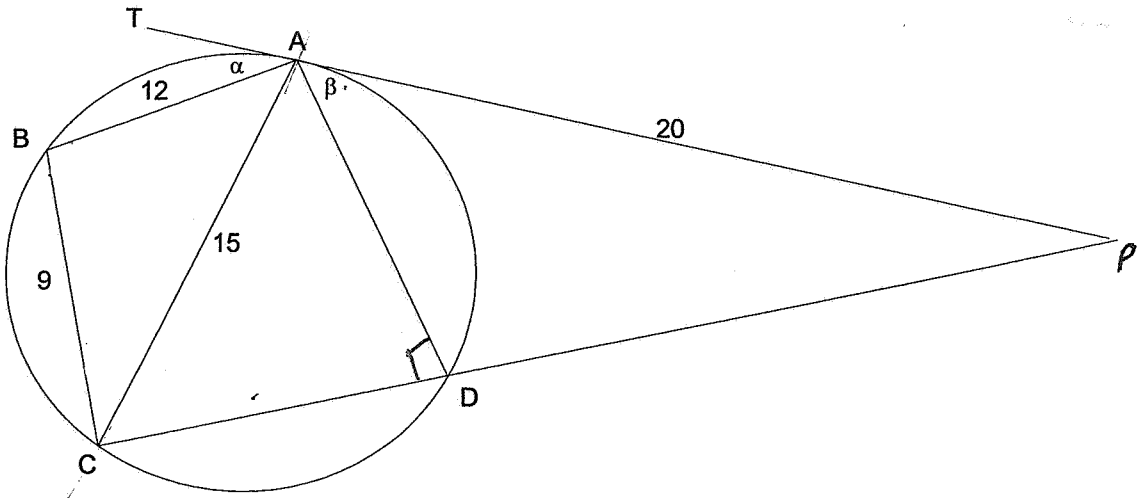
i) If the stone hits the water when t = 3, find θ. (2)

ii) Find the speed of the stone when it hits the water. (2)

iii) Find the angle between the direction the stone is travelling and the horizontal at the moment the stone hits the water. (1)

PTO FOR QUESTION 7b

b)



TAP is a tangent to the circle at A. CD produced meets TAP at P
AB = 12, AP = 20, AC = 15, BC = 9
Let ∠TAB = α and ∠DAP = β

i) Show that ∠ABC is a right angle. **(1)**

ii) Show that $\alpha = \cos^{-1}\frac{3}{5}$ **(2)**

iii) Show that $\beta = \tan^{-1}\frac{4}{3}$ **(2)**

iv) Hence show that $\alpha + \beta = \pi - \cos^{-1}(\frac{7}{25})$ **(2)**

END OF EXAM