CARINGBAH HIGH

OMNIA VINCIT DILIGENTIA

2009
TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Extension 1

General Instructions

- **Reading Time- 5 minutes**
- **Working Time – 2 hours**
- **Write using a black or blue pen**
- **Approved calculators may be used**
- **A table of standard integrals is provided at the back of this paper.**
- **All necessary working should be shown for every question.**
- **Begin each question on a fresh sheet of paper.**

Total marks (84)

- **Attempt Questions 1-7**
- **All questions are of equal value**

Question 1 (12 marks) Begin a NEW page. **MARKS**

a) Differentiate $\sin^{-1}(2x)$ **2**

b) Find $\int x\sqrt{1+x^2}\,dx$ using the substitution $u=1+x^2$ **2**

c) Evaluate $\int_0^2 \frac{dx}{4+x^2}$ **3**

d) Evaluate $\lim_{x\to 0}\frac{\sin 3x}{5x}$ **1**

e) Find the acute angle between the lines $3y=2x+8$ and $y=5x-9$ **2**

f) Find the remainder when the polynomial $P(x)=x^3-4x$ is divided by $x+3$. **1**

g) Given that $\log_3 7=m$, find an expression for $\log_3 21$. **1**

End of Question 1

Question 2 (12 marks) Begin a NEW page. **MARKS**

a) Solve $\cos\theta + \sqrt{3}\sin\theta = 1 \quad (0 \le \theta \le 2\pi)$ **4**

b) Solve $\frac{3x}{x-2} \le 1$ **3**

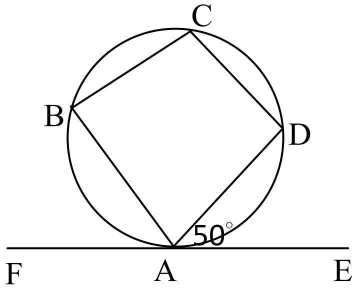
c) Find the coordinates of the point P which divides the interval AB externally in the ratio $1:2$ where A is $(-2,7)$ and B is $(1,5)$. **2**

d) Solve $\cos 2\theta = \cos\theta \quad (0 \le \theta \le 2\pi)$ **3**

End of Question 2

Question 3 (12 marks) Begin a NEW page. **MARKS**

a) ABCD is a cyclic quadrilateral and FAE is a tangent at A.
$\angle DAE = 50^\circ$ and $BD \parallel FE$



i) Calculate $\angle BAF$, giving reasons. **2**

ii) Calculate $\angle BCD$, giving reasons. **2**

b) Solve the equation $2x^3 - 21x^2 + 42x - 16 = 0$ given that the roots form consecutive terms of a geometric sequence. **3**

c) Evaluate $\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \sin x \cos^2 x \, dx$ using the substitution $u = \cos x$ **3**

d) A packet of peas initially at $24^\circ C$ is placed in a snap-freeze refrigerator which is maintained at an internal temperature of $-40^\circ C$. Assuming Newton's Law of Cooling, the temperature T° of the peas will be given by $T = -40 + 64e^{kt}$, where *k* is a constant and *t* is the time in seconds.

i) If after 5 seconds the temperature of the packet is $19^\circ C$, Find the value of k. **1**

ii) How long will it take for the packet's temperature to reduce to $0^\circ C$? Give your answer to the nearest second. **1**

End of Question 3

Question 4 (12 marks) Begin a NEW page. **MARKS**

a) The chord PQ of the parabola $x^2 = 4y$ subtends a right angle at the origin O.
If the coordinates of P and Q are $\left(2p, p^2\right)$ and $\left(2q, q^2\right)$ respectively:

i) Find the gradients of PO and QO. **1**

ii) Show $pq = -4$. **1**

iii) Find the equation of the locus of the midpoint M of PQ. **2**

b) A particle is moving along the x axis. Initially the particle is 2m to the right of the origin and moving with a velocity of $5\ m/s$.

i) If the particle's acceleration is given by $\ddot{x} = 2x^3 + 2x$, show that $\dot{x} = x^2 + 1$. **3**

ii) Hence find an expression for x in terms of t. **2**

c) Water flows from a hole in the base of a cylindrical tank at a rate given by $\frac{dh}{dt} = -k\sqrt{h}$ where k is a constant and h is the depth of the water in *mm* at any time *t* minutes. **3**

If the depth of the water falls from $100mm$ to $25mm$ in one minute, find how much longer it will take for the tank to empty.

End of Question 4

Question 5 (12 marks) Begin a NEW page. **MARKS**

a) Prove by Mathematical Induction that $7^n + 5$ is divisible by 6, for positive integral values of n. **3**

b) A missile is projected with a speed of $50m/s$, at an angle of elevation of 45°. It is aimed at a tall building which is at a horizontal distance of $200m$ from the point of projection.

i) By taking $g = 10\,m/s^2$ and neglecting air resistance, show that the equations of motion for this missile may be expressed as: **2**

$$x = 25\sqrt{2}t \quad \text{and} \quad y = -5t^2 + 25\sqrt{2}t$$

ii) Hence find the time of flight until the missile hits the building. **1**

iii) Find the height of the missile when it hits the building. **1**

c) Find m if: $\int_0^{\frac{\pi}{6}} \frac{\cos x\,dx}{1+\sin x} = \log_e m$. **2**

d) i) Find $\frac{d}{dx}\left[\tan^{-1} x + x\right]$ **1**

ii) Hence evaluate $\int_0^1 \frac{x^2+x+2}{x^2+1}dx$ **2**

(leave your answer in exact form)

End of Question 5

Question 6 (12 marks) Begin a NEW page. **MARKS**

a) For the curve with equation $y = \dfrac{x^2}{1-x^2}$

i) Show $\dfrac{dy}{dx} = \dfrac{2x}{\left(1-x^2\right)^2}$ **1**

ii) Find any stationary points and determine their nature. **2**

iii) Find the equations of any vertical asymptotes. **1**

iv) Find the equations of any horizontal asymptotes. **1**

v) Sketch the curve. **1**

b) A spherical balloon is being inflated and its volume increases at a constant rate of $50mm^3/s$. **3**

At what rate is its surface area increasing when the radius is $20mm$.

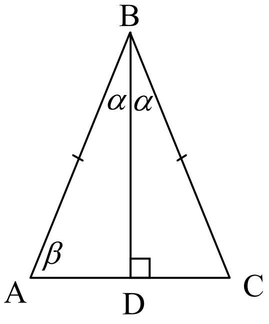
c) A particle moves such that $x = 3\cos 2t - 4\sin 2t$. **3**

Show this motion is Simple Harmonic Motion, and find the greatest speed of the particle.

End of Question 6

Question 7 (12 marks) Begin a NEW page. **MARKS**

a)



The triangle ABC is isosceles with $AB = BC$ and BD is perpendicular to AC. Let $\angle ABD = \angle CBD = \alpha$ and $\angle BAD = \beta$.

i) Show $\sin\beta = \cos\alpha$ **1**

ii) Using the sine rule in ΔABC show $\sin 2\alpha = 2\sin\alpha\cos\alpha$ **2**

iii) Given that $0 \le \alpha \le \frac{\pi}{2}$, express the limiting sum of the geometric series $\sin 2\alpha + \sin 2\alpha\cos^2\alpha + \sin 2\alpha\cos^4\alpha + \ldots$ in terms of α. **2**

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b) i) Evaluate: $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cos^2 x\,dx$ Leave your answer in exact form. **2**

ii) Hence, if the curve $y = 2 + \cos x$ is rotated about the x axis from $\frac{\pi}{4} \le x \le \frac{\pi}{2}$ find the volume of the solid of revolution. (leave your answer in exact form) **3**

c) Show without the use of a calculator: $\tan^{-1}(4) - \tan^{-1}\left(\frac{3}{5}\right) = \frac{\pi}{4}$. **2**

End of Question 7

End of Examination

STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x}\,dx = \ln x, \quad x > 0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}, \quad a \neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax, \quad a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax, \quad a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax\,dx = \frac{1}{a}\sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \ln\left(x+\sqrt{x^2-a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \ln\left(x+\sqrt{x^2+a^2}\right)$$

NOTE : $\ln x = \log_e x, \quad x > 0$

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① a) $\frac{d}{dx}\sin^{-1}(2x) = \frac{2}{\sqrt{1-4x^2}}$

b) $\int x\sqrt{1+x^2}\,dx$

$u = 1+x^2$

$\frac{du}{dx} = 2x$

$du = 2x\,dx$

$= \frac{1}{2}\int \sqrt{u}\;du$

$= \frac{1}{2} \times \frac{2}{3} u^{3/2} + C = \frac{1}{3}(1+x^2)^{3/2} + C$

c) $\int_0^2 \frac{dx}{4+x^2} = \frac{1}{2}\left[\tan^{-1}\left(\frac{x}{2}\right)\right]_0^2$

$= \frac{1}{2}\left[\tan^{-1}1 - \tan^{-1}0\right]$

$= \frac{1}{2}\left[\frac{\pi}{4} - 0\right] = \frac{\pi}{8}$

d) $\lim_{x\to 0} \frac{\sin 3x}{5x} = \lim_{x\to 0} \frac{3}{5}\left(\frac{\sin 3x}{3x}\right)$

$= \frac{3}{5}$

e) $y = \frac{2x}{3} + \frac{8}{3}$ $\quad$ $y = 5x - 9$

$m_1 = 2/3$ $\quad$ $m_2 = 5$

$\tan\theta = \left|\frac{2/3 - 5}{1 + 2/3 \times 5}\right| = 1$

$\theta = 45°$

f) $P(x) = x^3 - 4x$

Remainder $= P(-3) = (-3)^3 - 4(-3)$

$= -15$

g) $\log_3 7 = m$ $\quad$ $\log_3 21$

$= \log_3(7\times 3)$

$= \log_3 7 + \log_3 3$

$= m+1$

② a) $\cos\theta + \sqrt{3}\sin\theta = 1$

$2\left[\frac{1}{2}\cos\theta + \frac{\sqrt{3}}{2}\sin\theta\right] = 1$

$\cos(\theta - \pi/3) = \frac{1}{2}$

$\theta - \frac{\pi}{3} = \frac{\pi}{3}\ ,\ 2\pi - \frac{\pi}{3}$

$\theta = 0, 2\pi, \frac{2\pi}{3}$

b) $\frac{3x}{x-2} \le 1$

$3x(x-2) \le (x-2)^2$

$3x(x-2) - (x-2)^2 \le 0$

$(x-2)(2x+2) \le 0$

$-1 \le x \le 2$

But $x \ne 2$ $\quad \therefore\ -1 \le x < 2$

c) $A(-2, 7)$ $\quad$ $B(1, 5)$

$-1 : 2$

$x = \frac{-1\times 1 + 2\times -2}{-1+2}$ $\quad$ $y = \frac{-1\times 5 + 2\times 7}{-1+2}$

$\therefore P(-5, 9)$

d) $\cos 2\theta = \cos\theta$

$2\cos^2\theta - 1 = \cos\theta = 0$

Let $m = \cos\theta$

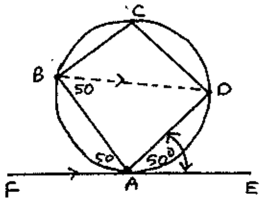
$2m^2 - m - 1 = 0$

$(m-1)(2m+1) = 0$

$\cos\theta = 1$ $\quad$ $\cos\theta = -1/2$

$\theta = 0, 2\pi, \frac{2\pi}{3}, \frac{4\pi}{3}$

③ a)



i) $\angle DBA = 50°$ (alt. segment thm)

$\therefore \angle BAF = 50°$ (alt ∠'s, BD ∥ FA)

ii) $\angle BAD = 80°$ (∠ sum straight line)

$\angle BCD = 100°$ (opp ∠'s cyclic quad)

b) $2x^3 - 21x^2 + 42x - 16 = 0$

Let roots be $\frac{\alpha}{\beta}, \alpha, \alpha\beta$

$\therefore\ -\frac{b}{a} = \frac{\alpha}{\beta} + \alpha + \alpha\beta = \frac{21}{2}$

$\frac{c}{a} = \frac{\alpha^2}{\beta} + \alpha^2 + \alpha^2\beta = 21$

$-\frac{d}{a} = \alpha^3 = 8$ $\quad \therefore \alpha = 2$

③ b) cont.

$\frac{2}{\beta} + 2 + 2\beta = \frac{21}{2}$

$4\beta^2 - 17\beta + 4 = 0$

$(\beta - 4)(4\beta - 1) = 0$

$\beta = 4, \frac{1}{4}$

$\therefore$ roots are $x = \frac{1}{2}, 2, 8$

c) $\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \sin x \cos^2 x\,dx$

$u = \cos x$

$\frac{du}{dx} = -\sin x$

$du = -\sin x\,dx$

$x = \frac{\pi}{3}$ $\quad u = 1/2$

$x = \frac{\pi}{2}$ $\quad u = 0$

$= -\int_{\frac{1}{2}}^{0} u^2\,du$

$= \left[\frac{u^3}{3}\right]_0^{1/2} = \frac{1}{24}$

d) $T = -40 + 64e^{kt}$

i) $t=5$, $T=19$ $\quad$ $19 = -40 + 64e^{5k}$

$\frac{59}{64} = e^{5k}$

$k = \frac{1}{5}\ln\left(\frac{59}{64}\right) \doteq -0.016269$

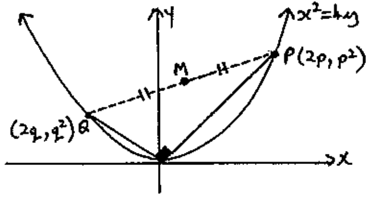
ii) $T=0$ $\quad$ $0 = -40 + 64e^{kt}$

$\frac{40}{64} = e^{kt}$

$t = \frac{\ln\left(\frac{40}{64}\right)}{k} \doteq 28.889$

$\therefore$ takes approx 29 sec.

④ a)



i) $m(PO) = \frac{p^2-0}{2p-0} = \frac{p}{2}$ $\quad$ $m(QO) = \frac{q^2-0}{2q-0} = \frac{q}{2}$

ii) as $\angle POQ = 90°$, then $m(PO) \times m(QO) = -1$

$\frac{p}{2} \times \frac{q}{2} = -1$

$pq = -4$

iii) Midpoint $PQ = \left(\frac{2p+2q}{2}, \frac{p^2+q^2}{2}\right)$

$= \left(p+q\ ,\ \frac{p^2+q^2}{2}\right)$

$x = p+q$ $\quad$ $y = \frac{1}{2}(p^2+q^2)$

$= \frac{1}{2}\left[(p+q)^2 - 2pq\right]$

$y = \frac{1}{2}\left[x^2 - 2(-4)\right]$

$y = \frac{1}{2}(x^2 + 8)$

b) $t=0$ $\quad x=2$ $\quad v=5$

i) $\ddot{x} = 2x^3 + 2x$

$\frac{d}{dx}\left(\frac{1}{2}v^2\right) = 2x^3 + 2x$

$\frac{1}{2}v^2 = \frac{2x^4}{4} + \frac{2x^2}{2} + C$

$v^2 = x^4 + 2x^2 + c_1$

$x=2$, $v=5$ $\quad$ $25 = 16 + 8 + c_1 \rightarrow c_1 = 1$

$v^2 = x^4 + 2x^2 + 1$

$= (x^2+1)^2$

$v = \pm(x^2+1)$

but as $x=2$, $v=5$ $\quad \therefore\ v = \dot{x} = x^2+1$

ii) $v = \frac{dx}{dt} = x^2+1$

$\frac{dt}{dx} = \frac{1}{x^2+1}$

$t = \tan^{-1}x + c_2$

$t=0$, $x=2$ $\quad$ $0 = \tan^{-1}2 + c_2$

$c_2 = -\tan^{-1}2$

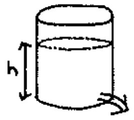
$\therefore\ t = \tan^{-1}x - \tan^{-1}2$

$\tan^{-1}x = t + \tan^{-1}2$

$x = \tan(t + \tan^{-1}2)$

$= \frac{\tan t + 2}{1 - 2\tan t}$

c)



$t=0$ $\quad h=100$

$t=1$ $\quad h=25$

$\frac{dh}{dt} = -k\sqrt{h}$

$\frac{dh}{dt} = \frac{1}{-k\sqrt{h}} = \frac{h^{-1/2}}{-k}$

$t = \frac{1}{-k}\cdot 2h^{1/2} + C$

$0 = \frac{1}{-k}\cdot 2(100)^{1/2} + C$

$C = \frac{20}{k}$

$\therefore\ t = \frac{2\sqrt{h}}{-k} + \frac{20}{k}$

$t=1, h=25$ $\quad$ $1 = \frac{2\sqrt{25}}{-k} + \frac{20}{k}$

$k = 10$

$\therefore\ t = -\frac{\sqrt{h}}{5} + 2$

④c) cont.

$$5t = -\sqrt{h} + 10$$
$$h = (10-5t)^2$$

empty when $h=0$ ie $0=(10-5t)^2$

$$t=2$$

$\therefore$ takes a <u>further</u> 1 min to empty

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⑤a) If $n=1$, $7^1+5=12$ which is divisible by 6

$\therefore$ True for $n=1$

Assume true for $n=k$

ie $7^k+5 = 6M$ where M is an integer

$$7^k = 6M-5$$

Aim to prove true for $n=k+1$

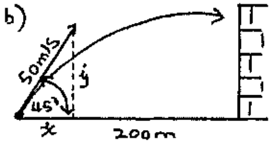
ie $7^{k+1}+5 = 7.7^k+5$
$=7(6M-5)+5$
$=42M-35+5$
$=42M-30$
$=6(7M-5)$
$=6Q$ where Q is an integer

$\therefore$ If true for $n=k$, then true for $n=k+1$

Proved true for $n=1$, thus true for $n=1+1=2$

Hence by the principle of mathematical induction, true for all integer n, $n\geq 1$

b)



$t=0$, $x=0$, $y=0$

$\cos 45 = \frac{\dot{x}}{50}$

$\dot{x} = 50\cos 45 = \frac{50}{\sqrt{2}}$
$= 25\sqrt{2}$

$\dot{y} = 25\sqrt{2}$

) $\ddot{x}=0$ $\qquad$ $\ddot{y}=-10$

$\dot{x}=c_1$ $\qquad$ $\dot{y}=-10t+c_2$

$t=0$, $\dot{x}=25\sqrt{2}$, $\dot{y}=25\sqrt{2}$

$\dot{x}=25\sqrt{2}$ $\qquad$ $\dot{y}=-10t+25\sqrt{2}$

$x=25t\sqrt{2}+c_3$ $\qquad$ $y=-5t^2+25t\sqrt{2}+c_4$

$=0$, $x=0$, $y=0$

$x=25t\sqrt{2}$ $\qquad$ $y=-5t^2+25t\sqrt{2}$

) hits building when $x=200$

ie $200 = 25t\sqrt{2}$

$t = \frac{8}{\sqrt{2}} = 4\sqrt{2}$ sec

iii) when $t=4\sqrt{2}$ $\quad y=-5(4\sqrt{2})^2+25\sqrt{2}(4\sqrt{2})$
$= 40$

Hits 40m off ground

c) $\int_0^{\pi/6} \frac{\cos x\,dx}{1+\sin x} = \left[\ln(1+\sin x)\right]_0^{\pi/6}$
$= \ln\left[1+\sin\frac{\pi}{6}\right] - \ln\left[1+\sin 0\right]$
$= \ln 1{\cdot}5 - 0$
$= \ln 1{\cdot}5$

$\therefore m = 1{\cdot}5$

d) i) $\frac{d}{dx}(\tan^{-1}x + x) = \frac{1}{1+x^2}+1$
$= \frac{2+x^2}{1+x^2}$

ii) $\int_0^1 \frac{x^2+x+2}{x^2+1}\,dx$

$= \int_0^1 \frac{2+x^2}{1+x^2} + \frac{x}{1+x^2}\,dx$

$= \left[\tan^{-1}x + x + \frac{1}{2}\ln(1+x^2)\right]$

$= \left[\tan^{-1}1 + 1 + \frac{1}{2}\ln(2)\right] - \left[\tan^{-1}0 + 0 + \frac{1}{2}\ln 1\right]$

$= \frac{\pi}{4} + 1 + \frac{1}{2}\ln 2$

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⑥ a) $y = \frac{x^2}{1-x^2}$

i) $y' = \frac{(1-x^2)\times 2x - x^2\times(-2x)}{(1-x^2)^2}$

$= \frac{2x-2x^3+2x^3}{(1-x^2)^2} = \frac{2x}{(1-x^2)^2}$

ii) Stat pts when $y'=0$

ie $2x=0$
$x=0$

at $x=0$, $y=0$

x	$-\frac{1}{2}$	0	$\frac{1}{2}$
y'	$-ve$	0	$+ve$

$\therefore$ min turning pt @ $(0,0)$

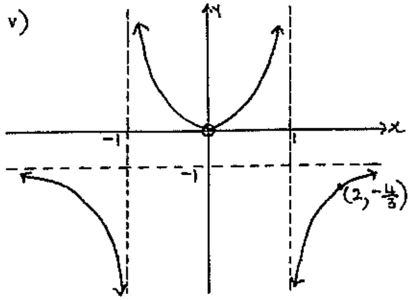
iii) $1-x^2=0$
$x \neq \pm 1$ $\quad\therefore$ vertical asymptotes at $x=1$ and $x=-1$

iv) $1-x^2 \overline{\big)\, x^2}$ quotient -1, x^2-1, remainder 1 $\quad\therefore y = \frac{x^2}{1-x^2} = -1 + \frac{1}{1-x^2}$

as $x\to\infty$ $\frac{1}{1-x^2}\to 0$

$\therefore y = \frac{x^2}{1-x^2} \to y=-1$

⑥ a) v)



b)

$\frac{dV}{dt} = 50$ mm^3/s

$V = \frac{4}{3}\pi r^3$ $\qquad$ $A = 4\pi r^2$

$\frac{dV}{dr} = 4\pi r^2$ $\qquad$ $\frac{dA}{dr} = 8\pi r$

$\therefore \frac{dr}{dt} = \frac{dr}{dV}\cdot\frac{dV}{dt}$ $\qquad$ $\frac{dA}{dt} = \frac{dA}{dr}\cdot\frac{dr}{dt}$

$= \frac{1}{4\pi r^2}\times 50$ $\qquad$ $= 8\pi r \times \frac{25}{2\pi r^2}$

$= \frac{25}{2\pi r^2}$ $\qquad$ $= \frac{100}{r}$

$\therefore$ when $r=20$ $\frac{dA}{dt} = 5$ mm^2/s

c) $x = 3\cos 2t - 4\sin 2t$

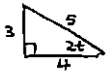
$\dot{x} = -6\sin 2t - 8\cos 2t$

$\ddot{x} = -12\cos 2t + 16\sin 2t$
$= -4(3\cos 2t - 4\sin 2t)$
$= -4x$ which is S.H.M.

Greatest speed at centre of motion

ie $0 = 3\cos 2t - 4\sin 2t$

$\frac{\sin 2t}{\cos 2t} = \frac{3}{4} = \tan 2t$



$\therefore$ Greatest speed $= \left|-6\times\frac{3}{5} - 8\times\frac{4}{5}\right| = 10$

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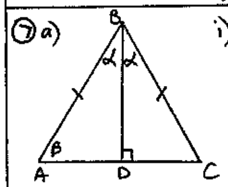
⑦a)



i) In ΔABD,

$\sin\beta = \frac{BD}{AB}$

$\cos\alpha = \frac{BD}{AB}$

$\therefore \sin\beta = \cos\alpha$

ii) In ΔABC $\quad \frac{AC}{\sin 2\alpha} = \frac{BC}{\sin\beta}$

$\sin 2\alpha = \frac{AC\sin B}{BC}$

$= \frac{2\,DC\sin B}{BC}$ $\quad$ $(AC=2DC)$

$= 2\sin\alpha\sin B$ $\quad$ $(\sin\alpha = \frac{DC}{BC})$

$= 2\sin\alpha\cos\alpha$ $\quad$ (from(i))

iii) $0\leq\alpha\leq\frac{\pi}{2}$ $\quad \sin 2\alpha + \sin 2\alpha\cos^2\alpha + \sin 2\alpha\cos^4\alpha + \ldots..$

$\rightarrow$ GP $\quad a = \sin 2\alpha \quad r = \cos^2\alpha$

$\therefore S_\infty = \frac{a}{1-r} = \frac{\sin 2\alpha}{1-\cos^2\alpha} = \frac{2\sin\alpha\cos\alpha}{\sin^2\alpha}$
$= 2\frac{\cos\alpha}{\sin\alpha}$
$= 2\cot\alpha$

b) i) $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cos^2 x\,dx = \frac{1}{2}\int_{\pi/4}^{\pi/2}(\cos 2x + 1)\,dx$

$= \frac{1}{2}\left[\frac{\sin 2x}{2} + x\right]_{\pi/4}^{\pi/2}$

$= \frac{1}{2}\left[\left(\frac{1}{2}\sin\pi + \frac{\pi}{2}\right) - \left(\frac{1}{2}\sin\frac{\pi}{2} + \frac{\pi}{4}\right)\right]$

$= \frac{1}{2}\left[\frac{\pi}{2} - \frac{1}{2} - \frac{\pi}{4}\right] = \frac{\pi}{8} - \frac{1}{4}$

ii) $y = 2+\cos x$

$y^2 = 4 + 4\cos x + \cos^2 x$

$V = \pi\int_{\pi/4}^{\pi/2} 4 + 4\cos x + \cos^2 x\,dx$

$= \pi\left[4x + 4\sin x\right]_{\pi/4}^{\pi/2} + \pi\int_{\pi/4}^{\pi/2}\cos^2 x\,dx$

$= \pi\left[\left(2\pi + 4\sin\frac{\pi}{2}\right) - \left(\pi + 4\sin\frac{\pi}{4}\right)\right] + \pi\left(\frac{\pi}{8} - \frac{1}{4}\right)$

$= \pi\left[\frac{9\pi}{8} + \frac{15}{4} - 2\sqrt{2}\right]$ units^3

c) LHS $= \tan^{-1}(4) - \tan^{-1}\left(\frac{3}{5}\right)$

Let $\tan^{-1}4 = x$ $\qquad$ $\tan^{-1}(3/5) = y$

$\tan x = 4$ $\qquad$ $\tan y = \frac{3}{5}$

$\tan(x-y) = \frac{\tan x - \tan y}{1 + \tan x.\tan y}$

$= \frac{4-\frac{3}{5}}{1+4\times\frac{3}{5}}$

$= 1$

$x - y = \tan^{-1}(1)$

$\therefore \tan^{-1}4 - \tan^{-1}\left(\frac{3}{5}\right) = \frac{\pi}{4}$

THE END!