Barker College

2005
TRIAL
HIGHER SCHOOL CERTIFICATE

Mathematics Extension 1

Staff Involved:

- GIC*
- AES*
- MRB
- VAB
- PJR
- GDH
- RMH

90 copies

PM FRIDAY 12 AUGUST

General Instructions

- Reading time – 5 minutes
- Working time – 2 hours
- Write using blue or black pen
- Make sure your Barker Student Number is on ALL pages
- Board-approved calculators may be used
- A table of standard integrals is provided on page 9
- ALL necessary working should be shown in every question
- Marks may be deducted for careless or badly arranged working

Total marks – 84

- Attempt Questions 1 – 7
- All questions are of equal value

Total marks – 84
Attempt Questions 1
ALL questions are of equal value

Answer each question on a SEPARATE sheet of paper

Marks

Question 1 (12 marks) **[START A NEW PAGE]**

(a) Evaluate $\lim\limits_{x \to 0} \dfrac{\sin 2x}{5x}$ **2**

(b) Find the acute angle between the lines $2x - y = 0$ and $x + 3y = 0$, giving the answer correct to the nearest minute. **3**

(c) Simplify $\dfrac{e^{\ell n(8+3x)}}{64 - 9x^2}$ **2**

(d) A is the point (–2, 1) and B is the point (x, y). The point P (13, –9) divides AB **externally** in the ratio 5 : 3.
Find the values of x and y for point B. **2**

(e) Solve the inequality $\dfrac{x}{x - 3} \le 3$ **3**

Marks

Question 2 (12 marks) [START A NEW PAGE]

(a) Evaluate $\int_0^2 2x\sqrt{1-\frac{x}{2}}\,dx$ using the substitution $u = 1 - \frac{x}{2}$ — 3

(b) Solve for $\theta : \cos\theta = \sin 2\theta$ for $0 \le \theta \le 2\pi$ — 3

(c) (i) Express $\sin x + \sqrt{3}\cos x$ in the form $R\sin(x+\alpha)$ where $R > 0$ and $0 \le \alpha \le \frac{\pi}{2}$ — 2

(ii) Hence, sketch $y = \sin x + \sqrt{3}\cos x$ for $-2\pi \le x \le 2\pi$ showing any x and y intercepts. — 2

(iii) Find the general solution to $\sin x + \sqrt{3}\cos x = \sqrt{2}$ — 2

Marks

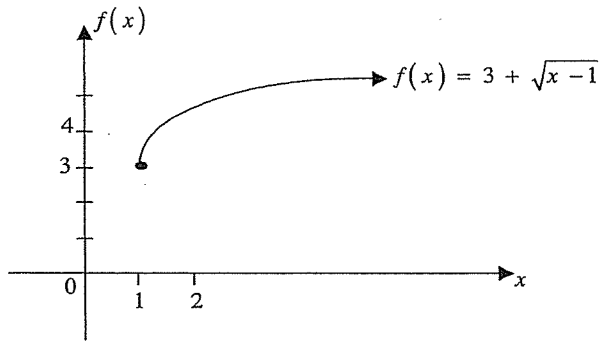
Question 3 (12 marks) [START A NEW PAGE]

(a) Find the domain and range of $y = 3\sin^{-1}(2x-1)$
(A sketch of the curve is **not** necessary, but may be helpful). — 2

(b) If $f(x) = (1+x^2)\tan^{-1}x$ find $f'(x)$ — 2

(c) Find $\int \frac{dx}{\sqrt{16-x^2}}$ — 1

(d) The function $f(x) = 3 + \sqrt{x-1}$ is sketched below.



(i) State the domain of $f(x)$ — 1

(ii) Explain why an inverse function, $f^{-1}(x)$, exists. — 1

(iii) Find $f^{-1}(x)$ — 2

(iv) (α) On what line will the curves $y = f(x)$ and $y = f^{-1}(x)$ intersect? — 1

(β) Hence, find the point of intersection of the graphs $y = f(x)$ and $y = f^{-1}(x)$ — 2

Marks

Question 4 (12 marks) **[START A NEW PAGE]**

(a) Evaluate $\int_0^{2\pi} \sin^2 2x \, dx$ **2**

(b) If $\tan A$ and $\tan B$ are the roots of the equation $3x^2 - 5x - 1 = 0$, find the value of $\tan(A + B)$. **3**

(c) Assuming that $r \neq 1$, prove by induction that

$$a + ar + ar^2 + ar^3 + \ldots\ldots + ar^{n-1} = \frac{a(r^n - 1)}{r - 1}$$ for all positive integers n **3**

(d) (i) Given that the curve $y = x\sin^{-1} x$ has only one stationary point, show that this stationary point occurs at (0, 0) and that it is a minimum turning point. **2**

(ii) Hence, or otherwise, sketch the curve $y = x\sin^{-1} x$ on the $x - y$ plane. **2**

Marks

Question 5 (12 marks) **[BEGIN A NEW PAGE]**

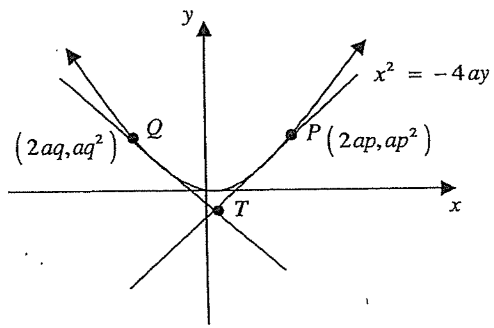
(a) A particle, initially at a fixed point O, is moving in a straight line. After time t seconds, it has displacement x metres from O and its velocity $v\,\text{ms}^{-1}$ is given by $v = 6 - 2x$.

(i) Find the acceleration of the particle at the origin. **2**

(ii) Show that $t = -\frac{1}{2}\log_e\left(1 - \frac{x}{3}\right)$ and hence find x as a function of t. **3**

(iii) What happens to x as t increases without bound? **1**

(b) $P(2ap, ap^2)$ and $Q(2aq, aq^2)$ are two points on the parabola $x^2 = 4ay$. The tangents at P and Q meet at T which is always on the parabola $x^2 = -4ay$.



(i) Derive the equations of the tangents to the parabola at P and Q. **2**

(ii) Show that T is the point $(a(p + q), apq)$ **2**

(iii) Show that $p^2 + q^2 = -6pq$ **2**

Marks

Question 6 (12 marks) **[START A NEW PAGE]**

(a) A particle is oscillating in simple harmonic motion such that its displacement x metres from the origin is given by the equation $\frac{d^2x}{dt^2} = -16x$, where t is time in seconds.

(i) Show that $x = a\cos(4t + \alpha)$ is a solution for the motion of this particle. (a and α are constants) **2**

(ii) Initially, the velocity is 4 ms^{-1} and displacement from the origin is 5 m. Show that the amplitude of the oscillation is $\sqrt{26}$ metres. **2**

(iii) What is the maximum speed of the particle? **2**

(b) In a particular equatorial African swamp, a colony of tsetse flies increases its population (P) according to the differential equation $\frac{dP}{dt} = k(P - 10000)$, where k is the growth rate of the colony. Initially, there were 15 000 tsetse flies and after six months there were 25 000 tsetse flies.

(i) Show that $P = 10000 + P_0e^{kt}$ is a solution of the above equation. (k and P_0 are constants) **2**

(ii) Determine P_0 and the growth rate k in exact form. **2**

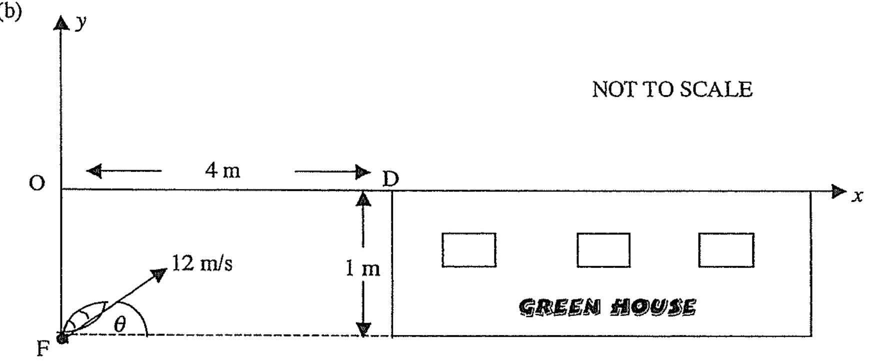
(iii) Determine the number of tsetse flies after one year. **2**

Marks

Question 7 (12 marks) **[START A NEW PAGE]**

(a) Find the term independent of x in the expansion of $\left(x + \frac{2}{x^2}\right)^{12}$ **3**

(b)



A football, lying at point F on level ground is 4 metres away from and 1 metre below the top of a flat-roofed long narrow green house. The football is kicked with an initial velocity of 12 m/s at an angle of projection θ.

(i) Using $g = -10$ ms^{-2}, show that the football's trajectory at time t seconds after being kicked may be defined by the equations $x = 12t\cos\theta$ and $y = -5t^2 + 12t\sin\theta - 1$ where x and y are the horizontal and vertical displacements, in metres, of the football from the origin O shown in the diagram. (Neglect air resistance). **3**

(ii) Given that $\theta = 30°$, how far from D will the football land on top of the green house? **3**

(iii) Find the range of values of θ, to the nearest degree, at which the football must be kicked so that it will land to the right of D. **3**

End of Paper

EXT 1 TRIAL 2005

1 a) $\lim\limits_{x\to 0} \dfrac{\sin 2x}{5x} = \lim\limits_{2x\to 0} \dfrac{\sin 2x}{2x} \times \dfrac{2}{5}$

$= 1 \times \dfrac{2}{5} = \dfrac{2}{5}$ ✓✓

b) $\tan(\alpha-\beta) = \dfrac{2 - -\frac{1}{3}}{1 + 2\times -\frac{1}{3}}$ ✓✓✓

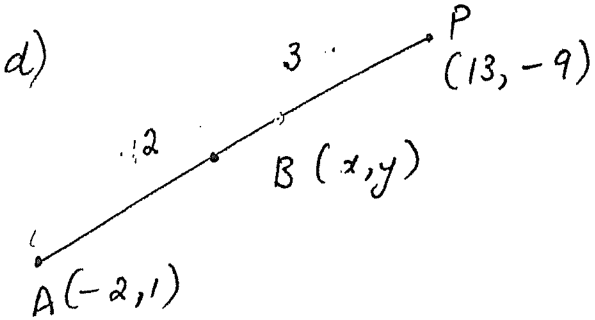
$= \dfrac{7}{3} \Big/ \dfrac{1}{3} = 7$

acute angle $= 81^\circ 52'$ (nearest minute)

c) $\dfrac{e^{\ln(8+3x)}}{64-9x^2} = \dfrac{8+3x}{(8+3x)(8-3x)}$

$= \dfrac{1}{8-3x}, \quad 3x \neq \pm 8$ ✓✓

d)



$x = \dfrac{2}{5} \times 15 + -2 = 4$ ✓✓

$y = \dfrac{2}{5} \times -10 + 1 = -3$

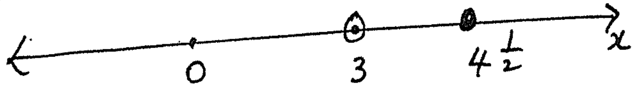
e) $\dfrac{x}{x-3} \le 3 \qquad x \neq 3$

Solve $x = 3(x-3)$

$\Rightarrow x = 3x - 9$

$\Rightarrow 2x = 9$

$\Rightarrow x = 4\frac{1}{2}$



Test $x < 3$ e.g. $x = 1$

$\dfrac{1}{1-3} \le 3$ True $\quad x < 3$

Test $3 < x < 4\frac{1}{2}$ e.g. $x = 4$

$\dfrac{4}{4-3} \le 3$ False $\quad x \notin (3, 4\frac{1}{2})$

Test $x > 4\frac{1}{2}$ e.g. $x = 5$

$\dfrac{5}{5-3} \le 3$ True $\quad x > 4\frac{1}{2}$

Solution $x < 3$ OR $x \ge 4\frac{1}{2}$ ✓✓✓

2. a) let $u = 1 - \dfrac{x}{2} \Rightarrow du =$

$\Rightarrow x = 2(1-u) \Rightarrow dx = -2du$

$\Rightarrow$ if $x = 0$: $u = 1$.
if $x = 2$: $u = 0$

i.e. $\displaystyle\int_0^2 2x\sqrt{1-\frac{x}{2}}\,dx = \int_1^0 4(1-u)\sqrt{u} \times -2du$

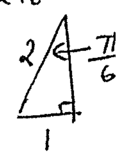
$\displaystyle = -8\int_1^0 (\sqrt{u} - u\sqrt{u})\,du$

$= 8\left[\dfrac{2}{3}u^{3/2} - \dfrac{2}{5}u^{5/2}\right]_0^1$

$= 8\left[\dfrac{2}{3} - \dfrac{2}{5} - (0-0)\right]$ ✓✓✓

$= 8 \times \dfrac{10-6}{15} = \dfrac{32}{15}$

b) $\cos\theta = \sin 2\theta$ for $0 \le \theta \le 2\pi$



$\Rightarrow \cos\theta = 2\sin\theta\cos\theta$

$\Rightarrow \cos\theta(1 - 2\sin\theta) = 0$

$\Rightarrow \cos\theta = 0$ OR $\sin\theta = \frac{1}{2}$

$\Rightarrow \theta = \dfrac{\pi}{2}$ OR $\dfrac{3\pi}{2}$ OR $\dfrac{\pi}{6}$ OR $\dfrac{5\pi}{6}$

✓✓✓

2c) i) $R\sin(x+\alpha)$

$= R\sin x\cos\alpha + R\cos x\sin\alpha$

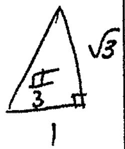
$= \sin x + \sqrt{3}\cos x$

$\Rightarrow R = \sqrt{1^2+(\sqrt{3})^2} = 2 \quad (R>0)$

and

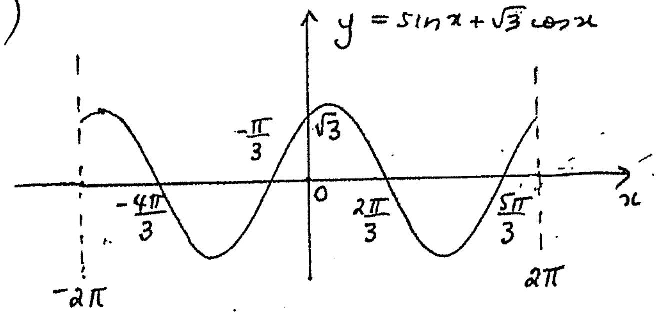
$\tan\alpha = \dfrac{\sin\alpha}{\cos\alpha} = \dfrac{\sqrt{3}}{1} \quad 0 \le \alpha \le \dfrac{\pi}{2}$

$\Rightarrow \alpha = \dfrac{\pi}{3}$



i.e. $\sin x + \sqrt{3}\cos x = 2\sin\left(x+\dfrac{\pi}{3}\right)$ ✓✓

ii)



✓✓

iii) i.e. Solve $2\sin\left(x+\dfrac{\pi}{3}\right) = \sqrt{2}$

$\Rightarrow \sin\left(x+\dfrac{\pi}{3}\right) = \dfrac{1}{\sqrt{2}}$

$\Rightarrow x+\dfrac{\pi}{3} = n\pi + (-1)^n\dfrac{\pi}{4} \quad n\in\mathbb{Z}$

$x = n\pi - \dfrac{\pi}{3} + (-1)^n\dfrac{\pi}{4}$ ✓✓

a) $y = 3\sin^{-1}(2x-1)$

Domain: $-1 \le 2x-1 \le 1$

$\Rightarrow \{x : 0 \le x \le 1\}$

Range: $\left\{y : -\dfrac{3\pi}{2} \le y \le \dfrac{3\pi}{2}\right\}$ ✓✓

b) $f(x) = (1+x^2)\tan^{-1}x$

$f'(x) = 2x\tan^{-1}x + \dfrac{(1+x^2)}{1+x^2}$

$= 1 + 2x\tan^{-1}x$ ✓✓

c) $\displaystyle\int \frac{dx}{\sqrt{16-x^2}} = \sin^{-1}\frac{x}{4} + C, \quad -4<x<4$ ✓

d) i) Domain $= \{x : x \ge 1\}$ ✓

ii) $f(x)$ is one to one ✓

iii) Let $y = f^{-1}(x)$ then

$x = 3 + \sqrt{y-1} \quad y \ge 1,\ x \ge 3$

gives $f^{-1}(x)$.

i.e. $(x-3)^2 = y-1$

$\Rightarrow y = (x-3)^2 + 1 \quad x \ge 3$

i.e. $f^{-1}(x) = x^2 - 6x + 10$ ✓✓

3d) iv) α) $y = x$ 1

β) Hence graphs intersect where

$x = (x-3)^2 + 1$

$\Rightarrow x = x^2 - 6x + 9 + 1$

$\Rightarrow x^2 - 7x + 10 = 0$

$\Rightarrow (x-5)(x-2) = 0$

$\Rightarrow x = 5$ or 2 but $x \ge 3$

Point of intersection:

$(5, 5)$

4a) $\displaystyle\int_0^{2\pi} \sin^2 2x\,dx$

$\displaystyle= \int_0^{2\pi} \frac{1-\cos 4x}{2}\,dx$

$\displaystyle= \frac{1}{2}\left[x - \frac{\sin 4x}{4}\right]_0^{2\pi}$

$= \frac{1}{2}\times[2\pi - 0 - (0-0)] = \pi$ ✓✓

Now $\cos 2A = 1 - 2\sin^2 A$

$\sin^2 A = \dfrac{1-\cos 2A}{2}$

b) $\tan A + \tan B = \alpha + \beta = \dfrac{5}{3}$

$\tan A \times \tan B = \alpha\beta = -\dfrac{1}{3}$

$\tan(A+B) = \dfrac{5}{3}\Big/\left(1+\dfrac{1}{3}\right) = \dfrac{5}{3}\Big/\dfrac{4}{3}$

$= \dfrac{5}{4}$

✓✓✓

c) Let $S_n = a + ar + a^2 + ar^3 + \dots + ar^{n-1}$

Show $S_n = \dfrac{a(r^n - 1)}{r-1}$ $\quad n \in Z^+$

Step 1 $\quad n = 1$

LHS: $S_1 = a$ $\quad$ RHS: $\dfrac{a(r^n-1)}{r-1} = \dfrac{a(r^1-1)}{r-1}$

$= a$

$= S_1$

as required

Step 2: Assume true for $n = k$

Show true for $n = k+1$

i.e. show that $S_{k+1} = \dfrac{a(r^{k+1}-1)}{r-1}$

Now $S_{k+1} = S_k + T_k$

$= \dfrac{a(r^k-1)}{r-1} + ar^{k+1-1}$ by assumption

$= \dfrac{a(r^k-1)}{r-1} + ar^k \dfrac{(r-1)}{r-1}$

$= \dfrac{ar^k - a + ar^{k+1} - ar^k}{(r-1)}$

$= \dfrac{-a + ar^{k+1}}{r-1}$

$= \dfrac{a(r^{k+1}-1)}{r-1}$ as required

Step 3 Show true for all n:

Step 1 showed statement true for $n=1$.

By Step 2 if true for $n=1$ then true for $n = 1+1 = 2$. Similarly if true for $n=2$ then true for $n = 2+1 = 3$ and so on.

i.e. Statement is true for all $n = 1, 2, 3 \dots$ all $n \in Z^+$.

4 d) i) $\quad y = x\sin^{-1}x$

$$\frac{dy}{dx} = \sin^{-1}x + \frac{x}{\sqrt{1-x^2}}$$

Stationary points occur when $\dfrac{dy}{dx} = 0$

i.e. when $\sin^{-1}x + \dfrac{x}{\sqrt{1-x^2}} = 0$

Let $x = 0 \Rightarrow \dfrac{dy}{dx} = \sin^{-1}0 + \dfrac{0}{\sqrt{1-0}}$

$= 0 + 0$

$= 0$

Hence $x = 0$ is a stationary pt.

Test nature find sign of $\dfrac{dy}{dx}$ for $x = 0 - \varepsilon$ and $x = 0 + \varepsilon$

x	-0.5	0	$x = 0.5$	
$\dfrac{dy}{dx}$	$-\frac{\pi}{6} - \frac{1}{\sqrt{3}}$	0	$\frac{\pi}{6} + \frac{1}{\sqrt{3}}$	exactly
	-1.1	0.0	$+1.1$	approx (1dp)
	falls	stat	rises	

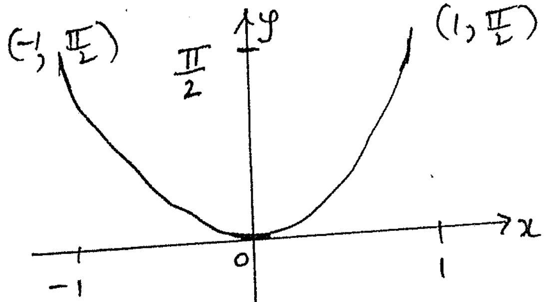
i.e. minimum at $x = 0, y = 0$ (0, 0)

(no other S.P. in domain $-1 \leq x \leq 1$ ✓✓

d) ii) Domain $-1 \leq x \leq 1$

Range $0 \leq y \leq \frac{\pi}{2}$

1 S.P. a min at $x = 0, y = 0$



$\dfrac{dy}{dx} \to -\infty$ as $x \to -1^+$

$\to +\infty$ as $x \to +1^-$ ✓✓

5a) $t = 0$, $x = 0$ $\quad v = 6 - 2x$

i) $\ddot{x} = \dfrac{d}{dx}\left(\frac{1}{2}v^2\right)$

$= \dfrac{d}{dx} \dfrac{(6-2x)^2}{2} = \dfrac{d}{dx} \dfrac{36 - 24x + 4x^2}{2}$

$= -12 + 4x$

$= -12$ when $x = 0$ ✓✓

ii) $\dfrac{dx}{dt} = v = 6 - 2x$

$\Rightarrow \dfrac{dt}{dx} = \dfrac{1}{6-2x}$

$\Rightarrow t = -\frac{1}{2}\ln(6-2x) + C$

5a) ii) ctd

when $x = 0$, $t = 0$ i.e.

$0 = -\frac{1}{2} \ln 6 + c$

$\Rightarrow t = -\frac{1}{2} \ln(6-2x) + \frac{1}{2} \ln 6$

$= -\frac{1}{2} \ln \frac{6-2x}{6}$

$= -\frac{1}{2} \ln\left(1 - \frac{x}{3}\right)$

$\Rightarrow \ln\left(1 - \frac{x}{3}\right) = -2t$

$\Rightarrow 1 - \frac{x}{3} = e^{-2t}$

$\Rightarrow \frac{x}{3} = 1 - e^{-2t}$

$\Rightarrow x = 3 - 3e^{-2t}$ ✓✓✓

✓

iii) $x \to 3$

b) i) $\frac{dy}{dx} = \frac{dy}{dp} \Big/ \frac{dx}{dp}$

$\left.\frac{dy}{dx}\right|_{TP} = 2ap/2a = p$

$\left.\frac{dy}{dx}\right|_{TQ} = 2aq/2a = q$

tangent TP: $y - ap^2 = p(x - 2ap)$

$\Rightarrow y - ap^2 = px - 2ap^2$

$\Rightarrow y = px - ap^2$

similarly tangent TQ

$\Rightarrow y = qx - aq^2$ ✓✓

ii) Solve TP, TQ simultaneously to find T

i.e. $px - ap^2 = qx - aq^2$

$\Rightarrow (p-q)x = a(p^2 - q^2)$

$\Rightarrow x = a\frac{(p+q)(p-q)}{p-q} \quad (p \neq q)$

$\Rightarrow \underline{x = a(p+q)}$

$\therefore y = pa(p+q) - ap^2$

$= ap^2 + apq - ap^2$

$= \underline{apq}$

i.e. T is $(a(p+q), apq)$ ✓✓

iii) Now T lies on $x^2 = -4ay$

$\therefore [a(p+q)]^2 = -4a(apq)$

$\Rightarrow a^2(p+q)^2 = -4a^2pq$

$\Rightarrow (p+q)^2 = -4pq \quad (a \neq 0)$

$\Rightarrow p^2 + 2pq + q^2 = -4pq$

$\Rightarrow p^2 + q^2 = -6pq$ ✓✓

6.i) Show that $\frac{d^2x}{dt^2} = -16x$

$x = a\cos(4t + \alpha)$

$\Rightarrow \frac{dx}{dt} = -4a\sin(4t+\alpha)$

$\Rightarrow \frac{d^2x}{dt^2} = -16a\sin(4t+\alpha)$

$= -16x$ as required ✓✓

ii) $t = 0$, $v = 4$

$\Rightarrow 4 = -4a\sin\alpha \Rightarrow \sin\alpha = -\frac{1}{a}$

$t = 0$, $x = 5$

$\Rightarrow 5 = a\cos\alpha \Rightarrow \cos\alpha = \frac{5}{a}$

$\Rightarrow \left(-\frac{1}{a}\right)^2 + \left(\frac{5}{a}\right)^2 = 1$

$\Rightarrow 1 + 25 = a^2$

$\Rightarrow a = \sqrt{26}$ ✓✓

iii) Max speed when $|\sin(4t+\alpha)| = 1$

i.e. $|v|_{max} = |-4 \times \sqrt{26} \times 1|$

$= 4\sqrt{26}$

b)i) $P = 10000 + P_0 e^{kt} \Rightarrow P_0 e^{kt} = P - 10000$

and

$\Rightarrow \frac{dP}{dt} = kP_0 e^{kt}$

$= k(P - 10000)$ as required ✓✓

ii) $t = 0 \quad P = 15000$ —— (1)

$t = 6 \quad P = 25000$ —— (2)

(1) $\Rightarrow 15000 = 10000 + P_0 e^{k \times 0}$

$\Rightarrow P_0 = 5000$

i.e. $\underline{P = 10000 + 5000e^{kt}}$

(2) $\Rightarrow 25000 = 10000 + 5000e^{6k}$

$\Rightarrow \frac{15000}{5000} = e^{6k}$

$\Rightarrow 6k = \ln 3$

$\Rightarrow k = \frac{\ln 3}{6}$ ✓✓

6 b) iii) $t = 12$

$\Rightarrow P = 10000 + 5000\, e^{\frac{\ln 3}{6}\times 2}$

$= 10000 + 5000\, e^{(\ln 3)\times 2}$

$= 10000 + 5000 \times 3^{2}$

$= \underline{\underline{55000}}$ flies after 1 year.

7a) $T_{r+1} = \binom{12}{r} x^{12-r} \left(\frac{2}{x^2}\right)^r$

$= \binom{12}{r} \times 2^r \times x^{12-r-2r}$

For term independent of x require x^0, i.e. $12 - 3r = 0$

$\underline{r = 4}$

5^{th} term is $\binom{12}{4} \times 2^4$

i.e. $\dfrac{\cancel{12}\times 11 \times \cancel{10}^{5} \times 9}{\cancel{4\times 3}\times \cancel{2}\times 1} \times 16$

$= 7920$ ✓✓✓

7b)

i) at $\underline{t = 0}$, $x = 0$, $y = -1$

$\dot{x} = 12\cos\theta$, $\dot{y} = 12\sin\theta$

given $\ddot{x}(t) = 0$, $\ddot{y}(t) = -10$

$\dot{x} = \int \ddot{x}(t)\,dt$

$= 0 + C$

$\Rightarrow C = 12\cos\theta$

$\Rightarrow \dot{x} = 12\cos\theta$

$\Rightarrow x = \int \dot{x}\,dt$

$= 12t\cos\theta + D$

$D = 0$

$\Rightarrow \underline{x = 12t\cos\theta}$

$\dot{y} = \int \ddot{y}(t)\,dt$

$= -10t + E$

$E = 12\sin\theta$

$\Rightarrow \dot{y} = -10t + 12\sin\theta$

$\Rightarrow y = \int(-10t + 12\sin\theta)\,dt$

$= -5t^2 + 12t\sin\theta + F$

$\Rightarrow -1 = F$

$\Rightarrow y = \underline{-5t^2 + 12t\sin\theta - 1}$ ✓✓✓

ii) Hits roof when $y = 0$

i.e. $y = 0 = -5t^2 + 12t\sin 30° - 1$

$\Rightarrow -5t^2 + 6t - 1 = 0$

$\Rightarrow 5t^2 - 6t + 1 = 0$

$\Rightarrow (5t - 1)(t - 1) = 0$

If ball hits roof does so on downward trajectory i.e. at greater value of t: $\underline{t = 1}$

i.e. $x = 12 \times 1 \times \cos 30°$

$= 6\sqrt{3}$ which is $(6\sqrt{3} - 4)$ m from D ✓✓✓

iii) To land to right of D without hitting greenhouse, ball must cross OD i.e. $x < 4$ for $-1 < y < 0$.

Limit is through D $(4, 0)$:

$x: 12t\cos\theta = 4 \Rightarrow t = \dfrac{4}{12\cos\theta}$

$y: -5t^2 + 12t\sin\theta - 1 = 0$ & in y

$\Rightarrow -5\left(\dfrac{1}{3\cos\theta}\right)^2 + 12\left(\dfrac{1}{3\cos\theta}\right)\sin\theta - 1 = 0$

$\Rightarrow -\dfrac{5}{9}\sec^2\theta + 4\tan\theta - 1 = 0$

$\Rightarrow -5(1 + \tan^2\theta) + 36\tan\theta - 9 = 0$

$\Rightarrow -5\tan^2\theta + 36\tan\theta - 14 = 0$

$\Rightarrow \tan\theta = \dfrac{36 \pm \sqrt{1016}}{10}$ ✓

$\theta > 22.48°$ and $< 81.62°$ (2dp) ✓✓

$22° < \theta < 82°$ nearest degree