BAULKHAM HILLS HIGH SCHOOL

2012

YEAR 12 TRIAL
HIGHER SCHOOL CERTIFICATE

Mathematics Extension 1

General Instructions

- Reading time, 5 minutes
- Exam time, 2 hours
- Write using black or blue pen
- Board approved calculators may be used
- A table of standard integrals is provided at the back of this question paper
- Write your student number at the top of each page of your answer booklet
- Show all necessary working in Questions 11 -14

Total Marks: 70

Attempt ALL questions

Section I **Pages 2-4**

(10 marks)
Questions 1-10

Section II **Pages 5-7**

(60 marks)
Questions 11-14
Worth 15 marks each

Section I – Multiple Choice
10 marks
Attempt Questions 1-10
Use the multiple choice answer sheet for Questions 1-10.

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1. This exam contains 10 Multiple Choice questions with four responses in each.
   If a candidate guesses all questions, what is the probability of correctly guessing exactly 7 answers?

   a) $^{10}C_4 \left(\frac{1}{4}\right)^7 \left(\frac{3}{4}\right)^3$

   b) $^{10}C_7 \left(\frac{1}{4}\right)^7 \left(\frac{3}{4}\right)^3$

   c) $\left(\frac{1}{4}\right)^7 \left(\frac{3}{4}\right)^3$

   d) $^{10}C_7 \left(\frac{1}{4}\right)^3 \left(\frac{3}{4}\right)^7$

2. Two roots of $4x^3 + 8x^2 + kx - 18 = 0$ are equal in magnitude but opposite in sign.
   Find the value of k.

   a) -2

   b) $+2$

   c) -9

   d) $+9$

3. For what value of x is $\sin x \cos x$ a maximum?

   a) $\frac{\pi}{4}$

   b) $\frac{\pi}{2}$

   c) $\frac{2\pi}{3}$

   d) $\frac{3\pi}{4}$

4. If $5x = 4t$ and $6y = 5t$ and $t > 0$, then

a) $x > y$

b) $y > x$

c) $x = y$

d) Cannot be determined from the information

p $$\frac{p+1}{p} \leq 1$$

a) $p > 0$

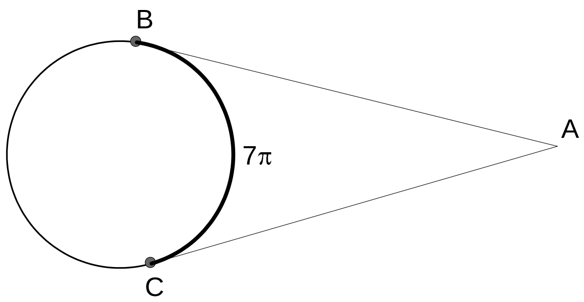
b) $p < 0$

c) $p \leq 0$

d) $-1 \leq p < 0$

6. AB and AC are tangents to a circle at points B and C respectively. Minor arc BC is 7π cm and the radius of the circle is 18cm.

What is the number of degrees in $\angle BAC$?



a) 70

b) 95

c) 100

d) 110

7. What is the period of the curve whose equation is $y = \frac{1}{3}(\cos^2 x - \sin^2 x)$?

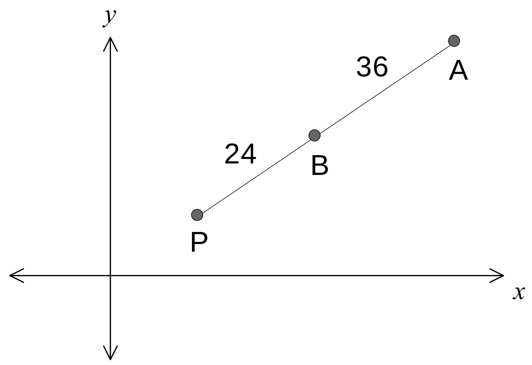
a) $\frac{\pi}{3}$

b) $\frac{2\pi}{3}$

c) π

d) 2π

8.



P divides *AB* externally in the ratio

a) 2 : 3

b) 2 : 5

c) 3 : 2

d) 5 : 2

9. A coin is biased so that there is a constant probability p (where $p \neq 0.5$) that the coin shows heads.

In 6 throws of the coin, the probability of 3 heads is twice the probability of 2 heads.

What is the value of p?

a) $\frac{3}{7}$

b) $\frac{3}{5}$

c) $\frac{1}{2}$

d) $\frac{1}{3}$

10. Find the value of $\tan(p+q)$ if $\tan p + \tan q + 1 = \cot p + \cot q = 6$.

a) $\frac{5}{6}$

b) $\frac{6}{5}$

c) 5

d) 30

END OF SECTION I

Section II - 60 marks
Attempt Questions 11-14

Answer each question on the appropriate answer sheet. Each answer sheet must show your Board of Studies number. Extra paper is available.
All necessary working should be shown in every question.

Marks

Question 11 (15 marks) Use a separate answer sheet

a) State the domain and range of $f(x) = 3\cos^{-1} 2x$. **2**

b) Evaluate $\int_0^{1\cdot 5} \frac{dx}{\sqrt{9 - x^2}}$ leaving your answer in exact form. **2**

c) Fully factorise $6x^3 + 17x^2 - 4x - 3$. **3**

d) Evaluate $\int_0^1 \frac{x\,dx}{\sqrt{1 + x}}$ using the substitution $u^2 = 1 + x$. **3**

e) P divides the interval from (-5, 6) to (4, 3) externally in the ratio 3 : 1. **3**
Find the coordinates of P.

f) A bag contains 6 red marbles, 5 white marbles and 3 blue marbles. **2**
Three marbles are drawn together at random from the bag.
What is the probability that exactly 2 are red?

Question 12 (15 marks) Use a separate answer sheet

$$\frac{d}{dx}(\tan^{-1} x + x)$$ **1**

$$\int_0^1 \frac{x^2 + 2}{x^2 + 1}\,dx$$ /

b) i) Express $\sqrt{3}\sin\theta - \cos\theta$ in the form $R\sin(\theta - \alpha)$ where α is acute. **2**

ii) Hence solve $\sqrt{3}\sin\theta - \cos\theta = 0$, for $0 \leq \theta \leq 2\pi$, leaving your answer in exact form. **2**

$$\left(2x + \frac{3}{x^2}\right)^9$$

i) Show that there is no term involving x^4. **3**

ii) Find the term independent of x. **2**

iii) Find the greatest coefficient. **2**

Marks

Question 13 (15 marks) Use a separate answer sheet

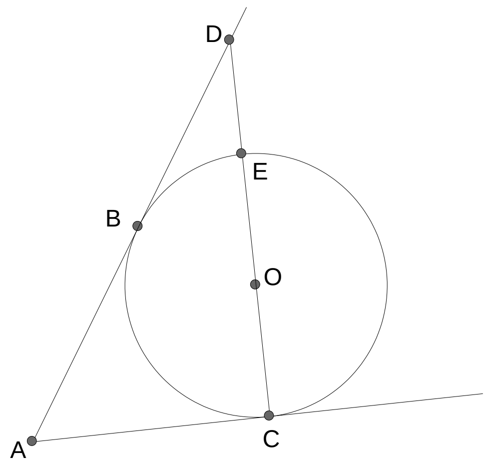
a) The acute angle between the tangents to the curve $y = \sin x$ and $y = \cos x$ at their point of intersection (where $0 < x < \frac{\pi}{2}$) is θ. **4**

Find the value of θ correct to the nearest degree.

b) The diagram shows two tangents, AB and AC drawn from a common point A to a circle centred at O. **4**

The diameter CE produced cuts the tangent AB at the point D.

Copy the diagram into your workbook.



Show that $\angle EDA + 2 \times \angle DEB = 270°$

c) A certain particle moves along the x axis according to $t = 2x^2 - 5x + 3$ where x is measured in metres and t in seconds.

Initially the particle is 1.5m to the right of O and moving away from O.

i) Prove that the velocity, v ms^{-1}, is given by $v = \dfrac{1}{4x - 5}$. **2**

ii) Find an expression for the acceleration in terms of x. **2**

iii) Find the velocity of the particle when $t = 6$ seconds. **3**

Marks

Question 14 (15 marks) Use a separate answer sheet

$f(x) = (1+x)^n \qquad \int_0^1 f(x)\,dx,$ **3**

$$\sum_{r=0}^{n} \frac{1}{r+1}\binom{n}{r} = \frac{2^{n+1}-1}{n+1}$$

b) i) Prove by mathematical induction that for all positive integers n, **3**

$\sin(n\pi + x) = (-1)^n \sin x$

ii) Let $S = \sin(\pi + x) + \sin(2\pi + x) + \sin(3\pi + x) + \ldots + \sin(n\pi + x)$ **2**

for $0 < x < \frac{\pi}{2}$ and for all positive integers n.

Show that $-1 < S \leq 0$.

c) The straight line $y = mx + b$ meets the parabola $x^2 = 4ay$ at the points $P\,(2ap, ap^2)$ and $Q\,(2aq, aq^2)$

$pq = \overline{}$ **2**

ii) Prove that $p^2 + q^2 = 4m^2 + \frac{2}{}$ **1**

iii) The equation of the normal to the parabola at P is given as **2**

$x + py = 2ap + ap^3$. **DO NOT PROVE THIS**

N is the point of intersection of the normals at P and Q with coordinates

$\left(- \quad (\; + \;),\; (2 + \quad ^2 + \quad + q^2)\right)$

Express these coordinates in terms of a, m and b.

iv) Suppose that PQ is free to move while maintaining a fixed gradient. **2**

Find the locus of N and show that this locus is a straight line which is a normal to the parabola.

End of Examination

STANDARD INTEGRALS

$$\int x^n dx = \frac{1}{n+1} x^{n+1}, \quad n \neq -1; \quad x \neq 0, \ \textit{if} \ \ n < 0$$

$$\int \frac{1}{x} dx = \ln x, \qquad x > 0$$

$$\int e^{ax} dx = \frac{1}{a} e^{ax}, \quad a \neq 0$$

$$\int \cos ax dx = \frac{1}{a} \sin ax, \quad a \neq 0$$

$$\int \sin ax dx = -\frac{1}{a} \cos ax, \quad a \neq 0$$

$$\int \sec^2 ax dx = \frac{1}{a} \tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax dx = \frac{1}{a} \sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2 + x^2} dx = \frac{1}{a} \tan^{-1} \frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1} \frac{x}{a}, \ a > 0, \ -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} dx = \ln\left(x + \sqrt{x^2 - a^2}\right), \ x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} dx = \ln\left(x + \sqrt{x^2 + a^2}\right)$$

NOTE: $\ln x = \log_e x, \ x > 0$

BHHS YEAR 12 MATHEMATICS EXTENSION 1 TRIAL

MULTIPLE CHOICE

(1) let p = prob of correctly guessing = $\frac{1}{4}$

q = prob of incorrectly guessing = $\frac{3}{4}$

For 7 correct responses

$^{10}C_7 p^7 q^3 = {}^{10}C_7 \left(\frac{1}{4}\right)^7 \left(\frac{3}{4}\right)^3$

$\therefore B$

(2) let roots be $\alpha, -\alpha, \beta$

$\alpha + (-\alpha) + \beta = -\frac{8}{4} = -2$

$\beta = -2$

$\alpha(-\alpha)\beta = \frac{18}{4}$

$-\alpha^2(-2) = \frac{9}{2}$

$\alpha^2 = \frac{9}{4}$

$\alpha(-\alpha) + \alpha\beta + (-\alpha\beta) = \frac{k}{4}$

$-\alpha^2 = \frac{k}{4}$

$-\frac{9}{4} = \frac{k}{4}$

$\therefore k = -9$

$\therefore C$

(3) $\sin x \cos x = \frac{1}{2}\sin 2x$

max when $\sin 2x = 1$

$\therefore 2x = \frac{\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}, \ldots$

$x = \frac{\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}, \ldots$

$\therefore A$

(4) $5x = 4t$

$t = \frac{5x}{4}$

sub in $6y = 5t$

$6y = 5 \times \frac{5x}{4}$

$6y = \frac{25x}{4}$

$24y = 25x$

$\frac{y}{x} = \frac{25}{24}$

$\therefore y > x$

$\therefore B$

(5) $p^2\left(\frac{p+1}{p}\right) \le 1p^2 \quad p \ne 0$

$p(p+1) \le p^2$

$p^2 + p \le p^2$

$p \le 0 \quad p \ne 0$

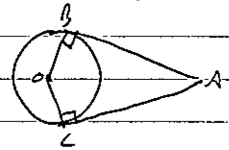
$\therefore p < 0$

$\therefore B.$

(6) $l = r\theta$

$7\pi = 18\theta$

$\theta = \frac{7\pi}{18} = 70^\circ$



OCBA is cyclic quadrilateral as opposite angles are supplementary

$\therefore \angle BAC = 180^\circ - 70^\circ$

$= 110^\circ$

$\therefore D$

(7) $y = \frac{1}{3}\cos 2x$

Period $= \frac{2\pi}{n}$

$= \frac{2\pi}{2}$

$= \pi$

$\therefore C$

(8) $\frac{PA}{PB} = \frac{60}{24}$

$= \frac{5}{2}$

ie $5:2$

$\therefore D$

(9) $P(3\text{heads}) = 2\,P(2\text{heads})$

$^6C_3 p^3 q^3 = 2\,{}^6C_2 p^2 q^4$ where $P(\text{tail}) = q = 1-p$

$20p = 30q$

$20p = 30(1-p)$

$20p = 30 - 30p$

$50p = 30$

$p = \frac{3}{5}$

$\therefore B$

(10) $\cot p + \cot q = 6$

$\frac{1}{\tan p} + \frac{1}{\tan q} = 6$

$\frac{\tan p + \tan q}{\tan p \tan q} = 6$

$\tan p \tan q = \frac{\tan p + \tan q}{6}$

$\tan(p+q) = \frac{\tan p + \tan q}{1 - \tan p \tan q}$

$= \frac{6-1}{1 - \frac{\tan p + \tan q}{6}}$

$= \frac{5}{1 - \frac{5}{6}}$

$= \frac{5}{\frac{1}{6}}$

$= 30$

$\therefore D$

MATHEMATICS EXTENSION 1 TRIAL 2012

SECTION II

11 a) Domain $= \{x : -1 \le 2x \le 1$

ie $D = -\frac{1}{2} \le x \le \frac{1}{2}$ ✓

Range $= \{y : 0 \le \frac{y}{3} \le \pi\}$

ie $R = 0 \le y \le 3\pi$ ✓

b) $\int_0^{1.5} \frac{dx}{\sqrt{9-x^2}} = \left[\sin^{-1}\frac{x}{3}\right]_0^{1.5}$ ✓

$= \sin^{-1}\frac{1}{2} - \sin^{-1} 0$

$= \frac{\pi}{6} - 0$

$= \frac{\pi}{6}$ ✓

c) $P(-3) = 6\times -27 + 17\times 9 + 12 - 3$

$= 0$

$\therefore x+3$ is a factor ✓

$6x^3 + 17x^2 - 4x - 3 = (x+3)(6x^2 - x - 1)$ ✓

$= (x+3)(2x-1)(3x+1)$ ✓

d) $\int_0^1 \frac{x\,dx}{\sqrt{1+x}}$

$u^2 = 1+x$

$x = u^2 - 1$

$\frac{dx}{du} = 2u$

when $x = 1$, $u = \sqrt{2}$ ✓

$x = 0$, $u = 1$

$\int_1^{\sqrt{2}} \frac{(u^2-1)\,2u\,du}{\sqrt{u^2}}$ ✓

$= 2\left[\frac{u^3}{3} - u\right]_1^{\sqrt{2}}$

$= 2\left[\left(\frac{2\sqrt{2}}{3} - \sqrt{2}\right) - \left(\frac{1}{3} - 1\right)\right]$

$= 2\left(\frac{2}{3} - \frac{\sqrt{2}}{3}\right)$ ✓

11 e) $(-5, 6)$ $(4, +3)$

$3 : -1$ ✓

$P: \left(\frac{-1\times -5 + 4\times 3}{3-1}, \frac{-1\times 6 + 3\times 3}{3-1}\right)$

$P = \left(\frac{17}{2}, \frac{3}{2}\right)$ ✓ ✓

11 f) $P(2\text{ red}, 1\text{ other})$

$= \frac{{}^6C_2 \times {}^8C_1}{{}^{14}C_3}$ — ① ✓ — ① ✓

$= \frac{30}{91}$

MULTIPLE CHOICE

1	B
2	C
3	A
4	B
5	B
6	D
7	C
8	D
9	B
10	D

12 a) i) $\frac{d}{dx}(\tan^{-1}x + x) = \frac{1}{1+x^2} + 1$ ✓

ii) $\int_0^1 \frac{x^2+2}{x^2+1}\,dx = \int_0^1 \frac{x^2+1+1}{x^2+1}\,dx$

$= \int_0^1 1 + \frac{1}{x^2+1}\,dx$ ✓

$= \left[\tan^{-1}x + x\right]_0^1$ ✓

$= (\tan^{-1}1 + 1) - (\tan^{-1}0 + 0)$

$= \frac{\pi}{4} + 1$ ✓

b i) $R = \sqrt{(\sqrt{3})^2 + 1^2}$

$= 2$ ✓

$\sqrt{3}\sin\theta - \cos\theta \equiv 2\sin(\theta - \alpha)$

$\sqrt{3}\sin\theta - \cos\theta \equiv 2(\sin\theta\cos\alpha - \cos\theta\sin\alpha)$

$\therefore \sqrt{3} = 2\cos\alpha$ & $1 = 2\sin\alpha$

$\cos\alpha = \frac{\sqrt{3}}{2}$ $\sin\alpha = \frac{1}{2}$

$\therefore \alpha$ in 1st quad.

$\alpha = \frac{\pi}{6}$ ✓

$\therefore \sqrt{3}\sin\theta - \cos\theta = 2\sin\left(\theta - \frac{\pi}{6}\right)$

ii) $\sqrt{3}\sin\theta - \cos\theta = 0$ $0 \le \theta \le 2\pi$

$2\sin\left(\theta - \frac{\pi}{6}\right) = 0$ ✓ $-\frac{\pi}{6} \le \theta - \frac{\pi}{6} \le \frac{11\pi}{6}$

$\sin\left(\theta - \frac{\pi}{6}\right) = 0$

$\theta - \frac{\pi}{6} = 0, \pi$ ✓

$\therefore \theta = \frac{\pi}{6}, \frac{\pi}{6}$

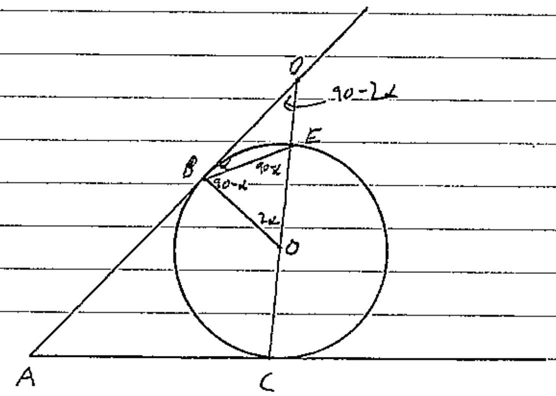
$$\tan\theta = \left|\frac{\frac{2}{\sqrt{2}}}{\frac{1}{2}}\right|$$

$$= \left|\frac{4}{\sqrt{2}}\right|$$

$$\theta = \tan^{-1}\left|\frac{4}{\sqrt{2}}\right|$$

$\theta = 71°$ (nearest degree) ✓

13 b)



Marks
4 correct solution
3 significant progress
2 Some progress
1 limited progress with no circle theorems

Join BE, BO

Let $\angle EBD = \alpha$

$\angle EBO = 90° - \alpha$ (tangent ⊥ radius) ✓

$OB = OE$ (equal radii)

$\triangle OBE$ is isosceles (2 sides equal)

$\angle OEB = 90° - \alpha$ (= ∠'s in isosceles $\triangle OBE$) ✓

$\angle BOE = 180° - (2(90° - \alpha))$ (∠ sum of $\triangle OBE$)

$= 2\alpha$

$\angle BDO = 90° - 2\alpha$ (∠ sum of $\triangle BDO$)

$\angle DEB = 90° - \alpha + 2\alpha$ (exterior angle of $\triangle EBD$) ✓

$= 90° + \alpha$

$\therefore \angle EDA + 2\angle DEB = 90° - 2\alpha + 2(90° + \alpha)$ ✓

$= 270° - 2\alpha + 2\alpha$

$\angle EDA + 2\angle DEB = 270°$ as required.

13 c) i) $t = 2x^2 - 5x + 3$

$\frac{dt}{dx} = 4x - 5$ ✓

$\frac{dx}{dt} = \frac{1}{4x-5}$

$v = \frac{1}{4x-5}$ ✓

ii) $a = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$

$a = \frac{d}{dx}\left(\frac{1}{2}\left(\frac{1}{4x-5}\right)^2\right)$ ✓

$= \frac{d}{dx}\left(\frac{1}{2}(4x-5)^{-2}\right)$

$= -1(4x-5)^{-3} \cdot 4$ ✓

$a = \frac{-4}{(4x-5)^3}$

iii) When $t = 6$

$6 = 2x^2 - 5x + 3$

$2x^2 - 5x - 3 = 0$

$(2x+1)(x-3) = 0$ ✓

$x = -\frac{1}{2}, \quad x = 3$

But when $t = 0$, $x = 1.5$ and moving right.

When $4x - 5 > 0$ ie $x > 1.25$, velocity is always positive

$\therefore x$ never passes through $x = -\frac{1}{2}$ ✓ with justification

$\therefore$ when $t = 6$, $x = 3$

$\therefore v = \frac{1}{4\times3-5}$

$= \frac{1}{7}$ ✓

velocity is $\frac{1}{7}\ ms^{-1}$.

12 c i) $\left(2x+\frac{3}{x^2}\right)^9 = \sum_{k=0}^{9} {}^9C_k (2x)^{9-k} (3x^{-2})^k$

General term $= {}^9C_k\, 2^{9-k} 3^k x^{9-k} x^{-2k}$

$= {}^9C_k\, 2^{9-k} 3^k x^{9-3k}$ ✓

For x^4: $9-3k=4$ ✓

$3k=5$

$k=\frac{5}{3}$

But k is an integer ✓

$\therefore$ No term involving x^4.

ii) Term independent of x:

$9-3k=0$

$3k=9$

$k=3$ ✓

$\therefore {}^9C_3\, 2^6\, 3^3 = 145152$ ✓

(1) either form

iii) let $t_k = {}^9C_k\, 2^{9-k} 3^k$ be coefficient where $k=0,1,\ldots,9$

Now $\frac{t_{k+1}}{t_k} = \frac{{}^9C_{k+1}\, 2^{8-k} 3^{k+1}}{{}^9C_k\, 2^{9-k} 3^k}$ ✓

$= \dfrac{\frac{9!}{(8-k)!(k+1)!}}{\frac{9!}{(9-k)!\,k!}} \cdot \frac{3}{2}$

$= \frac{9-k}{k+1} \cdot \frac{3}{2}$

Coefficients increasing when $\frac{t_{k+1}}{t_k} > 1$

ie $\frac{3(9-k)}{2(k+1)} > 1$

12 c) $27-3k > 2k+2$ $(k \geq 0)$

$25 > 5k$

$k < 5$

$\therefore$ Coefficients increasing for $k=0,1,2,3,4$ — ie $\frac{t_5}{t_4} > 1$

ie $t_0 < t_1 < t_2 < t_3 < t_4 < t_5$

when $k=4$, $t_{k+1} = {}^9C_5\, 2^4 . 3^5$ (or 489888) ✓

OR $\frac{t_{k+1}}{t_k} = \frac{n-k+1}{k}$ where $t_{k+1} = {}^9C_k\, 2^{9-k} 3^k$ ✓

$\frac{3}{2} \cdot \frac{9-k+1}{k} \geq 1$

$\frac{3}{2} \cdot \frac{10-k}{k} \geq 1$

$30-3k \geq 2k$

$30 \geq 5k$

$k \leq 6$

$\therefore$ True for $k=0,1,2,3,4,5,6$

$\therefore$ Greatest coeff $= {}^9C_6\, 2^3 3^6$

$= 489888$ ✓

NB there are in fact two greatest coefficients "equal"

Q13 a) $y=\sin x$ $\quad y=\cos x$

$\frac{dy}{dx}=\cos x$ $\quad \frac{dy}{dx}=-\sin x$

when $y=\sin x$ intersects $y=\cos x$

$\sin x = \cos x$

$\tan x = 1$

$x=\frac{\pi}{4}$ $\quad 0<x<\frac{\pi}{2}$ ✓

$\tan\theta = \left|\frac{m_1-m_2}{1+m_1m_2}\right|$ since $m_1=\cos\frac{\pi}{4} = \frac{1}{\sqrt2}$, $m_2 = -\sin\frac{\pi}{4}$, $m = \frac{-1}{\sqrt2}$ ✓

$= \left|\dfrac{\frac{1}{\sqrt2} - -\frac{1}{\sqrt2}}{1-\frac{1}{2}}\right|$ ✓

14 a) $\int_0^1 (1+x)^n \, dx = \left[\frac{(1+x)^{n+1}}{n+1}\right]_0^1$

$= \frac{2^{n+1}}{n+1} - \frac{1}{n+1}$ ✓

$= \frac{2^{n+1}-1}{n+1}$

Also $\int_0^1 f(x)\,dx = \int_0^1 \binom{n}{0} + \binom{n}{1}x + \binom{n}{2}x^2 + \dots + \binom{n}{n}x^n \, dx$ ✓

$= \left[\binom{n}{0}x + \binom{n}{1}\frac{x^2}{2} + \binom{n}{2}\frac{x^3}{3} + \dots + \binom{n}{n}\frac{x^{n+1}}{n+1}\right]_0^1$

$= \left(\binom{n}{0} + \binom{n}{1}\frac{1}{2} + \binom{n}{2}\frac{1}{3} + \dots + \binom{n}{n}\frac{1}{n+1}\right) - (0+0+0+\dots+0)$ ✓

$= \sum_{r=0}^{n} \frac{1}{r+1}\binom{n}{r}$

b i) For $n=1$, LHS $= \sin(\pi + x)$ RHS $= (-1)^1 \sin x$

$= -\sin x$ $= -\sin x$

$=$ LHS

$\therefore$ True for $n=1$. ✓

Assume true for $n=k$ $\sin(k\pi + x) = (-1)^k \sin x$

For $n=k+1$ we wish to prove $\sin((k+1)\pi + x) = (-1)^{k+1} \sin x$

LHS $= \sin((k+1)\pi + x)$

$= \sin((k\pi + x) + \pi)$

$= -\sin(k\pi + x)$ ✓

$= -(-1)^k \sin x$

$= (-1)^{k+1} \sin x$ as req'd. ✓

$\therefore$ If true for $n=k$ its true for $n=k+1$

But it is true for $n=1$, $\therefore$ true for $n=1+1=2$ and so on for all positive integers.

$\therefore$ By mathematical induction, $\sin(n\pi + x) = (-1)^n \sin x$

14 b ii) $S = \sin(\pi + x) + \sin(2\pi + x) + \sin(3\pi + x) + \dots + \sin(n\pi + x)$

$= -1 \sin x + (-1)^2 \sin x + (-1)^3 \sin x + \dots + (-1)^n \sin x$

This is a GP with $a = -\sin x$, $r = -1$, n terms

$\therefore S = \frac{-\sin x((-1)^n - 1)}{-1-1}$

$= \frac{\sin x((-1)^n - 1)}{2}$ ✓

For n even, $S = \frac{\sin x}{2}(1-1)$

$= 0$

For n odd, $S = \frac{\sin x(-1-1)}{2}$

$= -\sin x$

Now for $0 < x < \frac{\pi}{2}$ $0 < \sin x < 1$

$0 > -\sin x > -1$

ie $-1 < -\sin x < 0$ ✓

$\therefore$ For positive integers n, $-1 < S \leq 0$

14 c) i) $m_{PQ} = \frac{ap^2 - aq^2}{2ap - 2aq}$

$= \frac{a(p+q)(p-q)}{2a(p-q)}$

$m = \frac{p+q}{2}$

$\therefore$ Chord $y - ap^2 = \frac{p+q}{2}(x - 2ap)$

$y = \frac{(p+q)x}{2} - ap^2 - apq + ap^2$

$y = \left(\frac{p+q}{2}\right)x - apq$ ✓

y intercept of chord $y = mx + b$ is b.

y intercept of $y = \left(\frac{p+q}{2}\right)x - apq$ is $-apq$

ie $b = -apq$

$\therefore pq = \frac{-b}{a}$ ✓

14 c ii) $(p+q)^2 = p^2+q^2+2pq$

Using $m = \frac{p+q}{2}$ and $pq = \frac{-b}{a}$

$$(2m)^2 = p^2+q^2-\frac{2b}{a}$$ ✓

$$p^2+q^2 = 4m^2+\frac{2b}{a}$$

c iii) $x = -apq(p+q)$ $\quad$ $y = a(2+p^2+q^2+pq)$

$x = -a\left(\frac{-b}{a}\right)2m$ $\quad$ $y = a\left(2+4m^2+\frac{2b}{a}-\frac{b}{a}\right)$

$x = 2bm$ ✓ $\quad$ $y = a\left(4m^2+2+\frac{b}{a}\right)$ ✓

$\therefore$ N is $x = 2bm$, $y = 4am^2+2a+b$

iv) $x = 2bm$

$b = \frac{x}{2m}$

sub in y $\quad$ $y = 4am^2+2a+\frac{x}{2m}$

$$2my = 8am^3+4am+x$$ ✓

$$x+(-2m)y = -4am-8am^3$$

$$x+(-2m)y = 2a(-2m)+a(-2m)^3 \quad *$$

compared to normal $x+py = 2ap+ap^3$ means * is a normal as p has been replaced with $-2m$ as the parameter.

$\therefore$ Locus is a straight line which is a normal to the parabola at $(2a \times -2m, a(2m)^2)$ ie at $(-4am, 4am^2)$ ✓