BAULKHAM HILLS HIGH SCHOOL

2011

TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Extension 1

General Instructions

- Reading time – 5 minutes
- Working time – 2 hours
- Write using black or blue pen
- Board-approved calculators may be used
- A table of standard integrals is provided at the back of this paper
- All necessary working should be shown in every question
- Marks may be deducted for careless or badly arranged work

Total marks – 84

- Attempt Questions 1 – 7
- All questions are of equal value

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Attempt Questions 1 – 7
All questions are of equal value

Answer each question on a SEPARATE piece of paper clearly marked Question 1, Question 2, etc. Each piece of paper must show your student number.

Marks

<u>Question 1</u> ***(12 marks)*** Use a *separate* piece of paper

a) Let $f(x) = \sin^{-1}\left(\frac{x}{3}\right)$. What is the domain of $f(x)$? *1*

b) The value of $\tan 10^\circ$ is denoted by p. Find, in terms of p, the value of $\tan 55^\circ$. *1*

c) P divides the interval from $(-5, 6)$ to $(4, -3)$ internally in the ratio 3:1. Find P. *2*

d) Solve $\frac{2x}{x-2} \leq 3$ *3*

e) Differentiate $e^x \cos^{-1} x$ *2*

f) Use the substitution $u = \sqrt{x+2}$ to evaluate $\int_{-1}^{7} \frac{x}{\sqrt{x+2}}dx$ *3*

<u>Question 2</u> ***(12 marks)*** Use a *separate* piece of paper

a) Find the general solution to $2\sin x = \sqrt{3}$. Express your answer in terms of π. *2*

b) Find the coefficient of x^4 in the expansion of $(4+3x)^{10}$ *2*

c) Find $\int \sin^2 3x\,dx$ *2*

d) A function is defined by $f(x) = x^3 - 4x^2 - 7x + 10$.

(i) Show that $(x - 1)$ is a factor of $f(x)$. *1*

(ii) Hence, or otherwise, solve $x^3 - 4x^2 - 7x + 10 = 0$. *2*

e) Mr Ribbans has to leave home early for a before school lesson. He does not want to wake his wife so he does not turn on the lights. He goes to the sock drawer and chooses two socks. There are 10 black, 6 grey and 4 navy socks, however they are all mixed up.

(i) What is the probability that Mr Ribbans arrives at school with a pair of matching socks? *2*

(ii) To be on the safe side, Mr Ribbans takes a few more socks in his pocket so as to be sure that at least two socks match. What is the least number of socks he should take, in total, to ensure a matching pair? *1*

Marks

Question 3 ***(12 marks)*** Use a *separate* piece of paper

a) Find $\lim_{x \to 0} \dfrac{3x}{\sin 4x}$ *1*

b) $P\left(2p, p^2\right)$ and $Q\left(2q, q^2\right)$ are two points on the parabola $x^2 = 4y$. The chord PQ subtends a right angle at the origin, O. M is the midpoint of PQ.

(i) Show that $pq = -4$. *2*

(ii) Find the locus of M. *2*

c) The function $f(x)$ is defined by $f(x) = 3 + \sqrt{x}$.

(i) Find an expression for $f^{-1}(x)$ in terms of x. *2*

(ii) Find any points of intersection of the graphs $y = f(x)$ and $y = f^{-1}(x)$ *2*

d) The acceleration of a particle x metres from O at time t seconds is given by *3*

$$\frac{d^2x}{dt^2} = -e^{-2x}$$

If the particle's velocity is 1 metre per second when $x = 0$, find the exact velocity when $x = 4$ metres.

Question 4 ***(12 marks)*** Use a *separate* piece of paper

a) Express $3\cos x + 3\sin x$ in the form $R\cos(x - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$ *2*

b) The expression $T(x)$ is defined by $T(x) = \dfrac{1}{3\cos x + 3\sin x}$.

(i) Determine a value of x for which $T(x)$ is not defined. *1*

(ii) Find the smallest possible positive value of x satisfying $T(x) = \frac{\sqrt{6}}{9}$ *2*

c) The rise and fall of the water level in a tidal stretch of a river can be modelled, on a particular day, as simple harmonic motion with amplitude 1.5 metres and period 12 hours. The water is at its lowest level at 9 a.m.

(i) Show that the rise, x metres, in the water level, t hours after 9 a.m. can be described by the formula $x = -\frac{3}{2}\cos\left(\frac{\pi}{6}t\right)$. *2*

(ii) Find how fast the level is rising at 1 p.m. *2*

d) The roots of the equation $x^3 + ax^2 + bx + c = 0$, where $c \neq 0$, are in geometric progression. *3*
Prove that $b^3 = a^3c$.

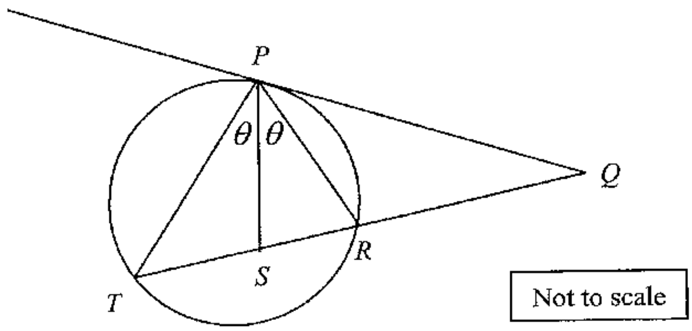
Marks

<u>Question 5</u> ***(12 marks)*** Use a *separate* piece of paper

a) (i) Show that the equation $e^x = x + 2$ has a solution in the interval $1 < x < 2$. *2*

(ii) By letting $x_0 = 1.5$, use one application of Newton's Method to approximate the solution, correct to 3 decimal places. *2*

b) *3*



In the diagram the vertices of triangle *PTR* lie on a circle. The tangent at *P* meets the secant *TR* produced at *Q*. The bisector of $\angle TPR$ meets *TR* at *S*.

Prove that $PQ = SQ$

c) Molten metal at a temperature of 1400°C is poured into moulds to form machine parts. After 15 minutes the metal has cooled to 995°C. If the temperature of the surroundings is 35°C, then the rate of cooling is approximately given by;

$$\frac{dT}{dt} = -k(T - 35)$$

where *k* is a positive constant.

(i) Show that a solution of this equation is $T = 35 + Ae^{-kt}$, where *A* is a constant. *1*

(ii) Find the value of *k*, correct to three decimal places. *2*

(iii) The metal can be taken out of the moulds when its temperature has dropped to 200°C. How long after the metal has been poured will this temperature be reached. Give your answer correct to the nearest minute. *2*

Marks

<u>Question 6</u> ***(12 marks)*** Use a *separate* piece of paper

a) Show that $\sin\left(2\tan^{-1}\frac{1}{2}\right) = \frac{4}{5}$ *2*

b) A farmer noticed that some the eggs laid by the hens had double yolks. He estimated the probability of this happening to be 0.05.
Eggs are packed in boxes of 12

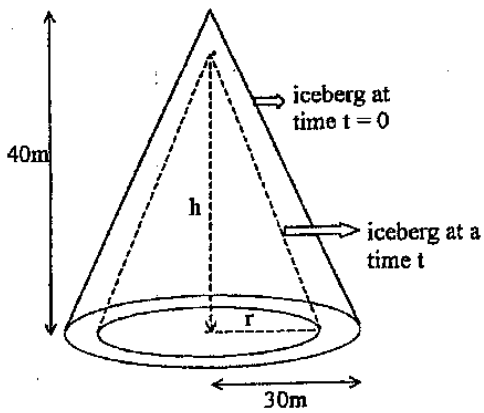
Find the probability that in a box, the number of eggs with double yolks will be;

(i) exactly one. Give your answer correct to four decimal places. *1*

(ii) at least one. Give your answer correct to four decimal places. *2*

(iii) A customer bought three boxes, find the probability that only two of the boxes contained at least one double yolk. Give your answer correct to three decimal places. *2*

c) A melting iceberg is decreasing in volume at the rate of $\frac{3\pi}{16}\,\text{m}^3/\text{hour}$, but it is always conical in shape and its semi vertical angle remains constant.



Initially the iceberg has a base radius 30 metres and height 40 metres as shown on the diagram. (Note: $V = \frac{1}{3}\pi r^2 h$)

(i) Find the rate at which the height is decreasing when the height of the block is 9 metres. *3*

(ii) Find the rate at which the base area is decreasing when the radius is 12 metres. *2*

Marks

<u>Question 7</u> ***(12 marks)*** Use a *separate* piece of paper

a) (i) By comparing the coefficients of x^k on both sides of the identity **2**

$$(1 + x)(1 + x)^n \equiv (1 + x)^{n+1}$$

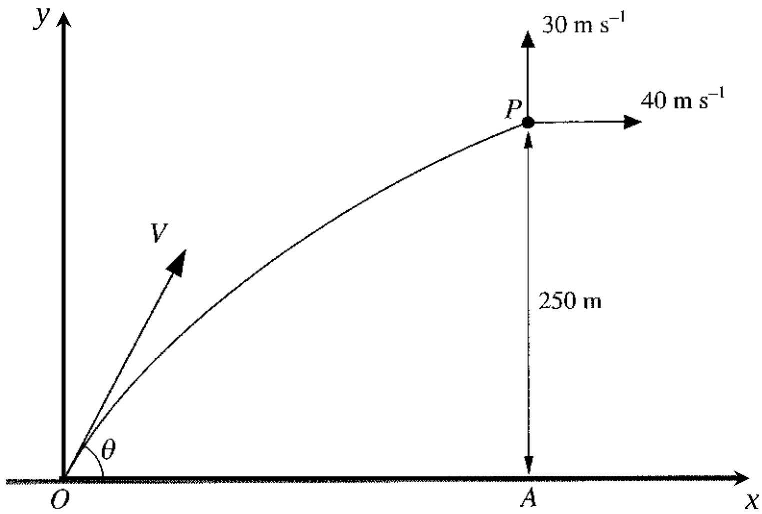
show that;

$$\binom{n}{k-1} + \binom{n}{k} = \binom{n+1}{k}$$

(ii) Use mathematical induction to prove that for integers $n \geq 3$, **4**

$$\binom{n+2}{3} - \binom{n}{3} = n^2$$

b) A particle is projected with speed V m/s, at an angle of elevation θ from a point O on horizontal ground. When the projectile is at a point P, vertically above a point A on the ground its height is 250 m and its velocity components are 40 m/s horizontally and 30 m/s vertically upwards.



The equations of motion are;

$$x = Vt\cos\theta$$
$$y = Vt\sin\theta - 4.9t^2$$

where t is the time in seconds. (Do NOT prove these equations of motion)

(i) Show that the time it takes the particle to reach P is approximately 4.71 seconds, correct to three significant figures. **3**

(ii) Show that $V = 86.0$ and $\theta = 62.3^\circ$, correct to three significant figures. **3**

~ END OF EXAMINATION ~

STANDARD INTEGRALS

$$\int x^n dx = \frac{1}{n+1}x^{n+1}, \ n \neq -1; x \neq 0, \text{ if } \ n < 0$$

$$\int \frac{1}{x} dx = \ln x, \ x > 0$$

$$\int e^{ax} dx = \frac{1}{a}e^{ax}, \ a \neq 0$$

$$\int \cos ax \, dx = \frac{1}{a}\sin ax, \ a \neq 0$$

$$\int \sin ax \, dx = -\frac{1}{a}\cos ax, \ a \neq 0$$

$$\int \sec^2 ax \, dx = \frac{1}{a}\tan ax, \ a \neq 0$$

$$\int \sec ax \tan ax \, dx = \frac{1}{a}\sec ax, \ a \neq 0$$

$$\int \frac{1}{a^2 + x^2} dx = \frac{1}{a}\tan^{-1}\frac{x}{a}, \ a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1}\frac{x}{a}, \ a > 0, \ -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} dx = \ln\left(x + \sqrt{x^2 - a^2}\right), \ \ x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} dx = \ln\left(x + \sqrt{x^2 + a^2}\right)$$

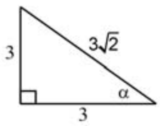
NOTE: $\ln x = \log_e x, \quad x > 0$

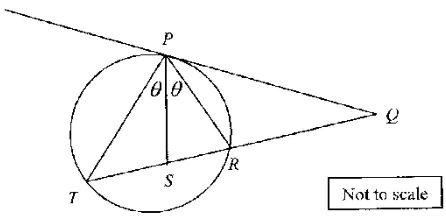
BAULKHAM HILLS HIGH SCHOOL
EXTENSION 1 TRIAL HSC 2011 SOLUTIONS

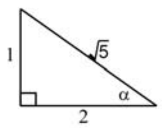
Solution	Marks	Comments
QUESTION 1		
1a) $-1 \le \frac{x}{3} \le 1$ $-3 \le x \le 3$	**1**	
1b) $\tan 55^\circ = \tan(45 + 10)$ $= \frac{\tan 45^\circ + \tan 10^\circ}{1 - \tan 45^\circ \tan 10^\circ}$ $= \frac{1 + p}{1 - p}$	**1**	
1 c) $(-5, 6)$ $(4, -3)$ 3 : 1 $P = \left(\frac{-5 \times 1 + 4 \times 3}{3 + 1}, \frac{6 \times 1 - 3 \times 3}{3 + 1}\right)$ $= \left(\frac{7}{4}, -\frac{3}{4}\right)$	**2**	**1 mark** • Uses a correct method
1d) $\frac{2x}{x - 2} \le 3$ $x - 2 \ne 0$ $x \ne 2$ $\frac{2x}{x - 2} = 3$ $2x = 3x - 6$ $x = 6$  $x < 2$ *or* $x \ge 6$	**3**	**3 marks** • Correct graphical solution on number line or algebraic solution, with correct working **2 marks** • Bald answer • Identifies the two correct critical points via a correct method • Correct conclusion to their critical points obtained using a correct method **1 mark** • Uses a correct method • Acknowledges a problem with the denominator. **0 marks** • Solves like a normal equation , with no consideration of the denominator.
1e) $f(x) = e^x \cos^{-1} x$ $f'(x) = \left(e^x\right)\left(\frac{-1}{\sqrt{1 - x^2}}\right) + \left(\cos^{-1} x\right)\left(e^x\right)$ $= \frac{e^x}{\sqrt{1 - x^2}} + e^x \cos^{-1} x$	**2**	**1 mark** • differentiates $\cos^{-1} x$ correctly.
1f) (v) $\int_{-1}^{7} \frac{x}{\sqrt{x + 2}} dx$ $u = \sqrt{x + 2}$ $du = \frac{dx}{2\sqrt{x + 2}}$ when $x = -1$, $u = 1$ when $x = 7$, $u = 3$ $= 2\int_{1}^{3} \left(u^2 - 2\right) du$ $= 2\left[\frac{u^3}{3} - 2u\right]_1^3$ $= 2\left(\frac{27}{3} - 6 - \frac{1}{3} + 2\right) = 9\frac{1}{3}$ $\left(= \frac{28}{3}\right)$	**3**	**2 marks** • correct primitive in terms of u • correct substitution into their primitive **1 mark** • correct integrand in terms of u • substitiutes correct limits

Solution	Marks	Comments
QUESTION 2		
2a) $2\sin x = \sqrt{3}$ $\sin x = \frac{\sqrt{3}}{2}$ $x = \pi k + (-1)^k \sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$, where k is an integer $x = \pi k + \frac{(-1)^k \pi}{3}$	**2**	**2 marks** • equivalent correct expressions **1 mark** • correct answer neglecting the condition for k • leaving answer in terms of $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$
2b) $\binom{10}{4} \times 4^6 \times (3x)^4$ $= 210 \times 4096 \times 81x^4$ $= 69672960x^4$	**2**	**1 mark** • Correct use of binomial theorem
2c) $\int \sin^2 3x dx = \frac{1}{2}\int (1 - \cos 6x) dx$ $= \frac{1}{2}\left(x - \frac{1}{6}\sin 6x\right) + c$ $= \frac{x}{2} - \frac{1}{12}\sin 6x + c$	**2**	**1 mark** • Correctly uses $\cos 2\theta$ identity • Integrates correctly from incorrect use of $\cos 2\theta$ identity
2d) (i) $f(1) = 1^3 - 4 \times 1^2 - 7 \times 1 + 10$ $= 1 - 4 - 7 + 10$ $= 0$ $\therefore (x - 1)$ is a factor	**1**	
2 d) (ii) $x^3 - 4x^2 - 7x + 10 = 0$ $(x-1)(x^2 - 3x - 10) = 0$ $(x-1)(x-5)(x+2) = 0$ $\therefore x = 1$, $x = 5$ or $x = -2$	**2**	**1 mark** • Correctly finds quadratic factor • Identifies another linear factor (or root)
2 e)(i) $P(\text{socks match}) = \frac{{}^{10}\mathbf{C}_2 + {}^{6}\mathbf{C}_2 + {}^{4}\mathbf{C}_2}{{}^{20}\mathbf{C}_2}$ $= \frac{33}{95}$	**2**	**1 mark** • Progress towards answer, involving the use of combinations or similar logic
2 e)(ii) As there are three different colours, four socks would ensure a matching pair.	**1**	

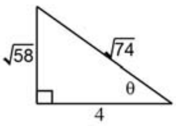
<table>
<tr><th>Solution</th><th>Marks</th><th>Comments</th></tr>
<tr><td colspan="3">QUESTION 3</td></tr>
<tr><td>3a) $\lim\limits_{x\to 0}\dfrac{3x}{\sin 4x} = \lim\limits_{x\to 0}\dfrac{3}{4}\times\dfrac{4x}{\sin 4x}$
$= \dfrac{3}{4}\times 1$
$= \dfrac{3}{4}$</td><td>1</td><td>0 marks
• $\dfrac{3}{4}\times\dfrac{x}{\sin x}$ or equivalent</td></tr>
<tr><td>3b) (i) $m_{OP}\times m_{OQ} = -1$
$\dfrac{p^2-0}{2p-0}\times\dfrac{q^2-0}{2q-0} = -1$
$p^2q^2 = -4pq$
$pq = -4$</td><td>2</td><td>1 mark
• Correctly finds the two slopes
• Identifies the condition for perpendicular lines</td></tr>
<tr><td>3 b)(ii) $M = \left(\dfrac{2p+2q}{2},\dfrac{p^2+q^2}{2}\right)$
$= \left(p+q,\dfrac{p^2+q^2}{2}\right)$
$x = p+q$
$y = \dfrac{p^2+q^2}{2}$
$= \dfrac{(p+q)^2-2pq}{2}$
$= \dfrac{x^2-2\times -4}{2}$
$= \dfrac{x^2+8}{2}$
$\therefore$ locus of M is $y = \dfrac{1}{2}x^2+4$</td><td>2</td><td>1 mark
• Correctly finds M
• Attempts to eliminate parameters from their answer for M</td></tr>
<tr><td>3 c) (i) $x = 3+\sqrt{y}$
$x-3 = \sqrt{y}$
$y = (x-3)^2,\ x\ge 3$</td><td>2</td><td>1 mark
• Correctly swaps the x and y pronumerals
• Does not consider domain (or range) of the solution</td></tr>
<tr><td>3 c)(ii) Point of intersection must lie on the line $y = x$
$x = (x-3)^2$
$x = x^2-6x+9$
$x^2-7x+9 = 0$
$x = \dfrac{7\pm\sqrt{13}}{2}$
But $x\ge 3$ $\therefore$ intersect at $\left(\dfrac{7+\sqrt{13}}{2},\dfrac{7+\sqrt{13}}{2}\right)$</td><td>2</td><td>1 mark
• Recognises the significance of $y = x$</td></tr>
<tr><td>3 d) $\dfrac{d^2x}{dt^2} = -e^{-2x}$
$\dfrac{d}{dx}\left(\dfrac{1}{2}v^2\right) = -e^{-2x}$
$\dfrac{1}{2}v^2 = \dfrac{1}{2}e^{-2x}+c$
when $x = 0$, $v = 1$
$\dfrac{1}{2} = \dfrac{1}{2}+c$
$c = 0$
$\therefore$ $v^2 = e^{-2x}$
When $x = 4$,
$v^2 = e^{-8}$
$v = \pm e^{-4}$
However : $v\ne 0$, i.e. particle does not stop and thus cannot change direction
$\therefore$ when $x = 4$, $v = e^{-4}$ m/s</td><td>3</td><td>2 marks
• $v = \pm e^{-4}$
• No justification for why v must be positive
1 mark
• Correctly integrates using $\dfrac{d}{dx}\left(\dfrac{1}{2}v^2\right)$</td></tr>
</table>

Solution	Marks	Comments
QUESTION 4		
4a)  $\alpha = \tan^{-1}\left(\frac{1}{1}\right)$ $= \tan^{-1}1$ $= \frac{\pi}{4}$ $3\cos x + 3\sin x$ $= 3\sqrt{2}\cos\left(x - \frac{\pi}{4}\right)$	**2**	**1 mark** • Correctly finds R and/or α
4 b)(i) $3\cos x + 3\sin x \neq 0$ $3\sqrt{2}\cos\left(x - \frac{\pi}{4}\right) \neq 0$ $x - \frac{\pi}{4} \neq \frac{\pi}{2}$ $x \neq \frac{3\pi}{4}$	**1**	**1 mark** • Any valid value of x
4 b)(ii) $\frac{1}{3\cos x + 3\sin x} = \frac{\sqrt{6}}{9}$ $3\sqrt{2}\cos\left(x - \frac{\pi}{4}\right) = \frac{9}{\sqrt{6}}$ $\cos\left(x - \frac{\pi}{4}\right) = \frac{3}{\sqrt{12}}$ $= \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{\sqrt{3}}$ $= \frac{\sqrt{3}}{2}$ $x - \frac{\pi}{4} = -\frac{\pi}{6}, \frac{\pi}{6}$ $x = \frac{\pi}{12}, \frac{5\pi}{12}$ $\therefore$ first positive value of x is $\frac{\pi}{12}$	**2**	**1 mark** • $\cos\left(x - \frac{\pi}{4}\right) = \frac{\sqrt{3}}{2}$
4c)(i) As problem starts at lowest amplitude of graph, it is most effectively modelled with a cosine curve which will obey SHM i.e. $x = -a\cos nt$ amplitude = 1.5 period = 12 i.e. $a = \frac{3}{2}$ i.e. $\frac{2\pi}{n} = 12$ $n = \frac{\pi}{6}$ $\therefore x = -\frac{3}{2}\cos\left(\frac{\pi}{6}t\right)$	**2**	**1 mark** • Correctly evaluating n • Explanation of why the situation is modelled by a negative cosine curve.
4 c)(ii) $\dot{x} = \frac{\pi}{4}\sin\left(\frac{\pi}{6}t\right)$ when $t = 4$, $\dot{x} = \frac{\pi}{4} \times \sin\frac{2\pi}{3}$ $= \frac{\pi\sqrt{3}}{8}$ $\therefore$ water level is rising at $\frac{\pi\sqrt{3}}{8}$ m/hour	**2**	**1 mark** • Correctly finds an expression for $\frac{dx}{dt}$
4 d) Let the roots be $\frac{\alpha}{r}$, α , αr $\alpha\beta\gamma = -c$ However α is a root of the equation $\alpha^3 = -c$ $\therefore$ $\alpha^3 + a\alpha^2 + b\alpha + c = 0$ $\alpha = \sqrt[3]{-c}$ $-c + a\left(\sqrt[3]{-c}\right)^2 + b\sqrt[3]{-c} + c = 0$ $a\sqrt[3]{(-c)^2} = -b\sqrt[3]{-c}$ $a^3c^2 = -b \times -c$ $a^3c = b$	**3**	**2 marks** • Significant progress towards a correct solution **1 mark** • solution correctly identifying the relationship between roots and coefficients

Solution	Marks	Comments
QUESTION 5		
5a)(i) $e^x = x + 2$ Let $f(x) = e^x - x - 2$ $\therefore$ looking for solutions to $f(x) = 0$ $f(1) = e^1 - 1 - 2$ $= -0.2817181715\ldots < 0$ $f(2) = e^2 - 2 - 2$ $= 3.389056099\ldots > 0$ As $f(x)$ is continuous and $f(1)$ and $f(2)$, a solution must lie in the interval $1 < x < 2$	**2**	**1 mark** • Shows the difference in signs without talking about continuity
5 a)(ii) $x_1 = x_0 - \dfrac{f(x_0)}{f'(x_0)}$ $= x_0 - \dfrac{e^x - x - 2}{e^x - 1}$ $= 1.5 - \dfrac{e^{1.5} - 1.5 - 2}{e^{1.5} - 1}$ $= 1.218042292\ldots$ $= 1.22$ (correct to 3 decimal places)	**2**	**1 mark** • Correctly uses Newton's Method
5 b)  $\angle RSP = \angle SPT + \angle PTS$ (exterior $\angle$ ΔSPT) $\angle RSP = \theta + \angle PTS$ $\angle SPQ = \angle SPR + \angle QPR$ (common $\angle$) $\angle QPR = \angle PTS$ (alternate segment theorem) $\therefore \angle SPQ = \theta + \angle QPR = \angle RSP$ ΔSPQ is isosceles (two $=$ $\angle$'s) $\therefore$ $PQ = SQ$ (= sides in isosceles ΔSPQ)	**3**	**2 marks** • Correct solution with poor reasoning • Significant progress towards solution with good reasoning. **1 mark** • Significant progress towards solution with poor reasoning. • Progress towards solution with good reasoning.
5 c)(i) $T = 35 + Ae^{-kt}$ $\dfrac{dT}{dt} = -kAe^{-kt}$ $= -k\left(Ae^{-kt} + 35 - 35\right)$ $= -k(T - 35)$	**1**	
5 c(ii) when $t = 0$, $T = 1400$ $\Rightarrow$ $T = 35 + 1365e^{-kt}$ when $t = 15$, $T = 995$ $995 = 35 + 1365e^{-15k}$ $e^{-15k} = \dfrac{960}{1365}$ $-15k = \log\dfrac{64}{91}$ $k = -\dfrac{1}{15}\log\dfrac{64}{91}$ $= 0.02346509488\ldots$ $= 0.023$ (to two decimal places)	**2**	**2 marks** • Leaving answer as an exact value • Do not penalise for rounding error **1 mark** • Correctly finding a value for A
5 c)(iii) when $T = 200$; $200 = 35 + 1365e^{-kt}$ $e^{-kt} = \dfrac{165}{1365}$ $-kt = \log\dfrac{11}{91}$ $t = -\dfrac{1}{k}\log\dfrac{11}{91}$ $= 90.04712083\ldots$ $= 90$ minutes	**2**	**1 mark** • Significant progress towards a correct solution..

Solution	Marks	Comments
QUESTION 6		
6a)  Let $\alpha = \tan^{-1}\frac{1}{2}$ $\sin\left(2\tan^{-1}\frac{1}{2}\right) = \sin 2\alpha$ $= 2\sin\alpha\cos\alpha$ $= 2 \times \frac{1}{\sqrt{5}} \times \frac{2}{\sqrt{5}}$ $= \frac{4}{5}$	**2**	**1 mark** • Correct use of the $\sin 2\alpha$ identity
6 b)(i) Let X = the number of eggs with double yolks $P(X = 1) = {}^{12}\mathbf{C}_1(0.95)^{11}(0.05)^1$ $= 0.341280054\ldots$ $= 0.3413$ (correct to 4 decimal places)	**1**	*Note: ignore rounding errors in all of 6 b)*
6 b)(ii) $P(X \geq 1) = 1 - P(X = 0)$ $= 1 - (0.95)^{12}$ $= 0.459639913\ldots$ $= 0.4596$ (correct to 4 decimal places)	**2**	**1 mark** • Solution involving the use of complementary events.
6 b)(iii) Let Y = the number of boxes containing at least one egg with a double yolk $P(Y = 2) = {}^{3}\mathbf{C}_2(0.5404)^1(.4596)^2$ $= 0.342449578\ldots$ $= 0.342$ (correct to 3 decimal places)	**2**	**1 mark** • Progress towards a correct solution using their answer to part (ii)
6 c(i) $\frac{dV}{dt} = -\frac{3\pi}{16}$ $V = \frac{1}{3}\pi r^2 h$ $\frac{h}{r} = \frac{4}{3}$ $r = \frac{3}{4}h$ $\therefore\ V = \frac{1}{3}\pi\left(\frac{3h}{4}\right)^2 h$ $= \frac{3\pi}{16}h^3 \quad \Rightarrow \quad \frac{dV}{dh} = \frac{9\pi}{16}h^2$ $\frac{dh}{dt} = \frac{dV}{dt} \times \frac{dh}{dV}$ $= -\frac{3\pi}{16} \times \frac{16}{9\pi h^2}$ $= -\frac{1}{3h^2}$ when $h = 9$; $\frac{dh}{dt} = -\frac{1}{243}$ $\therefore$ the iceberg's height is decreasing at a rate of $\frac{1}{243}$m/hour	**3**	**2 marks** • Correct expression for $\frac{dV}{dh}$ • Correct value for $\frac{dh}{dt}$ using their expression for $\frac{dV}{dh}$ **1 mark** • Finding an expression for r in terms of h
6 c)(ii) $A = \pi r^2 \quad \Rightarrow \quad \frac{dA}{dr} = 2\pi r$ $V = \frac{1}{3}\pi r^2 \times \frac{4}{3}r$ $= \frac{4}{9}\pi r^3 \quad \Rightarrow \quad \frac{dV}{dr} = \frac{4}{3}\pi r^2$ $\frac{dA}{dt} = \frac{dA}{dr} \times \frac{dr}{dV} \times \frac{dV}{dt}$ $= 2\pi r \times \frac{3}{4\pi r^2} \times -\frac{3\pi}{16}$ $= -\frac{9\pi}{32r}$ when $r = 12$; $\frac{dA}{dt} = -\frac{3\pi}{128}$ $\therefore$ the iceberg's base area is decreasing at a rate of $\frac{3\pi}{128}$ m/hour	**2**	**1 mark** • Solution involving $\frac{dA}{dt} = \frac{dA}{dr} \times \frac{dr}{dV} \times \frac{dV}{dt}$ or equivalent merit

Solution	Marks	Comments
QUESTION 7		
7 a)(i) Consider the expansion of $(1+x)(1+x)^n$ $(1+x)(1+x)^n=(1+x)\left[\binom{n}{0}+\binom{n}{1}x+\binom{n}{2}x^2+\ldots+\binom{n}{k-1}x^{k-1}+\binom{n}{k}x^k+\ldots\binom{n}{n}x^n\right]$ terms involving x^k are; $x\times\binom{n}{k-1}x^{k-1}+\binom{n}{k}x^k=\left[\binom{n}{k-1}+\binom{n}{k}\right]x^k$ Consider terms in the expansion of $(1+x)^{n+1}$ $T_{k+1}=\binom{n+1}{k}x^k$ $\therefore$ since $(1+x)(1+x)^n\equiv(1+x)^{n+1}$, equating coefficients gives $\binom{n}{k-1}+\binom{n}{k}=\binom{n+1}{k}$	**2**	**1 mark** • Considers terms in expansion or equivalent progress towards correct solution
7 a)(ii) When $n = 3$; $LHS=\binom{5}{3}-\binom{3}{3}$ $=10-1$ $=9$ $RHS=3^2$ $=9$ $LHS=RHS$ Hence the result is true for $n=3$ Assume the result is true for $n = k$ i.e. $\binom{k+2}{3}-\binom{k}{3}=k^2$ Prove the result is true for $n = k + 1$ i.e. Prove $\binom{k+3}{3}-\binom{k+1}{3}=(k+1)^2$ ***PROOF*** $\binom{k+3}{3}-\binom{k+1}{3}=\binom{k+2}{2}+\binom{k+2}{3}-\binom{k}{2}-\binom{k}{3}$ $=k^2+\binom{k+2}{2}-\binom{k}{2}$ $=k^2+\binom{k+1}{1}+\binom{k+1}{2}-\binom{k}{2}$ $=k^2+k+1+\binom{k}{1}+\binom{k}{2}-\binom{k}{2}$ $=k^2+k+1+k$ $=k^2+2k+1$ $=(k+1)^2$ Hence the result is true for $n = k + 1$,if it is true for $n = k$ Since the result is true for $n = 3$, then it is true for all $n\ge 3$ by induction.	**4**	There are 4 key parts of the induction; 1. Proving the result true for $n = 3$ 2. Clearly stating the assumption and what is to be proven 3. Using the assumption in the proof 4. Correctly proving the required statement **4 marks** • Successfully does all of the 4 key parts **3 marks** • Successfully does 3 of the 4 key parts **2 marks** • Successfully does 2 of the 4 key parts **1 mark** • Successfully does 1 of the 4 key parts

Solution	Marks	Comments
QUESTION 7 … continued		
7 b)(i) $y = Vt\sin\theta - 4.9t^2 \quad \Rightarrow \quad 250 = Vt\sin\theta - 4.9t^2$ $\dot{y} = V\sin\theta - 9.8t \quad \Rightarrow \quad 30 = V\sin\theta - 9.8t$ $V\sin\theta = 30 + 9.8t$ Solving simultaneously; $30t + 9.8t^2 - 4.9t^2 = 250$ $4.9t^2 + 30t - 250 = 0$ $t = \dfrac{-30 + \sqrt{5800}}{9.8} \quad , \quad t > 0$ $t = \dfrac{-30 + 10\sqrt{58}}{9.8}$ $= 4.709972557\ldots$ $= 4.71$ seconds (correct to 2 decimal places)	**3**	**2 marks** • Correctly finds a quadratic in terms of *t*, or equivalent merit **1 mark** • Finding an expression for the vertical velocity • Attempts to eliminate the parameters of *V* and θ in order to find a quadratic in terms of *t*.
7 b)(ii) $x = Vt\cos\theta$ $\dot{x} = V\cos\theta \quad \Rightarrow \quad V\cos\theta = 40$ $V\sin\theta = 30 + 9.8t$ $= 30 - 30 + 10\sqrt{58}$ $= 10\sqrt{58}$ $\tan\theta = \dfrac{\sqrt{58}}{4}$ $\theta = 62.29038921\ldots$ $= 62.3^\circ$ (correct to 3 sig figs)  $V\cos\theta = 40$ $V = \dfrac{40}{\cos\theta}$ $= 40 \times \dfrac{\sqrt{74}}{4}$ $= 10\sqrt{74}$ $= 86.02325267\ldots$ $= 86.0$ m/s (correct to 3 sig figs)	**3**	**2 marks** • Correctly finds *V* • Correctly finds θ **1 mark** • Progress towards finding either *V* or θ using a correct method.