YEAR 12
COMMON TEST 4
TRIAL HSC EXAMS
2007

EXTENSION 1 MATHEMATICS

Time allowed : 2 Hours

DIRECTIONS TO CANDIDATES

- Attempt ALL questions.
- ALL questions are of equal value.
- All necessary working should be shown in every question. Marks may be deducted for careless or badly arranged work.
- Board approved calculators may be used.
- Each question is to be started on a **new page** and you are to write your name, student number and teacher's name on each page.
- The marks allocated for each question are indicated.
- Standard integrals are listed on the last page.

Name : ________________________ Student Number: ______________

Teacher: ____________________________

Q	1	2	3	4	5	6	7	Total
Mark	/12	/12	/12	/12	/12	/12	/12	/84

Question 1 (12 marks)

(a) If $P(x) = x^3 - bx^2 - bx + 4$ is divisible by $x - 2$, find the value of b. **2**

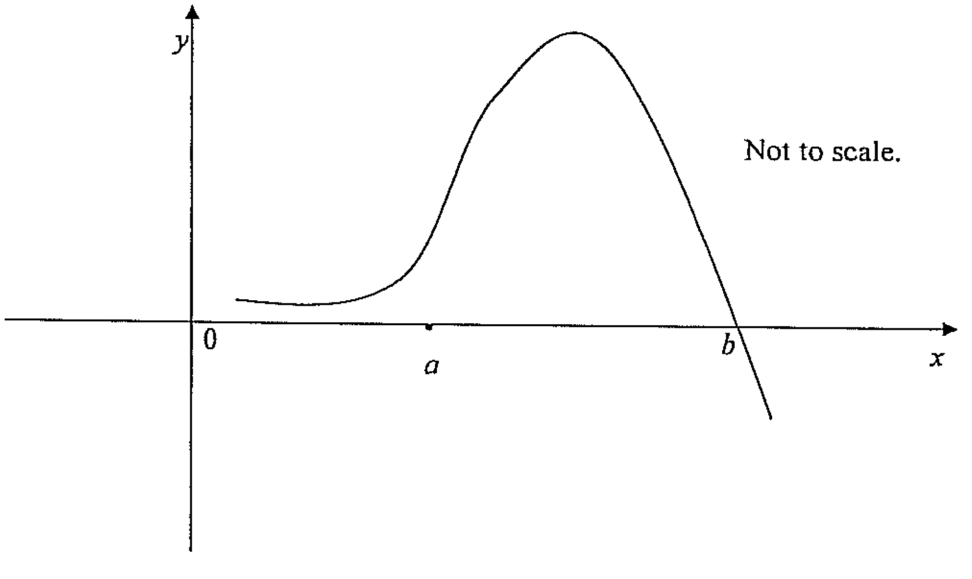
(b) Find $\lim_{x \to 0} \frac{\sin 3x}{x}$ **2**

(c) Write down the domain and range of $3\sin^{-1}\left(\frac{x}{2}\right)$. **2**

(d) Use the substitution $u = 2x + 1$ to find the exact value of **4**

$$\int_0^4 \frac{x}{\sqrt{2x+1}}\,dx$$

(e) Consider the graph of $y = f(x)$. The value a, shown on the graph is taken as the first approximation to the solution b of $y = f(x)$. **2**



Is the second approximation obtained by Newton's method a better approximation than a ? Justify your answer.

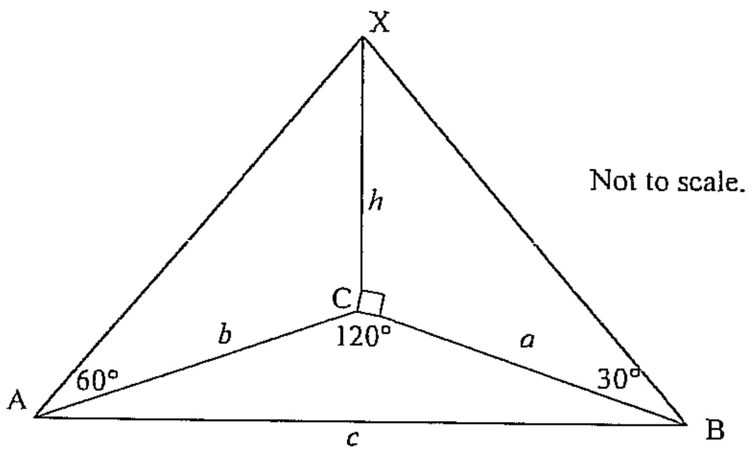
Question 2 (12 marks) Start a new page.

(a) Find all values of x which satisfy the inequality **3**

$$\frac{x+1}{x-1} > 2$$

(b) Find $\int \sin^2 2x \, dx$ **2**

(c)



A tetrahedron has base ΔABC in the horizontal plane with $\angle ACB = 120°$. XC is perpendicular to BC and AC. $\angle XBC = 30°$ and $\angle XAC = 60°$.

(i) Show that $a = h\sqrt{3}$. **1**

(ii) Hence show that $a = 3b$. **1**

(iii) Show that $c = b\sqrt{13}$. **2**

(d) Prove by induction that for all positive integers n **3**

$$\frac{1}{2 \times 3} + \frac{1}{3 \times 4} + \cdots + \frac{1}{(n+1)(n+2)} = \frac{n}{2(n+2)}$$

Question 3 (12 marks) Start a new page.

(a) P $(2ap, ap^2)$ and Q $(2aq, aq^2)$ are two distinct points on the parabola $x^2 = 4ay$.
The chord PQ passes through $(0, -a)$.

(i) Show that $pq = 1$. 3

(ii) The normals at P and Q meet at N where N has co-ordinates $(-apq(p+q), a(p^2+q^2+pq+2))$. (Do **NOT** prove this.) 2

Show that the equation of the locus of N is
$x^2 = a(y-a)$.

(b) Find all values of x, where $0 \le x \le 2\pi$, which satisfy the equation 3

$$\cos x + \sqrt{3}\sin x = 1.$$

(c) Assume that the rate at which a body warms in air is proportional to the difference between its temperature T and the constant temperature A of the surrounding air.
This can be expressed by $\frac{dT}{dt} = k(T - A)$.

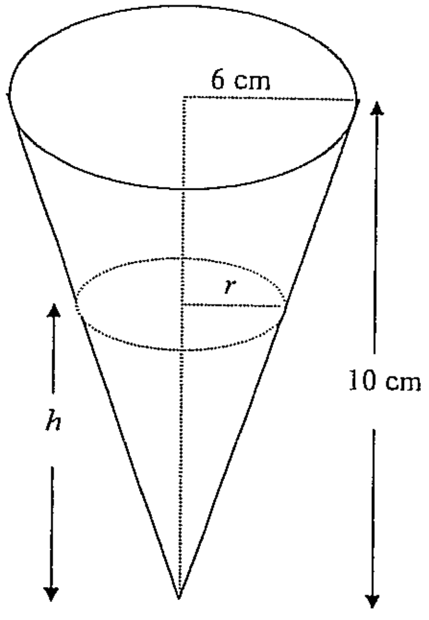
i) Show that $T = A + Ce^{kt}$, where C is a constant, is a solution of $\frac{dT}{dt} = k(T - A)$. 1

ii) A body warms from 5° to 10° in 20 minutes. The air temperature is 30°. 3
Find the temperature of the body after 35 minutes.

Question 4 (12 marks) Start a new page.

(a) Find a general solution to the equation $\sin 2x = 2\sin^2 x$ 3

(b) The diagram shows a conical drinking cup of height 10 cm and radius 6 cm. The cup is being filled with water at the rate of k cm^3 per second. The height of the water at time t seconds is h cm and the radius of the water's surface is r cm.



Not to scale.

i) Show that $r = \dfrac{3h}{5}$. 1

ii) Find the rate at which the height is increasing ie $\dfrac{dh}{dt}$. 3

Question 4 is continued on page 6.

Question 4 (cont)

c) The ΔABC has vertices A (2, 5) , B (-1 , 2) and C (3 , -1). The point

$$Y\left(\frac{3m+2n}{m+n}, \frac{5n-m}{m+n}\right) \text{ divides } AC \text{ in the ratio } m : n.$$

i) Find the point X which divides AB in the ratio $m : n$. **2**

ii) Show that XY is parallel to BC. **1**

iii) Determine $\dfrac{XY}{BC}$. **2**

Question 5 is on page 7.

Question 5 (12 marks) Start a new page.

(a) (i) How many different arrangements of the word REDEEMED are possible? **1**

(ii) An arrangement of the letters of the word REDEEMED is chosen at random
What is the probability that the Es are together ? **2**

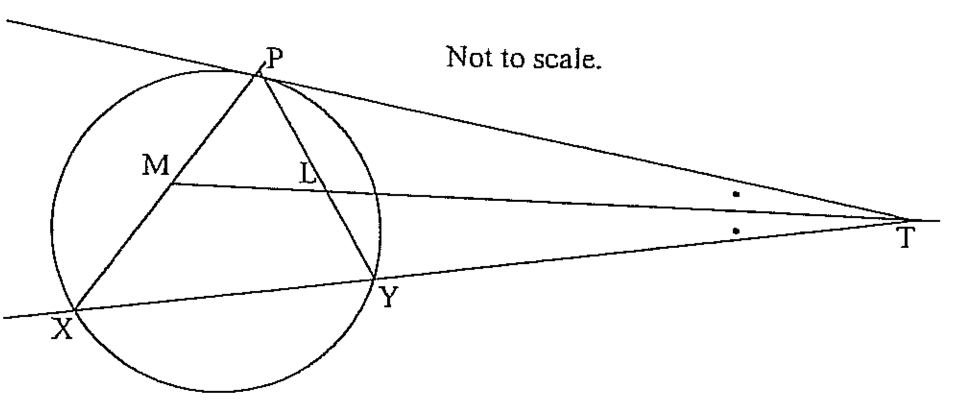
(b) i) Show that the curve $y = x^3 - 12x$ has stationary points at $(-2,16)$ and $(2,-16)$. **2**

ii) Find the largest domain including zero such that the function
$f(x) = x^3 - 12x$ has an inverse $f^{-1}(x)$. **1**

iii) On the same set of axes sketch the graphs of $y = f(x)$ and $y = f^{-1}(x)$ **1**

iv) Find the gradient of the tangent to the curve $y = f^{-1}(x)$ at the point $(11, -1)$. **1**

(c) TP is a tangent to the circle at P. XY is a chord of the same circle produced to T. MT bisects ∠PTX.



i) Show that ∠MPT = ∠LYT **2**

ii) Hence, or otherwise show that Δ MPL is isosceles. **2**

Question 6 (12 marks) Start a new page.

(a) A projectile is launched on a level surface with initial speed V (ms^{-1}) and at an angle θ to the horizontal. The equations of horizontal and vertical motion are given by

$$y = -\frac{1}{2}gt^2 + Vt\sin\theta \text{ and } x = Vt\cos\theta \quad , g \text{ is the acceleration due to gravity.}$$

(Do **NOT** prove this.)

(i) Show that the maximum height H reached by the projectile is $\dfrac{V^2\sin^2\theta}{2g}$. **2**

(ii) Show that the range R of the projectile (i.e. the horizontal distance from its starting position to where it lands) is $\dfrac{V^2\sin 2\theta}{g}$. **2**

A large indoor stadium is 300 metres long. A golf ball is to be hit from floor level from one end of the stadium to the other without hitting the roof. The roof is just high enough for the ball to reach a maximum height of 25 metres.

(iii) Show that if the ball is just going to miss the roof then $\theta = \tan^{-1}\left(\dfrac{1}{3}\right)$. **2**

(iv) Using this value of θ, and using $g = 9.8$, find the value of V so that the ball just lands at the other end of the stadium. **2**

(b) In a raffle 100 tickets are sold. Five tickets are drawn out which will each win a prize.

(i) How many different combinations of five winning tickets are there? **1**

(Leave your answer in unsimplified form)

(ii) Tom buys three tickets. What is the probability that all of Tom's tickets win prizes? **1**

(Leave your answer in unsimplified form)

(iii) What is the probability that at least one of Tom's tickets win prizes? **2**

(Leave your answer in unsimplified form)

Question 7 (12 marks) Start a new page.

a) The line $y = mx$ is a tangent to the curve $y = e^{kx}$, $k > 0$.

i) Show that $m = ke$. 2

ii) Determine the values of m such that the equation 2

$$e^{kx} = mx$$

will have 2 solutions , 1 solution or 0 solutions.

(b) Let $f(x) = \tan^{-1}(\tan x)$

(i) Show that $f'(x) = 1$ for all x where $f'(x)$ is defined. 1

ii) Sketch $y = f(x)$ for $-\pi \leq x \leq \pi$ indicating any discontinuities. 2

iii) For what values of x is $\tan^{-1}(\tan x) = x$. 1

iv) Show that if $-1 < x < 1$ then $2\tan^{-1} x = \tan^{-1}\left(\dfrac{2x}{1-x^2}\right)$. 2

v) Suppose $x > 1$. Write down a relationship between 2

$$2\tan^{-1} x \text{ and } \tan^{-1}\left(\frac{2x}{1-x^2}\right).$$

End of paper.

STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x}\,dx = \ln x, \quad x > 0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}, \quad a \neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax, \quad a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax, \quad a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax, \quad a \neq 0$$

$$\int \sec ax\,\tan ax\,dx = \frac{1}{a}\sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \ln\left(x+\sqrt{x^2-a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \ln\left(x+\sqrt{x^2+a^2}\right)$$

NOTE : $\ln x = \log_e x, \quad x > 0$

Question 1.

a) $P(2) = 8 - 4b - 2b + 4 = 0$

$4b + 2b = +12$

$6b = 12$

$b = 2$

b) $\lim_{x \to 0} \frac{\sin 3x}{x} = 3 \lim_{x \to 0} \frac{\sin 3x}{3x}$

$= 3$

c) $-1 \le \frac{x}{2} \le 1$

$-2 \le x \le 2$: domain

Range : $-\frac{3\pi}{2} \le y \le \frac{3\pi}{2}$

d) $\int_0^4 \frac{x}{\sqrt{2x+1}} \, dx$, $u = 2x+1 \Rightarrow \frac{u-1}{2} = x$

$\frac{du}{dx} = 2$

$x = 0$, $u = 1$

$x = 4$, $u = 9$

$= \frac{1}{2} \int_1^9 \frac{(u-1)\, du}{2u^{1/2}}$

$= \frac{1}{4} \int_1^9 u^{1/2} - u^{-1/2} \, du$

$= \frac{1}{4} \left[\frac{2u}{3} - 2u^{1/2} \right]_1^9$

$= \frac{1}{4} \left[\left(\frac{2}{3} \cdot 9 - 2 \cdot 3 \right) - \left(\frac{2}{3} - 2 \right) \right]$

$= \frac{1}{4} \left[(18 - 6) + 1\frac{1}{3} \right]$

$= 3\frac{2}{3}$

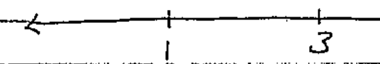
e) No, the x intercept of the tangent at a is further from b than a is.

Question 2

a) $\frac{x+1}{x-1} = 2$, $x \neq 1$

$x + 1 = 2x - 2$

$3 = x$



test $x = 0$, LHS = -1

LHS $\not>$ RHS

$\therefore$ soln $1 < x < 3$

b) $\int \sin^2 2x \, dx$

~~$\cos^2 x + \sin$~~

$\cos 2x = \cos^2 x - \sin^2 x$

$= 1 - \sin^2 x - \sin^2 x$

$1 - 2\sin^2 x$

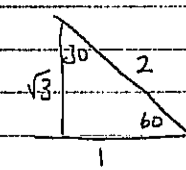
$\sin^2 x = \frac{1}{2} - \frac{1}{2} \cos 2x$

$= \int \frac{1}{2} - \frac{1}{2} \cos 4x \, dx$

$= \frac{x}{2} - \frac{1}{2 \cdot 4} \sin 4x + C$

$= \frac{x}{2} - \frac{1}{8} \sin 4x + C$

c) i) $\tan 30 = \frac{h}{a}$



$a = h \cot 30$

$a = h\sqrt{3}$

ii) $\tan 60 = \frac{h}{b}$

$b\sqrt{3} = h$

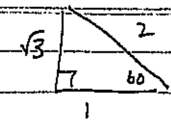
$a = b\sqrt{3} \cdot \sqrt{3}$

$a = 3b$

iii) $c^2 = a^2 + b^2 - 2ab\cos C$

$c^2 = (3b)^2 + b^2 - 2\cdot 3b\cdot b\cos 120^\circ$

$= 10b^2 + 6b^2\cdot\frac{1}{2}$



$c^2 = 13b^2$

$c = b\sqrt{13}$

d) $n=1$, LHS $= \frac{1}{2\cdot 3} = \frac{1}{6}$

RHS $= \frac{1}{2(1+2)} = \frac{1}{6}$

LHS = RHS

assume true for $n=k$ ie $\frac{1}{2\cdot 3} + \dots \frac{1}{(k+1)(k+2)} = \frac{k}{2(k+2)}$

prove true for $n=k+1$ ie $\frac{1}{2\cdot 3} + \dots \frac{1}{(k+2)(k+3)} = \frac{k+1}{2(k+3)}$

$S_{k+1} = S_k + T_{k+1}$

$= \frac{k}{2(k+2)} + \frac{1}{(k+2)(k+3)}$

$= \frac{k(k+3)+2}{2(k+2)(k+3)}$

$= \frac{k^2+3k+2}{2(k+2)(k+3)}$

$= \frac{(k+2)(k+1)}{2(k+2)(k+3)}$

$= \frac{k+1}{2(k+3)}$

Q3

a) i) $\frac{y-aq^2}{x-2aq} = \frac{ap^2-aq^2}{2ap-2aq}$

$= \frac{a(p-q)(p+q)}{2a(p-q)}$

$= \frac{p+q}{2}$

$2y - 2aq^2 = (x-2aq)(p+q)$

passes through $(0,-a)$

$-2a - 2aq^2 = -2aq(p+q)$

$1 + q^2 = qp + q^2$

$pq = 1$

ii) $x = -apq(p+q)$; $y = a(p^2+q^2+pq+2)$

$x = -a(p+q)$ $\quad y = a(p^2+q^2+3)$

$x^2 = a^2(p+q)^2$

$\frac{x^2}{a^2} = p^2+2pq+q^2$

$\frac{x^2}{a^2} - 2 = p^2+q^2$

sub into y: $y = a\left(\frac{x^2}{a^2} - 2 + 3\right)$

$y = a\left(\frac{x^2}{a^2} + 1\right)$

$y = \frac{x^2}{a} + a$

$ay = x^2 + a^2$

$x^2 = ay - a^2$

$x^2 = a(y-a)$

b) $\cos x + \sqrt{3}\sin x = 1$

$\cos x + \sqrt{3}\sin x = r\cos(x \mp \alpha)$

$= r\cos x\cos\alpha + r\sin x\sin\alpha$

$r\cos\alpha = 1$, $r\sin\alpha = \sqrt{3}$

$\frac{r\sin\alpha}{r\cos\alpha} = \tan\alpha = \sqrt{3}$ $\qquad r^2 = 4$

$r = 2$

$\alpha = \frac{\pi}{3}$

$2\cos\left(x - \frac{\pi}{3}\right) = 1$

$\cos\left(x - \frac{\pi}{3}\right) = \frac{1}{2}$

$x - \frac{\pi}{3} = \frac{\pi}{3}, \frac{5\pi}{3}$

$x = \frac{2\pi}{3}, 2\pi, 0$

c) i) $T = A + Ce^{kt}$

$\frac{dT}{dt} = Cke^{kt}$

$= kA + Cke^{kt} - kA$

$= kT - kA$

$= k(T - A)$

ii) when $t = 0$, $T = 5°$, $A = 30°$

$t = 20$, $T = 10°$

when $t = 35$, $T = ?$

when $t = 0$, $A = 30°$, $T = 5$

$5 = 30 + Ce^0$

$C = -25$

$T = 30 - 25e^{kt}$

when $t = 20$, $T = 10°$

$10 = 30 - 25e^{20k}$

$-20 = -25e^{20k}$

$\frac{4}{5} = e^{20k}$

$\ln\left(\frac{4}{5}\right) = 20k$

$k = \frac{1}{20}\ln\left(\frac{4}{5}\right)$

when $t = 35$, $T = ?$

$T = 30 - 25e^{kt}$

$T = 30 - 25e^{35k}$

~~$T = 29.99° \approx 30°$~~

$T = 13.082059$

$\doteqdot 13.1°$

Question 4

a) $\sin 2x = 2\sin^2 x$

$2\sin x\cos x = 2\sin^2 x$

$\sin^2 x - \sin x\cos x = 0$

$\sin x(\sin x - \cos x) = 0$

$\sin x = 0$; $\sin x - \cos x = 0$

$x = 0$; $\sin x = \cos x$

$\tan x = 1$

$x = \frac{\pi}{4}$

b) i) $\frac{6}{10} = \frac{r}{h}$

$r = \frac{3h}{5}$

ii) $\frac{dV}{dt} = k$, $V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi\left(\frac{3h}{5}\right)^2 h$

$V = \frac{3\pi h^3}{25}$

$\frac{dV}{dh} = \frac{9\pi h^2}{25}$

$\frac{dh}{dt} = \frac{dh}{dV}\cdot\frac{dV}{dt}$

$= \frac{25}{9\pi h^2}\cdot k$

$\frac{dh}{dt} = \frac{25k}{9\pi h^2}$

$\frac{1}{\pi} = \frac{25k}{25\cdot 9\pi}$.

$k = 9$

c) i) $A(2,5)$, $B(-1,2)$

$m : n$

$X\left(\frac{-m+2n}{m+n}, \frac{2m+5n}{m+n}\right)$

ii) $m_{BC} = \frac{2+1}{-1-3}$

$= -\frac{3}{4}$

$m_{XY} = \frac{5n-m-(2m+5n)}{3m+2n-(2n-m)}$

$= \frac{-3m}{4m}$

$= -\frac{3}{4}$

iii) $XY = \sqrt{(-m+2n-3m-2n)^2 + (2m+5n-5n+m)^2}$

$= \sqrt{16m^2 + 9m^2}$

$= \sqrt{25m^2}$

$= 5m$

$BC = \sqrt{(-1-3)^2 + (2+1)^2}$

$= \sqrt{16+9}$

$= \sqrt{25}$

$= 5$

$\frac{XY}{BC} = \frac{5m}{5}$

$= m$

Question 5

a) i) $\frac{8!}{2!4!} = 840$

ii) $P(\text{E's together}) = \frac{5!}{2!\,840}$

$= \frac{120}{1680}$

$= \frac{1}{14}$

b) i) $y = x^3 - 12x$

$y' = 3x^2 - 12$, $y'' = 6x$

stat pts when $y' = 0$

ie $3x^2 - 12 = 0$

$x^2 - 4 = 0$

$(x-2)(x+2) = 0$

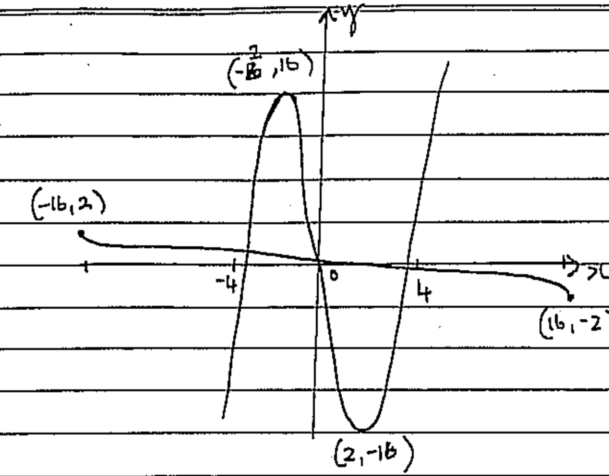
$x = \pm 2$

when $x = 2$, $y = 8 - 24 = -16$ $(2, -16)$

$x = -2$, $y = -8 + 24 = 16$ $(-2, 16)$

ii) $-2 \le x \le 2$

iii)



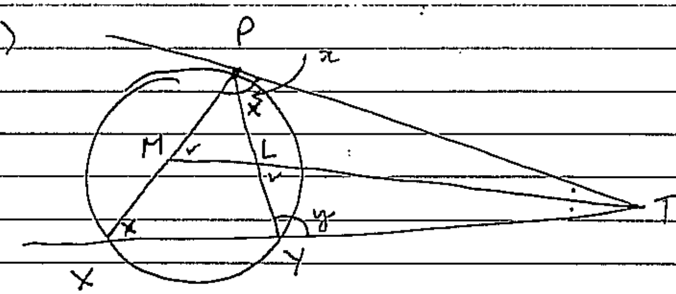
iv) at $x = -1$, $f'(x) = 3x^2 - 12$

$f'(-1) = 3 - 12$

$= -9$

gradient of tangent on $f^{-1}(x)$ is $-\frac{1}{9}$.

c)



i) $\angle LPT = \angle PXY$ ($\angle$ in alt segment)

$\angle TYL = \angle YXM + \angle YPM$ (ext $\angle$ of Δ)

$\angle MPT = \angle LPT + \angle MPY$

$= \angle PXY + \angle MPY$

$\therefore \angle LYT = \angle MPT$

ii) $\Delta MPT \parallel\!| \Delta LYT$ (equiangular)

$\angle YLT = \angle LMP$ (corr. $\angle$'s of $\parallel\!|$ Δ's)

$\angle YLT = \angle PLM$ (vertically opp $\angle$'s)

ΔPML is isosceles

Question 6

a)

$y = -\frac{1}{2}gt^2 + Vt\sin\theta \quad , \quad x = Vt\cos\theta$

i) when $y = 0$

$0 = -\frac{1}{2}gt^2 + Vt\sin\theta$

$0 = t\left(V\sin\theta - \frac{1}{2}gt\right)$

$t = 0, \quad \frac{1}{2}gt = V\sin\theta$

$t = \frac{2V}{g}\sin\theta$ ← time of flight

max ht when $t = \frac{V}{g}\sin\theta$

$H = y = -\frac{1}{2}g\left(\frac{V}{g}\sin\theta\right)^2 + V\left(\frac{V}{g}\sin\theta\right)\sin\theta$

$H = -\frac{1}{2}\frac{V^2}{g}\sin^2\theta + \frac{V^2}{g}\sin^2\theta$

$H = \frac{V^3}{2g}\sin^2\theta$

ii) when $t = \frac{2V}{g}\sin\theta$, $y = 0$ & $x = R$.

$x = R = V\left(\frac{2V}{g}\sin\theta\right)\cos\theta$

$= \frac{V^2}{g}\sin 2\theta$

iii) if $H = 25 = \frac{V^2\sin^2\theta}{2g}$

$50g = V^2\sin^2\theta$

$\sin^2\theta = \frac{50g}{V^2}$

when $y = H$,

$x = Vt\cos\theta$

$150 = V\left(\frac{V}{g}\sin\theta\right)\cos\theta$

$150 = \frac{V^2}{g}\sin\theta\cos\theta$

$\frac{150g}{V^2} = \sin\theta\cos\theta$

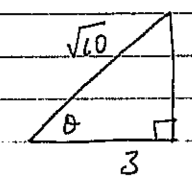
$\frac{\sin^2\theta}{\sin\theta\cos\theta} = \frac{50g}{V^2} \div \frac{150g}{V^2}$

$\frac{\sin\theta}{\cos\theta} = \frac{50g}{V^2} \times \frac{V^2}{150g}$

$\tan\theta = \frac{1}{3}$

$\theta = \tan^{-1}\left(\frac{1}{3}\right)$

iv) $R = \frac{V^2}{g}\sin 2\theta$



$300 = \frac{V^2}{g}\sin 2\theta$

$300g = V^2 . 2 . \frac{1}{\sqrt{10}} . \frac{3}{\sqrt{10}}$

$4900 = V^2$

$V = 70$ m/s.

b) i) $^{100}C_5$

ii) $\dfrac{^3C_0 \cdot {}^{97}C_2}{^{100}C_5}$ $\left(= \dfrac{^{97}C_2}{^{100}C_5}\right)$

iii) $1 - P(\text{winning none})$

$= 1 - \dfrac{^{97}C_5}{^{100}C_5}$

Question 7

i) $y' = ke^{kx}$, $y = mx$ — at x,

$m = ke^{kx}$

$\dfrac{m}{k} = e^{kx}$

$\ln\left(\dfrac{m}{k}\right) = kx$

$x = \dfrac{1}{k}\ln\left(\dfrac{m}{k}\right)$

now $y = \dfrac{m}{k}\ln\left(\dfrac{m}{k}\right)$

$\dfrac{m}{k}\ln\left(\dfrac{m}{k}\right) = e^{\ln(m/k)}$ ~~$\ln(\frac{m}{k})$~~

$= \dfrac{m}{k}$

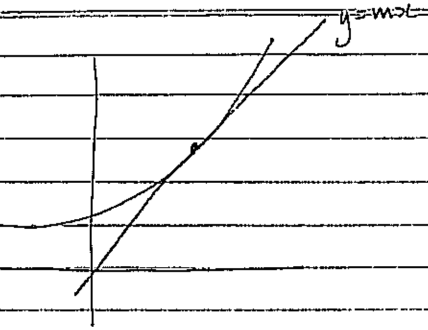
$\ln\left(\dfrac{m}{k}\right) = \dfrac{m}{k} \cdot \dfrac{k}{m}$

$\ln\left(\dfrac{m}{k}\right) = 1$

$\dfrac{m}{k} = e$

$m = ke$

ii)



now when $y = mx$ is a tangent there is 1 soln

ie $mx = ke^{kx}$

$\Rightarrow m = ke$

to have 2 solns $y = mx$ must be a chord

ie $mx > ke^{kx}$

so the y value on ~~$mx=y$~~ $y = mx$ > than y value on

$mx > e^{kx}$

ie $m > ke$

to have 0 solns $y = m$ must not touch $y = e^{kx}$

ie $m < ke$.

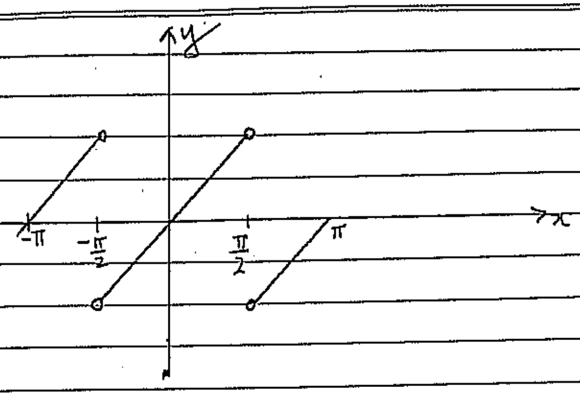
b) $f(x) = \tan^{-1}(\tan x)$

$f'(x) = \dfrac{1}{1 + \tan^2 x} \cdot \sec^2 x$

$= \dfrac{\sec^2 x}{\sec^2 x}$

$= 1$, $\tan\frac{\pi}{2}$ is not defined.

ii)



iii) $-\frac{\pi}{2} < x < \frac{\pi}{2}$ from graph

iv) if $-1 < x < 1$, let $\tan^{-1} x = \alpha$

$x = \tan\alpha$

$$\tan 2\alpha = \frac{2\tan\alpha}{1-\tan^2\alpha}$$

$$= \frac{2x}{1-x^2}$$

also $-1 < x < 1$

$-1 < \tan\alpha < 1$

$-\frac{\pi}{4} < \alpha < \frac{\pi}{4}$

$-\frac{\pi}{2} < 2\alpha < \frac{\pi}{2}$

since $\tan^{-1}(\tan x) = x$ for $-\frac{\pi}{2} < x < \frac{\pi}{2}$

$\Rightarrow \tan^{-1}(\tan 2\alpha) = 2\alpha$ as $-\frac{\pi}{2} < 2\alpha < \frac{\pi}{2}$

now $\tan 2\alpha = \frac{2x}{1-x^2}$ $\therefore$ $\tan^{-1}\left(\frac{2x}{1-x^2}\right) = 2\alpha$

and $\alpha = \tan^{-1} x$ $\tan^{-1}\left(\frac{2x}{1-x^2}\right) = 2\tan^{-1} x$

v) if $x > 1$ then $\frac{\pi}{4} < \alpha < \frac{\pi}{2}$ $(\alpha = \tan^{-1} x)$

$\frac{\pi}{2} < 2\alpha < \pi$

from the graph in ii) it can be seen that

$\tan^{-1}(\tan x) = x - \pi$

let $x = 2\alpha$, $\tan^{-1}(\tan 2\alpha) = 2\alpha - \pi$

now $\tan 2\alpha = \frac{2x}{1-x^2}$ & $\alpha = \tan^{-1} x$

$\therefore$ $\tan^{-1}\left(\frac{2x}{1-x^2}\right) = 2\tan^{-1} - \pi$