# CHELTENHAM GIRLS HIGH SCHOOL

KD

## TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATIONS 1999

# MATHEMATICS

## 3/4 UNIT (Common)

Name: ____________________ ________

Class: 12M Exam. No.: ______ ______

Time allowed: 2 hours.
All questions may be attempted.
All questions are of equal value. (Marks for part questions shown at left.)
All necessary working should be shown. Marks may not be awarded for careless or badly arranged work.
Approved calculators may be used.
Write your name, class and number on this page and exam number on each answer page.
Hand in your question sheet and your answers in bundles clearly marked A, B and C.
This paper does not necessarily reflect the format or content of the HSC.
This paper constitutes 40% of your school assessment.

Standard integrals.

$\int x^n\,dx = \frac{1}{n+1}x^{n+1},\quad n \neq -1;\quad x \neq 0, \text{ if } n < 0$

$\int \frac{1}{x}\,dx = \log_e x,\ x > 0$

$\int e^{ax}\,dx = \frac{1}{a}e^{ax},\ a \neq 0$

$\int \cos ax\,dx = \frac{1}{a}\sin ax,\ a \neq 0$

$\int \sin ax\,dx = -\frac{1}{a}\cos ax,\ a \neq 0$

$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax,\ a \neq 0$

$\int \sec ax \tan ax\,dx = \frac{1}{a}\sec ax,\ a \neq 0$

$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a},\ a \neq 0$

$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\frac{x}{a},\ a > 0,\ -a < x < a$

$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \log_e\{x+\sqrt{x^2-a^2}\},\ |x| > |a|$

$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \log_e\{x+\sqrt{x^2+a^2}\}$

## Part A - Start a new sheet

### Question 1

3 (a) Write $\frac{\sqrt{5}-1}{\sqrt{5}+2}$ in the form $a + b\sqrt{5}$ where a and b are rational.

3 (b) Write down the exact value (irrational denominators accepted) of

(i) tan 60°

(ii) tan 45°

(iii) tan 105°

3 (c) Use the substitution $u = x^2 + 2$ to evaluate $\int_0^1 \frac{x}{(x^2+2)^2}\,dx$.

3 (d) Find all real numbers such that $x^2 + 4x > 5$.

### Question 2

4 (a) Given $\frac{x^3 + 3x^2 + 9x - 1}{x^2 + 9} = Q(x) - \frac{28}{x^2+9}$,

(i) by dividing or otherwise find the polynomial Q(x).

(ii) Hence find $\int \frac{x^3 + 3x^2 + 9x - 1}{x^2 + 9}\,dx$.

3 (b) Use the method of mathematical induction to prove that $7^n - 5^n$ is even for all positive integer n.

3 (c) Find the smallest term of the sequence 3, 12, 21, ... which is greater than 1999.

2 (d) Given that the quadrilateral ABCD is cyclic, show that the sum of the tangents of the angles is zero.
(i.e. Prove that tan A + tan B + tan C + tan D = 0. )

### Question 3

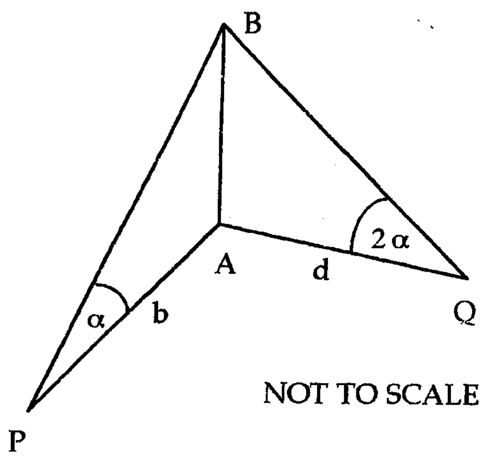
3 (a) Consider the equation $2x^3 + x^2 - 15x - 18 = 0$.
One of the roots of this equation is positive and equals the product of the other 2 roots. Find the roots of this equation.

2 (b) Solve the inequation $x - 1 < \frac{3}{x+1}$.

Question 3 is continued on page 3.

Question 3 continued.

(c)




From a point P, distant b metres due south of a tower AB, the angle of elevation of the top of the tower B is $\alpha$ . From a point Q, distant d metres due east of the tower, the angle of elevation of the top of the tower is $2\alpha$ .

2 (i) Show that $b \tan \alpha = d \tan 2\alpha$ .

3 (ii) Find the height of the tower in terms of d and b.

2 (iii) If the distance PQ is $d\sqrt{10}$ metres, find $\alpha$ .

**Part B - Start a new sheet**

Question 4

2 (a) Find the exact value of $\int_{\frac{\pi}{6}}^{0} \cos x \, dx$

3 (b) Find the volume (1 decimal place) of the hemisphere formed by rotating the curve $y = \sqrt{4 - x^2}$ between $x = 0$ and 2 about the x-axis.

4 (c) On the same diagram sketch the following functions in $0 \le x \le 2\pi$.

(i) $y = \sin x$

(ii) $y = 3 \sin 2x$

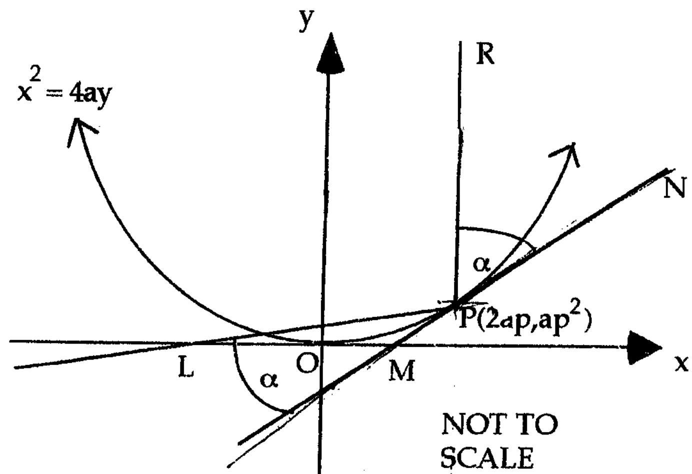
3 (d) A circular piece of paper is cut along 2 radii to form a sector which has area $\frac{1}{3}$ of the circle. If the perimeter of the sector is 100cm, find the radius, correct to 1 decimal place.

Question 5 begins on page 4

Question 5

3 (a) It is known that $f(x) = \frac{1}{2}x - \sin x$ has a root of $f(x) = 0$ near $x = 2$. By using one application of Newton's method find a better approximation to two decimal places.

(b)




The parabola $x^2 = 4ay$ is shown in the sketch above.
The tangent at P $(2ap, ap^2)$ cuts the x-axis at M and passes through the point N.

PR is parallel to the axis of the parabola and makes $\angle RPN = \alpha°$.
PL is such that it cuts the x-axis at L and $\angle LPM = \alpha°$.

2 (i) Show that $\tan \alpha° = \frac{1}{p}$ .

2 (ii) Show that the gradient of LP is $\frac{p^2-1}{2p}$ .

2 (iii) Show that the line LP passes through the focus of the parabola.

3 (c) A particle is projected such that at any time 't', the equation of the trajectory is given by $x = 36t$ and $y = 15t - \frac{1}{2}gt^2$.
Find the angle of projection to the nearest minute.

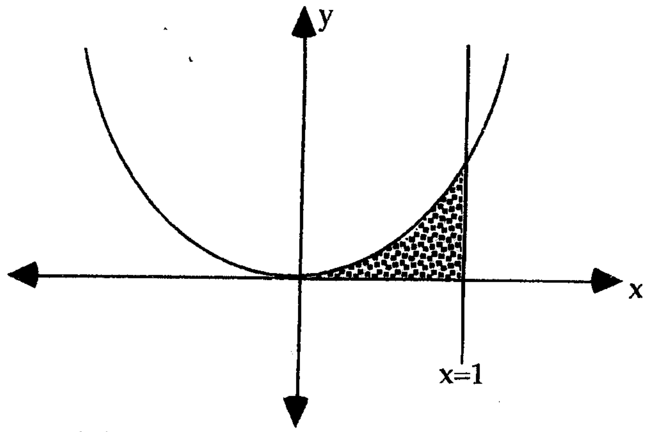
**Part C - Start a new sheet**

Question 6

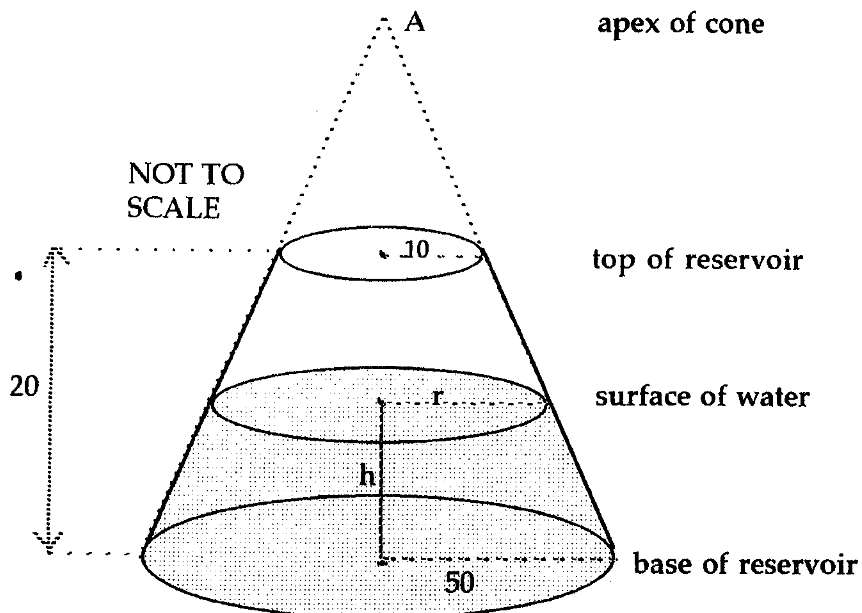
4 (a) The shaded area in the diagram on page 5 represents the area bounded by the curve $y = \frac{x^2}{\sqrt{4-x^2}}$ , the x-axis and the line $x = 1$. Using the substitution $x = 2\sin\theta$, find the shaded area, in terms of $\pi$.

See page 5 for diagram.

Diagram for Question 6 a)




(b) A reservoir containing water is in the shape of a truncated right circular cone of height 20m, as shown below. The radii of the top and the base circles are 10m and 50m respectively. The water from the reservoir is being emptied at the rate of $\frac{\pi}{6}$ $m^3/min$. At any moment during this process, the volume of water remaining is given by $V = \frac{\pi}{3}\left[62\,500 - r^2(25 - h)\right]$ where r is the radius of the upper surface of the water and h is its depth as shown in the diagram.




2 (i) Show that the apex A of the cone would be 5m above the top of the reservoir.

3 (ii) Show that the volume of the water remaining can be expressed by $V = \frac{4\pi}{3}\left[15\,625 - (25 - h)^3\right]$

3 (iii) Find the rate at which the water depth is decreasing when the depth of the water is 15m.