CARINGBAH HIGH SCHOOL

2010

TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Extension 1

General Instructions

Reading time - 5 minutes

Working time - 2 hours

Write using black or blue pen.

Board-approved calculators may be used.

A table of standard integrals is provided at the back of this paper.

Total marks - 84

Attempt Questions 1 - 7

All questions of equal value.

All necessary working should be shown in every question.

Start each question on a new page.

Question 1 (12 marks) **Marks**

(a) Factorise $2x^3 + 54$. **2**

(b) Let $f(x) = e^x - 1$. What is the range of $f(x)$? **1**

(c) Given that $\log_a b = 3 \cdot 4$ and $\log_a c = 4 \cdot 5$, find $\log_a \left(\frac{b}{c}\right)$. **1**

(d) Differentiate $e^{3x} \sin x$ **2**

(e) Find the exact value of $\int_0^{\frac{\pi}{4}} \cos^2 x \, dx$. **3**

(f) Using the substitution $u = x^2 + 1$, or otherwise, find the **3**

exact value of $\int_0^3 \frac{2x}{x^2 + 1} \, dx$.

Question 2 (12 marks) Start a new page.

(a) The function $f(x) = x^3 - \log_e(x+1)$ has a root near $x = 1$. **3**

Use one application of Newton's method to obtain another approximation to this root. Give your answer correct to two decimal places.

(b) The polynomial $q(x) = x^3 + 3x^2 + ax + b$ has a factor of $(x-2)$ and a remainder of -9 when divided by $(x+1)$. **3**

Find the values of a and b.

(c) (i) Express $\sqrt{3}\sin x - \cos x$ in the form $A\sin(x-\theta)$ where $0 \le \theta \le \frac{\pi}{2}$. **2**

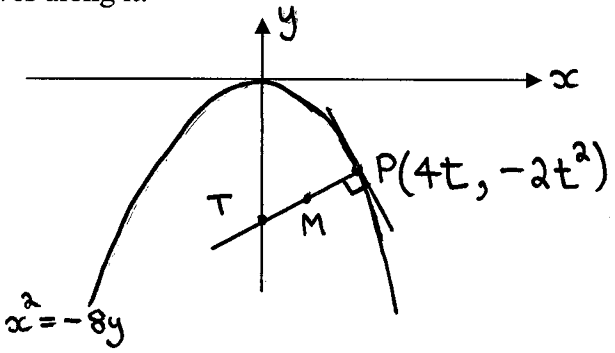
(ii) Hence, or otherwise, solve $\sqrt{3}\sin x - \cos x = 1$ for $0 \le x \le 2\pi$. Give your answer in exact form. **2**

(d) Evaluate $\int_0^{\ln 3} e^{2x}\, dx$. **2**

Question 3 (12 marks) Start a new page.

(a) Find $\lim_{x \to 0} \frac{\sin 5x}{2x}$ **1**

(b) The diagram shows the parabola $x^2 = -8y$ and the point $P(4t, -2t^2)$ which moves along it.



(i) Show that the equation of the normal at P is $y = \frac{x}{t} - 2t^2 - 4$. **2**

(ii) This normal meets the y-axis at T. The midpoint of PT is M. Find the coordinates of T and M. **2**

(iii) Find the Cartesian equation of the locus of M. **1**

(c) (i) On the same set of axes sketch the graphs of $y = \cos 2x$ and $y = -\frac{1}{2}x$ for $0 \le x \le \pi$. (Show at least 2 points which lie on $y = -\frac{1}{2}x$.) **2**

(ii) Use the graph to determine the number of solutions there are to the equation $\cos 2x = -\frac{1}{2}x$ for $0 \le x \le \pi$. **1**

(d) Use mathematical induction to prove that $7^n - 1$ is divisible by 3, for all integers $n \ge 1$. **3**

Question 4 (12 marks) Start a new page.

(a) A particle is moving in a straight line with its acceleration as a function of x given by $\ddot{x} = 3x^2$. It is initially 1 metre to the right of the origin and is travelling with a velocity of $-\sqrt{2}$ metres per second.

(i) Show that $\dot{x} = -\sqrt{2x^3}$ **2**

(ii) Hence show that $x = \dfrac{2}{\left(t+\sqrt{2}\right)^2}$ **3**

(b) (i) Find the x-coordinates of stationary points on the curve **3**

$$y = x - 2\sin x \quad \text{for } 0 \le x \le 2\pi .$$

(ii) Write down the y-coordinates of these stationary points in exact form. **1**

(iii) Find the nature of each of the stationary points. **1**

(iv) Find the coordinates of those points on the curve corresponding to $x = 0$ and 2π. **1**

(v) Hence sketch the curve for $0 \le x \le 2\pi$. (You do not have to find any other intercepts with the axes or points of inflexion.) **1**

Question 5 (12 marks) Start a new page.

(a) The displacement x metres of a particle moving in simple harmonic motion is given by $x = 5\sin(\pi t + \frac{\pi}{2})$ where the time t is in seconds.

(i) What is the period of the oscillation? **1**

(ii) Show that the particle's oscillation starts 5 metres to the right of the equilibrium position. **1**

(iii) What is the speed of the particle as it moves through the equilibrium position? **2**

(iv) Show that the acceleration of the particle is proportional to the displacement from the equilibrium position. **2**

(b) Show that for $0 < x < 1$, $\frac{d}{dx}\left[\sin^{-1}\sqrt{(1-x^2)}\right] = \frac{-1}{\sqrt{(1-x^2)}}$ **3**

(c) (i) Sketch the graph of $y = \tan x$ for $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$. **1**

(ii) By using (i), or otherwise, find those values of x satisfying $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$ for which the geometric series **2**

$1 + \sqrt{3}\tan x + 3\tan^2 x + 3\sqrt{3}\tan^3 x + \ldots$ has a limiting sum.

Question 6 (12 marks) Start a new page.

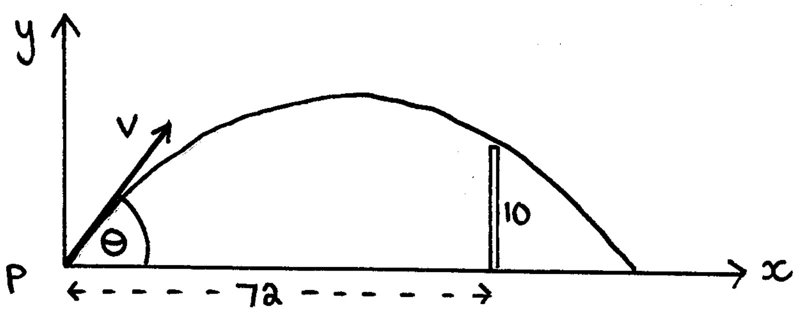
(a) Consider the function $f(x) = x^2 - 4x + 7$.

(i) Write down the largest positive domain for which $f(x)$ has an inverse function $f^{-1}(x)$. **1**

(ii) Find the inverse function $f^{-1}(x)$. **2**

(iii) Sketch $y = f^{-1}(x)$ and $y = f(x)$ on the same diagram. **1**

(b)



A ball is hit with initial velocity v from a point P on the ground at an angle of θ with the horizontal.

(i) The equations of the motion of the ball are:

$\ddot{x} = 0$ and $\ddot{y} = -10$ (i.e. acceleration due to gravity is $10 m/s^2$)

Using calculus, show that the position of the ball at time t is given by: **3**

$$x = vt\cos\theta \quad \text{and} \quad y = vt\sin\theta - 5t^2$$

(ii) After 2 seconds the ball just clears a wall which is 10 metres high and 72 metres from P. **3**

Calculate the angle of projection θ (to the nearest degree) and the initial velocity v.

(iii) How far from P does the ball land? **2**

Question 7 (12 marks) Start a new page.

(a) Find $\int \frac{1}{\sqrt{49 - x^2}} \, dx$. **1**

(b) Nicole is staying in the Blue Mountains. She takes a cup of coffee whose temperature is $80^0 C$ outside where the air temperature is $5^0 C$.

The temperature of the coffee, $T^0 C$, after t minutes outside satisfies the equation

$$\frac{dT}{dt} = -k(T - 5).$$

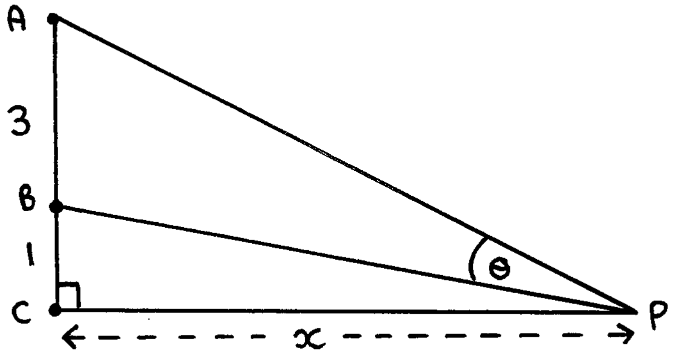
(i) Show that $T = 5 + 75e^{-kt}$ satisfies both the above equation and the initial condition. **2**

(ii) After 10 minutes outside the temperature of the coffee is $57^0 C$. After how many minutes (to the nearest minute) will the coffee's temperature be $20^0 C$? **3**

Question 7 continues on page 9.

Question 7 (continued)

(c)



In the diagram, the goal posts A and B in a children's soccer game are 3 metres apart. Goal post B is 1 metre from the corner post C.
A player stands at P on the boundary line x metres from C.
The goal posts A and B subtend an angle of θ at P.

(i) Show that $\theta = \tan^{-1}\dfrac{4}{x} - \tan^{-1}\dfrac{1}{x}$. **2**

(ii) Show that θ is a maximum when $x = 2$. **2**

(iii) Deduce that the maximum angle subtended at P is $\theta = \tan^{-1}\dfrac{3}{4}$. **2**

End of paper

STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x}\,dx = \ln x, \quad x > 0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}, \quad a \neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax, \quad a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax, \quad a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax\,dx = \frac{1}{a}\sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \ln\left(x+\sqrt{x^2-a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \ln\left(x+\sqrt{x^2+a^2}\right)$$

NOTE : $\ln x = \log_e x, \quad x > 0$

CHS EXT. 1 TRIAL HSC 2010

1. (a) $2(x^3+27) = 2(x+3)(x^2-3x+9)$

(b) $y > -1$

(c) $\log_a\left(\frac{b}{c}\right) = \log_a b - \log_a c$

$= 3\cdot4 - 4\cdot5$

$= -1\cdot1$

(d) $u = e^{3x}$ $\quad v = \sin x$

$u' = 3e^{3x}$ $\quad v' = \cos x$

PRODUCT RULE : $y' = vu' + uv'$

$= \sin x . 3e^{3x} + e^{3x} . \cos x$

$= e^{3x}(3\sin x + \cos x)$

(e) $\cos 2x = 2\cos^2 x - 1$

$\therefore \cos^2 x = \frac{1}{2}(1+\cos 2x)$

$\int_0^{\frac{\pi}{4}} \cos^2 x = \frac{1}{2}\int_0^{\frac{\pi}{4}} 1+\cos 2x \, dx$

$= \frac{1}{2}\left[x + \frac{1}{2}\sin 2x\right]_0^{\frac{\pi}{4}}$

$= \frac{1}{2}\left[\left(\frac{\pi}{4}+\frac{1}{2}\sin\frac{\pi}{2}\right)-\left(0+\frac{1}{2}\sin 0\right)\right]$

$= \frac{\pi}{8} + \frac{1}{4}$

(f) $x=0 : u = 1$

$x = 3 : u = 10$

$\frac{du}{dx} = 2x$

$\therefore du = 2x\,dx$

$\therefore \int_0^3 \frac{2x\,dx}{x^2+1} = \int_1^{10} \frac{du}{u}$

$= [\ln u]_1^{10}$

$= \ln 10 - \ln 1$

$= \ln 10$

2. (a) $f'(x) = 3x^2 - \frac{1}{x+1}$

$f'(1) = 3 - \frac{1}{2} = 2\cdot5$

and $f(1) = 1 - \ln 2 \doteqdot 0\cdot307$

$z_2 = z_1 - \frac{f(z_1)}{f'(z_1)}$ NEWTON'S METHOD

$= 1 - \frac{0\cdot307}{2\cdot5}$

$\doteqdot 0\cdot88$

(b) $(x-2)$ factor $\therefore q(2)=0$

i.e. $2a+b=-20$ (1)

rem. $= 9$ when $\div$ by $(x+1)$ $\therefore q(-1)=-9$

i.e. $-a+b=-11$ (2)

Solving (1) and (2) : $a=-3$, $b=-14$

(c) (i) $\sqrt{(\sqrt{3})^2+1^2} = 2$

$\therefore \sqrt{3}\sin x - \cos x = 2\left(\frac{\sqrt{3}}{2}\sin x - \frac{1}{2}\cos x\right)$

$= 2(\cos\theta\sin x - \sin\theta\cos x)$

[where $\cos\theta = \frac{\sqrt{3}}{2}$ and $\sin\theta = \frac{1}{2}$ i.e. $\theta = \frac{\pi}{6}$

$= 2\sin(x-\theta)$

$= 2\sin\left(x-\frac{\pi}{6}\right)$

(ii) $\sqrt{3}\sin x - \cos x = 1$

$2\sin\left(x-\frac{\pi}{6}\right) = 1$

$\sin\left(x-\frac{\pi}{6}\right) = \frac{1}{2}$

$\left(x-\frac{\pi}{6}\right) = \frac{\pi}{6}, \frac{5\pi}{6}$

$x = \frac{\pi}{3}, \pi$

(d) $\frac{1}{2}\left[e^{2x}\right]_0^{\ln 3} = \frac{1}{2}\left[e^{2\ln 3} - e^0\right]$

$= \frac{1}{2}\left[e^{\ln 9} - 1\right]$

$= \frac{1}{2}[9-1]$

$= 4$

3. (a) $\lim_{x\to0}\frac{\sin 5x}{5x} \times \frac{5}{2} = 1 \times \frac{5}{2}$

$= 2\frac{1}{2}$

3 (b) (i) $y = -\frac{1}{8}x^2$

$y' = -\frac{1}{4}x$

At $(4t, -2t^2)$: $m_{TAN} = -\frac{1}{4} \times 4t = -t$

$\therefore m_{NORM} = \frac{1}{t}$

EQUN NORM: $y + 2t^2 = \frac{1}{t}(x - 4t)$

$y = \frac{x}{t} - 2t^2 - 4$

(ii) $T(0, -2t^2 - 4)$

$M: x = \frac{4t+0}{2} = 2t$, $y = \frac{-2t^2 + -2t^2 - 4}{2} = -2t^2 - 2$

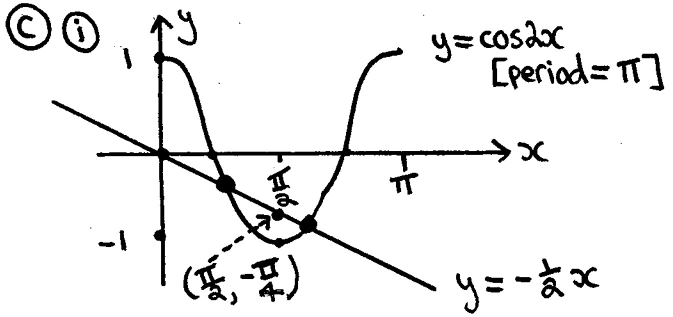
i.e. $M(2t, -2t^2 - 2)$

(iii) sub $t = \frac{x}{2}$ into $y = -2t^2 - 2$

$y = -2 \times \frac{x^2}{4} - 2$

$= -\frac{1}{2}x^2 - 2$

(c) (i)



(ii) 2 points of intersection.

$\therefore \cos 2x = -\frac{1}{2}x$ has 2 solns

(d) STEP 1: Prove true for $n=1$

$7^1 - 1 = 6$ which is divis. by 3

STEP 2: Assume true for $n=k$

i.e. $7^k - 1 = 3M$ (M an integer)

[OR $7^k = 3M+1$]

Hence prove true for $n = k+1$

i.e. $7^{k+1} - 1$ also divis. by 3

Now $7^{k+1} - 1 = 7.7^k - 1$

$= 7.(3M+1) - 1$ by our assumption

$= 21M + 7 - 1$

$= 21M + 6$

$= 3(7M+2)$ which is divis. by 3

i.e. if true for $n=k$ then true for $n=k+1$

STEP 3: we assumed true for k and hence proved true for $n=k+1$. Since true for $n=1$ then by the Principle of Mathematical Induction true for all positive integers.

4. (a) (i) $\frac{d}{dx}\left(\frac{1}{2}v^2\right) = 3x^2$

$\frac{1}{2}v^2 = x^3 + c$

sub $x=1, v=-\sqrt{2}$: $1 = 1 + c \Rightarrow c = 0$

$\frac{1}{2}v^2 = x^3$

$v^2 = 2x^3$

$v = \pm\sqrt{2x^3}$ but take −ve since $v = -\sqrt{2}$ initially

$v = -\sqrt{2x^3}$

(ii) $\frac{dx}{dt} = -\sqrt{2x^3}$

$\frac{dt}{dx} = -\frac{1}{\sqrt{2x^3}} = -\frac{1}{\sqrt{2}}x^{-3/2}$

$t = -\frac{1}{\sqrt{2}} \cdot -2x^{-\frac{1}{2}} + c$

$= \frac{\sqrt{2}}{\sqrt{x}} + c$

sub $t=0, x=1$: $0 = \sqrt{2} + c$

$c = -\sqrt{2}$

i.e. $t = \frac{\sqrt{2}}{\sqrt{x}} - \sqrt{2}$

$t + \sqrt{2} = \frac{\sqrt{2}}{\sqrt{x}}$

$(t + \sqrt{2})^2 = \frac{2}{x}$

$x = \frac{2}{(t+\sqrt{2})^2}$

(b) (i) $\frac{dy}{dx} = 1 - 2\cos x$

Stationary points $\frac{dy}{dx} = 0$:

$1 - 2\cos x = 0$

$\cos x = \frac{1}{2}$

$x = \frac{\pi}{3}, \frac{5\pi}{3}$

4 (b) (ii) $x = \frac{\pi}{3}$, $y = \frac{\pi}{3} - 2 \times \frac{\sqrt{3}}{2} = \frac{\pi}{3} - \sqrt{3}$

$x = \frac{5\pi}{3}$, $y = \frac{5\pi}{3} - 2 \times -\frac{\sqrt{3}}{2} = \frac{5\pi}{3} + \sqrt{3}$

(iii) $\frac{d^2y}{dx^2} = 2\sin x$

$\left(\frac{\pi}{3}, \frac{\pi}{3} - \sqrt{3}\right)$: $\frac{d^2y}{dx^2} = 2\sin\frac{\pi}{3} = \sqrt{3} > 0$

$\therefore$ min. turning point

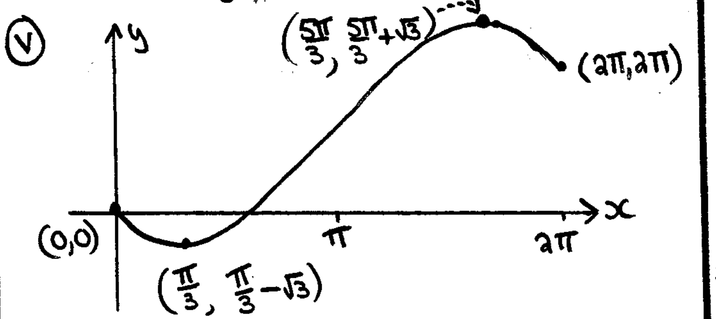
$\left(\frac{5\pi}{3}, \frac{5\pi}{3} + \sqrt{3}\right)$: $\frac{d^2y}{dx^2} = 2\sin\frac{5\pi}{3} = -\sqrt{3} < 0$

$\therefore$ max. turning point

(iv) $x = 0$: $y = 0 - 2\sin 0 = 0$ i.e. $(0,0)$

$x = 2\pi$: $y = 2\pi - 2\sin 2\pi = 2\pi$ i.e. $(2\pi, 2\pi)$

(v)



5. (a) (i) period $= \frac{2\pi}{n} = \frac{2\pi}{\pi} = 2$ s

(ii) sub $t = 0$: $x = 5\sin\frac{\pi}{2}$
$= 5 \times 1$
$= 5$ i.e. 5 m right of $x = 0$.

(iii) when $x = 0$: $5\sin\left(\pi t + \frac{\pi}{2}\right) = 0$

$\pi t + \frac{\pi}{2} = 0, \pi, \ldots$

$\pi t = -\frac{\pi}{2}, \frac{\pi}{2}, \ldots$

$t = -\frac{1}{2}, \frac{1}{2}, \ldots$ ignore $t = -\frac{1}{2}$

i.e. thru $x = 0$ when $t = \frac{1}{2}$ s

$v = \dot{x} = 5\pi\cos\left(\pi t + \frac{\pi}{2}\right)$

when $t = \frac{1}{2}$: $v = 5\pi\cos\left(\frac{\pi}{2} + \frac{\pi}{2}\right)$
$= 5\pi\cos\pi$
$= -5\pi$ m/s

i.e. speed $= 5\pi$ m/s

(iv) $a = \ddot{x} = -5\pi^2\sin\left(\pi t + \frac{\pi}{2}\right)$
$= -\pi^2 \times 5\sin\left(\pi t + \frac{\pi}{2}\right)$
$= -\pi^2 x$

i.e. acceleration proportional to displacement

(b) $\frac{1}{\sqrt{1-\left(\sqrt{1-x^2}\right)^2}} \times \frac{-x}{\sqrt{1-x^2}}$

$= \frac{1}{\sqrt{1-(1-x^2)}} \times \frac{-x}{\sqrt{1-x^2}}$

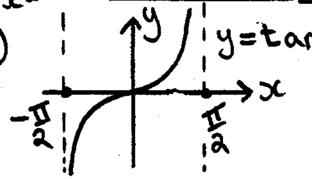
$= \frac{1}{\sqrt{x^2}} \times \frac{-x}{\sqrt{1-x^2}}$

$= \frac{-1}{\sqrt{1-x^2}}$

N.B. $\sqrt{x^2} = x$ for $0 < x < 1$

since $\frac{d}{dx}(1-x^2)^{\frac{1}{2}}$
$= \frac{1}{2}(1-x^2)^{-\frac{1}{2}} \cdot -2x$
$= \frac{-x}{\sqrt{1-x^2}}$

(c) (i)



(ii) series has limiting sum if $-1 < r < 1$.

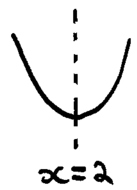
Here $r = \sqrt{3}\tan x$

i.e. $-1 < \sqrt{3}\tan x < 1$

$-\frac{1}{\sqrt{3}} < \tan x < \frac{1}{\sqrt{3}}$

$\therefore -\frac{\pi}{6} < x < \frac{\pi}{6}$

6. (a) (i) axis of symmetry: $x = -\frac{-4}{2} = 2$



$\therefore$ DOMAIN $x \geq 2$

(ii) swap x, y: $x = y^2 - 4y + 7$

$x - 7 = y^2 - 4y$

$x - 7 + 4 = y^2 - 4y + 4$

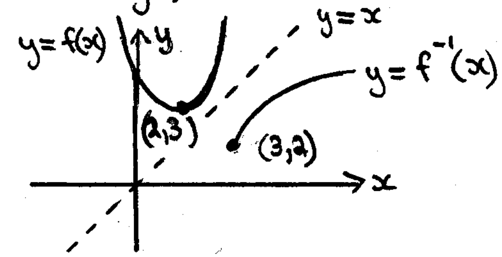
$x - 3 = (y-2)^2$

$y - 2 = \pm\sqrt{x-3}$

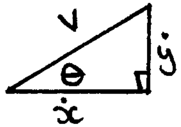
$y = 2 \pm \sqrt{x-3}$

TAKE $y = 2 + \sqrt{x-3}$ SINCE RANGE OF $f^{-1}(x)$ IS $y \geq 2$

(iii)



6. (b) (i) INITIALLY



$\dot{x} = v\cos\theta$

$\dot{y} = v\sin\theta$

HORIZONTAL

$\ddot{x} = 0$

$\dot{x} = c$

$\therefore \dot{x} = v\cos\theta$

$x = vt\cos\theta + K$

sub $t=0, x=0 \Rightarrow K=0$

$x = vt\cos\theta$

VERTICAL

$\ddot{y} = -10$

$\dot{y} = -10t + c$

sub $t=0, \dot{y} = v\sin\theta$

$\Rightarrow c = v\sin\theta$

$\dot{y} = v\sin\theta - 10t$

$y = vt\sin\theta - 5t^2 + K$

sub $t=0, y=0 \Rightarrow K=0$

$y = vt\sin\theta - 5t^2$

(ii) sub $t=2, x=72$ into $x = vt\cos\theta$

$72 = 2v\cos\theta$ (1)

sub $t=2, y=10$ into $y = vt\sin\theta - 5t^2$

$10 = 2v\sin\theta - 20$

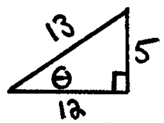
$30 = 2v\sin\theta$ (2)

(2) ÷ (1): $\dfrac{2v\sin\theta}{2v\cos\theta} = \dfrac{30}{72}$

$\tan\theta = 5/12$

$\theta \doteqdot 23°$

since $\tan\theta = 5/12$ $\therefore \cos\theta = \dfrac{12}{13}$



$[\sin\theta = \dfrac{5}{13}]$

so (1): $72 = 2v \times \dfrac{12}{13}$

$v = 39$ m/s

(iii) $y = 39t \times \dfrac{5}{13} - 5t^2 = 15t - 5t^2$

hits ground when $y=0$: $15t - 5t^2 = 0$

$5t(3-t) = 0$

$t = 0, 3.$

$\therefore$ dist. from P: $x = 39 \times 3 \times \dfrac{12}{13}$

$= 108$ m

7. (a) $\sin^{-1}\dfrac{x}{7} + c$

(b) (i) $T = 5 + 75e^{-kt}$

$\dfrac{dT}{dt} = -k \times 75e^{-kt}$

$= -k(T-5)$ since $T - 5 = 75e^{-kt}$

also when $t=0$: $T = 5 + 75e^0 = 80$

(ii) sub $t=10, T=57$: $57 = 5 + 75e^{-10k}$

$52 = 75e^{-10k}$

$\dfrac{52}{75} = e^{-10k}$

$k = \ln\left(\dfrac{52}{75}\right) \div -10$

$\doteqdot 0.0366$

sub $T=20$: $20 = 5 + 75e^{-0.0366t}$

$15 = 75e^{-0.0366t}$

$0.2 = e^{-0.0366t}$

$t = \ln 0.2 \div -0.0366$

$\doteqdot 44$ min

(c) (i) Let $\angle BPC = \alpha$

In $\triangle ACP$: $\tan(\theta + \alpha) = \dfrac{4}{x}$

$\therefore \theta + \alpha = \tan^{-1}\dfrac{4}{x}$

In $\triangle BCP$: $\tan\alpha = \dfrac{1}{x}$

$\therefore \alpha = \tan^{-1}\dfrac{1}{x}$

Now $\theta = (\theta + \alpha) - \alpha = \tan^{-1}\dfrac{4}{x} - \tan^{-1}\dfrac{1}{x}$

(ii) $\dfrac{d\theta}{dx} = \dfrac{1}{1 + \left(\frac{4}{x}\right)^2} \times \dfrac{-4}{x^2} - \dfrac{1}{1 + \left(\frac{1}{x}\right)^2} \times \dfrac{-1}{x^2}$

$= \dfrac{-4}{x^2 + 16} + \dfrac{1}{x^2 + 1}$

when $x = 2$: $\dfrac{d\theta}{dx} = \dfrac{-4}{20} + \dfrac{1}{5} = 0$

$\therefore$ stationary point when $x = 2$.

$\dfrac{d^2\theta}{dx^2} = \dfrac{8x}{(x^2+16)^2} - \dfrac{2x}{(x^2+1)^2}$

when $x = 2$: $\dfrac{d^2\theta}{dx^2} = \dfrac{16}{400} - \dfrac{4}{25}$

$= -3/25 < 0$

$\therefore \theta$ max. when $x = 2$.

(iii) So max. $\theta = \tan^{-1}2 - \tan^{-1}\dfrac{1}{2}$

let $x = \tan^{-1}2$ $\therefore \tan x = 2$

and $y = \tan^{-1}\dfrac{1}{2}$ $\therefore \tan y = \dfrac{1}{2}$

i.e. $\theta = x - y$

$\tan\theta = \dfrac{\tan x - \tan y}{1 + \tan x \tan y}$

$= \dfrac{2 - \frac{1}{2}}{1 + 2 \times \frac{1}{2}}$

$\doteqdot 3/4$

$\therefore \theta = \tan^{-1} 3/4$