Barker College

2001
YEAR 12
TRIAL HSC
EXAMINATION

# MATHEMATICS EXTENSION 1

DATE : PM Friday 17 August

**General Instructions**

- Working time – 2 hours
- Write using blue or black pen
- Write your Barker Student Number at the top of each page of your answers
- Board-approved calculators may be used
- A Table of Standard Integrals is provided at the back of this paper

**Total marks (84)**

- Attempt Questions 1 - 7
- Begin EACH Question on a NEW PAGE
- Only write on ONE side of the page
- ALL necessary working MUST be shown in every question

**Question 1** (12 marks) Start a NEW page. **Marks**

(a) Evaluate $\lim_{x \to 0} \left( \frac{\sin 3x}{2x} \right)$ **1**

(b) Use the table of standard integrals to evaluate $\int_0^2 \frac{4}{\sqrt{x^2 + 16}} \, dx$ **2**

(c) Differentiate $e^{2x} \sin x$ **2**

(d) The interval AB has end points A(-1, 3) and B(2, -3). **2**

Find the coordinates of the point P which divides the interval AB externally in the ratio 1 : 2.

(e) Find the acute angle between the lines $x - y + 3 = 0$ and $2x + y + 1 = 0$. **2**

Give your answer correct to the nearest minute.

(f) Solve the inequality $\frac{x^2 - 4}{x} < 3$ **3**

**Question 2** (12 marks) Start a NEW page. **Marks**

(a) Evaluate $\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right) - \tan^{-1}(1)$, giving your answer in exact form. **2**

(b) Use the substitution $u^2 = x + 4$ (where $u > 0$) to evaluate $\int_{-3}^{0} \frac{x}{\sqrt{x + 4}}\, dx$ **4**

(c) Find the value of the term independent of $x$ in the expansion $\left(2x + \frac{1}{x^2}\right)^6$ **3**

(d) The region enclosed by the curve $y = \tan x$, the $x$-axis and the ordinate at $x = \frac{\pi}{4}$ is rotated about the $x$-axis. **3**

Find the exact volume of the solid of revolution that is formed.

**Question 3** (12 marks) Start a NEW page. **Marks**

(a) Prove that $\dfrac{\cos 2\theta}{\sin\theta} + \dfrac{\sin 2\theta}{\cos\theta} = \cos ec\theta$ **3**

(b) For the function $f(x) = 3\sin^{-1}(2x)$

(i) State the domain. **1**

(ii) State the range. **1**

(iii) Sketch the function, clearly labeling the domain and range. **1**

(iv) Find the equation of the tangent to the curve at the point where it crosses the x-axis. **2**

(c) (i) Prove that $\sin^2\theta = \dfrac{1}{2} - \dfrac{1}{2}\cos 2\theta$ **2**

(i1) Hence, or otherwise, evaluate $\int_0^{\frac{\pi}{3}} \sin^2\theta \; d\theta$ **2**

**Question 4** (12 marks) Start a NEW page. **Marks**

(a) A post $HD$ stands vertically at one corner of a flat rectangular field $ABCD$.

The angles of elevation of the top $H$ of the post $HD$ from the nearest corners $A$ and $C$ are $30^\circ$ and $45^\circ$ respectively.
Let the height of the post $HD$ be $h$ metres and let $AD = a$ metres.

(i) By drawing a diagram which shows the above information, find the length of $BD$ in terms of $h$. **3**

(ii) Hence, find the angle of elevation of $H$ from the corner $B$ (correct to the nearest minute). **1**

(b) Prove, by Mathematical Induction, that $7^n + 5$ is divisible by 3, where $n$ is any positive integer. **4**

(c) The point $P(2ap,\ ap^2)$ is a point on the parabola $x^2 = 4ay$ with focus $S(0,\ a)$.

(i) Find $M$, the mid-point of the chord $OP$, where $O$ is the origin. **1**

(ii) Find the gradient of the chord $OP$. **1**

(iii) Hence, find the point $A$ on the parabola where the tangent is parallel to the chord $OP$. **1**

(iv) Show that $A$ is equidistant from $M$ and the $x$-axis. **1**

**Question 5** (12 marks) Start a NEW page. **Marks**

(a) Solve the equation $100e^{-3t} = 20e^{2t}$ **2**

(b) (i) Express $\cos 3\theta + \sin 3\theta$ in the form $R\cos(3\theta - \alpha)$, where $R > 0$ and $\alpha$ is an acute angle in radians. **2**

(ii) Hence, or otherwise, find all the values of $\theta$ in the range $0 \leq \theta \leq \pi$ for which $\cos 3\theta + \sin 3\theta = 1$. **2**

(c) A particle moves in a straight line such that its displacement $x$ centimetres at any time $t$ seconds is given by $x = \cos 3t + \sin 3t$.

(i) Prove that the motion is Simple Harmonic Motion. **2**

(ii) State the period of the motion. **1**

(iii) Find the initial position of the particle. **1**

(iv) Find the velocity of the particle when it first returns to its initial position. **2**
*(You may use your results from Part (b))*

**Question 6** (12 marks) Start a NEW page. **Marks**

(a) Consider the function $f(x) = \dfrac{x+1}{x^2+3}$

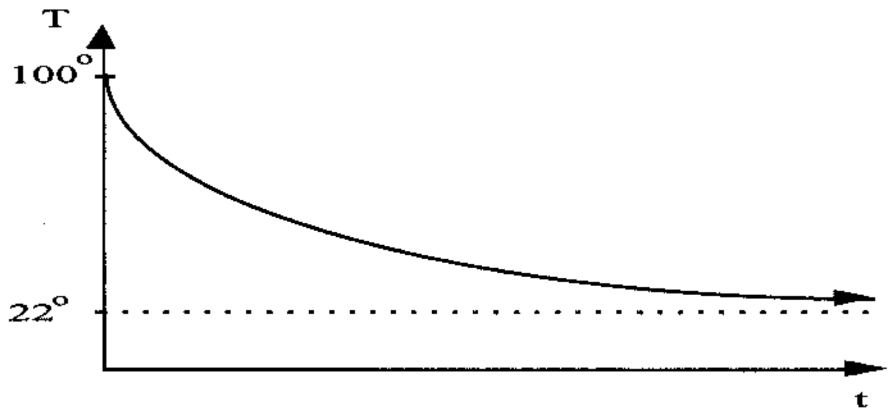
(i) Find the points where the curve crosses the $x$-axis and the $y$-axis. **1**

(ii) Find the coordinates of any stationary points on the curve $y = f(x)$ and, without finding the second derivative, determine their nature. **3**

(iii) Describe the behaviour of $y = f(x)$ for large positive and negative values of $x$. **1**

(iv) Sketch the curve $y = f(x)$, using an appropriate scale and showing all the information above. Label the axes and any critical points. **1**

(b) The graph shown below represents the relationship between $T$, the temperature in $C^\circ$ of a cooling cup of coffee, and $t$, the time in minutes.




The rate of cooling of this coffee is given by $\dfrac{dT}{dt} = -k(T - A)$,

where $k$ and $A$ are constants and $k > 0$.

(i) Show that $T = A + Be^{-kt}$ is a solution to the differential equation $\dfrac{dT}{dt} = -k(T - A)$, given that $B$ is a constant. **1**

(ii) By examining the graph when $t = 0$ and $t \to \infty$, find the values of $A$ and $B$ **2**

(iii) If the temperature of the coffee is $50^\circ C$ after 90 minutes, show that $k = -\dfrac{1}{90}\ln\left(\dfrac{14}{39}\right)$. **2**

(iv) Hence, find the rate at which the coffee is cooling after 90 minutes. **1**

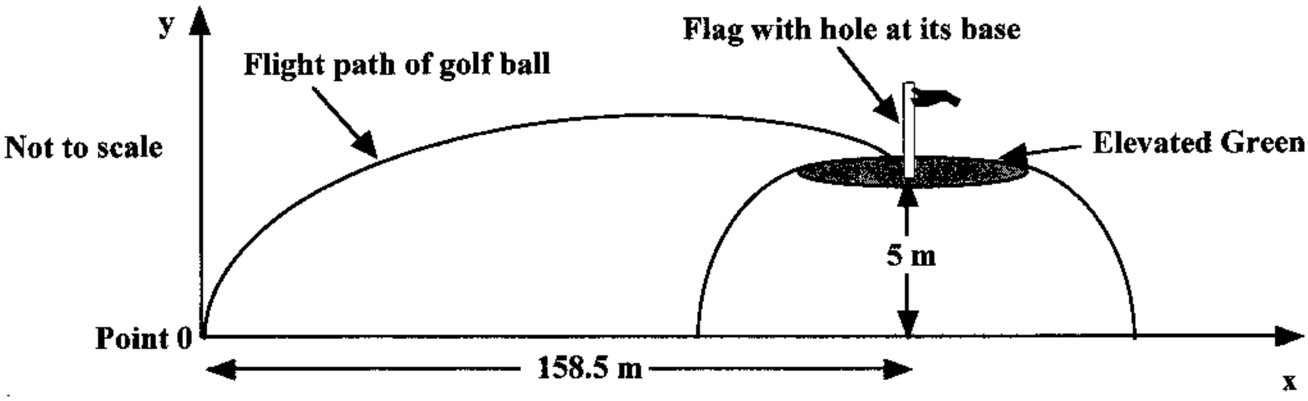
**Question 7** (12 marks) Start a NEW page. **Marks**

(a) Use the binomial expansion of $(1 + x)^{2n}$ to show that

(i) $1 - 2\binom{2n}{1} + 4\binom{2n}{2} - \ldots + \binom{2n}{2n}4^n = 1$ **2**

(ii) $\binom{2n}{1} - 4\binom{2n}{2} + 12\binom{2n}{3} - \ldots - n\binom{2n}{2n}4^n = -2n$ **2**

(b) A Mathematics teacher hits a golf ball from a point 0 towards a flat, elevated green as shown in the diagram below. The hole at the base of the flag is situated in the centre of the green.




The golf ball is projected from the point 0 with a initial velocity of $V\ ms^{-1}$ and at an angle of $\alpha$ to the horizontal. You may assume the only force acting on the golf ball in flight is gravity, which you may approximate to be $10ms^{-2}$.

(i) Taking the point 0 as the origin, show that the parametric equations of the flight path of the golf ball are given by $x = Vt\cos\alpha$ and $y = -5t^2 + Vt\sin\alpha$. **2**

(ii) If the angle of projection is given by $\alpha = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$, find the value of $V\ ms^{-1}$ **3**

which will enable the teacher to hit the golf ball directly into the hole on the green. The hole is situated 158.5 metres horizontally and 5 metres vertically from the point 0. Give your answer correct to the nearest integer.

(iii) Using your value of $V\ ms^{-1}$ from Part (ii), find the alternative angle of projection (from the point 0) that would enable the teacher to still hit the golf ball directly into the hole. **3**

**E N D O F P A P E R**

JM

17th Aug 2001 ~~EXAM~~ SOLUTIONS YR 12 EXTENS 1. p.1

2001,

YR11 3U TRIAL HSC SOLUTIONS (EXTENSION 1)

QUESTION 1

a) $\lim_{x\to 0}\left(\frac{\sin 3x}{2x}\right)$

$= \lim_{x\to 0} \frac{3}{2}\times\frac{\sin 3x}{3x}$

$= \frac{3}{2}\times 1$

$= \frac{3}{2}$ ✓

b) $\int_0^2 \frac{4}{\sqrt{x^2+16}}\,dx$

$= 4\left[\ln\{x+\sqrt{x^2+4^2}\}\right]_0^2$ ✓

$= 4\left[\ln(2+\sqrt{20}) - \ln(0+4)\right]$

$\doteq 1.9248473$ ✓

c) $\frac{d}{dx}\left(e^{2x}\sin x\right)$

$= e^{2x}\times\cos x + \sin x \times 2e^{2x}$ ✓

$= e^{2x}\cos x + 2e^{2x}\sin x$ ✓

$= e^{2x}(\cos x + 2\sin x)$

d) $A(-1,3)$, $B(2,-3)$

$k:l = 1:-2$.

$x = \frac{kx_2+lx_1}{k+l}$, $y = \frac{ky_2+ly_1}{k+l}$

$x = \frac{1\times 2 - 2\times -1}{-1}$, $y = \frac{1\times -3 - 2\times 3}{-1}$

$x = \frac{+4}{-1}$, $y = \frac{-3-6}{-1}$

$x = -4$ ✓, $y = 9$ ✓

$\therefore$ The point is $(-4,9)$

e) $x-y+3=0 \rightarrow m_1 = 1$

$2x+y+1=0 \rightarrow m_2 = -2$.

$\tan\theta = \left|\frac{m_1-m_2}{1+m_1m_2}\right|$

$= \left|\frac{1+2}{1-2}\right|$ ✓

$= |-3|$

$\tan\theta = 3$

$\therefore \theta = 71°34'$ ✓

f) $\frac{x^2-4}{x} < 3$

$x(x^2-4) < 3x^2$ ✓

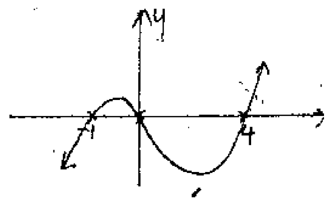
$x^3-4x < 3x^2$

$x^3-3x^2-4x < 0$

$x(x^2-3x-4) < 0$

$\therefore x(x-4)(x+1) < 0$ ✓

f) cont...




$\therefore x < -1$ or $0 < x < 4$ ✓

QUESTION 2

a) $\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right) - \tan^{-1}(1)$

$= \frac{5\pi}{6} - \frac{\pi}{4}$ ✓

$= \frac{7\pi}{12}$

b) $u^2 = x+4$

$\frac{dx}{du} = 2u$

$dx = 2u\,du$

$x=-3$, $u=1$

$x=0$, $u=2$.

$\therefore I = \int_1^2 \frac{u^2-4}{\sqrt{u^2}}\,2u\,du$

$= \int_1^2 \frac{u^2-4}{u}\times 2u\,du$ ✓

$= 2\left[\frac{u^3}{3}-4u\right]_1^2$ ✓

p.2

$= 2\left[\left(\frac{8}{3}-8\right)-\left(\frac{1}{3}-4\right)\right]$

$I = \frac{-10}{3}$ ✓

c) $\left(2x+\frac{1}{x^2}\right)^6$

$(k+1)$th term $= \binom{6}{k}(2x)^{6-k}(x^{-2})^k$

$\therefore x^{6-k}.x^{-2k} = x^0$ ✓

$6-k-2k=0$

$6-3k=0$

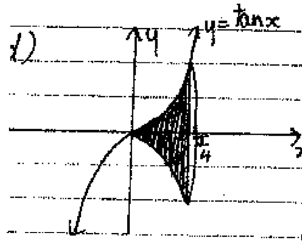
$k=2$ ✓

$\therefore$ the third term

Its value is

$\binom{6}{2}2^4 = 240.$ ✓

d)




$V = \pi\int_0^{\frac{\pi}{4}} y^2\,dx$

$= \pi\int_0^{\frac{\pi}{4}} \tan^2 x\,dx$ ✓

$= \pi\int_0^{\frac{\pi}{4}} (\sec^2 x - 1)\,dx$

$= \pi\left[\tan x - x\right]_0^{\frac{\pi}{4}}$ ✓

$= \pi\left[\left(\tan\frac{\pi}{4}-\frac{\pi}{4}\right)-(\tan 0 - 0)\right]$

$= \pi\left[1-\frac{\pi}{4}\right]$ ✓

$V = \frac{\pi(4-\pi)}{4}\,u^3$

QUESTION 3

a) $\frac{\cos 2\theta}{\sin\theta} + \frac{\sin 2\theta}{\cos\theta} = \operatorname{cosec}\theta$

LHS $= \frac{\cos^2\theta-\sin^2\theta}{\sin\theta} + \frac{\sin 2\theta}{\cos\theta}$

$= \frac{1-2\sin^2\theta}{\sin\theta} + \frac{2\sin\theta\cos\theta}{\cos\theta}$ ✓ ✓

$= \frac{1-2\sin^2\theta+2\sin^2\theta}{\sin\theta}$

$= \frac{1}{\sin\theta}$ ✓

$= \operatorname{cosec}\theta$

$=$ RHS

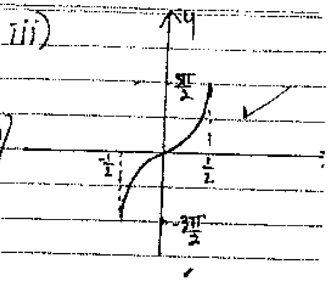
b) $f(x) = 3\sin^{-1}(2x)$

i) $-1 \le 2x \le 1$

$\therefore -\frac{1}{2} \le x \le \frac{1}{2}$. ✓

ii) $-\frac{3\pi}{2} \le y \le \frac{3\pi}{2}$ ✓

iii)




iv) $f'(x) = 3\times\frac{1}{\sqrt{1-(2x)^2}}\times 2$

$= \frac{6}{\sqrt{1-4x^2}}$ ✓

Crosses x-axis at $(0,0)$

$f'(0) = \frac{6}{\sqrt{1}} = 6$

$\therefore$ eqn tangent is

$y-y_1 = m(x-x_1)$

$y-0 = 6(x-0)$

$\therefore y = 6x$ ✓

c) i) RHS $= \frac{1}{2} - \frac{1}{2}\cos 2\theta$

$= \frac{1}{2}(1-\cos 2\theta)$

$= \frac{1}{2}(1-(1-2\sin^2\theta))$ ✓

$= \frac{1}{2}\times 2\sin^2\theta$ ✓

$= \sin^2\theta$

$=$ LHS

i) $\int_0^{\frac{\pi}{3}} \sin^2\theta \; d\theta$

$= \frac{1}{2}\int_0^{\frac{\pi}{3}} 1-\cos 2\theta \; d\theta$

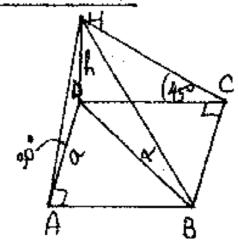
$= \frac{1}{2}\left[\theta - \frac{1}{2}\sin 2\theta\right]_0^{\frac{\pi}{3}}$ ✓

$= \frac{1}{2}\left[\left(\frac{\pi}{3} - \frac{1}{2}\sin\frac{2\pi}{3}\right) - \left(0 - \frac{1}{2}\sin 0\right)\right]$ ✓

$= \frac{1}{2}\left(\frac{\pi}{3} - \frac{1}{2}\times\frac{\sqrt{3}}{2}\right) = \frac{1}{2}\left(\frac{\pi}{3} - \frac{\sqrt{3}}{4}\right)$

QUESTION 4

i)




$\tan 30° = \frac{h}{a}$

$h = a\tan 30°$

$h = \frac{a}{\sqrt{3}}$

$\therefore a = h\sqrt{3}$ ✓

$\tan 45° = \frac{h}{CD}$

$CD = \frac{h}{1} = h$ ✓

$\therefore (BD)^2 = (h\sqrt{3})^2 + (h)^2$

$= h^2\times 3 + h^2$

$= 4h^2$ ✓

$\therefore BD = 2h$ units

ii) $\tan\alpha = \frac{h}{2h} = \frac{1}{2}$

$\therefore \alpha = 26°34'$ ✓

b) Step 1

Prove true for $n=1$

$7^1 + 5 = 12$, which is divisible by 3

$\therefore$ true for $n=1$

Step 2

Assume true for $n=k$.

ie $\frac{7^k+5}{3} = M$ where $M$ is an integer ✓

ie assume $7^k+5 = 3M$.

Prove $7^{k+1}+5$ is divis. by 3.

$7^{k+1}+5$

$= 7.7^k + 5$

$= 7(7^k+5) - 30$ ✓

$= 7.3M - 30$ ✓

$= 3(7M-10)$, which is divis. by 3.

$\therefore$ if true for $n=k$, it's true for $n=k+1$.

Step 3

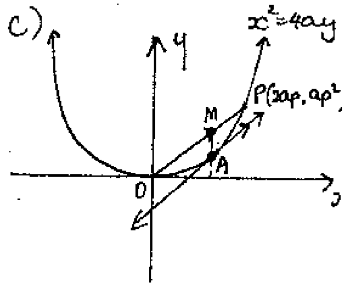
But it's true for $n=1$

$\therefore$ true for $n=1+1=2$;

it's true for $n=2+1=3$ ✓

and so on for all +ve integer $n$.

c)




i) $M = \left(\frac{2ap+0}{2}, \frac{ap^2+0}{2}\right)$

$= \left(ap, \frac{ap^2}{2}\right)$ ✓

ii) $m_{OP} = \frac{ap^2-0}{2ap-0}$

$= \frac{p}{2}$ ✓

iii) $y = \frac{x^2}{2a}$

$\frac{dy}{dx} = \frac{2x}{2a} = \frac{x}{a}$

$\frac{x}{2a} = \frac{p}{2}$ $\quad\therefore x = ap$

$\therefore x = ap$, $y = \frac{a^2p^2}{4a} = \frac{ap^2}{4}$

$\therefore A = \left(ap, \frac{ap^2}{4}\right)$ ✓

iv)

dist $MA = \frac{ap^2}{2} - \frac{ap^2}{4} = \frac{ap^2}{4}$

dist A to x-axis $= \frac{ap^2}{4}$ ✓

$\therefore$ equidistant.

QUESTION 5

a) $100e^{-3t} = 20e^{2t}$

$\frac{100}{20} = \frac{e^{2t}}{e^{-3t}}$

$5 = e^{5t}$ ✓

$5t\ln e = \ln 5$

$\therefore t = \frac{\ln 5}{5}$ ✓

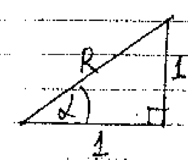
b)

i) $\cos 3\theta + \sin 3\theta = R\cos(3\theta - \alpha)$

$= R\cos 3\theta\cos\alpha + R\sin 3\theta\sin\alpha$

$\therefore R\cos\alpha = 1 \rightarrow \cos\alpha = \frac{1}{R}$

$R\sin\alpha = 1 \rightarrow \sin\alpha = \frac{1}{R}$




$R^2 = 2$

$R = \sqrt{2}$ ✓

$\therefore \tan\alpha = 1$

$\therefore \alpha = \frac{\pi}{4}$ ✓

$\therefore R\cos(3\theta - \alpha)$ ✓

$= \sqrt{2}\cos\left(3\theta - \frac{\pi}{4}\right)$

ii)

$\sqrt{2}\cos\left(3\theta - \frac{\pi}{4}\right) = 1$

$\cos\left(3\theta - \frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$

$\therefore 3\theta - \frac{\pi}{4} = -\frac{\pi}{4}, \frac{\pi}{4}, \frac{7\pi}{4}, \frac{9\pi}{4}, \frac{15\pi}{4}, \ldots$ ✓

$\therefore 3\theta = 0, \frac{\pi}{2}, 2\pi, \frac{5\pi}{2}, \ldots$

$\therefore \theta = 0, \frac{\pi}{6}, \frac{2\pi}{3}, \frac{5\pi}{6}$ ✓

c) $x = \cos 3t + \sin 3t$

i) $\dot{x} = -3\sin 3t + 3\cos 3t$ ✓

$\ddot{x} = -9\cos 3t - 9\sin 3t$

$= -9(\cos 3t + \sin 3t)$

$\ddot{x} = -9x$ ✓

$\ddot{x} = -n^2 x$ $\therefore$ SHM

ii) Period $= \frac{2\pi}{3}$ ✓

iii)

$t=0$, $x = \cos 0 + \sin 0$

$x = 1$ unit right of O. ✓

iv) Returns to initial position when $x=1$

$\therefore \cos 3t + \sin 3t = 1$

From b)

$\sqrt{2}\cos\left(3t - \frac{\pi}{4}\right) = 1$

$\therefore t = \frac{\pi}{6}$ sec

$t = \frac{\pi}{6}$, $\dot{x} = -3\sin 3\times\frac{\pi}{6} + 3\cos 3\times\frac{\pi}{6}$

$\dot{x} = -3\sin\frac{\pi}{2} + 3\cos\frac{\pi}{2}$

$= -3\times 1 + 3\times 0$

$\dot{x} = -3$ cm/s ✓

QUESTION 6

a) $f(x) = \frac{x+1}{x^2+3}$

i) Crosses y-axis at $\frac{1}{3}$

Crosses x-axis when $\frac{x+1}{x^2+3} = 0$

$x+1 = 0 \rightarrow x = -1$

✓

ii) $f'(x) = \frac{(x^2+3)\times 1 - (x+1)\times 2x}{(x^2+3)^2}$

$= \frac{x^2+3-2x^2-2x}{(x^2+3)^2}$

$= \frac{3-2x-x^2}{(x^2+3)^2}$ ✓

$= \frac{(3+x)(1-x)}{(x^2+3)^2}$

For stat pts $f'(x)=0$.

$\therefore \frac{(3+x)(1-x)}{(x^2+3)^2} = 0$

$\therefore x=-3$ OR $1$. ✓

When $x=-3$, $y=\frac{1}{6}$ $(-3,\frac{1}{6})$

$x=1$, $y=\frac{1}{2}$ $(1,\frac{1}{2})$

Test:

| $x$ | -4 | -3 | 0 | 1 | 2 |
|---|---|---|---|---|---|
| $f'(x)$ | $\frac{-5}{361}$ | 0 | $\frac{1}{3}$ | 0 | $\frac{-5}{49}$ |

\ — / ‾ \

$\therefore$ min. turning pt at $(-3,\frac{1}{6})$ ✓

Max. tur. pt at $(1,\frac{1}{2})$

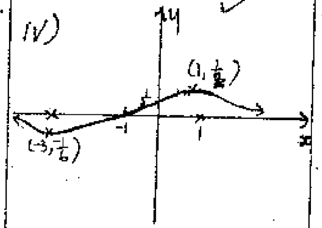
i) $\lim_{x\to\pm\infty} \frac{x+1}{x^2+3}$

$= \lim_{x\to\pm\infty} \frac{\frac{x}{x^2}+\frac{1}{x^2}}{\frac{x^2}{x^2}+\frac{3}{x^2}}$

$= \lim_{x\to\pm\infty} \frac{\frac{1}{x}+\frac{1}{x^2}}{1+\frac{3}{x^2}}$

$=0$. ✓

iv) ✓




b)
i) $T=A+Be^{-kt}$

$Be^{-kt}=T-A$.

$\frac{dT}{dt}=-kBe^{-kt}$

$\therefore \frac{dT}{dt}=-k(T-A)$ ✓

ii) $t=0$, $T=100°$.

As $t\to\infty$, $T\to 22°$.

$\therefore 100 = A+Be^{0}$

$\therefore A+B=100$.

As $t\to\infty$, $e^{-kt}\to 0$

$\therefore T\to A$.

ie $A\to 22$. ✓

$\therefore 22+B=100$

$B=78$. ✓

iii) $T=50°$, $t=90$.

$T=A+Be^{-kt}$

$T=22+78e^{-kt}$

$50=22+78e^{-90k}$

$78e^{-90k}=28$

$e^{-90k}=\frac{28}{78}=\frac{14}{39}$ ✓

$-90k\ln e=\ln\left(\frac{14}{39}\right)$ ✓

$\therefore k=\frac{-1}{90}\ln\left(\frac{14}{39}\right)$

iv) $\frac{dT}{dt}=-k(T-A)$ from i).

$\therefore \frac{dT}{dt}=-\frac{1}{90}\ln\left(\frac{14}{39}\right)(50-22)$ ✓

$\frac{dT}{dt}=-0.3187°C/\text{min}$

QUESTION 7

a) $(1+x)^{2n}$

i) $(1+x)^{2n}=\binom{2n}{0}x^0+\binom{2n}{1}x^1+\binom{2n}{2}x^2+\dots+\binom{2n}{2n}x^{2n}$ *

Let $x=-2$ on B.S. of *

$\therefore (-1)^{2n}=(1)x^0+\binom{2n}{1}(-2)+\binom{2n}{2}(-2)^2+\dots+\binom{2n}{2n}(-2)^{2n}$ ✓

ie $[(-1)^2]^n=1-2\binom{2n}{1}+4\binom{2n}{2}\dots+(4)^n\binom{2n}{2n}$ ✓

$\therefore 1 = 1-2\binom{2n}{1}+4\binom{2n}{2}+\dots+4^n\binom{2n}{2n}$ as required.

ii) Differentiate B.S. of * wrt $x$

$\therefore 2n(1+x)^{2n-1}=0+\binom{2n}{1}+2x\binom{2n}{2}+3x^2\binom{2n}{3}+\dots(2n)\binom{2n}{2n}x^{2n-1}$ ✓

Now let $x=-2$

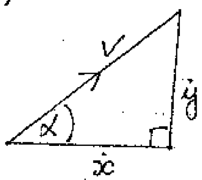
$\therefore 2n(-1)^{2n-1}=\binom{2n}{1}-4\binom{2n}{2}+12\binom{2n}{3}+\dots+(2n)\binom{2n}{2n}(-2)^{2n-1}$

$n[[(-1)^2]^n.(-1)^{-1}]=\binom{2n}{1}-4\binom{2n}{2}+12\binom{2n}{3}+\dots+(2n)\binom{2n}{2n}[[(-2)^2]^n.(-2)^{-1}]$

$\therefore 2n(-1)=\binom{2n}{1}-4\binom{2n}{2}+12\binom{2n}{3}+\dots+(2n)\binom{2n}{2n}4^n.\frac{-1}{2}$ ✓

$\therefore -2n=\binom{2n}{1}-4\binom{2n}{2}+12\binom{2n}{3}+\dots-n\binom{2n}{2n}4^n$ as required.

b)




$\cos\alpha = \frac{\dot x}{V}$

$\dot x = V\cos\alpha$
$\dot y = V\sin\alpha$.

$\therefore t=0, x=0, y=0, \dot x = V\cos\alpha, \dot y = V\sin\alpha$.

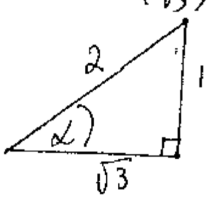
**Horiz.**

$\ddot x = 0$
$\dot x = C_1$
say $\boxed{\dot x = V\cos\alpha}$
$x = \int V\cos\alpha \, dt$
$x = Vt\cos\alpha + C_2$
$t=0, x=0 \therefore C_2 = 0$
$\boxed{x = Vt\cos\alpha}$ ✓

**Vertic.**

$\ddot y = -10$
$\dot y = -10t + C_3$
$t=0, \dot y = V\sin\alpha$
$\therefore C_3 = V\sin\alpha$
$\boxed{\dot y = -10t + V\sin\alpha}$
$y = \int -10t + V\sin\alpha \, dt$
$y = -5t^2 + Vt\sin\alpha + C_4$
$t=0, y=0 \therefore C_4 = 0$
$\boxed{y = -5t^2 + Vt\sin\alpha}$ ✓

) $\alpha = \tan^{-1}\left(\frac{1}{\sqrt3}\right)$




$\therefore \cos\alpha = \frac{\sqrt3}{2}$

$\sin\alpha = \frac{1}{2}$.

$x = Vt\cos\alpha \longrightarrow t = \frac{x}{V\cos\alpha}$

sub. into $y = -5t^2 + Vt\sin\alpha$
$= -5\times\frac{x^2}{V^2\cos^2\alpha} + V\times\frac{x}{V\cos\alpha}\times\sin\alpha$

$\therefore y = \frac{-5x^2\sec^2\alpha}{V^2} + x\tan\alpha$

$y = \frac{-5x^2}{V^2}(1+\tan^2\alpha) + x\tan\alpha$ ✓ *

Using $x = 158\cdot5$, $y = 5$, $\tan\alpha = \frac{1}{\sqrt3}$ then

$5 = \frac{-5\times158\cdot5^2}{V^2}\left(1+\frac{1}{3}\right) + 158\cdot5\times\frac{1}{\sqrt3}$

$5V^2 = -5\times158\cdot5^2\times\frac{4}{3} + \frac{158\cdot5V^2}{\sqrt3}$

$15\sqrt3V^2 = -5\times158\cdot5^2\times4\times\sqrt3 + 158\cdot5V^2\times$

$\therefore V^2(158\cdot5\times3 - 15\sqrt3) = 5\times158\cdot5^2\times4\times\sqrt{}$

$\therefore V^2 = \frac{20\sqrt3\times158\cdot5^2}{158\cdot5\times3 - 15\sqrt3}$

$V^2 = 1935\cdot980031$   ✓ for some working

$\therefore V = 43\cdot999$ m/s
ie $V \doteqdot 44$ m/s. ✓

iii) Using * in part ii) above

$y = \frac{-5x^2}{V^2}(1+\tan^2\alpha) + x\tan\alpha$

$5 = \frac{-5\times158\cdot5^2}{44^2}(1+\tan^2\alpha) + 158\cdot5\tan$ ✓

$\longrightarrow$

iii) Cont...

$5\times44^2 = -5\times158\cdot5^2(1+\tan^2\alpha) + 158\cdot5\times44^2\times\tan\alpha$

$5\times44^2 = -5\times158\cdot5^2 - 5\times158\cdot5^2\tan^2\alpha + 158\cdot5\times44^2\times\tan\alpha$

~~$\therefore 158\cdot5\times44^2\tan\alpha - 5\times158\cdot5^2$~~

$5\times158\cdot5^2\tan^2\alpha - 158\cdot5\times44^2\tan\alpha + (5\times44^2 + 5\times158\cdot5^2) = 0$. ✓

$\therefore \tan\alpha = \frac{158\cdot5\times44^2 \pm \sqrt{158\cdot5^2\times44^4 - 4\times5\times158\cdot5^2\times(5\times44^2+5\times158\cdot5^2)}}{2\times158\cdot5^2\times5}$

$\tan\alpha = 1\cdot865562615$ OR $0\cdot577339593$

$\therefore \alpha = 61\cdot8°$ OR $30°$ ✓

$\therefore$ the alternative $\alpha$ is $61\cdot8°$.

THE END!