ASCHAM SCHOOL

MATHEMATICS EXTENSION 1

TRIAL EXAMINATION

2006

Time : 2 hours + 5 minutes reading time

Instructions:

Attempt all questions

All questions are of equal value

All necessary working should be shown for every question.

Full marks may not be awarded for careless or badly arranged work

A Table of Standard Integrals is provided

Approved calculators may be used

Each question should be answered in a separate booklet

Do not use whiteout, part marks may be awarded for scored out work if it is legible

Question 1 (12 marks)

(a) Find $\int \frac{1}{\sqrt{4-x^2}}dx$ [1]

(b) Sketch the region in the number plane defined by $y>|x|-1$ [2]

(c) Find the domain and range of $y=\sqrt{x^2-9}$ [2]

(d) Find $\lim_{x\to 0}\frac{x}{\sin 2x}$ [2]

(e) The parametric equation of a function is

$$x=2t^2,\ \ y=4-t$$

Find the cartesian equation. [1]

(f) A $(x,10)$ and B $(6,y)$. The point P (5, 4) divides AB externally in the ratio 3:1. Find x and y [2]

(g) Find $\frac{d}{d\theta}\left(\cos^3 2\theta\right)$ [2]

Question 2 (12 marks) **Begin a new booklet**

(a) (i) Show $\frac{d}{dx}\left(x\sqrt{1-x^2}+\sin^{-1}x\right)=2\sqrt{1-x^2}$ [2]

(ii) Hence evaluate $\int_0^{\frac{1}{2}}\sqrt{1-x^2}\,dx$ [2]

(b) Using the substitution $u=\log_e x$, evaluate $\int_e^{e^2}\frac{1}{x\log_e x}dx$ [3]

(c) The polynomial equation $3x^3-2x^2+3x-4=0$ has roots α,β,λ.

Find the exact value of $\frac{1}{\alpha\beta}+\frac{1}{\alpha\lambda}+\frac{1}{\beta\lambda}$ [2]

(d) Consider the polynomial $P(x)=x^3+ax^2+bx+2$ which has factors $x+1$ and $x-2$. Find the values of a and b. [3]

Question 3 (12 marks) **Begin a new booklet**

(a) Find the general solution of $\cos x\cos 27^\circ + \sin x\sin 27^\circ = \cos 2x$ [3]

(b) Drinks for a barbeque have been left in the sun and their temperature has risen to 30°C. They are placed in the freezer where the temperature is maintained at -5°C. After t minutes, the temperature T°C of the drinks is changing so that $\frac{dT}{dt} = -k(T+5)$

(i) Prove that $T = Ae^{-kt} - 5$ satisfies the differential equation, and find the value of A. [2]

(ii) After 20 minutes the temperature of the drinks has fallen to 20°C. How long after they are put in the fridge will it take before the drinks begin to freeze? Assume that freezing point is 0°C. [2]

(c) (i) Prove using calculus that the equation $x^3 + 2x + 4 = 0$ has only one real root α [2]

(ii) Show that $-2 < \alpha < -1$ [1]

(iii) Starting with an initial approximation $\alpha = -1$, use one application of Newton's method to find a further approximation for α. [2]

Question 4 (12 marks) **Begin a new booklet**

(a) A particle is moving along the x-axis. Its speed v m/s at position x metres is given by

$$v = \sqrt{5x - x^2}$$

Find the acceleration when $x = 2$. [2]

(b) A particle moves along the x-axis according to the equation

$$x = \cos 2t - \sqrt{3}\sin 2t$$

where x metres is the displacement after t seconds from the origin O.

(i) Express x in the form $R\cos(2t+\alpha)$ where $R > 0$ and $0 \le \alpha \le \frac{\pi}{2}$ [2]

(ii) Prove that the particle moves in simple harmonic motion. [2]

(iii) Find the amplitude and period of the motion. [2]

(iv) Determine whether the particle is initially moving towards O or away from O, and whether it is initially speeding up or slowing down. Justify your answers. [2]

(v) Find the time at which the particle first returns to its starting point. [2]

Question 5 (12 marks) **Begin a new booklet**

(a) (i) From a lighthouse L, the bearing of ships A and B are 035° and 145° respectively. Show this on a diagram and find ∠ALB. [1]

(ii) Lighthouse LT is 120 metres high. The angle of elevations from ships A and B to the top of the lighthouse are 40° and 50° respectively. Find the distance between the ships. [3]

(b) (i) Show that $f(x) = \sin^{-1}(\cos x)$ is an even function. [1]

(ii) Differentiate $f(x) = \sin^{-1}(\cos x)$ and hence find the gradient for $0 < x < \pi$. [2]

(iv) Evaluate $f(0)$, $f(-\pi)$ and $f(\pi)$ [1]

(iii) Sketch $f(x)$ for $-\pi \le x \le \pi$ [1]

(c) Use mathematical induction to prove that

$$1^3 + 2^3 + 3^3 + \ldots\ldots + n^3 = \frac{n^2(n+1)^2}{4} \quad \text{for integer } n \ge 1$$ [3]

Question 6 (12 marks) **Begin a new booklet**

(a) Given that $\sin^{-1}x$ and $\cos^{-1}x$ are acute,

(i) Show that $\sin(\sin^{-1}x - \cos^{-1}x) = 2x^2 - 1$ [2]

(ii) Solve the equation $\sin^{-1}x - \cos^{-1}x = \sin^{-1}(0.5)$ [2]

(b) A particle is projected from a point O with velocity V m/s at an angle θ above the horizontal. At time t seconds it has horizontal and vertical components x metres and y metres respectively from O. The acceleration due to gravity is g m/s^2.

(i) Given the equations below, derive equations for horizontal displacement x and vertical displacement y [2]

$$\ddot{x} = 0, \qquad \ddot{y} = -g$$
$$\dot{x} = V\cos\theta, \quad \dot{y} = V\sin\theta - gt$$

(ii) Hence show that the equation of the path is

$$y = x\tan\theta - \frac{gx^2}{2V^2}\left(1 + \tan^2\theta\right)$$

[2]

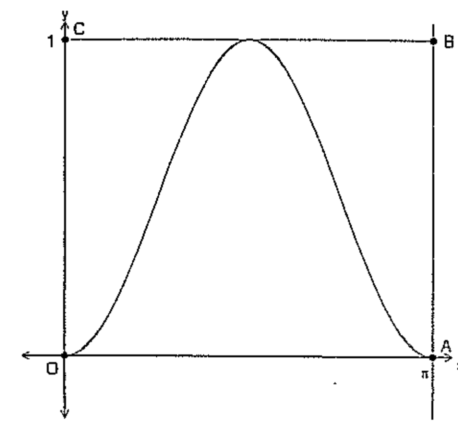
(c) A particle is projected from O with velocity 60 m/s at an angle α above the horizontal. T seconds later, another particle is also projected from O with velocity 60 m/s at an angle β above the horizontal, where $\beta < \alpha$. The two particles collide 240 metres horizontally and 80 metres vertically from O. Taking $g = 10\text{m/s}^2$, and using the results from (b):

(i) Show that $\tan\alpha = 2$ and $\tan\beta = 1$ [2]

(ii) Find the value of T in simplest exact form. [2]

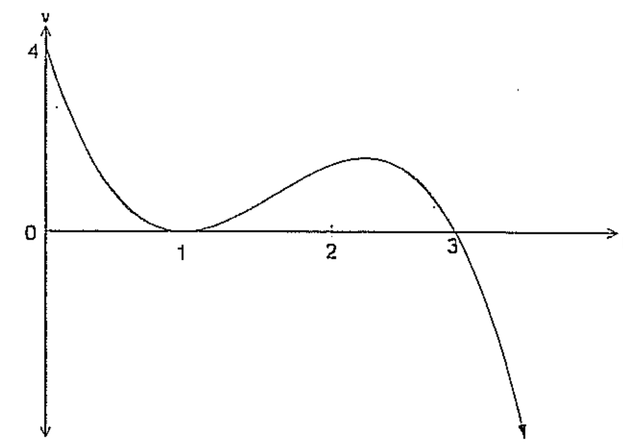
Question 7 (12 marks) **Begin a new booklet**

(a)



The rectangle OABC has vertices O (0, 0), A $(\pi,0)$, B $(\pi,1)$, C (0, 1).
The curve $y = \sin^2 x$ is shown. Use calculus methods to show that the area under the curve is half the area of rectangle OABC [3]

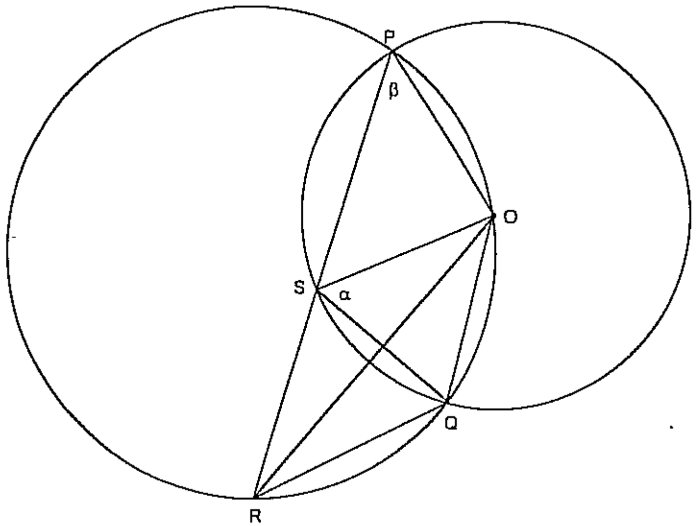
(b) A particle P moves along a straight line. A velocity-time graph for P is shown below.



(i) Between what times does the particle travel to the right? [1]

(ii) Sketch a displacement-time graph for P given that the particle starts 2 metres to the left of O. [2]

(c)



O is a point on the larger circle. The smaller circle has centre O. The circles intersect at P and Q. PR is a chord of the larger circle that cuts the smaller circle at S.

Copy the diagram into your answer booklet (about half a page)

Let $\angle SPO = \beta$, $\angle OSQ = \alpha$

(i) Explain why $\angle PSO = \beta$ [1]

(ii) Prove that $\angle SQR = 180 - (\alpha + \beta)$ [2]

(iii) Prove that $SQ \perp OR$ [3]

End of Examination

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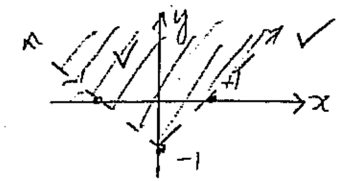
1 a) $\int \frac{1}{\sqrt{4-x^2}}\,dx = \sin^{-1}\frac{x}{2} + C$. (1)

b) (2)



c) $x^2 - 9 \geq 0$ (2)

D : $x \leq -3$ or $x \geq 3$. ✓

R : $y \geq 0$. ✓

d) $\lim_{x \to 0} \frac{x}{\sin 2x} = \lim_{2x \to 0} \frac{2x}{\sin 2x} \cdot \frac{1}{2}$ ✓ (2)

$= \frac{1}{2}$. ✓

e) $x = 2t^2$ $\quad y = 4 - t$ (1)

$t = 4 - y$

$\therefore x = 2(4-y)^2$. ✓

f) $A(x, 10)$ $\quad B(6, y)$ (2)

$-3 : 1$

$5 = \frac{-3 \times 6 + x}{-3+1}$ ½ $\quad 4 = \frac{-3 \times y + 10}{-3+1}$ ½

$-10 = -18 + x$ $\quad -8 = -3y + 10$

$x = 8$ ½ $\quad 3y = 18$

$y = 6$. ½

g) $\frac{d}{d\theta}(\cos^3 2\theta) = 3\cos^2 2\theta\,(-\sin 2\theta).2$ ½ ½ ½ (2)

$= -6\cos^2 2\theta \sin 2\theta$. ½

2 a) (i)
$$\frac{d}{dx}\left(x\sqrt{1-x^2}+\sin^{-1}x\right)$$
$$= x\cdot\tfrac{1}{2}(1-x^2)^{-\frac{1}{2}}\cdot(-2x) + \sqrt{1-x^2}\cdot 1 + \frac{1}{\sqrt{1-x^2}} \quad \tfrac{1}{2}\ \tfrac{1}{2}\ \tfrac{1}{2}$$
$$= \frac{-x^2}{\sqrt{1-x^2}} + \sqrt{1-x^2} + \frac{1}{\sqrt{1-x^2}}$$
$$= \frac{-x^2+1-x^2+1}{\sqrt{1-x^2}}$$
(2)
$$= \frac{2(1-x^2)}{\sqrt{1-x^2}} \quad \tfrac{1}{2}$$
$$= 2\sqrt{1-x^2}$$

(ii)
$$\int_0^{\frac{1}{2}}\sqrt{1-x^2}\,dx = \frac{1}{2}\left[x\sqrt{1-x^2}+\sin^{-1}x\right]_0^{\frac{1}{2}} \quad ✓$$
$$= \frac{1}{2}\left[\left(\frac{1}{2}\sqrt{\frac{3}{4}}+\sin^{-1}\frac{1}{2}\right)-0\right]$$
(2)
$$= \frac{1}{2}\left[\frac{\sqrt{3}}{2}+\frac{\pi}{6}\right] \quad ✓$$
$$= \frac{1}{12}\left[3\sqrt{3}+\pi\right]$$

(b)
$$\int_e^{e^2}\frac{1}{x\log_e x}\,dx$$
let $u = \log_e x$, $du = \frac{dx}{x}$ $\tfrac{1}{2}$

$$= \int_1^2 \frac{1}{u}\,du \quad ✓$$
when $x = e$, $u = 1$; $x = e^2$, $u = 2$ ✓
$$= \left[\ln u\right]_1^2$$
$$= \ln 2 - \ln 1$$
$$= \ln 2. \quad \tfrac{1}{2}$$

Q2 cont.

(c) $3x^3-2x^2+3x-4=0$

roots α, β, γ.

$$\frac{1}{\alpha\beta}+\frac{1}{\alpha\gamma}+\frac{1}{\beta\gamma}$$
(2)
$$= \frac{\gamma+\beta+\alpha}{\alpha\beta\gamma} \quad \tfrac{1}{2}$$
$$= \frac{2/3}{4/3} \quad \tfrac{1}{2}\ \tfrac{1}{2}$$
$$= \frac{1}{2}. \quad \tfrac{1}{2}$$

(d) $P(x) = x^3+ax^2+bx+2$

$(x+1)$; $x-2$ are factors

$\therefore P(-1) = 0$ ✓

$P(2) = 0$

$-1+a-b+2=0$

$a-b=-1$ (1)

(3)

$8+4a+2b+2=0$

$2a+b=-3$ (2)

(1) + (2)

$3a = -4$

$a = -4/3$. ✓

$\rightarrow$ (1)

$-4/3 - b = -1$

$b = 1 - 4/3$

$= -1/3$. ✓

3 a). $\cos x \cos 27 + \sin x \sin 27 = \cos 2x$

$\cos(x - 27) = \cos 2x$ ✓

$x - 27^\circ = \pm 2x + 360n$ ½, $n \in \{\text{integers}\}$ ½

$x = (-27 + 360n)^\circ$ or $x = \frac{27 + 360n}{3}$

$x = (360n - 27)^\circ$ ½ or $x = (9 + 120n)^\circ$ ½

b) $\frac{dT}{dt} = -k(T+5)$

when $t = 0$, $T = 30$,

(i) $T = Ae^{-kt} - 5$ or $Ae^{-kt} = T + 5$ ①.

$\frac{dT}{dt} = A.(-k)e^{-kt}$

$= -k\,Ae^{-kt}$ sub in ①. ✓

$= -k(T+5)$

2

∴ $T = Ae^{-kt} - 5$ is a soln of $\frac{dT}{dt} = -k(T+5)$

When $t = 0$; $T = 30$

∴ $30 = Ae^{0} - 5$

$A = 35$. ✓

(ii) when $t = 20$, $T = 20$

∴ $20 = 35e^{-k \times 20} - 5$

$\frac{25}{35} = e^{-20k}$

$-20k = \ln \frac{5}{7}$

$k = -\frac{1}{20} \ln 5/7$ ✓

2

$(= \frac{1}{20} \ln 7/5)$

when $T = 0$.

$0 = 35e^{\frac{1}{20} \ln 5/7 . t} - 5$

$\frac{5}{35} = e^{1/20 \ln 5/7 . t}$

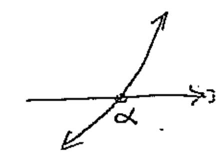
$t = \frac{20 \ln 1/7}{\ln 5/7}$ ½

$\doteqdot 115.66.$ ½

∴ After about 116 mins the drinks begin to freeze.

(c) (i) Let $f(x) = x^3 + 2x + 4$

$f'(x) = 3x^2 + 2 > 0$ for all x ✓



2. ∴ $f(x)$ increases for all x ✓

∴ $x^3 + 2x + 4 = 0$ has only 1 root α.

(ii) $f(-2) = (-2)^3 + 2\times(-2) + 4$

$= -8$

< 0 ½

$f(-1) = (-1)^3 + 2\times(-1) + 4$

1

$= 1$

> 0 ½

$f(\alpha) = 0$

$f(-2) < f(\alpha) < f(-1)$

∴ $-2 < \alpha < -1$ since $f(x)$ is increasing

(iii) $x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$ ✓ let $x_0 = -1$

$f'(-1) = 3\times(-1)^2 + 2$

$= 5$

$= -1 - \frac{1}{5}$

2

$= -\frac{6}{5}$ ✓

∴ A further approx. for α is $-6/5$.

Q4. a) $V = \sqrt{5x - x^2}$

$\ddot{x} = \frac{d}{dx}\left(\frac{1}{2}V^2\right)$ ½

$= \frac{d}{dx}\left(\frac{1}{2}(5x - x^2)\right)$

$= \frac{1}{2}(5 - 2x)$ ½

$= \frac{5}{2} - x.$

$= \frac{5}{2} - 2 \quad \text{when } x = 2$ ½

$\therefore$ When $x = 2$, acceleration is $\frac{1}{2}$ m/s². ½

(2)

b) $x = \cos 2t - \sqrt{3}\sin 2t.$

(i) $R\cos(2t + \alpha) = R\cos 2t\cos\alpha - R\sin 2t\sin\alpha$

and $x = \cos 2t - \sqrt{3}\sin 2t.$

Equating coef:

$R\cos\alpha = 1$ ✓

$R\sin\alpha = \sqrt{3}$

Dividing $\tan\alpha = \sqrt{3}$

$\alpha = 60^\circ$ ½

Squaring & adding

$R^2(\sin^2\alpha + \cos^2\alpha) = \sqrt{3}^2 + 1^2$

$R^2 = 4$

$R = 2 \quad (R > 0)$ ½

$\therefore \quad x = 2\cos\left(2t + \frac{\pi}{3}\right).$

(2)

(ii) $\dot{x} = -2\sin\left(2t + \frac{\pi}{3}\right) \times 2$

$= -4\sin\left(2t + \frac{\pi}{3}\right)$ ✓

$\ddot{x} = -8\cos\left(2t + \frac{\pi}{3}\right)$ ✓

$= -4x$

$\therefore$ Particle is in SHM since in form $\ddot{x} = -n^2 x$.

(2)

(iii) amplitude $= 2$ ✓ (from $x = 2\cos(2t + \frac{\pi}{3})$)

$n^2 = 4$

$n = 2 \quad (n > 0)$

$\therefore$ Period $= \frac{2\pi}{n}$

$= \pi.$ ✓

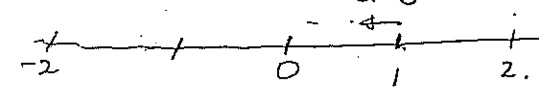
(2)

(iv) when $t = 0$, $x = 2\cos\frac{\pi}{3}$

$= 1$ ✓ $\therefore$ 1 unit to right of 0.

$\dot{x} = -4\sin\frac{\pi}{3}$

$= -2\sqrt{3}$ $\therefore$ moving $\leftarrow$ ½



$\therefore$ Initially particle moves towards 0. ½

when $t = 0$ $\ddot{x} = -4x$

$= -4$ $\therefore$ force acts $\leftarrow$ ½

$\therefore$ Particle is speeding up when $t = 0$. ½

(2)

(v) Returns to its starting point when $x = 1$

$2\cos\left(2t + \frac{\pi}{3}\right) = 1$ ½

$\cos\left(2t + \frac{\pi}{3}\right) = \frac{1}{2}$

$2t + \frac{\pi}{3} = \frac{\pi}{3}, \frac{5\pi}{3}$ ½

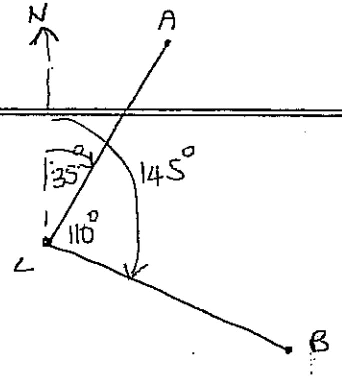
$2t = 0, \frac{4\pi}{3}$

$t = 0, \frac{2\pi}{3}$ ½

$\therefore$ Returns to start after $\frac{2\pi}{3}$ secs ½

(2)

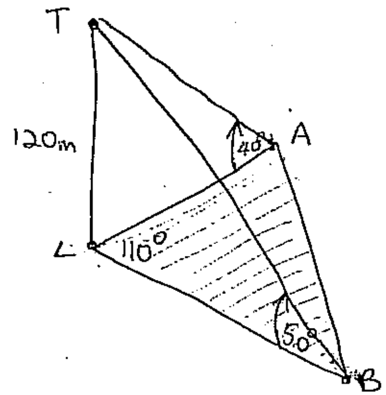
Q5 a) (i)



(1)

$\angle ALB = 110°$

(ii)



$AL = 120 \cot 40°$

$BL = 120 \cot 50°$

$AB^2 = AL^2 + BL^2 - 2AL.BL \cos A\hat{L}B$

$= (120 \cot 40)^2 + (120 \cot 50)^2 - 2 \times 120 \cot 40 \times 120 \cot 50 \times \cos 110°$

$= 120^2 \left[\cot^2 40 + \cot^2 50 - 2 \cot 40 \cot 50 \cos 110\right)$

$= 40441.033...$

$AB = 201.099$

$\therefore$ Distance between the ships is 201 m (to n. m).

b) (i) $f(x) = \sin^{-1}(\cos x)$

$f(-x) = \sin^{-1}(\cos(-x))$

$= \sin^{-1}(\cos x)$

$= f(x)$

(ii) $f(x) = \sin^{-1}(\cos x)$

$f'(x) = \dfrac{1}{\sqrt{1-\cos^2 x}} \times -\sin x$

(2)

$= \dfrac{-\sin x}{\sqrt{\sin^2 x}}$

$= -\dfrac{\sin x}{\sin x}$

$= -1$

$\therefore$ Gradient $= -1$ for $0 < x < \pi$ where $\sin x > 0$

(iii) $f(0) = \sin^{-1}(\cos 0)$

$= \sin^{-1} 1$

$= \frac{\pi}{2}$ $\quad \frac{1}{2}$

$f(-\pi) = \sin^{-1}(\cos(-\pi))$

$= \sin^{-1}(-1)$

(1)

$= -\frac{\pi}{2}$

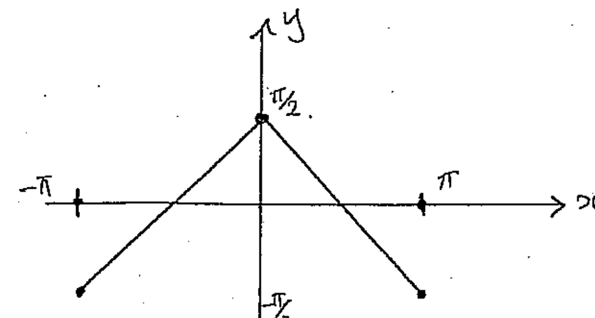
$f(\pi) = \sin^{-1}(\cos \pi)$

$= \sin^{-1}(-1)$

$= -\frac{\pi}{2}$ $\quad \frac{1}{2}$

(iv)

(1)



5 cont.

(c) Let $P(n)$ be $1^3+2^3+3^3+\cdots+n^3 = \frac{n^2(n+1)^2}{4}$ integer $n \geq 1$

$P(1)$ is that $1^3 = \frac{1^2(1+1)^2}{4} = 1$

$1^3 = 1$ $\therefore P(1)$ is true $\frac{1}{2}$

Assume $P(k)$: $1^3+2^3+\cdots k^3 = \frac{k^2(k+1)^2}{4}$ is true $\frac{1}{2}$

and Prove $P(k+1)$ is true

ie Prove $1^3+2^3+\cdots+(k+1)^3 = \frac{(k+1)^2(k+1+1)^2}{4}$

$$LHS = 1^3+2^3+\cdots+k^3+(k+1)^3$$
$$= \frac{k^2(k+1)^2}{4} + (k+1)^3 \checkmark$$
$$= (k+1)^2\left[\frac{k^2}{4} + k+1\right]$$
$$= \frac{(k+1)^2}{4}\left[k^2+4k+4\right]$$
$$= \frac{(k+1)^2(k+2)^2}{4} \checkmark$$
$$= RHS$$

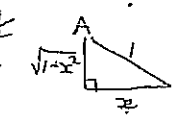
$\therefore$ If $P(k)$ is true then $P(k+1)$ is true

Since $P(1)$ is true the result is proved by Mathematical induction

(3)

Q6(a) $\sin^{-1}x$ and $\cos^{-1}x$ are acute angles.

(i) $\sin(\sin^{-1}x - \cos^{-1}x) = \sin(A-B)$ where $\sin A = x$, $\cos B = x$.



$$= \sin A\cos B - \cos A \sin B. \quad \frac{1}{2}$$
$$= x.x - \sqrt{1-x^2}.\sqrt{1-x^2} \checkmark$$
$$= x^2 - (1-x^2)$$
$$= 2x^2 - 1. \quad \frac{1}{2}$$

(2)

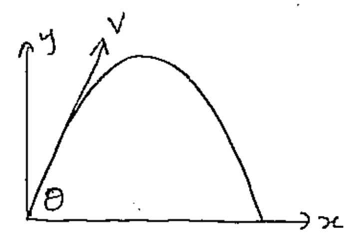
(ii) $\sin^{-1}x - \cos^{-1}x = \sin^{-1}(0.5)$.

taking sine of both sides

$$\sin(\sin^{-1}x - \cos^{-1}x) = \sin(\sin^{-1}0.5)$$
$$2x^2 - 1 = 0.5 \checkmark$$
$$2x^2 = 1.5$$
$$x^2 = \frac{3}{4}$$
$$x = \pm\frac{\sqrt{3}}{2}. \quad \frac{1}{2}$$
$$= \frac{\sqrt{3}}{2} \quad \text{since } \sin^{-1}x \text{ is acute} \quad \frac{1}{2}$$

(2)

(b)



$\ddot{x} = 0$ $\quad \ddot{y} = -g$

$\dot{x} = V\cos\theta$ $\quad \dot{y} = V\sin\theta - gt$

(i) $x = \int V\cos\theta \, dt$

$= Vt\cos\theta + c_1$

when $t=0, x=0 \Rightarrow c_1 = 0$

$x = Vt\cos\theta$ ✓ (1)

$y = \int V\sin\theta - gt \, dt$

$= Vt\sin\theta - \frac{g}{2}t^2 + c_2$

when $t=0, y=0 \Rightarrow c_2 = 0$

$y = Vt\sin\theta - \frac{g}{2}t^2$. ✓ (2)

(2)

(ii) from ① $t = \frac{x}{V\cos\theta}$ $\frac{1}{2}$

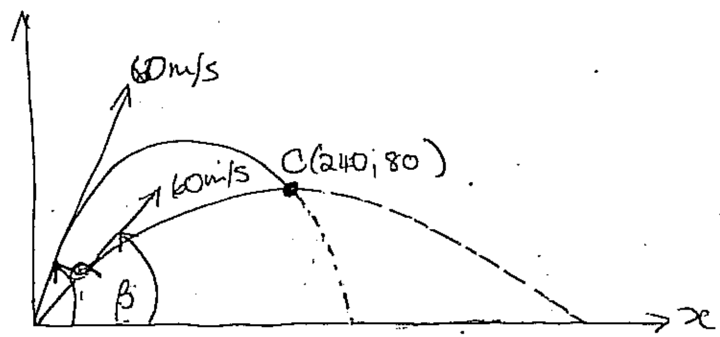
$\rightarrow$ ②

$$y = V\frac{x}{V\cos\theta}\cdot\sin\theta - \frac{g}{2}\frac{x^2}{V^2\cos^2\theta}$$ ✓

② $$y = x\tan\theta - \frac{gx^2}{2V^2}\sec^2\theta.$$ $\frac{1}{2}$

$$y = x\tan\theta - \frac{gx^2}{2V^2}(1+\tan^2\theta).$$ ③

(c)



Collide at point $C(240, 80)$.

(i) For 1^{st} particle which passes through $x=240$, $y=80$

$\rightarrow$ ③

$$80 = 240\tan\alpha - \frac{10\times240^2}{2\times60^2}(1+\tan^2\alpha)$$ ✓

$\div 80$

② $1 = 3\tan\alpha - (1+\tan^2\alpha)$

$\tan^2\alpha - 3\tan\alpha + 2 = 0$

$(\tan\alpha - 2)(\tan\alpha - 1) = 0$

$\therefore \tan\alpha = 2$ or 1 $\frac{1}{2}$

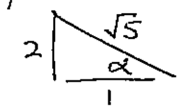
Similarly for the second particle

$\tan\beta = 2$ or 1.

Since $\beta < \alpha$ $\tan\alpha = 2$, $\tan\beta = 1$ $\frac{1}{2}$

(ii) find T, the time between projections when $x = 240$

$240 = 60t\cos\alpha$ $\frac{1}{2}$ for 1^{st} particle

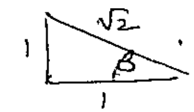


$240 = 60t\cdot\frac{1}{\sqrt5}$

② $t = \frac{240\sqrt5}{60}$

$= 4\sqrt5.$ $\frac{1}{2}$

$240 = 60t\cos\beta$ for 2^{nd} particle



$t = \frac{240}{60}\sqrt2$

$= 4\sqrt2.$ $\frac{1}{2}$

$\therefore T = 4\sqrt5 - 4\sqrt2$

$= 4(\sqrt5 - \sqrt2)$ secs. $\frac{1}{2}$

7a) Area under curve

$\cos 2x = 1 - 2\sin^2 x$

$2\sin^2 x = 1 - \cos 2x$

$= \int_0^\pi \sin^2 x\,dx$

$= \frac{1}{2}\int_0^\pi 1 - \cos 2x\,dx$ ✓

③ $= \frac{1}{2}\left[x - \frac{\sin 2x}{2}\right]_0^\pi$

$= \frac{1}{2}\left[(\pi - 0) - (0 - 0)\right]$

$= \pi/2\ u^2.$ ✓

Area of rectangle OABC $= l\times b$

$= \pi\times1$

$= \pi\ u^2$ ✓

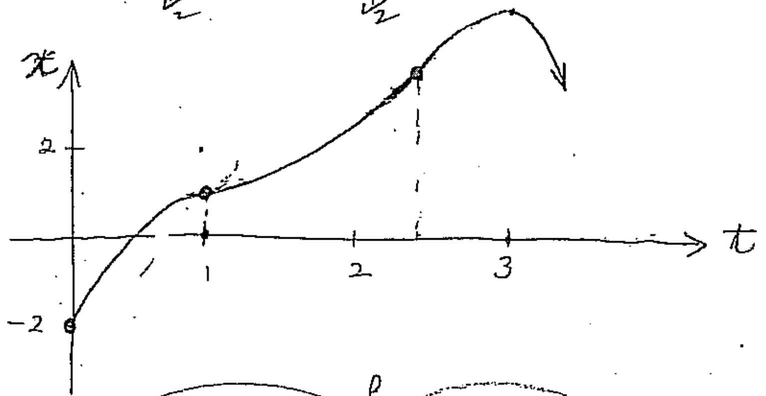
$\therefore$ Area under curve is half the area of rectang

(b) (i) Particle travels to the right when $v > 0$.

① $0 \le t < 3$, $t \ne 1$

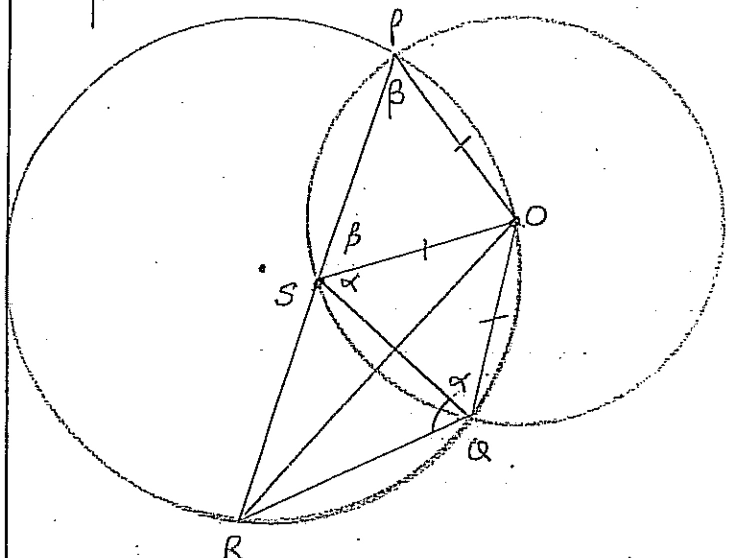
$\frac{1}{2}$, $\frac{1}{2}$

(ii)



②

(c).



① (i) $\angle SPO = \beta$ $\quad OQ = OP = OS$ (radii)

$\therefore \angle PSO = \beta$ ($\angle$s opp equal sides in isos $\triangle POS$) ✓

(ii) $\angle OSQ = O\hat{Q}S = \alpha$ (base $\angle$s of isos $\triangle OSQ$, $OS = OQ$ radii) ✓

$S\hat{P}O + O\hat{Q}R = 180$ (opp $\angle$s of cyclic quad $POQR$) ✓

② $\therefore \beta + S\hat{Q}R + \alpha = 180$

$S\hat{Q}R = 180 - (\alpha + \beta)$.

(iii) $R\hat{S}Q = 180 - (\alpha + \beta)$ (straight $\angle$ at S) ✓

$\therefore R\hat{S}Q = S\hat{Q}R = 180 - (\alpha + \beta)$.

③ $\therefore RS = RQ$ (sides opp = $\angle$s in $\triangle RSQ$) ✓

$\therefore OQRS$ is a kite (2 prs adj sides =)

$\therefore SQ \perp OR$ (diagonals of a kite intersect at right $\angle$s. ✓