Blacktown Boys' High School

2020

HSC Trial Examination

Mathematics Extension 1

General Instructions

- Reading time – 10 minutes
- Working time – 2 hours
- Write using black pen
- Calculators approved by NESA may be used
- A reference sheet is provided for this paper
- All diagrams are not drawn to scale
- In Questions 11 – 14, show relevant mathematical reasoning and/or calculations

Total marks: 70

Section I – 10 marks (pages 3 – 7)

- Attempt Questions 1 – 10
- Allow about 15 minutes for this section

Section II – 60 marks (pages 8 – 12)

- Attempt Questions 11 – 14
- Allow about 1 hour and 45 minutes for this section

Assessor: X. Chirgwin

Student Name: ____________________________________

Teacher Name: ____________________________________

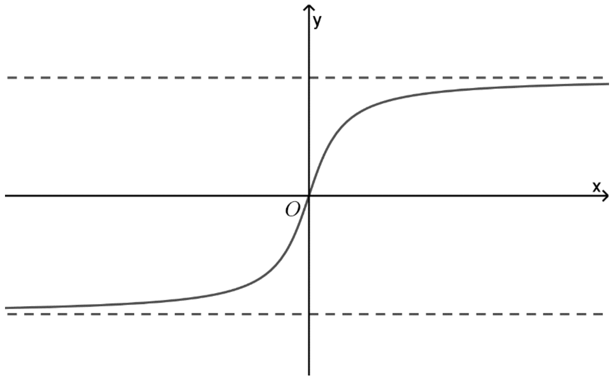
Students are advised that this is a trial examination only and cannot in any way guarantee the content or format of the 2020 Higher School Certificate Examination.

Section I

10 marks
Attempt Questions 1–10
Allow about 15 minutes for this section

Use the multiple choice answer sheet for Questions 1–10.

Q1. Part of the graph of $y = 3\tan^{-1} x$ is shown below.



The equation of its asymptotes are

A. $y = \pm 3$

B. $y = \pm \frac{\pi}{2}$

C. $y = \pm 3\pi$

D. $y = \pm \frac{3\pi}{2}$

Q2. The polynomial $2x^3 - 5x^2 - 16x + 24$ has zeroes α, β and γ.

What is the value of $\alpha\beta\gamma(\alpha + \beta + \gamma)$?

A. -30

B. -20

C. 20

D. 30

Q3. What is the angle between the vectors $\begin{pmatrix}-3\\2\end{pmatrix}$ and $\begin{pmatrix}1\\18\end{pmatrix}$?

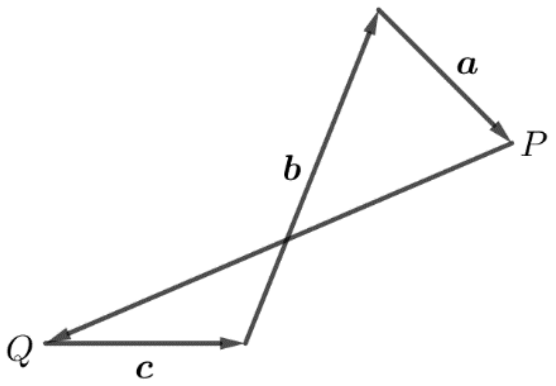
A. $\theta = \cos^{-1}\left(\frac{39}{65}\right)$

B. $\theta = \cos^{-1}\left(-\frac{39}{65}\right)$

C. $\theta = \cos^{-1}\left(\frac{33}{65}\right)$

D. $\theta = \cos^{-1}\left(-\frac{33}{65}\right)$

Q4. Express the vector $\overrightarrow{PQ}$ in terms of $\boldsymbol{a}, \boldsymbol{b}$ and $\boldsymbol{c}$.



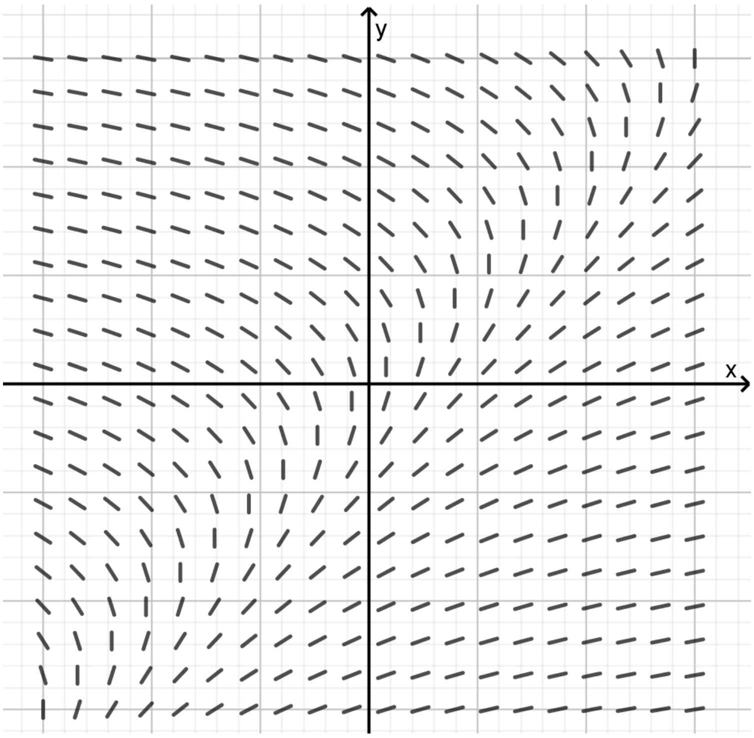
A. $-\boldsymbol{a}-\boldsymbol{b}-\boldsymbol{c}$

B. $\boldsymbol{a}+\boldsymbol{b}+\boldsymbol{c}$

C. $-\boldsymbol{a}+\boldsymbol{b}+\boldsymbol{c}$

D. $\boldsymbol{a}-\boldsymbol{b}-\boldsymbol{c}$

Q5. The differential equation that is best represented by the direction field below is



A. $\frac{dy}{dx} = x - y$

B. $\frac{dy}{dx} = \frac{1}{x - y}$

C. $\frac{dy}{dx} = y - x$

D. $\frac{dy}{dx} = \frac{1}{y - x}$

Q6. Which expression is equivalent to $\frac{\tan 5x - \tan 4x}{1 + \tan 5x \tan 4x}$?

A. $\frac{\tan x}{1 + \tan 20x}$

B. $\frac{\tan 5x}{1 + \tan 5x}$

C. $\tan 9x$

D. $\tan x$

Q7. Which expression is equal to $\int \cos^2 6x\, dx$?

A. $\dfrac{\cos^3 6x}{18} + c$

B. $\dfrac{1}{24}(12x + \cos 12x) + c$

C. $\dfrac{1}{24}(12x + \sin 12x) + c$

D. $\dfrac{1}{24}(12x - \sin 12x) + c$

Q8. By letting $u = \tan x$, $\int_0^{\frac{\pi}{3}} \tan^2 x \sec^2 x\, dx$ can be expressed as

A. $\int_0^{\frac{\pi}{3}} u^2 du$

B. $\int_0^{\sqrt{3}} u^2 du$

C. $\int_0^{\sqrt{3}} u du$

D. $\int_0^{\sqrt{3}} (u^4 + u^2) du$

Q9. A private institution course regulation requires that the same number of students achieve each grade from A to E where possible. What is the smallest number of students required to ensure at least one particular grade is awarded 7 times?

A. 30

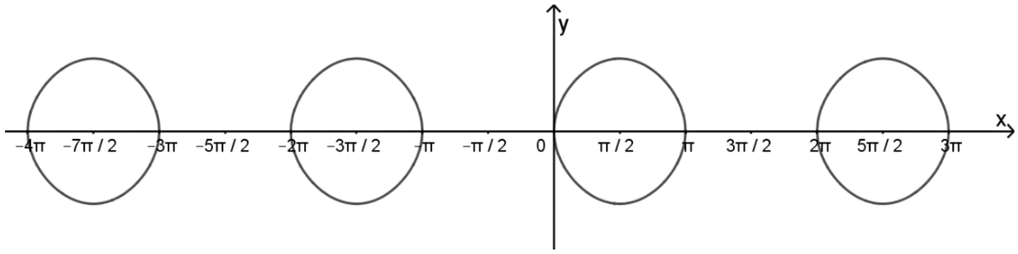
B. 31

C. 35

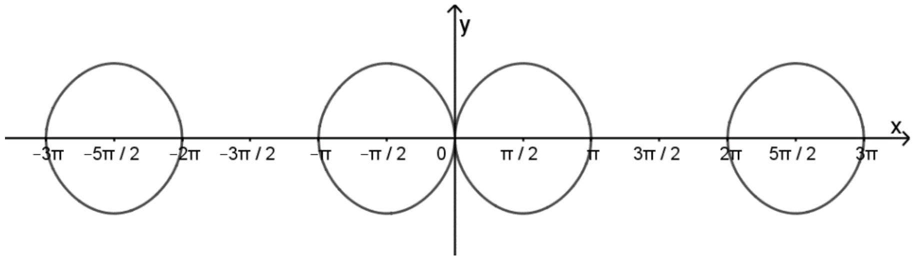
D. 36

Q10. Which graph best represents $y^2 = 3\sin|2x|$?

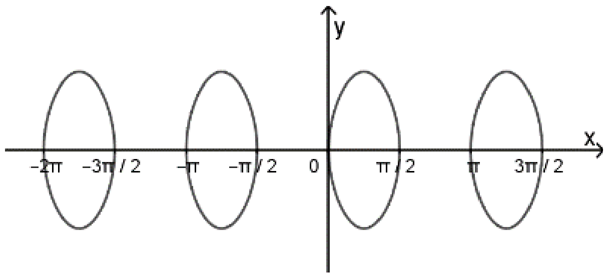
A.



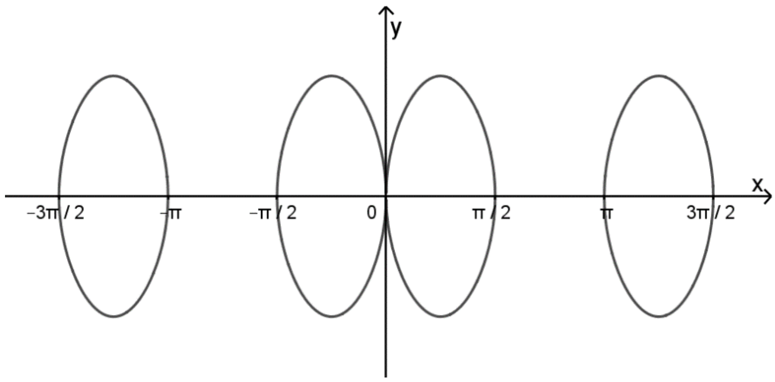
B.



C.



D.



End of Section I

Section II
60 Marks
Attempt Questions 11–14
Allow about 1 hour and 45 minutes for this section

Answer each question on a SEPARATE writing booklet. Extra writing booklets are available.

In Questions 11–14, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) Use a SEPARATE writing booklet.

a) Differentiate $y = 2\cos^{-1}(5x)$ **1**

b) Using the substitution $u = 2 - x^4$, find $\int x^3(2 - x^4)^{10}dx$ **3**

c) Solve $\dfrac{4}{3x+1} \leq 1$ **3**

d) A function $f(x)$ is given by $x^2 - 4x + 3$.

i) Explain why the domain of the function $f(x)$ must be restricted if $f(x)$ is to have an inverse functions. **1**

ii) Find the equation of the inverse function $f^{-1}(x)$ if the domain of $f(x)$ is restricted to $x \geq 2$. **2**

iii) State the domain and range of $f^{-1}(x)$, given the restriction in part ii). **2**

iv) On the same diagram, sketch the graphs of $y = f(x)$ and $y = f^{-1}(x)$, showing clearly the coordinates of the end points and the intercepts on the coordinate axes. **3**

End of Questions 11

Question 12 (15 marks) Use a SEPARATE writing booklet.

a) Bags of lollipops are supposed to contain 50 lollipops. Production records indicate that 92% of bags contain 50 lollipops. A batch of 15 bags is sampled. If more than 2 bags do not contain exactly 50 lollipops, production is stopped.

i) Find the probability, correct to four decimal places, that exactly 2 of the 15 bags selected do not contain 50 lollipops. **1**

ii) Find the probability, correct to four decimal places, that production is stopped. **2**

b) Two vectors are given by $\underset{\sim}{a} = 3\underset{\sim}{i} + m\underset{\sim}{j}$ and $\underset{\sim}{b} = -10\underset{\sim}{i} + n\underset{\sim}{j}$ where $m, n > 0$.

i) If $\left|\underset{\sim}{a}\right| = 5$ and $\underset{\sim}{a}$ is perpendicular to $\underset{\sim}{b}$, find the values of m and n. **2**

ii) Hence find the unit vector $\underset{\sim}{\hat{b}}$. **1**

c) Use the substitution $t = \tan\frac{x}{2}$ to show that $\sec x + \tan x = \tan\left(\frac{\pi}{4} + \frac{x}{2}\right)$ **3**

d) The polynomial $P(x) = 2x^3 + ax^2 + b$ has a double zero at $x = -5$.

i) Find the values of a and b. **2**

ii) Hence factorise $P(x)$ completely. **1**

e) Using the substitution $u = \sqrt{x}$ find $\int_0^3 \frac{1}{\sqrt{x}(9+x)}\,dx$ **3**

End of Questions 12

Question 13 (15 marks) Use a SEPARATE writing booklet.

a) Using mathematical induction, prove the following is true for all positive integers n **3**

$$1 + 6 + 11 + \cdots + (5n - 4) = \frac{1}{2}n(5n - 3)$$

b) Metal Fatigue is a phenomenon where a piece of steel will fail when repeatedly subjected to force F. The endurance limit is the force below which the steel will not break even if subjected to an infinite number of application of that force. Let the number of applications be n.

The force and the number of applications are related by the differential

equation $\dfrac{dF}{dn} = -k(F - F_0)$ where k and F_0 are constants.

i) Show that $F = 275e^{-k(n-1)} + F_0$ is a solution to the differential equation. **1**

ii) If $F = 350$ when $n = 1$, find the value of F_0. **1**

iii) Find the exact value of k if $F = 80$ when $n = 200$. **2**

iv) Find the endurance limit. **1**

c) i) Given that $5\cos\theta - 5\sqrt{3}\sin\theta = A\cos(\theta + \alpha)$, find values of A **2**

and α where $A > 0$ and $0 < \alpha < \dfrac{\pi}{2}$.

ii) Hence find the maximum value of $5\cos\theta - 5\sqrt{3}\sin\theta + 15$. **1**

d) Solve the differential equation $\dfrac{dy}{dx} = \dfrac{1}{(2-y)\sqrt{2-x^2}}$ given that when **4**

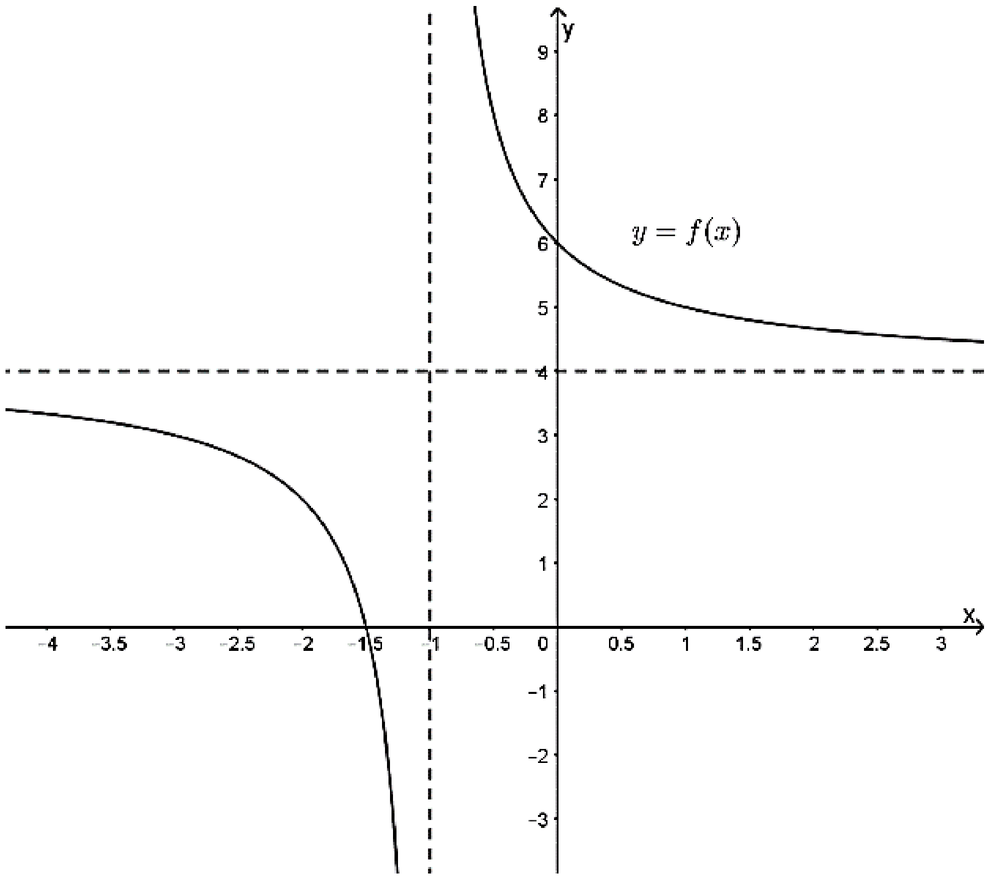
$x = 1, y = 0$. Express y as a function of x.

End of Questions 13

Question 14 (15 marks) Use a SEPARATE writing booklet.

a) The diagram below is a sketch of the graph of the function $y = f(x)$.

$$f(x) = \frac{2}{x+1} + 4$$



Draw a separate half-page graph of each of the following functions, showing all asymptotes and intercepts.

i) $y = |f(x)|$ **1**

ii) $y = f(|x|)$ **1**

iii) $y = \dfrac{1}{f(x)}$ **2**

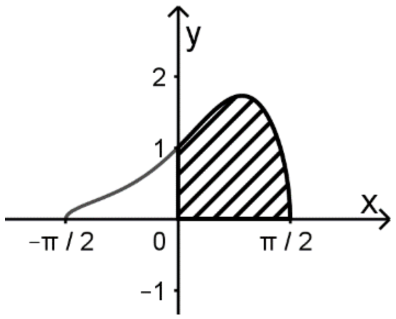
iv) $y^2 = f(x)$ **3**

Question 14 continues on next page

Question 14 (continued)

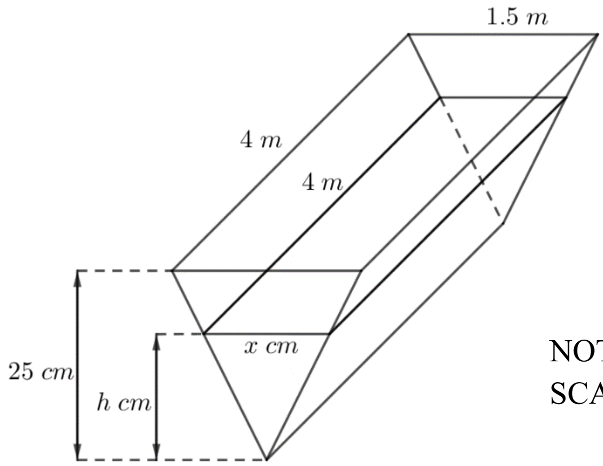
b) i) Show that $\frac{d}{dx}\left(e^{2\sin x}\right) = 2\cos x\, e^{2\sin x}$ **1**

ii) The region bounded by the graph $y = \sqrt{\cos x}\, e^{\sin x}$ and the x-axis between $x = 0$ and $x = \frac{\pi}{2}$ is rotated about the x-axis to form a solid. **3**



Using the result from part i), find the exact volume of this solid.

c) An open flat topped water trough in the shape of a triangular prism is being emptied through a hole in its base at a constant rate of 18000 cm^3 per second. Its top measures 1.5 metres by 4 metres and its triangular end has a vertical height of 25 centimetres. When the water depth is h centimetres the water surface measures x centimetres by 4 metres.



NOT TO SCALE

i) Show that when the water depth is h centimetres, the volume $V\ cm^3$ of water in the trough is given by $V = 1200h^2$ **2**

ii) Find the rate at which the depth of water is changing when $h = 20\ cm$. **2**

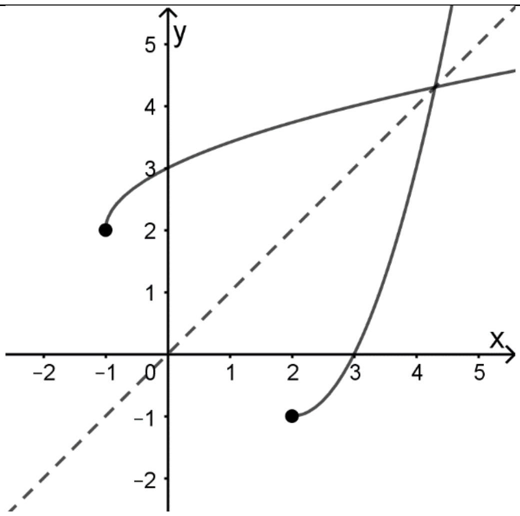
End of Paper

2020 Mathematics Extension 1 AT4 Trial Solutions

Section 1		
Q1	**D** Asymptote for $y = \tan^{-1} x$ is $y = \pm\frac{\pi}{2}$ Asymptote for $y = 3\tan^{-1} x$ is $y = \pm\frac{3\pi}{2}$	1 Mark
Q2	**A** $\alpha+\beta+\gamma = -\frac{-5}{2} = \frac{5}{2}$ $\alpha\beta\gamma = -\frac{24}{2} = -12$ $\alpha\beta\gamma(\alpha+\beta+\gamma) = \frac{5}{2} \times -12 = -30$	1 Mark
Q3	**C** $\cos\theta = \frac{-3\times 1 + 2\times 18}{\sqrt{(-3)^2+2^2}\times\sqrt{1^2+18^2}}$ $\cos\theta = \frac{33}{65}$ $\theta = \cos^{-1}\left(\frac{33}{65}\right)$	1 Mark
Q4	**A** $\boldsymbol{c}+\boldsymbol{b}+\boldsymbol{a} = \overrightarrow{QP}$ $\overrightarrow{PQ} = -\boldsymbol{a}-\boldsymbol{b}-\boldsymbol{c}$	1 Mark
Q5	**B** The vertical line segments on the line $y = x$, so option B and D are possibilities. Positive gradients for $x = 0$ and $y < 0$ then gives option B.	1 Mark
Q6	**D** $\frac{\tan 5x - \tan 4x}{1+\tan 5x \tan 4x}$ $= \tan(5x - 4x)$ $= \tan x$	1 Mark
Q7	**C** $\cos(2\times 6x) = 2\cos^2 6x - 1$ $\cos 12x + 1 = 2\cos^2 6x$ $\cos^2 6x = \frac{1}{2}(\cos 12x + 1)$ $\int \cos^2 6x\, dx$ $= \frac{1}{2}\int(\cos 12x + 1)dx$ $= \frac{1}{2}\left(\frac{1}{12}\sin 12x + x\right) + c$ $= \frac{1}{24}\sin 12x + \frac{1}{2}x + c$ $= \frac{1}{24}(\sin 12x + 12x) + c$	1 Mark

Q8	**B** $I = \int_0^{\frac{\pi}{3}} \tan^2 x \sec^2 x \, dx$ $u = \tan x$ $du = \sec^2 x \, dx$ $x = \frac{\pi}{3}, \quad u = \sqrt{3}$ $x = 0, \quad u = 0$ $I = \int_0^{\sqrt{3}} u^2 du$	1 Mark
Q9	**B** There are 5 grades from A to E. At least one grade awarded 7 times implies each group has at least 6. $5 \times 6 + 1 = 31$ ∴The least possible number of students in this group would be 31 students.	1 Mark
Q10	**D** $y^2 = 3\sin\|2x\|$ $y = \pm\sqrt{3\sin\|2x\|}$ $\|2x\|$ has the graph reflecting along the y axis, so option B and D are possibilities. $\sin 2x$ has period π, therefore the only option is D.	1 Mark

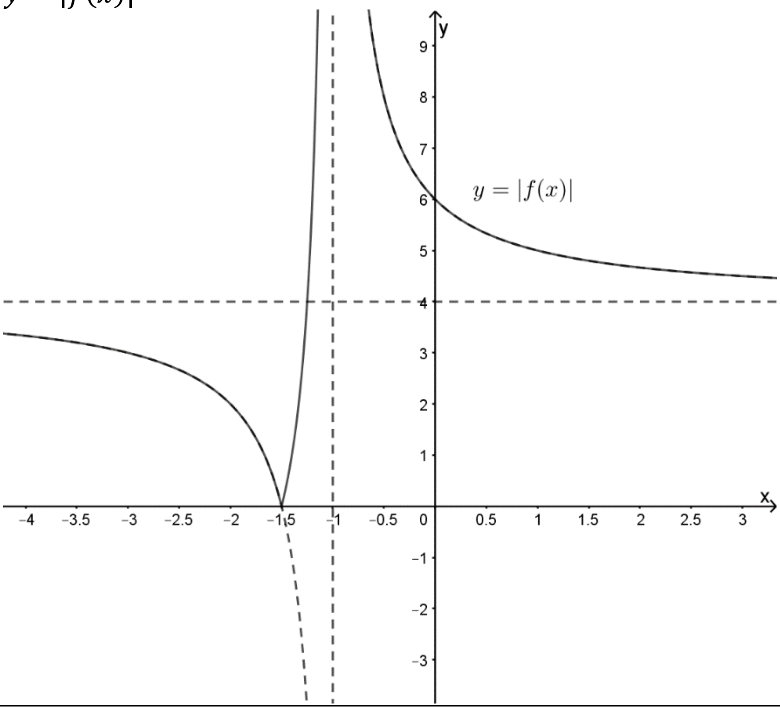
Section 2		
Q11a	$\frac{d}{dx}(2\cos^{-1}5x)$ $= 2 \times -\frac{5}{\sqrt{1-25x^2}}$ $= -\frac{10}{\sqrt{1-25x^2}}$	1 Mark Correct solution
Q11b	$I = \int x^3(2-x^4)^{10}dx$ $u = 2 - x^4$ $du = -4x^3dx$ $I = -\frac{1}{4}\int (2-x^4)^{10} \times -4x^3dx$ $I = -\frac{1}{4}\int u^{10}du$ $I = -\frac{1}{4} \times \frac{u^{11}}{11} + c$ $I = -\frac{u^{11}}{44} + c$ $I = -\frac{(2-x^4)^{11}}{44} + c$	3 Marks Correct solution 2 Marks Correct primitive function in terms of u 1 Mark Correct substitution
Q11c	$\frac{4}{3x+1} \leq 1 \qquad x \neq -\frac{1}{3}$ $4(3x+1) \leq (3x+1)^2$ $4(3x+1) - (3x+1)^2 \leq 0$ $(3x+1)[4-(3x+1)] \leq 0$ $(3x+1)(-3x+3) \leq 0$ $(3x+1)(x-1) \geq 0$ $x < -\frac{1}{3}, x \geq 1$	3 Marks Correct solution 2 Marks Identifies both important values 1 Mark Multiplies both sides by the square of the denominator
Q11di	$f(x) = x^2 - 4x + 3$ is a parabola. For each value of y value of $f(x)$ except the turning point, there are two x values. A horizontal line will cut the graph twice. If x and y are swapped, each x in the domain will have two y values, so the inverse will not be a function.	1 Mark Correct solution
Q11dii	$f(x) = x^2 - 4x + 3 \qquad x \geq 2$ $f(x) = (x^2 - 4x + 4) - 1$ $f(x) = (x-2)^2 - 1$ For $f^{-1}(x)$, swap x and y $x = (y-2)^2 - 1 \qquad y \geq 2$ $x + 1 = (y-2)^2$ $y - 2 = \pm\sqrt{x+1}$ $-\sqrt{x-1}$ is discarded as $y \geq 2$ $y = 2 + \sqrt{x+1}$ $\therefore f^{-1}(x) = 2 + \sqrt{x+1}$	2 Marks Correct solution 1 Mark Swaps x and y, and attempts to find inverse function

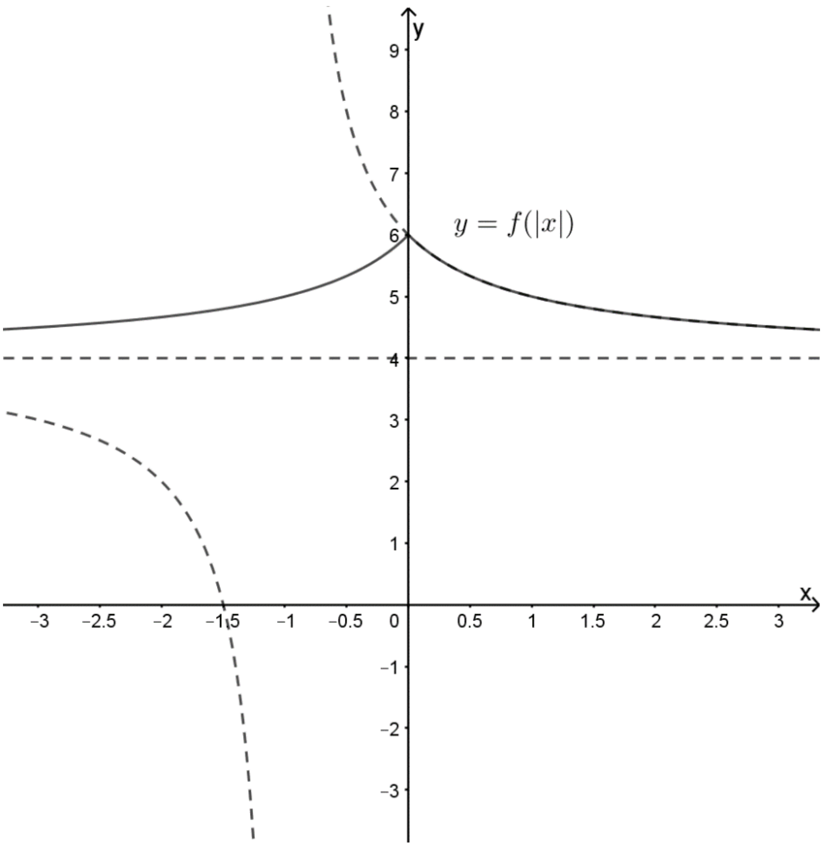
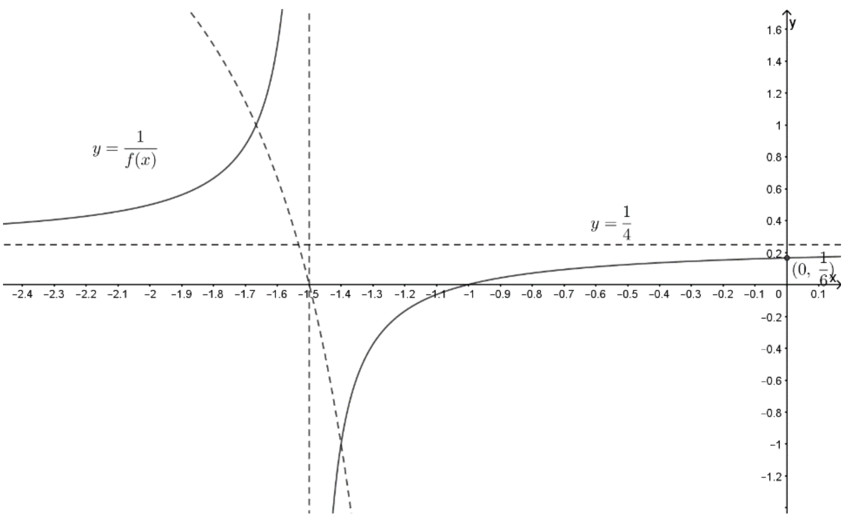
Q11diii	Domain: $x \geq -1$ as $x + 1 \geq 0$ Range: $y \geq 2$ as $\sqrt{x+1} \geq 0$	2 Marks Correct solution 1 Mark Correct domain or range
Q11div		3 Marks Correct solution 2 Marks Provides both graphs with most key features shown 1 Mark Provides one correct graph
Q12ai	$P(X = 2), X \sim B(15, 0.18)$ ${}^{15}C_2 \times 0.08^2 \times 0.92^{13}$ $= 0.227306 \ldots$ $= 0.2273$	1 Mark Correct solution
Q12aii	Production is stopped $= P(X > 2)$ $1 - P(X \leq 2)$ $= 1 - \left(P(X = 0) + P(X = 1) + P(X = 2)\right)$ $= 1 - \left({}^{15}C_0 \times 0.08^0 \times 0.92^{15} + {}^{15}C_1 \times 0.08^1 \times 0.92^{14} + {}^{15}C_2 \times 0.08^2 \times 0.92^{13}\right)$ $= 0.112965 \ldots$ $= 0.1130$	2 Marks Correct solution 1 Mark Shows that $P(X > 2)$ $= 1 - P(X \leq 2)$
Q12bi	$\underset{\sim}{a} = 3\underset{\sim}{i} + m\underset{\sim}{j}$ $\underset{\sim}{b} = -10\underset{\sim}{i} + n\underset{\sim}{j}$ $\lvert\underset{\sim}{a}\rvert = 5$ $3^2 + m^2 = 5^2$ $m = \sqrt{5^2 - 3^2} \quad (m > 0)$ $m = 4$ Since $\underset{\sim}{a}$ is perpendicular to $\underset{\sim}{b}$ $\underset{\sim}{a} \cdot \underset{\sim}{b} = 0$ $3 \times -10 + 4 \times n = 0$ $4n = 30$ $n = \frac{30}{4}$ $n = \frac{15}{2}$	2 Marks Correct solution 1 Mark Finds the correct value of m

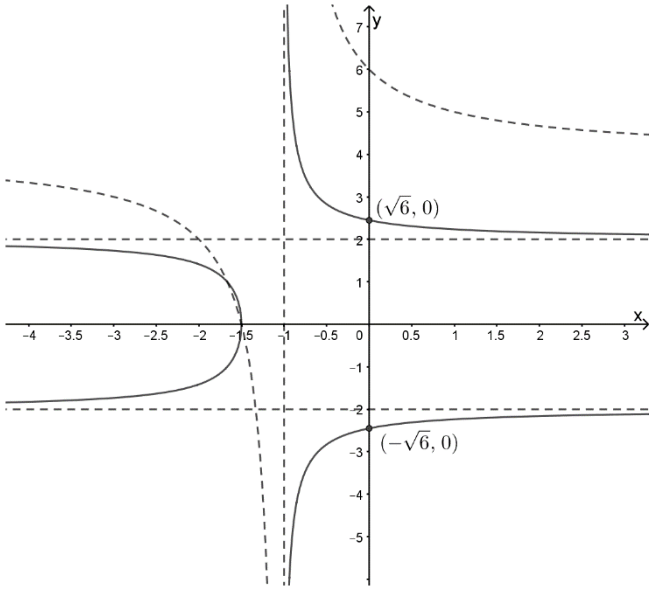
Q12bii	$\hat{\underset{\sim}{b}} = \dfrac{\underset{\sim}{b}}{\left\|\underset{\sim}{b}\right\|}$ $\hat{\underset{\sim}{b}} = \dfrac{-10\underset{\sim}{i} + \frac{15}{2}\underset{\sim}{j}}{\sqrt{(-10)^2 + \left(\frac{15}{2}\right)^2}}$ $\hat{\underset{\sim}{b}} = \dfrac{-10\underset{\sim}{i} + \frac{15}{2}\underset{\sim}{j}}{\sqrt{(-10)^2 + \left(\frac{15}{2}\right)^2}}$ $\hat{\underset{\sim}{b}} = \dfrac{-10\underset{\sim}{i} + \frac{15}{2}\underset{\sim}{j}}{\frac{25}{2}}$ $\hat{\underset{\sim}{b}} = \frac{1}{5}\left(-4\underset{\sim}{i} + 3\underset{\sim}{j}\right)$	1 Mark Correct solution
Q12c	$t = \tan\frac{x}{2}$ $LHS = \sec x + \tan x$ $LHS = \dfrac{1+t^2}{1-t^2} + \dfrac{2t}{1-t^2}$ $LHS = \dfrac{t^2+2t+1}{1-t^2}$ $LHS = \dfrac{(t+1)^2}{(1+t)(1-t)}$ $LHS = \dfrac{1+t}{1-t}$ $LHS = \dfrac{\tan\frac{\pi}{4} + \tan\frac{x}{2}}{1 - \tan\frac{\pi}{4}\tan\frac{x}{2}}$ $LHS = \tan\left(\frac{\pi}{4} + \frac{x}{2}\right)$ $LHS = RHS$	3 Marks Correct solution 2 Marks Makes significant progress and uses compound angle formula correctly 1 Mark Correct substitution of t formula for $\sec x + \tan x$
Q12di	$P(x) = 2x^3 + ax^2 + b$ $P'(x) = 6x^2 + 2ax$ Double root at $x = -5$ $P'(-5) = 0$ $6(-5)^2 + 2a(-5) = 0$ $150 = 10a$ $a = 15$ $P(-5) = 0$ $2(-5)^3 + 15(-5)^2 + b = 0$ $125 + b = 0$ $b = -125$	2 Marks Correct solution 1 Mark Uses multiple root theorem to obtain the correct value for a
Q12dii	$P(x) = 2x^3 + 15x^2 - 125$ $P(x) = (x+5)(x+5)(2x-5)$	1 Mark Correct solution
Q12e	$u = \sqrt{x}$ $du = \frac{1}{2}x^{-\frac{1}{2}}dx = \frac{1}{2\sqrt{x}}dx$ $x = 3 \quad u = \sqrt{3}$ $x = 0 \quad u = 0$	3 Marks Correct solution 2 Marks Correct integration 1 Mark

	$I = \int_0^3 \frac{1}{\sqrt{x}(9+x)}\,dx$ $I = 2\int_0^3 \frac{1}{(9+x)} \times \frac{1}{2\sqrt{x}}\,dx$ $I = 2\int_0^{\sqrt{3}} \frac{1}{(9+u^2)}\,du$ $I = 2 \times \left[\frac{1}{3}\tan^{-1}\frac{u}{3}\right]_0^{\sqrt{3}}$ $I = \frac{2}{3}\left(\tan^{-1}\frac{\sqrt{3}}{3} - \tan^{-1} 0\right)$ $I = \frac{2}{3} \times \frac{\pi}{6}$ $I = \frac{\pi}{9}$	Obtains correct integrand in terms of u
Q13a	$1 + 6 + 11 + \cdots + (5n-4) = \frac{1}{2}n(5n-3)$ 1. Prove statement is true for $n = 1$ $LHS = 1$ $RHS = \frac{1}{2} \times 1 \times (5 \times 1 - 3)$ $RHS = 1$ $LHS = RHS$ ∴Statement is true for $n = 1$ 2. Assume statement is true for $n = k$ (k some positive integer) i.e. $1 + 6 + 11 + \cdots + (5k-4) = \frac{1}{2}k(5k-3)$ 3. Prove statement is true for $n = k+1$ i.e. $1 + 6 + 11 + \cdots + (5k-4) + (5(k+1)-4)$ $= \frac{1}{2}(k+1)(5(k+1)-3)$ $1 + 6 + 11 + \cdots + (5k-4) + (5k+1) = \frac{1}{2}(k+1)(5k+2)$ $LHS = 1 + 6 + 11 + \cdots + (5k-4) + (5k+1)$ $LHS = \frac{1}{2}k(5k-3) + (5k+1)$ (from step 2) $LHS = \frac{1}{2}k(5k-3) + (5k+1)$ $LHS = \frac{1}{2}(5k^2 - 3k + 10k + 2)$ $LHS = \frac{1}{2}(5k^2 + 7k + 2)$ $LHS = \frac{1}{2}(k+1)(5k+2)$ $LHS = RHS$ ∴Statement is true by mathematical induction for all positive integers.	3 Marks Correct solution 2 Marks Makes significant progress 1 Mark Proves initial case

Q13bi	$F = 275e^{-k(n-1)} + F_0$ $\frac{dF}{dn} = -k \times 275e^{-k(n-1)}$ $\frac{dF}{dn} = -k \times \left(275e^{-k(n-1)} + F_0 - F_0\right)$ $\frac{dF}{dn} = -k \times (F - F_0)$	1 Mark Correct solution
Q13bii	$F = 350, n = 1$ $350 = 275e^{-k(1-1)} + F_0$ $350 = 275e^{0} + F_0$ $F_0 = 350 - 275$ $F_0 = 75$	1 Mark Correct solution
Q13biii	$F = 80, n = 200, F_0 = 75$ $80 = 275e^{-k(200-1)} + 75$ $5 = 275e^{-199k}$ $\frac{5}{275} = e^{-199k}$ $\frac{1}{55} = e^{-199k}$ $\ln\left(\frac{1}{55}\right) = -199k$ $k = \frac{\ln\left(\frac{1}{55}\right)}{-199}$ $k = \frac{\ln 55}{199}$	2 Marks Correct solution 1 Mark Makes significant progress
Q13biv	As $n \to \infty, e^{-\infty} \to 0$ $F = 275e^{-k(n-1)} + F_0$ $F \to 0 + 75$ $F \to 75$ $\therefore$The endurance limit is 75.	1 Mark Correct solution
Q13ci	$5\cos\theta - 5\sqrt{3}\sin\theta = A\cos(\theta + \alpha)$ $5\cos\theta - 5\sqrt{3}\sin\theta = A\cos\theta\cos\alpha - A\sin\theta\sin\alpha$ $A\cos\alpha = 5 \quad (1)$ $A\sin\alpha = 5\sqrt{3} \quad (2)$ $(1)^2 + (2)^2$ $A^2\cos^2\alpha + A^2\sin^2\alpha = 5^2 + \left(5\sqrt{3}\right)^2$ $A^2 = 100$ $A = 10$ $(1) \div (2)$ $\tan\alpha = \frac{5\sqrt{3}}{5}$ $\alpha = \frac{\pi}{3}$	2 Marks Correct solution 1 Mark Finds the correct value for A or α
Q13cii	$5\cos\theta - 5\sqrt{3}\sin\theta = 10\cos\left(\theta + \frac{\pi}{3}\right)$ $5\cos\theta - 5\sqrt{3}\sin\theta + 15 = 10\cos\left(\theta + \frac{\pi}{3}\right) + 15$ Maximum value of $10\cos\left(\theta + \frac{\pi}{3}\right)$ is 10 Maximum value of $5\cos\theta - 5\sqrt{3}\sin\theta + 15$ is $10 + 15 = 25$	1 Mark Correct solution

Q13d	$\frac{dy}{dx}=\frac{1}{(2-y)\sqrt{2-x^2}}$ $(2-y)dy=\frac{1}{\sqrt{2-x^2}}dx$ $\int(2-y)dy=\int\frac{1}{\sqrt{2-x^2}}dx$ $2y-\frac{y^2}{2}=\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)+c$ $x=1, y=0$ $2\times 0-\frac{0^2}{2}=\sin^{-1}\left(\frac{1}{\sqrt{2}}\right)+c$ $0=\frac{\pi}{4}+c$ $c=-\frac{\pi}{4}$ $2y-\frac{y^2}{2}=\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)-\frac{\pi}{4}$ $y^2-4y=\frac{\pi}{2}-2\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)$ $y^2-4y+4=\frac{\pi}{2}-2\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)+4$ $(y-2)^2=\frac{\pi}{2}+4-2\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)$ $y-2=\pm\sqrt{\frac{\pi}{2}+4-2\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)}$ $y=2\pm\sqrt{\frac{\pi}{2}+4-2\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)}$ Since $x=1, y=0$ $\therefore y=2-\sqrt{\frac{\pi}{2}+4-2\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)}$	4 Marks Correct solution 3 Marks Makes significant progress 2 Marks Correct primitive function 1 Mark Shows split $(2-y)dy=\frac{1}{\sqrt{2-x^2}}dx$
Q14ai	$y=\lvert f(x)\rvert$  	1 Mark Correct solution

Q14aii	$y = f(\|x\|)$ 	1 Mark Correct solution
Q14aiii	$y = \frac{1}{f(x)}$ Horizontal asymptote becomes $y = \frac{1}{4}$ Vertical asymptote becomes $x = -1.5$ y intercept becomes $y = \frac{1}{6}$ 	2 Marks Correct solution 1 Mark Correct graph with some key features shown

Q14aiv	$y^2 = f(x)$ Horizontal asymptote becomes $y = 2$ y intercepts becomes $y = \pm\sqrt{6}$  	3 Marks Correct solution 2 Marks Correct graph with most key features shown 1 Mark Correct graph of $y = \sqrt{f(x)}$
Q14bi	$\frac{d}{dx}\left(e^{f(x)}\right) = f'(x)e^{f(x)}$ $\frac{d}{dx}\left(e^{2\sin x}\right) = 2\cos x\, e^{2\sin x}$	1 Mark Correct solution
Q14bii	$V = \pi\int_0^{\frac{\pi}{2}}\left(\sqrt{\cos x}\, e^{\sin x}\right)^2 dx$ $V = \pi\int_0^{\frac{\pi}{2}} \cos x\, e^{2\sin x} dx$ $V = \frac{\pi}{2}\int_0^{\frac{\pi}{2}} 2\cos x\, e^{2\sin x} dx$ $V = \frac{\pi}{2}\left[e^{2\sin x}\right]_0^{\frac{\pi}{2}}$ $V = \frac{\pi}{2}\left(e^{2\sin\frac{\pi}{2}} - e^{2\sin 0}\right)$ $V = \frac{\pi}{2}\left(e^2 - 1\right)\, units^3$	3 Marks Correct solution 2 Marks Makes significant progress 1 Mark Provides correct integrand for volume of revolution
Q14ci	By similar triangles $\frac{x}{150} = \frac{h}{25}$ $x = 6h$ $V = \frac{1}{2} \times x \times h \times 400$ $V = \frac{1}{2} \times 6h \times h \times 400$ $V = 1200h^2$	2 Marks Correct solution 1 Mark Finds x in terms of h
Q14cii	$\frac{dh}{dt} = ?$ $\frac{dh}{dt} = \frac{dh}{dV} \times \frac{dV}{dt}$	2 Marks Correct solution 1 Mark Finds $\frac{dh}{dV}$

	$\frac{dV}{dt} = -18000\ cm^3/s$ $V = 1200h^2$ $\frac{dV}{dh} = 2400h$ $\frac{dh}{dV} = \frac{1}{2400h}$ When $h = 20\ cm$ $\frac{dh}{dt} = \frac{1}{2400 \times 20} \times -18000$ $\frac{dh}{dt} = -\frac{3}{8} = -0.375\ cm/s$ ∴ water level is falling at a rate of $0.375\ cm/s$	