STUDENT NUMBER: ____________________

TEACHER'S NAME: ____________________

BAULKHAM HILLS HIGH SCHOOL

TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

2006

MATHEMATICS
EXTENSION 1

Time allowed – Two hours
(Plus five minutes reading time)

GENERAL INSTRUCTIONS:

- Attempt **ALL** questions.
- Start each of the 7 questions on a new page.
- All necessary working should be shown.
- Write your student number at the top of each page of answer sheets.
- At the end of the exam, staple your answers in order behind the cover sheet.

QUESTION 1

Marks

(a) Find $\int \frac{dx}{9+16x^2}$. **2**

(b) The line $y = mx$ makes an angle of 45^0 with the line $y = 3x - 4$. Find the possible values of m. **3**

(c) Find the ratio in which $P(-15, -10)$ divides the interval AB where $A = (3, -1)$, $B = (9,2)$. **2**

(d) Find $\lim_{x \to o} \frac{\sin 5x}{2x}$. **1**

(e) Use the substitution $u = x + 1$ to find $\int_0^8 \frac{2x\,dx}{\sqrt{x+1}}$. **4**

QUESTION 2

(a) Find $\frac{d}{dx}\left(\sin^{-1} 2x^3\right)$ **2**

(b) Sketch $y = 3\cos^{-1} 2x$, showing clearly the domain and range. **2**

(c) A curve has parametric equations:

$x = \cos 2\theta$.

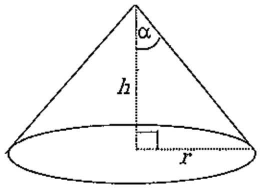
$y = \sin\theta + 1$. **2**

Find its Cartesian equation.

(d) Find $\int \sin^2 3x\,dx$. **2**

QUESTION 2 (Continued) **Marks**

(e) Sand is poured at the rate of $5\text{cm}^3/\text{min}$ into a heap in the shape of a right circular cone whose semi – vertex angle is α where $\tan\alpha = \frac{4}{3}$.



(i) Show that the volume of the cone of sand is given by:
$V = \frac{16\pi}{27}h^3$. **2**

(ii) Find the rate at which the height is increasing at the instant when the height is 12 cm. **2**

QUESTION 3

(a) In the expansion of $\left(2x+\frac{1}{x^2}\right)^{21}$, find the term independent of x. **3**

(b) When $P(x)$ is divided by $x^2 - 4$, the remainder is $2x + 3$. Find the remainder when $P(x)$ is divided by $x - 2$. **2**

(c) Prove by mathematical induction that $5^{2n} - 1$ is a multiple of 24 for all integers $n \geq 1$. **3**

a b c d

(d) α, β, γ are the roots of the equation $x^3 - 2x^2 + kx + 16 = 0$.

(i) If two of the roots are equal but opposite in sign, find the value of k. **2**

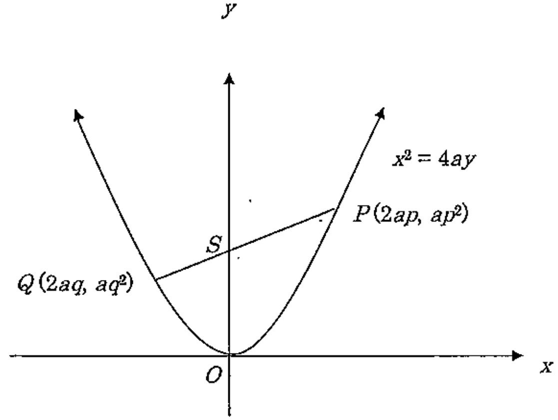
(ii) Find the value of $\alpha^2 + \beta^2 + \gamma^2$ **2**

QUESTION 4 **Marks**

(a) Find the exact value of $\tan\left(2\cos^{-1}-\frac{7}{25}\right)$. **3**

(b) The equation $4\cos\frac{\pi}{2}x - 6 + x = 0$ has a root near 3.5. Use one application of Newton's method to find a second approximation to the root. Give answer to 3 significant figures. **3**

(c)



$P(2ap, ap^2)$ and $Q(2aq, aq^2)$ are 2 variable points on the parabola $x^2 = 4ay$. S is the focus.

(i) If PQ is a focal chord, show that $pq = -1$. **2**

(ii) R is the midpoint of PQ. Find the coordinates of R and hence find the equation of the locus of R. **3**

(iii) Find the length of SP in terms of p. **1**

QUESTION 5

Marks

(a) Prove the identity $\dfrac{2\tan\theta}{1+\tan^2\theta} = \sin 2\theta$. 3

(b) Find $\dfrac{d}{dx}\left(x\tan^{-1}2x - \dfrac{1}{4}\ln\left(1+4x^2\right)\right)$. 2

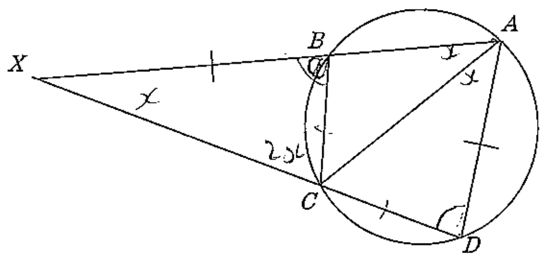
Hence find $\displaystyle\int_0^{\frac{1}{2}} \tan^{-1} 2x\,dx$ in exact form. 2

(c) In the figure below, it is given that AC bisects $\angle BAD$ and $BX = AD$.

Prove that:

(i) $\Delta BCX \equiv \Delta ACD$. 3

(ii) ΔACX is isosceles. 2



QUESTION 6

Marks.

(a) A particle moves along the x axis and its velocity v m/s at the position x metres is given by $v^2 = 30+4x-2x^2$

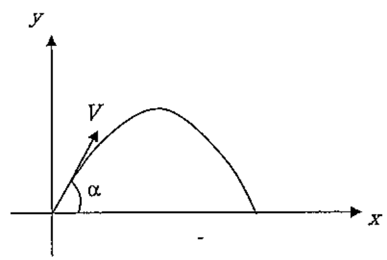
(i) Prove that the motion is simple harmonic. 2

(ii) Find the centre, and period of the motion. 2

(iii) What is the amplitude of the motion? 2

(iv) Find the maximum speed. 1

(b)



A stone is projected with velocity V at an angle α. It hits a target 40m from the point of projection on the ground. On its path, it passes through a point 10 m above the ground and 25 m from the point of projection. [Take $g = 10\text{m/s}^2$]

(i) Given that

$x = V\cos\alpha\, t,\ y = \dfrac{-gt^2}{2} + V\sin\alpha\, t.$

Find the Cartesian equation of the trajectory. 2

(ii) Show that $\alpha = \tan^{-1}\dfrac{16}{15}$. 3

QUESTION 7 **Marks**

(a) (i) Without calculus, sketch the graph of the function.

$$f(x) = x - \frac{1}{x} \qquad \text{for } x < 0.$$ **1**

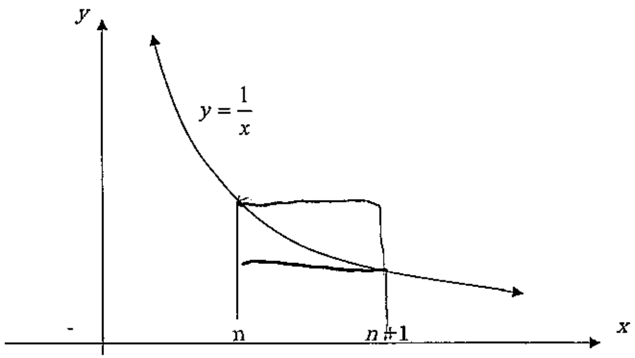
(ii) Find the inverse function $f^{-1}(x)$. **2**

(iii) Sketch the inverse function $f^{-1}(x)$ on the same axes in (i). **1**

(b) Using the expansion of $x(1 + x)^n$, show that : **3**

$$\binom{n}{0} + 2\binom{n}{1} + 3\binom{n}{2} + \dots\dots\dots + (n+1)\binom{n}{n} = (n+2)2^{n-1}.$$

(c)



(i) Copy the graph of $y = \frac{1}{x}$ above and use it to show that: **2**

$$\frac{1}{n+1} < \int_n^{n+1} \frac{1}{x}\,dx < \frac{1}{n}.$$

(ii) Deduce that $\left(1+\frac{1}{n}\right)^n < e < \left(1+\frac{1}{n}\right)^{n+1}$. **3**

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Q1

a) $\displaystyle\int \frac{dx}{9+16x^2}$

$= \displaystyle\int \frac{dx}{16\left(\frac{9}{16}+x^2\right)}$

$= \frac{1}{16}\cdot\frac{4}{3}\tan^{-1}\frac{4x}{3} + C$ (1) (1)

b) $\tan 45^\circ = \left|\frac{m-3}{1+3m}\right|$ (1)

$1 = \frac{m-3}{1+3m}$ or $-1 = \frac{m-3}{1+3m}$

$1+3m = m-3$ $\quad$ $-1-3m = m-3$

$m = -2$ (1) $\quad$ $m = \frac{1}{2}$ (1)

c) Let the ratio be $m:n$

$A(3,-1)$ $\quad$ $B(9,2)$
$\quad m \quad\quad n$

$(-15,-10) = \left(\frac{3n+9m}{m+n}, \frac{-n+2m}{m+n}\right)$

$-15 = \frac{3n+9m}{m+n}$ (1)

$-24m = 18n$ $\quad \therefore \frac{m}{n} = -\frac{3}{4}$ (1)

$\therefore$ divides externally in the ratio 3 : 4

d) $\displaystyle\lim_{x\to 0}\frac{\sin 5x}{2x} = \lim_{x\to 0}\frac{\sin 5x}{5x}\cdot\frac{5}{2}$

$= \frac{5}{2}$ (1)

e) $\displaystyle\int_0^8 \frac{2x\,dx}{\sqrt{x+1}}$ $\quad$ Let $u = x+1$, $du = dx$; $x=0,\ u=1$; $x=8,\ u=9$ (1)

$= \displaystyle\int_1^9 \frac{2(u-1)\,du}{u^{1/2}}$ (1)

$= \displaystyle\int_1^9 2u^{\frac{1}{2}} - 2u^{-\frac{1}{2}}\,du$

$= \left[2\cdot\frac{2}{3}u^{3/2} - 2\cdot 2u^{\frac{1}{2}}\right]_1^9$ (1)

$= \left(\frac{4}{3}\cdot 9^{3/2} - 4\cdot 9^{\frac{1}{2}}\right) - \left(\frac{4}{3} - 4\right)$

$= 26\frac{2}{3}$ (1)

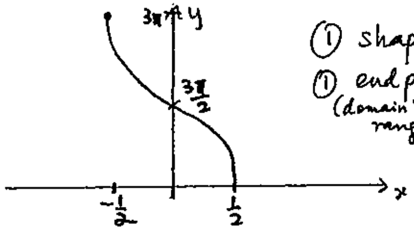
Q2

a) $\frac{d}{dx}\left(\sin^{-1}2x^3\right)$

$= \frac{1}{\sqrt{1-4x^6}}\cdot 6x^2$ (1) (1)

b) $\mathbb{D}: -1 \le 2x \le 1$ $\quad$ $\mathbb{R}: 0 \le y \le 3\pi$

$-\frac{1}{2} \le x \le \frac{1}{2}$



(1) shape

(1) end points (domain + range)

(c) $x = \cos 2\theta$

$y = \sin\theta + 1$

$\cos 2\theta = 1 - 2\sin^2\theta$ (1)

$x = 1 - 2(y-1)^2$ (1)

(d) $\displaystyle\int \sin^2 3x\,dx$

$= \frac{1}{2}\displaystyle\int 1 - \cos 6x\,dx$ (1)

$= \frac{1}{2}\left[x - \frac{\sin 6x}{6}\right] + C$ (1)

(e)(i) $\tan\alpha = \frac{r}{h} = \frac{4}{3}$ $\quad \therefore r = \frac{4}{3}h$ (1)

$V = \frac{1}{3}\pi r^2 h$

$= \frac{1}{3}\pi\left(\frac{16}{9}h^2\right).h$

$= \frac{16}{27}\pi h^3$ (1)

(ii) $\frac{dV}{dh} = \frac{16}{9}\pi h^2$

$\frac{dh}{dt} = \frac{dh}{dV}\times\frac{dV}{dt}$

$= \frac{9}{16\pi h^2}\times 5$ cm/min (1)

$h = 12$, $\frac{dh}{dt} = \frac{9}{16\pi 12^2}\times 5$ cm/min

$= \frac{5}{256\pi}$ cm/min (1)

Q3

(a) $\left(2x + \frac{1}{x^2}\right)^{21}$

$T_{r+1} = \binom{21}{r}(2x)^{21-r}\cdot\left(\frac{1}{x^2}\right)^r$

$= \binom{21}{r}2^{21-r}\cdot x^{21-3r}$ (1)

For term indep of x: $21 - 3r = 0$

$r = 7$ (1)

$\therefore T_8 = \binom{21}{7}\cdot 2^{14}$

$= 1905131520$ (1)

(b) $P(x) = (x^2-4)Q(x) + 2x + 3$ (1)

when divided by $x-2$, remainder $= P(2)$

remainder $= (4-4)Q(2) + 4 + 3$

$= 7$ (1)

(c) test $n = 1$ $\quad 5^2 - 1 = 24$ is a multiple of 24.

$\therefore$ true for $n = 1$ (1)

Assume true for $n = k$

$5^{2k} - 1 = 24N$

when $n = k+1$

$5^{2(k+1)} - 1 = 5^{2k}.5^2 - 1$

$= 25(24N+1) - 1$

$= 25.24N + 24$

$= 24(25N+1)$ which is a multiple of 24 (2)

$\therefore$ If true for $n=k$, it will be true for $n=k+1$. Since true for $n=1$, it will be true for $n = 2, 3, \ldots$

d)(i) If $\beta = -\alpha$

sum of roots $= \alpha - \alpha + \gamma = 2$

$\therefore \gamma = 2$ (1)

sub $x=2$

$\therefore 2^3 - 2.2^2 + 2k + 16 = 0$

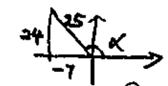
$k = -8$ (1)

(ii) $\Sigma\alpha^2 = (\Sigma\alpha)^2 - 2\Sigma\alpha\beta$ (1)

$= 2^2 - 2(k) = 20$ (1)

Q4

a) $\tan\left(2\cos^{-1}-\frac{7}{25}\right)$
$\quad\quad\quad\ \alpha$



(1)

$= \frac{2\tan\alpha}{1-\tan^2\alpha}$ (1)

$= \frac{2\left(-\frac{24}{7}\right)}{1-\left(-\frac{24}{7}\right)^2} = \frac{336}{527}$ (1)

(b) $f(x) = 4\cos\frac{\pi}{2}x - 6 + x$

$f'(x) = -4\cdot\frac{\pi}{2}\sin\frac{\pi}{2}x + 1$ — (1)

$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$

$= 3.5 - \frac{\left(4\cos\frac{\pi}{2}\times 3.5 - 6 + 3.5\right)}{-2\pi\sin\frac{\pi}{2}(3.5) + 1}$ (1)

$= 3.5 - \frac{0.328427}{5.44288}$

$= 3.44$ (1)

c)(i) PQ is focal chord

$m_{PS} = m_{QS}$

$\frac{ap^2-a}{2ap} = \frac{aq^2-a}{2aq}$ (1)

$q(p^2-1) = p(q^2-1)$

$pq(p-q) + (p-q) = 0$

$(p-q)(pq+1) = 0$

$p \ne q$ $\quad \therefore pq = -1$ (1)

(ii) $R = \left(a(p+q), \frac{a(p^2+q^2)}{2}\right)$ (1)

Locus is $x = a(p+q)$

$y = \frac{a}{2}(p^2+q^2)$ (1)

Using $(p+q)^2 = p^2 + q^2 + 2pq$

$\left(\frac{x}{a}\right)^2 = \frac{2y}{a} - 2$

or $x^2 = 2a(y - a)$ (1)

(iii) SP = distance of P from directrix

$= ap^2 + a$ (1)

Q5

(a) $LHS = \dfrac{\frac{2\sin\theta}{\cos\theta}}{\sec^2\theta} \longrightarrow (1)$

$= \dfrac{2\sin\theta}{\cos\theta} \times \cos^2\theta \longrightarrow (1)$

$= 2\sin\theta\cos\theta$

$= \sin 2\theta \quad \Big\} — (1)$

$= RHS$

(b) $\dfrac{d}{dx}\left(x\tan^{-1}2x - \frac{1}{4}\ln(1+4x^2)\right)$ (1)

$= x\cdot\dfrac{1}{1+4x^2}\cdot 2 + \tan^{-1}2x\cdot 1 - \dfrac{1}{4}\cdot\dfrac{8x}{1+4x^2}$

$= \dfrac{2x}{1+4x^2} + \tan^{-1}2x - \dfrac{2x}{1+4x^2}$ (1)

$= \tan^{-1}2x$

$\therefore \displaystyle\int_0^{\frac{1}{2}} \tan^{-1}2x\,dx$

$= \left[x\tan^{-1}2x - \frac{1}{4}\ln(1+4x^2)\right]_0^{\frac{1}{2}}$ (1)

$= \left(\frac{1}{2}\tan^{-1}1 - \frac{1}{4}\ln 2\right) - \left(0 - \frac{1}{4}\ln 1\right)$

$= \dfrac{\pi}{8} - \dfrac{1}{4}\ln 2$ (1)

(c)(i) To prove $\Delta BCX \equiv \Delta ACD$

Proof: $\angle BAC = \angle DAC$ (AC bisects $\angle BAD$)

(1) $\therefore BC = DC$ (equal chords subtend equal $\angle$s at circum)

(1) $\angle XBC = \angle ADC$ (ext $\angle$ of cyclic quad = int opp. $\angle$)

$BX = AD$ (given)

$\therefore \Delta BCX \equiv \Delta ACD$ (SAS)

(ii) $\angle CXB = \angle CAD$ (matching $\angle$s of congt. Δs) (1)

but $\angle CAD = \angle CAB$ (given)

$\therefore \angle CXB = \angle CAB$ (1)

$\therefore \Delta ACX$ is isosceles (2 equal $\angle$s)

Q6

a) $v^2 = 30 + 4x - 2x^2$

(i) $\ddot{x} = \dfrac{d}{dx}\left(\frac{1}{2}v^2\right)$

$= \dfrac{d}{dx}(15 + 2x - x^2)$ (1)

$= 2 - 2x$

$= -2(x-1)$ of the form $-n^2(x-c)$ (1)

$\therefore$ motion is S.H.

(ii) centre is $x = 1$ (1)

period $= \dfrac{2\pi}{n} = \dfrac{2\pi}{\sqrt{2}} = \sqrt{2}\pi$ (1)

(iii) when $v = 0$ $\quad 30 + 4x - 2x^2 = 0$

$15 + 2x - x^2 = 0$

$(3+x)(5-x) = 0$

$\therefore x = -3$ or 5 (1)

amplitude $= 4$ (1)

(iv) max speed when $\ddot{x} = 0$

ie at $x = 1$ (1)

max speed $= \sqrt{30+4-2} = 4\sqrt{2}$ m/s

(b)(i) $x = V\cos\alpha\, t \quad \therefore t = \dfrac{x}{V\cos\alpha}$ (1)

$y = \dfrac{-gt^2}{2} + V\sin\alpha\, t$

$= -\dfrac{g}{2}\left(\dfrac{x^2}{V^2\cos^2\alpha}\right) + V\sin\alpha\cdot\dfrac{x}{V\cos\alpha}$

$y = x\tan\alpha - \dfrac{gx^2}{2V^2\cos^2\alpha}$ (1)

(ii) $x = 40,\ y = 0$

$\therefore 0 = 40\tan\alpha - \dfrac{g\,\cancel{(1600)}^{800}}{\cancel{2}V^2\cos^2\alpha}$ (1)

$\dfrac{g}{V^2\cos^2\alpha} = \dfrac{\tan\alpha}{20}$

Also $x = 25,\ y = 10$

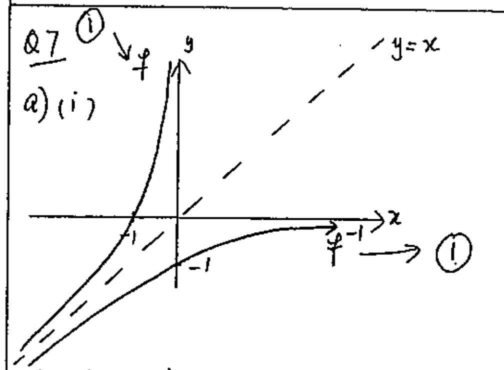
$10 = 25\tan\alpha - \dfrac{625\,g}{2\,V^2\cos^2\alpha}$ (1)

$= 25\tan\alpha - \dfrac{625}{2}\cdot\dfrac{\tan\alpha}{20}$

$10 = \dfrac{75}{8}\tan\alpha \quad \therefore \tan\alpha = \dfrac{80}{75} = \dfrac{16}{15}$

Q7

a)(i)



$\longrightarrow (1)$

(ii) For inverse function,

let $x = y - \dfrac{1}{y}$

$y^2 - xy - 1 = 0$

$y = \dfrac{x \pm \sqrt{x^2 - 4.1.(-1)}}{2}$

$y = \dfrac{x \pm \sqrt{x^2+4}}{2} \longrightarrow (1)$

Since $R_{f^{-1}}$ is $y < 0$

$f^{-1}(x) = \dfrac{x - \sqrt{x^2+4}}{2} \longrightarrow (1)$

(b) $x(1+x)^n = x\left[\binom{n}{0} + \binom{n}{1}x + \binom{n}{2}x^2 + \dots + \binom{n}{n}x^n\right]$

$= \binom{n}{0}x + \binom{n}{1}x^2 + \binom{n}{2}x^3 + \dots + \binom{n}{n}x^{n+1}$

differentiate both sides

$x.n(1+x)^{n-1} + (1+x)^n.1 = \binom{n}{0} + 2\binom{n}{1}x + 3\binom{n}{2}x^2 + \dots + (n+1)\binom{n}{n}x^n$

Sub $x = 1$

$n.2^{n-1} + 2^n = \binom{n}{0} + 2\binom{n}{1} + 3\binom{n}{2} + \dots + (n+1)\binom{n}{n}$

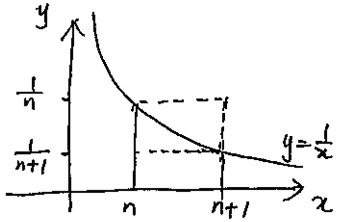
$\therefore 2^{n-1}[n+2] = \dots + (n+1)\binom{n}{n}$

(1) differentiate

(1) Sub $x = 1$

(1) result

(c)(i)



(1)

from the graph

area of lower rectangle < area under curve from n to n+1 < area of upper rectangle

ie $\dfrac{1}{n+1}\times 1 < \displaystyle\int_n^{n+1}\dfrac{1}{x}\,dx < \dfrac{1}{n}\times 1$ } (1)

(ii) From $\displaystyle\int_n^{n+1}\frac{1}{x}\,dx < \frac{1}{n}$

$[\ln x]_n^{n+1} < \dfrac{1}{n}$ (1)

$\ln\dfrac{n+1}{n} < \dfrac{1}{n}$

$1 + \dfrac{1}{n} < e^{\frac{1}{n}}$

$\left(1+\dfrac{1}{n}\right)^n < e$ (1)

or

From $\displaystyle\int_n^{n+1}\frac{1}{x}\,dx > \frac{1}{n+1}$

$\ln\dfrac{n+1}{n} > \dfrac{1}{n+1}$

$\left(1+\dfrac{1}{n}\right) > e^{\frac{1}{n+1}}$

$\left(1+\dfrac{1}{n}\right)^{n+1} > e$ (1)

$\therefore \left(1+\dfrac{1}{n}\right)^n < e < \left(1+\dfrac{1}{n}\right)^{n+1}$

6 b(i) $\ddot{y} = -10$

$\dot{y} = -10t + c_3 \quad c_3 = V\sin\theta$

$\dot{y} = -10t + V\sin\theta$

$y = -5t^2 + Vt\sin\theta + c_4$

when $t=0$ $y=50 \Rightarrow c_4 = 50$

$\therefore y = -5t^2 + Vt\sin\theta + 50$ 1

(ii) when $t=5$ $x=100$ and $y=0$

$\therefore 100 = 5V\cos\theta$

$V\cos\theta = 20$ --(1) 1

$0 = -125 + 5V\sin\theta + 50$

$\therefore 75 = 5V\sin\theta$

$15 = V\sin\theta$ --(2) 1

(2) ÷ (1) $\frac{3}{4} = \tan\theta$

$\theta = 36°52'$ 1

$\therefore V\cos 36°52' = 20$

$V = \frac{20}{\cos 36°52'}$

$= 25$ m/s. 1

(iii) At impact $V = \sqrt{(\dot{x})^2 + (\dot{y})^2}$

when $t=5$ $V=25$

$\dot{x} = 25 \cos 36°52'$

$= 20$ m/s

$\dot{y} = -10(5) + 25 \sin 36°52'$

$= 25$ m/s.

1

$\therefore V = \sqrt{(20)^2 + (35)^2}$

$\boxed{12}$ $= \sqrt{25}$ m/s (40.3) 1 (8)

7(a)

(i) $P(\text{Winning}) = \frac{3}{6} \times \frac{3}{6} = \frac{1}{4}$ 1

$P(\text{Losing}) = P(B) + P(\text{other, Black})$

$= \frac{1}{6} + \frac{5}{6} \times \frac{1}{6}$

$= \frac{11}{36}$ 1

(ii) $P(\text{Win first go}) = \frac{1}{4}$

$P(\text{Win 2nd Go}) = P(\text{Draw then win})$

1 $\leftarrow = \frac{4}{9} \times \frac{1}{4}$

$P(\text{Win 3rd Go}) = P(\text{Draw, Draw, Win})$

$= \frac{4}{9} \times \frac{4}{9} \times \frac{1}{4}$

$= \left(\frac{4}{9}\right)^2 \times \frac{1}{4}$

$\therefore P(\text{Win 1st, 2nd, 3rd..}) =$

$\frac{1}{4} + \frac{1}{4} \times \frac{4}{9} + \frac{1}{4} \times \left(\frac{4}{9}\right)^2 =$ 1

(iii) $P(\text{Wins}) =$

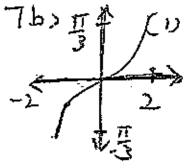
$\frac{1}{4}\left(1 + \frac{4}{9} + \left(\frac{4}{9}\right)^2 + \left(\frac{4}{9}\right)^3 + \cdots\right)$ 1

$S_\infty = \frac{a}{1-r}$

$= \frac{1}{4}\left(\frac{1}{1-\frac{4}{9}}\right)$

$= \frac{1}{4} \times \frac{9}{5} = \frac{9}{20}$. 1 (6)

7b) (i)



$y = a \sin^{-1} bx$.

range $-\frac{\pi}{3} \leftrightarrow \frac{\pi}{3}$

Now $y = \sin^{-1} x$

range $-\frac{\pi}{2} \leftrightarrow \frac{\pi}{2}$.

$\frac{\pi}{3}$ is $\frac{2}{3}$'s $\frac{\pi}{2}$ $\therefore a = \frac{2}{3}$ 1

$\frac{y}{a} = \sin^{-1} bx$

$x = \frac{1}{b} \sin \frac{y}{a}$

range is $-2 \leftrightarrow 2$ (twice that of $y = \sin^{-1} x$

$\therefore \frac{1}{b} = 2$

$b = \frac{1}{2}$ 1

$a = \frac{2}{3} \quad b = \frac{1}{2}$

(ii) $y = \frac{2}{3} \sin^{-1} \frac{x}{2}$

$y' = \frac{2}{3} \cdot \frac{\frac{1}{2}}{\sqrt{1 - \left(\frac{x}{2}\right)^2}}$

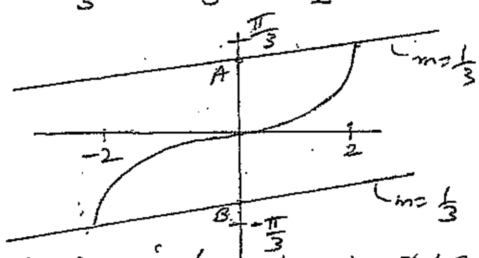
$= \frac{2}{3} \cdot \frac{\frac{1}{2}}{\sqrt{\frac{4-x^2}{4}}}$ 1

$= \frac{2}{3} \cdot \frac{1}{2} \cdot \frac{\sqrt{4}}{\sqrt{4-x^2}}$

$= \frac{2}{3\sqrt{4-x^2}}$

when $x = 0$ $y' = \frac{1}{3}$ 1

(iii) $\frac{x}{3} + c = \frac{2}{3} \sin^{-1} \frac{x}{2}$



$\therefore$ Require to solve $y = \frac{x}{3} + c$ & $y = \frac{2}{3} \sin^{-1} \frac{x}{2}$ simultaneously

c will lie between A and B.

1 mark for recognising need to solve $y = \frac{x}{3} + c$ & $y = \frac{2}{3} \sin^{-1} \frac{x}{2}$ simultaneously

(6)

At A

$y = \frac{x}{3} + A$ passes through $(2, \frac{\pi}{3})$.

$\therefore \frac{\pi}{3} = \frac{2}{3} + A$

$\therefore A = \frac{\pi}{3} - \frac{2}{3}$

At B $y = \frac{x}{3} + B$ passes through $(-2, -\frac{\pi}{3})$

$-\frac{\pi}{3} = -\frac{2}{3} + B$

$\therefore B = \frac{2-\pi}{3}$

$\therefore \frac{2-\pi}{3} \leq c \leq \frac{\pi - 2}{3}$. 1