BAULKHAM HILLS HIGH SCHOOL

TRIAL 2014

YEAR 12 TASK 4

Mathematics Extension 1

General Instructions

- Reading time – 5 minutes
- Working time – 120 minutes
- Write using black or blue pen
- Board-approved calculators may be used
- Show all necessary working in Questions 11-14
- Marks may be deducted for careless or badly arranged work

Total marks – 70
Exam consists of 11 pages.

This paper consists of TWO sections.

<u>Section 1</u> – Page 2-4 (10 marks)
Questions 1-10

- Attempt Question 1-10

<u>Section II</u> – Pages 5-10 (60 marks)

- Attempt questions 11-14

Table of Standard Integrals is on page 11

Section I - 10 marks
Use the multiple choice answer sheet for question 1-10

1. Given the equation $A = 10e^{-kt}$, what is the value of k given that $A = 3.6$ and $t = 5$.

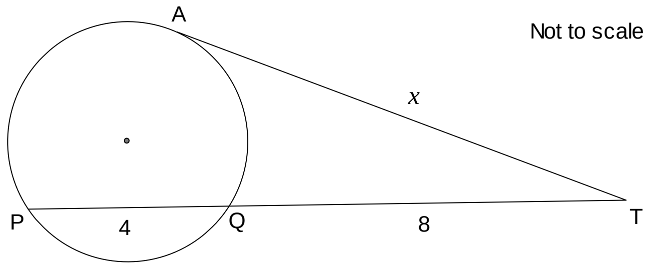
(A) -0.717

(B) -0.204

(C) 0.204

(D) 0.717

2.



In the diagram above, TA is a tangent and PQ is a chord produced to T. The value of x is

(A) 12

(B) $2\sqrt{3}$

(C) $4\sqrt{2}$

(D) $4\sqrt{6}$

3. How many distinct permutations of the letter of the word "$D\ I\ V\ I\ D\ E$" are possible in a straight line when the word begins and ends with the letter D

(A) 12

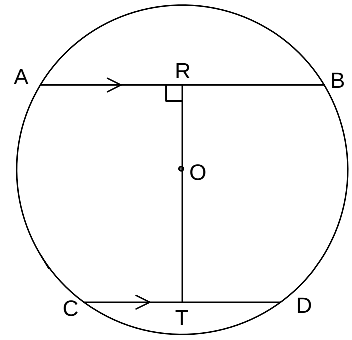
(B) 180

(C) 360

(D) 720

4.	The coordinates of the point that divides the interval joining $(-7,5)$ and $(-1,-7)$ externally in the ratio $1:3$ are (A) $(-10,8)$ (B) $(-10,11)$ (C) $(2,8)$ (D) $(2,11)$
5.	What is the domain and range of $y = 2\cos^{-1}\frac{3x}{2}$? (A) $D = \left\{x: -\frac{2}{3} \leq x \leq \frac{2}{3}\right\}$, $R = \{y: 0 \leq y \leq 2\pi\}$ (B) $D = \left\{x: -\frac{3}{2} \leq x \leq \frac{3}{2}\right\}$, $R = \{y: 0 \leq y \leq 2\pi\}$ (C) $D = \left\{x: -\frac{2}{3} \leq x \leq \frac{2}{3}\right\}$, $R = \left\{y: 0 \leq y \leq \frac{\pi}{2}\right\}$ (D) $D = \left\{x: -\frac{3}{2} \leq x \leq \frac{3}{2}\right\}$, $R = \left\{y: 0 \leq y \leq \frac{\pi}{2}\right\}$
6.	Which of the following is the general solution of $3\tan^2 x - 1 = 0$, where n is an integer? (A) $n\pi \pm \frac{\pi}{6}$ (B) $n\pi \pm \frac{\pi}{3}$ (C) $2n\pi \pm \frac{\pi}{6}$ (D) $2n\pi \pm \frac{\pi}{3}$
7.	The displacement of a particle moving in simple harmonic motion is given by $x = 3\cos \pi t$ where t is the time in seconds. The period of oscillation is: (A) π (B) $\frac{2\pi}{3}$ (C) 2 (D) 3

8. AB and CD are parallel chords in a circle, which are 10cm apart.
$OR \perp AB,\ AB = 14$cm and $CD = 12$cm.



Find the diameter of the circle to 1 decimal place

(A) 4.4cm

(B) 8.2cm

(C) 14.8cm

(D) 16.5cm

9. The domain of $f(x) = \log_e[(x-4)(5-x)]$ is

(A) $4 \leq x \leq 5$

(B) $x \leq 4,\ x \geq 5$

(C) $4 < x < 5$

(D) $x < 4, x > 5$

10. Which of the following represents the derivate of $y = \sin^{-1}\left(\frac{1}{x}\right)$?

(A) $\frac{1}{x\sqrt{x^2-1}}$

(B) $\frac{1}{\sqrt{x^2-1}}$

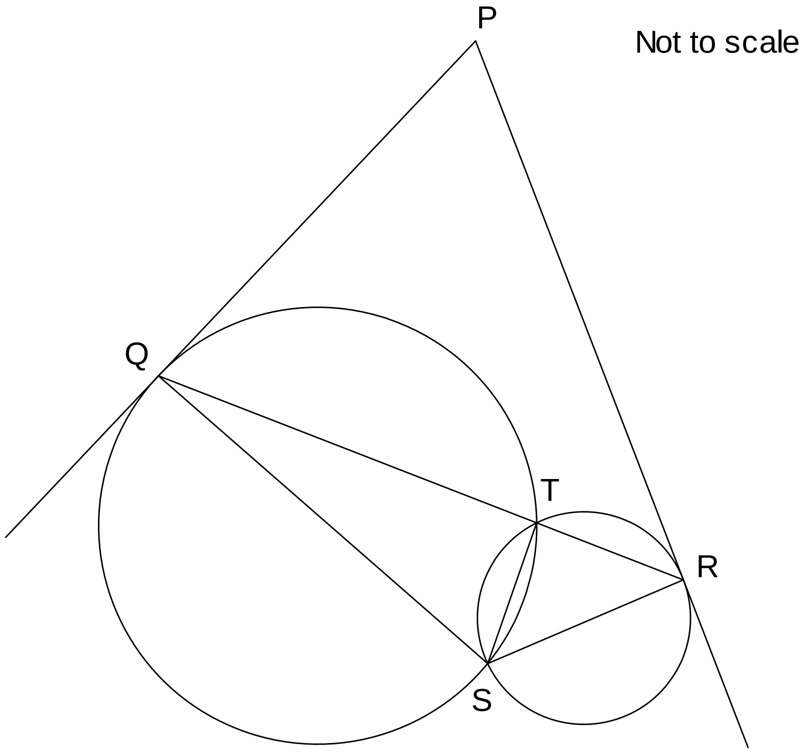
(C) $\frac{-1}{x\sqrt{x^2-1}}$

(D) $\frac{-1}{\sqrt{x^2-1}}$

End of Section 1

Section II – Extended Response
All necessary working should be shown in every question.

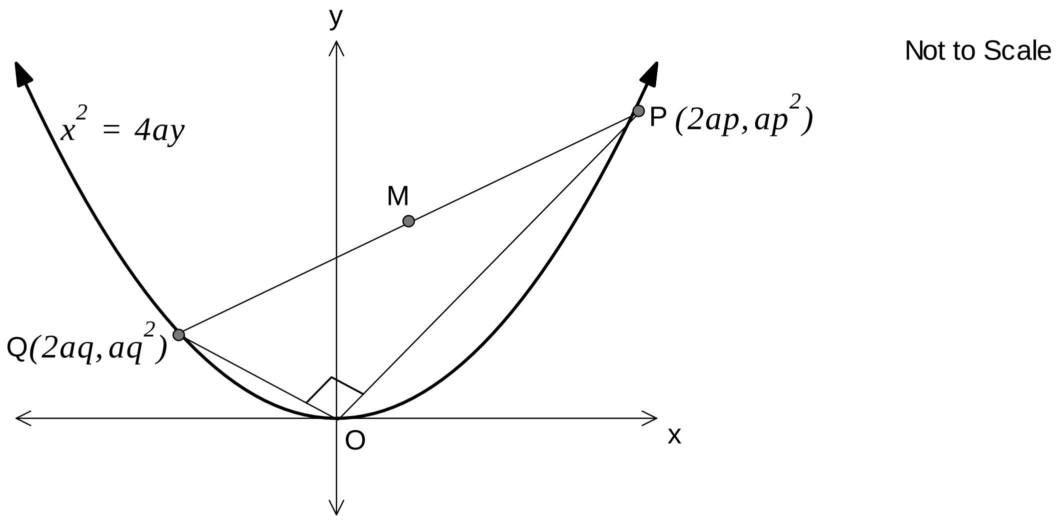
Question 11 **(15 marks)** - Start on the appropriate page in your answer booklet		**Marks**
a)	Evaluate $\int_0^{\frac{\pi}{4}} \cos^2 4x \; dx$	**3**
b)	Find $\int \frac{dx}{x(\log_e x)^{11}}$, using the substitution $u = \log_e x$	**2**
c)	Prove the identity $$\frac{1+\sin 2x+\cos 2x}{\cos x+\sin x} = 2\cos x$$	**2**
d)	Solve for x $$\frac{4}{x-1} \leq 3$$	**3**
e)	(i) Show that a root of the continuous function $f(x) = x^3 - \ln(x+1)$ lies between 0.8 and 0.9.	**1**
	(ii) Hence use the halving the interval method to find the value of the root correct to 1 decimal place.	**1**
f)	(i) Find $\frac{d}{dx}\left[\tan^{-1} x + \tan^{-1}\left(\frac{1}{x}\right)\right]$	**2**
	(ii) Hence sketch $y = \tan^{-1} x + \tan^{-1}\left(\frac{1}{x}\right)$ for $-2 \leq x \leq 2$	**1**
	End of Question 11	

Question 12 (15 marks) - Start on the appropriate page in your answer booklet		**Marks**
a)	When a polynomial $P(x)$ is divided by $x^2 - 4$ the remainder is $2x + 3$. What is the remainder when $P(x)$ is divided by $x - 2$	**2**
b)	In the given diagram, PQ and PR are tangents and Q, T, R are collinear.  Copy or trace the diagram in to your writing booklet. Prove that the points P, Q, S, R are concyclic.	**3**

Question 12 continues on the following page

Question 12 (continued)

c)



Points P$(2ap, ap^2)$ and $Q(2aq, aq^2)$ lies on the parabola $x^2 = 4ay$.
The chord PQ subtends a right angle at the origin.

(i) Prove $pq = -4$ **2**

(ii) Find the equation of the locus of M, the midpoint of PQ. **3**

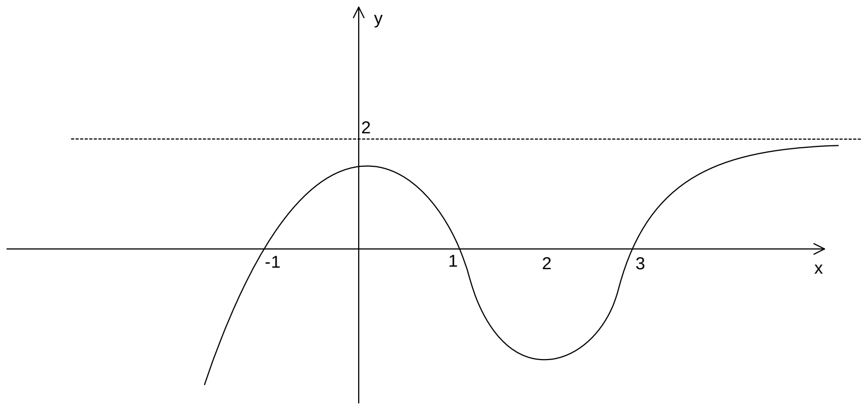
d) Find the coefficient of x^4 in the expression of $\left(x - \frac{2}{x}\right)^{12}$ **2**

e) Prove by mathematical induction **3**

$$1 \times 2 + 2 \times 2^2 + 3 \times 2^3 + \cdots + n \times 2^n = (n-1)2^{n+1} + 2$$

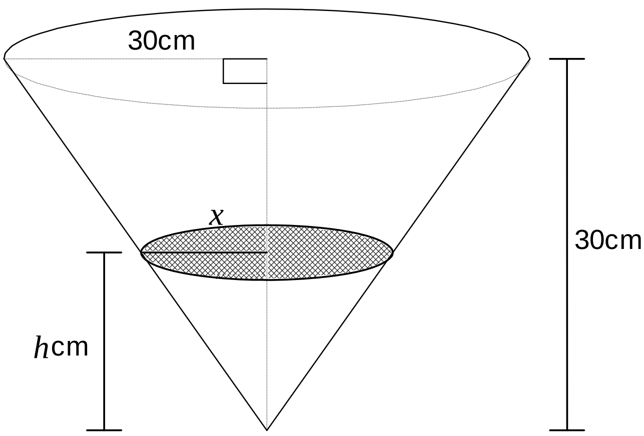
for positive integers $n \geq 1$

End of Question 12

Question 13 (15 marks) - Start on the appropriate page in your answer booklet		**Marks**
a)	(i) Express $\sqrt{3}\sin x - \cos x$ in the form $R\sin(x-\alpha)$ where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$.	**2**
	(ii) Hence state the least value of $\sqrt{3}\sin x - \cos x$ and the smallest positive value of x for this least value to occur.	**2**
b)	In the cubic equation $3x^3 - (2k-4)x^2 + 5x + k^2 = 0$ the sum of the roots is equal to twice their product. Find the values of k.	**3**
c)	Find the number of arrangements of the letters of the word $PENCILS$ if there are 3 letters between E and I.	**2**
d)	Below is the graph of a function $y = f(x)$  Copy the diagram in your booklet, and on the same set of axes sketch a possible graph for $y = f'(x)$.	**2**
e)	It is estimated that the rate of increase in the population of a particular species of bird is given by the equation $$\frac{dP}{dt} = kP(L-P)$$ where k and L are positive constants.	
	(i) Verify that for any positive constant c, the expression $P = \dfrac{Lc}{c + e^{-kLt}}$ satisfies the above differential equation.	**3**
	(ii) What can be deduced about P as t increases?	**1**
	End of Question 13	

Question 14 (15 marks) - Start on the appropriate page in your answer booklet

a)



Water is poured into a conical vessel at a constant rate of 24cm^3/s.
The depth of water is $h$cm at any time t seconds.

(i) Show that the volume of water is given by $V = \frac{1}{3}\pi h^3$. **1**

(ii) Find the rate at which the depth of water is increasing when $h = 16$cm. **2**

(iii) Hence find that rate of increase of the area of surface of the liquid when $h = 16$. **1**

b) The acceleration of a particle is given by the equation $\frac{d^2x}{dt^2} = 8x(x^2 + 1)$, where x is the displacement in centimetres from a fixed point O, after t seconds.
Initially the particle is moving from O with speed 2cm/s in a negative direction.

(i) Prove the general result $\ddot{x} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$. **2**

(ii) Hence show that the speed is given by $2(x^2 + 1)$ cm/s. **2**

(iii) Find an expression for x in terms of t. **2**

Question 14 continues on the following page

Question 14 (continued)

c) A projectile is fired from the origin with velocity V with an angle of elevation θ, where $\theta \neq \frac{\pi}{2}$.

YOU MAY ASSUME

$$x = Vt\cos\theta\,,\;\; y = -\frac{1}{2}gt^2 + Vt\sin\theta$$

Where x and y are the horizontal and vertical displacements from O, t seconds after firing

(i) Show the equation of flight can be expressed as **2**

$$y = x\tan\theta - \frac{1}{4h}x^2(1+\tan^2\theta) \quad \text{where } h = \frac{V^2}{2g}$$

(ii) Show that a point (X, Y) can be hit by firing at 2 different angles θ_1 and θ_2 provided $X^2 < 4h(h - Y)$. **2**

(iv) Show that no point above the x-axis can be hit by firing at 2 different angles θ_1 and θ_2 satisfying both $\theta_1 < \frac{\pi}{4}$ and $\theta_2 < \frac{\pi}{4}$. **1**

End of Paper.

STANDARD INTEGRALS

$$\int x^n dx = \frac{1}{n+1} x^{n+1}, \ n \neq -1; \ x \neq 0, \ if \ n < 0$$

$$\int \frac{1}{x} dx = \ln x, \quad x > 0$$

$$\int e^{ax} dx = \frac{1}{a} e^{ax}, \ a \neq 0$$

$$\int \cos axdx = \frac{1}{a} \sin ax, \ a \neq 0$$

$$\int \sin axdx = -\frac{1}{a} \cos ax, \ a \neq 0$$

$$\int \sec^2 axdx = \frac{1}{a} \tan ax, \ a \neq 0$$

$$\int \sec ax \tan axdx = \frac{1}{a} \sec ax, \ a \neq 0$$

$$\int \frac{1}{a^2 + x^2} dx = \frac{1}{a} \tan^{-1} \frac{x}{a}, \ a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1} \frac{x}{a}, \ a > 0, \ -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} dx = \ln(x + \sqrt{x^2 - a^2}), \ x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} dx = \ln(x + \sqrt{x^2 + a^2})$$

NOTE: $\ln x = \log_e x, \ x > 0$

EXT I SOLUTIONS

1) $3.6 = 10e^{-5k}$

$0.36 = e^{-5k}$

$k = 0.204$ [C]

2) $x^2 = 12 \times 8$

$= 96$

$x = \sqrt{96}$

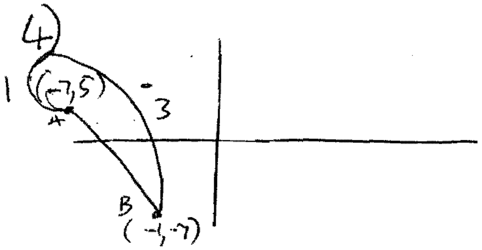
$= 4\sqrt{6}$ [D]

3) [D] $\frac{4!}{2!}$ [D]

$\frac{4 \times 3 \times 2}{2} = 12$

[A]

4)



$X = \frac{1 \times -1 + -3 \times -7}{-2}$

$= \frac{-1 + 21}{-2}$

$= -10$

$Y = \frac{1 \times -7 + -3 \times 5}{-2}$

$= \frac{-7 + -15}{-2} = 11$

[B]

5) $y = 2\cos^{-1}\frac{3x}{2}$

$\frac{y}{2} = \cos^{-1}\frac{3x}{2}$

$-1 \le \frac{3x}{2} \le 1$

$-\frac{2}{3} \le x \le 2/3$

$0 \le \frac{y}{2} \le \pi$

$0 \le y \le 2\pi$

[A]

6) $3\tan^2 x - 1 = 0$

$\tan^2 x = \frac{1}{3}$

$\tan x = \pm\frac{1}{\sqrt{3}}$

$x = \pi/6,\ \pi - \frac{\pi}{6},\ \pi + \frac{\pi}{6}$

$2\pi - \pi/6,\ 2\pi + \pi/6$

$= n\pi \pm \pi/6$

[A]

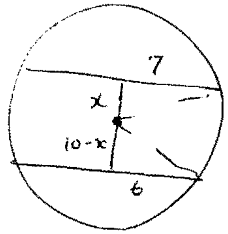
7) $x = 3\cos\pi t$

Period $= \frac{2\pi}{\pi}$

$= 2$

[C]

8/



$7^2 + x^2 = (10-x)^2 + 6^2$

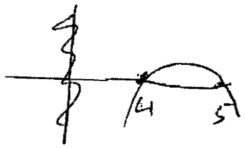
$x = \frac{80}{27}$

$r = \sqrt{7^2 + \left(\frac{80}{27}\right)^2}$

$= 8.24\ldots$

$\therefore d = 16.5$ [D]

(9) $(x-4)(5-x) > 0$



$4 < x < 5$ [C]

10. $y = \sin^{-1}\left(\frac{1}{x}\right)$

$\frac{dy}{dx} = \frac{1}{\sqrt{1-\left(\frac{1}{x}\right)^2}} \cdot \frac{-1}{x^2}$

$= \frac{-1}{x^2\sqrt{1-\frac{1}{x^2}}}$

$= \frac{-1}{x\sqrt{x^2-1}}$ [C]

Q11

a) i) $\cos 2x = 2\cos^2 x - 1$

$\cos^2 x = \frac{1}{2}[\cos 2x + 1]$

$\cos^2 4x = \frac{1}{2}[\cos 8x + 1]$.

$\int_0^{\pi/4} \cos^2 4x\, dx = \frac{1}{2}\int_0^{\pi/4} \cos 8x + 1\, dx$ (1)

$= \frac{1}{2}\left[\frac{1}{8}\sin 8x + x\right]_0^{\pi/4}$ (1)

$= \frac{1}{2}\left[\left(0 + \frac{\pi}{4}\right) - 0\right]$

$= \pi/8$ (1)

b) $\int \frac{dx}{x(\log x)^{11}}$

$u = \log x$

$\frac{du}{dx} = \frac{1}{x}\, dx$

$= \int \frac{du}{u^{11}}$ (1)

$= \int u^{-11}$

$= \frac{u^{-10}}{-10} + c$ (1)

$= \frac{1}{-10}(\log x)^{-10} + c$

(1)

c) $LHS = \frac{1 + 2\sin x\cos x + 2\cos^2 x - 1}{\cos x + \sin x}$ (1)

$= \frac{2\cos x(\sin x + \cos x)}{\cos x + \sin x}$

$= 2\cos x$

$= RHS$

d) $\frac{4}{x-1} \leq 3.$

$4 \leq 3x - 3$

$\frac{7}{3} \leq x$

$x < 1,\ x \geq 2\frac{1}{3}$

if $x \leq 1,\ x \geq 2\frac{1}{3}$ (2 marks)

if $1 < x \leq 2\frac{1}{3}$ (2 marks)

11e) (i) $f(x) = x^2 - \ln(x+1)$

$f(0.8) = -0.075$

$f(0.9) = 0.046$

$\therefore f(0.8)\ \&\ f(0.9)$ opp. sign $\therefore$ Root exists (1)

(ii) Let 1st APPROX $= 0.85$.

$f(0.85) = -0.001$

$\therefore$ Root lies between $0.85\ \&\ 0.9$

$\therefore$ Root $= 0.9$ to 1 dec place. (1)

11f)

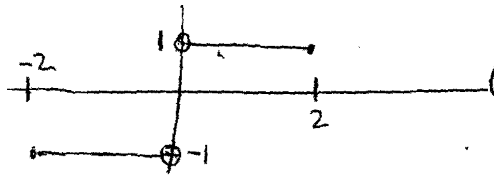
c) $\frac{d}{dx}\left[\tan^{-1} x + \tan^{-1}\frac{1}{x}\right]$

$= \frac{1}{1+x^2} + \frac{1}{1+\left(\frac{1}{x}\right)^2} \cdot \frac{-1}{x^2}$

(1) $= \frac{1}{1+x^2} - \frac{1}{x^2+1}$

$= 0$ (1)

$\therefore y = \tan^{-1} x + \tan^{-1}\left(\frac{1}{x}\right)$ is horizontal



(1)

12a) $P(x) = (x^2-4)(Q(x)) + 2x+3$

$P(2) = 4+3$

$= 7.$

$\therefore$ Rem $= 7.$

b) $\angle PRT = \angle TSR = x^\circ$ ($\angle$ between tang & chord = $\angle$ in ALT segment) (1)

SIMILARLY

$\angle PQR = \angle QST = y^\circ.$

$\angle QSR = x^\circ + y^\circ.$

$\angle QPR = 180 - x - y$ ($\angle$ sum of Δ) (1)

$\therefore \angle QSR + \angle QPR = 180^\circ$ (1)

$\therefore$ P, Q, S, R are CONCYCLIC.

c) (i) $m_{OP} = \frac{ap^2-0}{2ap-0}$

$= \frac{p}{2}$

Similarly $m_{OQ} = \frac{q}{2}$ (1)

$m_{OP} \times m_{OQ} = -1$ since lines $\perp$.

ie $\frac{p}{2} \times \frac{q}{2} = -1$

$\therefore pq = -4.$ (1)

(ii) Midpoint $\left(\frac{2ap+2aq}{2}, \frac{ap^2+aq^2}{2}\right)$

$= \left[a(p+q), \frac{a(p^2+q^2)}{2}\right]$ (1)

$x = a(p+q)$

$\therefore p+q = \frac{x}{a}$ $\quad y = \frac{a\left[(p+q)^2 - 2pq\right]}{2}$ (1)

$y = \frac{a\left[\frac{x^2}{a^2} - 2(-4)\right]}{2}$

$y = \frac{x^2}{2a} + 4a.$ (1) or equivalent

12d) $T_{k+1} = {}^{12}C_k\, x^k \left(\frac{-2}{x}\right)^{12-k}$ (1)

$= {}^{12}C_k\, x^k (-2)^{12-k} x^{k-12}$

$= {}^{12}C_k (-2)^{12-k} x^{2k-12}$

$\therefore 2k-12 = 4$

$k = 8$

$\therefore$ coeff $= {}^{12}C_8 (-2)^4 = 7920$ (1)

e) Prove True $n=1$

Is $1 \times 2 = (1-1)2^2 + 2$

$= 2$ (1)

Yes.

2 ticks – 1 mark
3 ticks – 2 marks
4 ticks – 3 marks

ASSUME TRUE $n=k$.

$1\times 2 + 2\times 2^2 + 3\times 2^3 + \dots k\times 2^k = (k-1)2^{k+1} + 2.$

Prove True $n = k+1.$

$\dots 2 + \dots k\times 2^k + (k+1)\times 2^{k+1} = k \cdot 2^{k+2} + 2.$ (1)

$LHS = (k-1)2^{k+1} + 2 + (k+1)\times 2^{k+1}$

$= 2^{k+1}(2k) + 2$

$= 2^{k+1} \cdot 2^1 \times k + 2$

$= 2^{k+2} \cdot k + 2$ (1)

$= RHS.$ $\therefore$ true for $n=k+1$ if true for $n=k$. $\therefore$ true for $n=1, n=2$ & for all n by... (1)

13\. a)(i) $\sqrt{3}\sin x - \cos x = R\sin(x+\alpha)$

$= R\sin x\cos\alpha + R\cos x\sin\alpha$

$\therefore R\cos\alpha = \sqrt{3}$

$R\sin\alpha = -1$

$\therefore \tan\alpha = \frac{-1}{\sqrt{3}}$

$\therefore \alpha = \frac{5\pi}{6}, \frac{11\pi}{6}$ (1)

but $R>0$, $\cos\alpha>0$, $\sin\alpha<0$

$\therefore \alpha$ in Quad 4. $\therefore \alpha = \frac{5\pi}{6}$.

$R^2 = (-1)^2 + (\sqrt{3})^2$

$= 4$ (1)

$R = 2$

$\therefore \sqrt{3}\sin x - \cos x = 2\sin(x + \frac{6\pi}{6})$.

(ii) $\therefore$ Least value of $\sqrt{3}\sin x - \cos x = -2$. (1)

$\sin(x + \frac{11\pi}{6}) = -1$

$x + \frac{11\pi}{6} = \frac{3\pi}{2}, \frac{7\pi}{2}$

but $x > 0$ $\quad x = \frac{21\pi}{6} - \frac{11\pi}{6} \Rightarrow x = \frac{5\pi}{3}$ (1)

13b) $\alpha+\beta+\gamma = \frac{2K-4}{3}$

$\alpha\beta\gamma = \frac{-K^2}{3}$

$\therefore 2K-4 = -2K^2$ (1)

$2K^2+2K-4=0$

$K^2+K-2=0$

$(K+2)(K-1)=0$

$\therefore K = -2, 1$ (1)

13c) E _ _ _ I

(1) 3 letters between E & I in $^5P_3$ ways 60

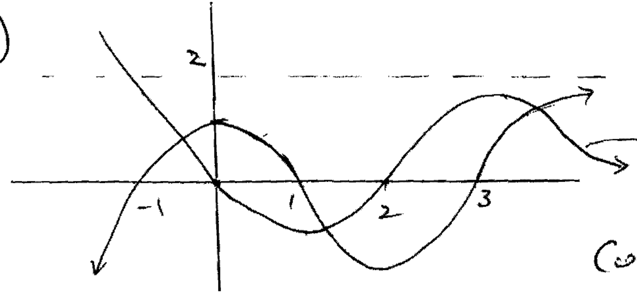
no. ways $60 \times 3! \times 2$

$= 720$ ways

(2) correct

(1) progress

13d)



Correct $0 \le x \le 2$ (1)

Correct the rest $x<0$ $x>2$ (1)

e) $\frac{dP}{dt} = KP(L-P)$

i) $P = \frac{LC}{C+e^{-KLt}} = LC(C+e^{-KLt})^{-1}$

$\frac{dP}{dt} = -LC(C+e^{-KLt})^{-2} \cdot -KLe^{-KLt}$

$= \frac{KL^2Ce^{-KLt}}{(C+e^{-KLt})^2}$

CONSIDER: $\frac{dP}{dt} = KP(L-P)$

$= K \cdot \frac{LC}{C+e^{-KLt}}\left(L - \frac{LC}{C+e^{-KLt}}\right)$

$= \frac{KLC}{(C+e^{-KLt})^2} \cdot LC + Le^{-KLt} - LC$

$= \frac{KL^2Ce^{-KLt}}{(C+e^{-KLt})^2} = \frac{dP}{dt}$

(1) - Differentiating

(1) Sign. further progress

(1) fully correct.

ii) as $t \to \infty$ $\quad P \to L$ (1)

14a) i) by SIM △'s

$\frac{r}{30} = \frac{h}{30}$ ① (matching sides in ||| △'s)

$\therefore r = h$.

$V = \frac{1}{3}\pi r^2 h$

$= \frac{1}{3}\pi h^2 . h$

$= \frac{1}{3}\pi h^3$.

ii) $\frac{dV}{dt} = \frac{dh}{dt}\cdot\frac{dV}{dh}$

$24 = \frac{dh}{dt}\cdot \pi h^2$ ①

$h = 16$

$\frac{dh}{dt} = \frac{24^3}{\pi . 16^2 32}$

$= \frac{3}{32\pi}$ cm/s. ①

iii) $S = \pi r^2$

$= \pi h^2$.

$\frac{dS}{dt} = \frac{dS}{dh}\cdot\frac{dh}{dt}$

$= 2\pi h \cdot \frac{3}{32\pi}$ ①

$= 3$ cm²/s.

b)i) $\frac{d}{dx}\left(\frac{1}{2}v^2\right) = \frac{d(\frac{1}{2}v^2)}{dv}\cdot\frac{dv}{dx}$

$= v\frac{dv}{dx}$

$= \frac{dx}{dt}\cdot\frac{dv}{dx}$

$= \frac{dv}{dt}$

$= \ddot{x}$

$\ddot{x} = 8x(x^2+1)$

ii) $\frac{1}{2}v^2 = \int 8x^3 + 8x\,dx$

$\frac{1}{2}v^2 = \frac{8x^4}{4} + 4x^2 + C$ ①

$x = 0,\ v = -2$

$2 = C$.

$\frac{1}{2}v^2 = 2x^4 + 4x^2 + 2$

$v^2 = 4x^4 + 8x^2 + 4$

$= 4(x^4 + 2x^2 + 1)$

$v^2 = 4(x^2+1)^2$

$v = \pm 2(x^2+1)$.

speed $= 2(x^2+1)$ cm/s ①

iii) $v = \pm 2(x^2+1)$

but when $x = 0$, $v = -2$.

$\therefore v = -2(x^2+1)$. ①

$\frac{dx}{dt} = -2(x^2+1)$

$\frac{dt}{dx} = \frac{1}{-2(x^2+1)}$.

$t = -\frac{1}{2}\int \frac{1}{x^2+1}\,dx$

$t = -\frac{1}{2}\tan^{-1}(x) + C$

$t = 0,\ x = 0,\ \therefore C = 0$

$t = -\frac{1}{2}\tan^{-1}(x)$

$-2t = \tan^{-1}(x) \implies x = \tan(-2t)$ ①

$= -\tan(2t)$

14c) $x = Vt\cos\theta$.

$\therefore t = \frac{x}{V\cos\theta}$.

$y = -\frac{1}{2}g\left(\frac{x}{V\cos\theta}\right)^2 + \frac{Vx}{V\cos\theta}\sin\theta$.

simplify to $\tan\theta$ ①

$= -\frac{gx^2}{2V^2\cos^2\theta} + x\tan\theta$.

$= x\tan\theta - \frac{gx^2}{2V^2}\sec^2\theta$

$= x\tan\theta - \frac{gx^2}{2V^2}(1+\tan^2\theta)$

clearly show relationship $\frac{gx^2}{V^2}$ to $\frac{1}{h}$ ①

but $V^2 = 2gh$

$y = x\tan\theta - \frac{gx^2}{4gh}(1+\tan^2\theta)$

$= x\tan\theta - \frac{x^2}{4h}(1+\tan^2\theta)$ ①

ii) If Particle to Pass thru (X, Y)

$$Y = X\tan\theta - \frac{X^2}{4h}(1+\tan^2\theta)$$

$$\therefore X^2\tan^2\theta - 4hX\tan\theta + (4hY + X^2) = 0.$$ (1)

FOR DIFFERENT ROOTS $\Delta > 0$

$$\therefore 16h^2X^2 - 4X^2(4hY+X^2) > 0$$

$$4X^2(4h^2 - 4hY - X^2) > 0$$

$$\therefore \text{since } 4X^2 > 0, \quad 4h^2 - 4hY - X^2 > 0$$

$$\therefore 4h(h-Y) > X^2.$$ (1)

identifies $\tan\theta$ as the variable & $y \neq 0$

uses $\Delta > 0$ to find 2 solutions

iii) If $\tan\theta_1$, $\tan\theta_2$ are roots of quadratic eqn

$$X^2\tan^2\theta - 4hX\tan\theta + (4hY + X^2) = 0$$

$$\therefore \tan\theta_1 \cdot \tan\theta_2 = \frac{4hY + X^2}{X^2}$$

$$= 1 + \frac{4hY}{X^2}$$

$$> 1$$

$$\therefore \tan\theta_1 \text{ or } \tan\theta_2 > 1$$

$$\therefore \theta_1 \text{ or } \theta_2 > \pi/4.$$ (1)