Baulkham Hills 1998 3U Trial HSC

Question 1

(a) Solve

$$\frac{x}{x-3} > 10$$

(b) Solve for $0° \leq x \leq 360°$ $\sin x + \cos x + 1 = 0$

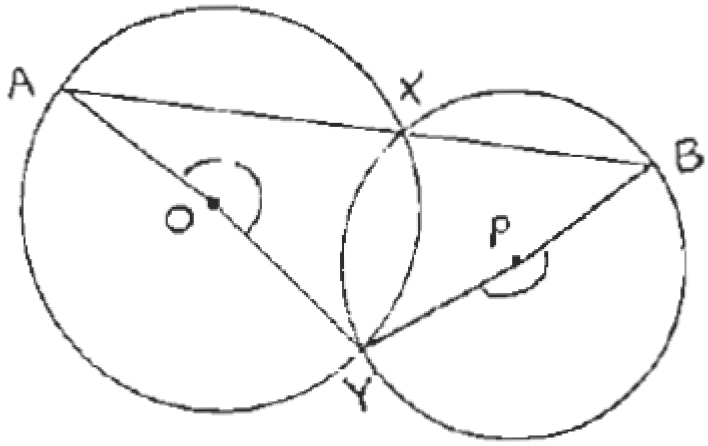
(c) Find the acute angle between the lines $5x + 4y + 3 = 0$ and $3x + 8y - 1 = 0$

(d) If A and B are the points $(-3, -4)$ and $(2, -1)$, find the co-ordinates of the point P dividing AB externally in the ratio $4:7$

(e) Show that $(x - 3)$ is a factor of $2x^3 - 11x^2 + 12x + 9$ and hence find the factors of this polynomial

Question 2

(a) O and P are the centres of the circles; AXB is a straight line. Prove that $\angle AOY = \angle BPY$



(b) $P\ (2ap, ap^2)$ and $Q\ (2aq, ap^2)$ are two points on the parabola $x^2 = 4ay$. PQ subtends a right angle at the vertex O

i. Show that $pq = -4$
ii. Prove that the equation of the normal at P is given by $x + py = 2ap + ap^3$
iii. Write down the equation of the normal at Q, and hence determine the point of intersection of these normal.
iv. Find the equation of the locus of R, and describe it geometrically

Question 3

(a) Find $\int x\sqrt{3 + x^2}\, dx$, using the substitution $u = 3 + x^2$

(b) Find

$$\int_0^x 2\sin^2 x\ dx$$

Question 3 (continued)

(c) Assume that the rate at which the body warms in air is proportional to the difference between its temperature and the constant temperature A of the surrounding air.
This rate can be expressed by the differential equation

$$\frac{dT}{dt} = K(T - A)$$

where t is time in minutes, and K is a constant

i. Show that $T = A + Ce^{kt}$, where C is a constant, is a solution of the differential equation.
ii. A cooled body warms from 10°C to 15°C in 20 minutes. The air temperature around the body is 28°C. Find the temperature of the body after a further 20 minutes have elapsed. Give your answer to the nearest degree
iii. By referring to the equation for T, explain the behaviour of T as t becomes large.

Question 4

(a) The acceleration of a body P is given by $a = 18x(x^2 + 1)$ where x cm is the displacement at time t sec. Initially P starts from the origin with velocity 3 cm/s

i. Show that $v = 3(x^2 + 1)$
ii. Find x in terms of t

(b) A ball projected from a horizontal plane with initial velocity V m/s and an angle of projection of α where $\tan \alpha = \frac{3}{4}$.
The ball just clears a wall which is 27m high and 96m from the point of projection. Let g, the acceleration due to gravity be 10m/s^2

i. Show that the horizontal and vertical displacements are given by
$x = \frac{4}{5}Vt$ and $y = \frac{3}{5}Vt - 5t^2$

ii. Find the time to reach the wall in seconds

iii. Show that the speed of projection is 40 m/s

iv. Find the greatest height to which the ball will rise above the plane.

Question 5

(a) A particle moves along the x-axis with acceleration $\ddot{x} = 4\cos 2t$. If the particle is initially at rest at the origin O, find expressions for

i. the velocity v in terms of t
ii. the position x in terms of t
iii. Express $\ddot{x}$ in terms of x and hence show that the motion is simple harmonic
iv. Find the centre and period of the motion
v. Sketch the graph of x in terms of t for $0 \leq t \leq \pi$

(b)

i. Write down a primitive function of $e^{f(x)} \cdot f'(x)$

ii. Hence evaluate

$$\int_0^1 \frac{e^{\cos^{-1} x}}{\sqrt{1 - x^2}} dx$$

(Leave your answer in exact form)

(c) Find the inverse function f^{-1} of the function f, defined by $f(x) = 2\log_e x + 3$
Express the result in the form y in terms of x

Question 6

(a) Prove by induction that $n(n+3)$ is divisible by 2 for all positive integers n

(b) Find the term independent of x in the expansion of $\left(2x^2+\frac{1}{x}\right)^{12}$

(c) Find the relationship between p, q, r if the roots of the equation $x^3+px^2+qx+r=0$ are in an arithmetic progression.

Question 7

(a) Use Newton's method once, and a first approximation of $x=2$ to solve $x^2-2-\sqrt{x}=0$ to 2 decimal places.

(b) A right circular cone with vertex downwards and a semi-vertical angle 60° is being filled with water.

i. Show that when the height of the water in the cone is h cm, then the volume of water is πh^3 cm^3

ii. If the height of the water is increasing at the constant rate of $\frac{1}{2} cm/s$, find the rate of the increase of the volume when the height is 6 cm.

(c)

i. Prove that

$$\frac{2}{(x^2+1)(x^2+3)}=\frac{1}{x^2+1}-\frac{1}{x^2+3}$$

ii. Hence determine the value of

$$\int_{-1}^{\sqrt{3}}\frac{dx}{(x^2+1)(x^2+3)}$$