Student Name:_______________________

Student Number: ____________________

Teacher: ___________________

2015 TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Extension 1

General Instructions

- Reading time - 5 minutes
- Working time - 2 hours
- Write using black or blue pen Black pen is preferred
- Board-approved calculators may be used
- A table of standard integrals is provided at the back of this paper
- In Questions 11-14, show relevant mathematical reasoning and/or calculations

Total marks – 70

Section I - pages 2 - 5

10 marks

- Attempt Questions 1-10
- Allow about 15 minutes for this section

Section II - pages 6 - 9

60 marks

- Attempt Questions 11-14
- Allow about 1 hour 45 minutes for this section

MC	Question 11	Question 12	Question 13	Question 14	Total
/10	/15	/15	/15	/15	/70

Section I

10 marks
Attempt Questions 1-10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1-10

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1 The polynomial $P(x) = x^3 + 2x + k$ has $(x-2)$ as a factor.
What is the value of k?

(A) -12

(B) -10

(C) 10

(D) 12

2 How many different ways can 4 people be chosen from a group of 20 people?

(A) 4,845

(B) 160,000

(C) 116,280

(D) 240,000

3 The functions $y = x$ and $y = x^3$ meet at the point $(1,1)$.
What is the acute angle between the tangents to these functions at this point to the nearest degree?

(A) 10°

(B) 27°

(C) 45°

(D) 63°

4 A stone is thrown at an angle of α to the horizontal. The position of the stone at time t seconds is given by $x = Vt\cos\alpha$ and $y = Vt\sin\alpha - \frac{1}{2}gt^2$ where g m/s^2 is the acceleration due to gravity and V m/s is the initial velocity of projection.

At what time does the stone reach its maximum height?

(A) $\dfrac{V\sin\alpha}{g}$

(B) $\dfrac{g\sin\alpha}{V}$

(C) $\dfrac{V^2\sin^2\alpha}{2g}$

(D) $\dfrac{g\sin^2\alpha}{2V^2}$

5 What is the derivative of $\cos^{-1}\left(\frac{x}{3}\right)$?

(A) $\dfrac{-1}{3\sqrt{9-x^2}}$

(B) $\dfrac{1}{3\sqrt{9-x^2}}$

(C) $\dfrac{-1}{\sqrt{9-x^2}}$

(D) $\dfrac{1}{\sqrt{9-x^2}}$

6 The velocity $v\ ms^{-1}$ of a particle moving in simple harmonic motion along the x axis is given by $v^2 = 32 + 8x - 4x^2$

What is the amplitude, A, and the period, T, of the motion?

(A) $A = 2$ and $T = \frac{\pi}{2}$

(B) $A = 2$ and $T = \pi$

(C) $A = 3$ and $T = \frac{\pi}{2}$

(D) $A = 3$ and $T = \pi$

7 What is the domain and range of $y = 2\cos^{-1}(x-1)$?

(A) Domain: $0 \le x \le 2$. Range: $0 \le y \le \pi$

(B) Domain: $-1 \le x \le 1$. Range: $0 \le y \le \pi$

(C) Domain: $0 \le x \le 2$. Range: $0 \le y \le 2\pi$

(D) Domain: $-1 \le x \le 1$. Range: $0 \le y \le 2\pi$

8 What is the value of $\int_e^{e^2} \frac{1}{x\log_e x}\, dx$? Use the substitution $u = \log_e x$.

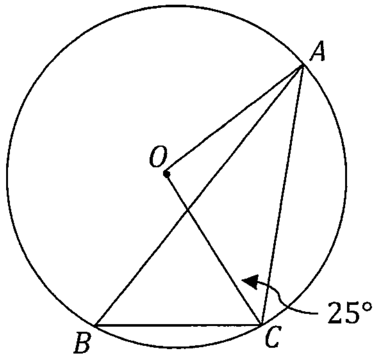
(A) $\log_e 0.5$

(B) $\log_e 2$

(C) $\log_e 4$

(D) 1

9 The points A, B and C lie on a circle with centre O, as shown in the diagram.
The size of $\angle OCA = 25°$.



NOT TO SCALE

What is the size of $\angle ABC$?

(A) 25°

(B) 50°

(C) 65°

(D) 85°

10 What is the solution to the inequality $\dfrac{3}{x-4} \leq 1$?

(A) $x \leq 4$ or $x \geq 7$

(B) $x < 4$ or $x \geq 7$

(C) $4 \leq x \leq 7$

(D) $4 < x \leq 7$

Section II

60 marks
Attempt Questions 11-14
Allow about 1 hour and 45 minutes for this section

Answer each question in a SEPARATE writing booklet. Extra writing booklets are available.

In Questions 11-14, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) Use a SEPARATE writing booklet.

(a) The acceleration of a particle is defined in terms of its position x, by the equation $a = 2x + 4$. If $v = 5$ when $x = 2$, show that the velocity v is given by $v^2 = 2x^2 + 8x + 1$. **2**

(b) The equation $\log_e x - \frac{1}{x} = 0$ has an approximate solution $x_0 = 1$. **2**

Use one application of Newton's method to find another approximate solution to $\log_e x - \frac{1}{x} = 0$.

(c) In a class of 27 students, there are 14 boys and 13 girls.

(i) In how many ways could a captain and vice-captain be chosen from the class? **1**

(ii) In how many ways could two boys and two girls be chosen for the student council? **2**

(iii) The class elected Henry and Olivia for the student council. In how many ways could another boy and another girl be chosen? **1**

(d) The polynomial $P(x) = x^3 + bx^2 + cx + d$ has roots 0, 3 and -3.

(i) What are the values of b, c and d? **2**

(ii) Without using calculus, sketch the graph of $y = P(x)$. **2**

(iii) Hence or otherwise, solve the inequality $\frac{x^2 - 9}{x} > 0$ **1**

(e) The function $f(x)$ is given by $f(x) = 4\tan^{-1} x$. Find the gradient of the tangent to the curve where the function $y = f(x)$ cuts the y-axis. **2**

End of Question 11

Question 12 (15 marks) Use a SEPARATE writing booklet.

(a) The point $C(-3,8)$ divides the interval AB externally in the ratio $k:1$. Find the value of k if A is the point $(6,-4)$ and B is the point $(0,4)$. **2**

(b) The point $P(2ap, ap^2)$ is on the parabola $x^2 = 4ay$. The normal at P cuts the x-axis at S and the y-axis at T.

(i) Prove that the equation of the normal to the parabola at P is given by $x + py = 2ap + ap^3$. **2**

(ii) Hence show that S is the point $(ap(2+p^2), 0)$ and that T is the point $(0, a(2+p^2))$. **2**

(iii) Find the values of p if P is the midpoint of ST. **2**

(c)

(i) Show that $\sin(x+\frac{\pi}{4}) = \frac{\sin x + \cos x}{\sqrt{2}}$. **2**

(ii) Hence or otherwise, solve $\frac{\sin x + \cos x}{\sqrt{2}} = \frac{\sqrt{3}}{2}$ for $0 \le x \le 2\pi$. **2**

(d) After time t years the number N of animals in a national park decreases according to the equation:

$$\frac{dN}{dt} = -0.09(N-100)$$

(i) Verify that $N = 100 + Ae^{-0.09t}$ is a solution of the above equation, where A is a constant. **1**

(ii) After one year the number of animals in the national park is 400. Find the time taken for the number of animals to reach 200. Answer correct to three significant figures. **2**

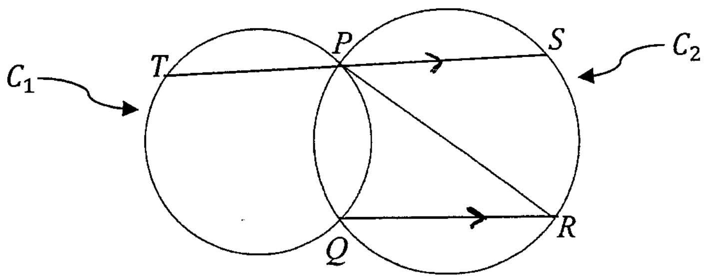
End of Question 12

Question 13 (15 marks) Use a SEPARATE writing booklet.

(a) (i) Find the horizontal asymptote of the graph $y = \frac{3x^2}{x^2+2}$. **1**

(ii) Without the use of calculus, sketch the graph $y = \frac{3x^2}{x^2+2}$, showing the asymptote found in part (i). **2**

(b)



(i) PR is a diameter of circle C_2. A straight line through P, parallel to QR meets circle C_1 at T and circle C_2 at S. Prove that QS is a diameter of circle C_2. **2**

(ii) Prove that the circles have equal radii if TQ is parallel to PR. **2**

(c) Let $f(x) = \frac{2-e^{-x}}{3}$

(i) Find the range of $f(x)$. **1**

(ii) Find the inverse function $f^{-1}(x)$. **2**

(d) A particle moves in a straight line under simple harmonic motion. Its displacement (x metres) from a fixed point O at any time (t seconds) is given by: $x = 4\cos^2 t - 1$.

(i) Find a expression for acceleration in terms of x. **2**

(ii) Sketch $x = 4\cos^2 t - 1$ for $0 \le t \le \pi$. **2**
Clearly show the times when the particle passes through O.

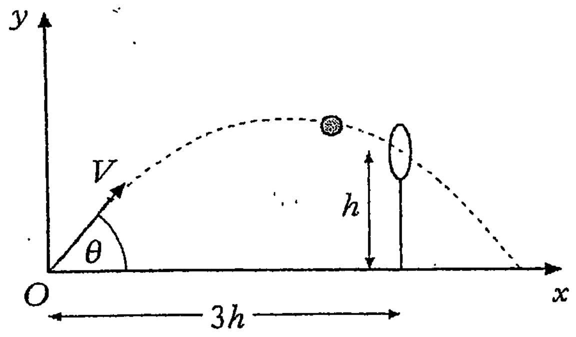
(iii) Find the time when the acceleration of the particle is greatest for $0 \le t \le \pi$. **1**

End of Question 13

Question 14 (15 marks) Use a SEPARATE writing booklet.

(a) Use mathematical induction to prove that $3^n - 2n - 1$ is divisible by 4 for all positive integers greater than 1. **3**

(b)



A football is kicked from O with a velocity of $V = \sqrt{\frac{9gh}{2}}$ at an angle θ to the horizontal. The ball passes through the centre of the hoop which is at point $(3h, h)$ as shown in the diagram above.

With the above axes, you may assume that the position of the projectile is given by

$$x = Vt\cos\theta$$
$$y = Vt\sin\theta - \frac{gt^2}{2}$$

where t is the time, in seconds, after throwing, and g is the acceleration due to gravity.

(i) Show that the Cartesian equation of the path of the ball is given by $y = x\tan\theta - \frac{x^2}{9h}(1 + \tan^2\theta)$. **2**

(ii) Determine the two possible values of $\tan\theta$. **2**

(iii) When $\tan\theta$ takes the larger of these values, find an expression for the range of the football in terms of h. **3**

(c)

(i) Prove that $\sqrt{\frac{1-\sin 2\theta}{1+\sin 2\theta}} = \frac{1-\tan\theta}{1+\tan\theta}$ for $0 \le \theta \le \frac{\pi}{4}$. **2**

(ii) Hence, show that the exact value of $\tan\left(\frac{\pi}{12}\right) = 2 - \sqrt{3}$. **3**

End of paper

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STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x}\,dx = \ln x, \quad x > 0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}, \quad a \neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax, \quad a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax, \quad a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax\,dx = \frac{1}{a}\sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \ln\left(x+\sqrt{x^2-a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \ln\left(x+\sqrt{x^2+a^2}\right)$$

NOTE : $\ln x = \log_e x, \quad x > 0$

Year 12 Mathematics Extension 1 Trial Examination 2015

Section I - Answer Sheet

Student Name/Number ______________________________

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample: $2 + 4 =$ (A) 2 (B) 6 (C) 8 (D) 9

A ○ B ● C ○ D ○

- If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ● B ⊗ C ○ D ○

- If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word correct and drawing an arrow as follows.

correct ↙

A ● B ⊗ C ○ D ○

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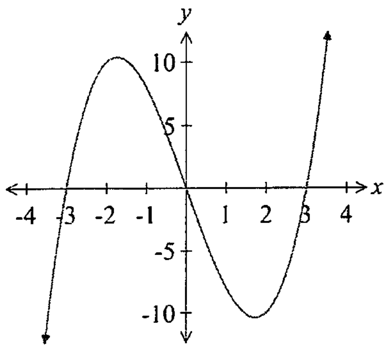
1. A ○ B ○ C ○ D ○
2. A ○ B ○ C ○ D ○
3. A ○ B ○ C ○ D ○
4. A ○ B ○ C ○ D ○
5. A ○ B ○ C ○ D ○
6. A ○ B ○ C ○ D ○
7. A ○ B ○ C ○ D ○
8. A ○ B ○ C ○ D ○
9. A ○ B ○ C ○ D ○
10. A ○ B ○ C ○ D ○

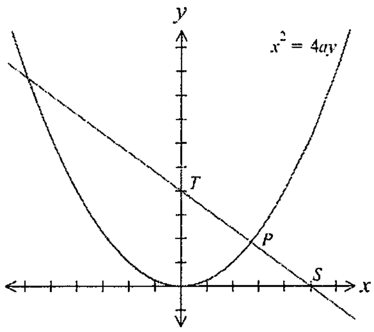
Cheltenham Girls High School
HSC Mathematics Extension 1 Trial Examination
Worked solutions and marking guidelines

SectionI		
	Solution	Criteria
1	$P(x)=x^3+2x+k$ $P(2)=2^3+2\times2+k=0$ ($x-2$ is a factor of $P(x)$) $8+4+k=0$ $k=-12$	1 Mark: A
2	${}^{20}C_4=4,845$	1 Mark: A
3	$y=x$ $m_1=1$ $y=x^3$, $\frac{dy}{dx}=3x^2$ At (1,1) $m_2=3$ $\tan\theta=\left\|\frac{m_1-m_2}{1+m_1m_2}\right\|$ $=\left\|\frac{1-3}{1+1\times3}\right\|$ $=\frac{2}{4}=\frac{1}{2}$ $\theta=26.56505118...\approx27^\circ$	1 Mark: B
4	$y=Vt\sin\alpha-\frac{1}{2}gt^2$ $\dot{y}=V\sin\alpha-gt$ Maximum height when $\dot{y}=0$ $0=V\sin\alpha-gt$ $t=\frac{V\sin\alpha}{g}$	1 Mark: A
5	$\frac{d\left(\cos^{-1}\frac{x}{3}\right)}{dx}=\frac{-1}{\sqrt{1-\left(\frac{x}{3}\right)^2}}\times\frac{1}{3}$ $=\frac{-1}{\sqrt{9-x^2}}$	1 Mark: C
6	$A=3$ and $T=\pi$	1 Mark: D

	Solution	Criteria
7	Domain: $-1 \le (x-1) \le 1$ or $0 \le x \le 2$. Range: $0 \le \cos^{-1}(x-1) \le \pi$ or $0 \le y \le 2\pi$	1 Mark: C
8	$u = \log_e x$ and $du = \frac{1}{x} dx$ $u = \log_e e = 1 \qquad u = \log_e e^2 = 2$ $\int_e^{e^2} \frac{1}{x \log x} dx = \int_1^2 \frac{1}{u} du$ $= [\log_e u]_1^2$ $= \log_e 2 - \log_e 1$ $= \log_e 2$	1 Mark: B
9	$180° - (25° + 25°)$ $= 180° - 50°$ $= 130°$ $< ABC = 130° \div 2$ $= 65°$	1 Mark: C
10	$\frac{3}{x-4} \le 1$ $3(x-4) \le (x-4)^2, \quad x \ne 4$ $3(x-4) - (x-4)^2 \le 0$ $(x-4)[3-(x-4)] \le 0, \quad x \ne 4$ $(x-4)(7-x) \le 0, \quad x \ne 4$ $x < 4$ or $x \ge 7$	1 Mark: B

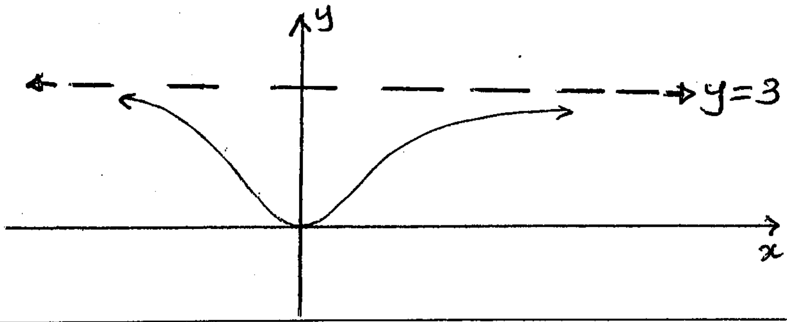
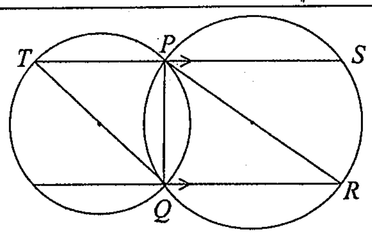
Section II		
	Solution	Criteria
11(a)	$a = 2x + 4$ $\frac{d\left(\frac{v^2}{2}\right)}{dx} = 2x + 4$ $\frac{v^2}{2} = \int (2x+4)\ dx$ $v^2 = 2\int (2x+4)\ dx$ $v^2 = 2x^2 + 8x + C$, C is a constant To find C, When $v = 5, x = 2$ $25 = 8 + 16 + C$ $C = 1$ $\therefore v^2 = 2x^2 + 8x + 1$	2 Marks: Correct answer. 1 Mark: Uses the substitution and simplifies the integral.
11(b)	$f(x) = \log_e x - x^{-1}$ $\quad f'(x) = \frac{1}{x} + \frac{1}{x^2}$ $f(1) = \log_e 1 - \frac{1}{1} = -1$ $\quad f'(1) = \frac{1}{1} + \frac{1}{1^2} = 2$ $x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$ $= 1 - \left(\frac{-1}{2}\right) = 1.5$	2 Marks: Correct answer. 1 Mark: Finds $f(1), f'(1)$ or shows some understanding of Newton's method
11(c) (i)	Order is important. (Select the captain then vice-captain) $^{27}P_2 = 702$	1 Mark: Correct answer.
11(c) (ii)	Order is not important $^{14}C_2 \times {}^{13}C_2 = 91 \times 78$ $= 7098$	2 Marks: Correct answer. 1 Mark: Shows understanding
11(c) (iii)	Order is not important $^{13}C_1 \times {}^{12}C_1 = 13 \times 12$ $= 156$	1 Mark: Correct answer.
11(d) (i)	$P(x) = x^3 + bx^2 + cx + d$ $P(0) = 0^3 + b \times 0^2 + c \times 0 + d = 0, \therefore d = 0$ $P(3) = 3^3 + b \times 3^2 + c \times 3 = 0, \therefore 27 + 9b + 3c = 0$ $P(-3) = (-3)^3 + b \times (-3)^2 + c \times (-3) = 0, \therefore -27 + 9b - 3c = 0$ By inspection $b = 0$ and $c = -9$ Therefore $b = 0,\ c = -9$ and $d = 0$	2 Marks: Correct answer. 1 Mark: Uses the factor theorem or shows some understanding.

11(d) (ii)	$y = x^3 - 9x = x(x^2 - 9)$ 	2 Marks: Correct answer.
11(d) (iii)	$x^2 \times \left(\dfrac{x^2 - 9}{x}\right) > 0 \times x^2$ $x(x^2 - 9) > 0$ From the graph the solution is $-3 < x < 0$ or $x > 3$	1 Mark: Correct answer.
11(e)	$f(x) = 4\tan^{-1} x$ $f'(x) = \dfrac{4}{1 + x^2}$ The curve cuts the y-axis when $x = 0$ $f'(0) = \dfrac{4}{1 + 0^2} = 4$ Slope of the tangent is 4.	2 Marks: Correct answer. 1 Mark: Differentiates the inverse function.

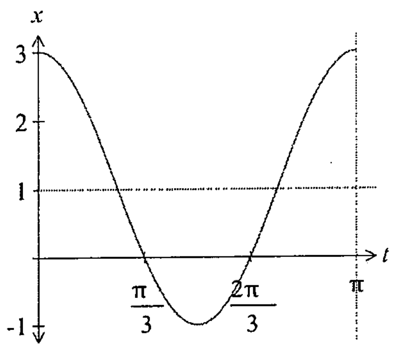
	Solution	Criteria
12(a)	$x = \frac{mx_2 + nx_1}{m+n}$ or $y = \frac{my_2 + ny_1}{m+n}$ $-3 = \frac{k \times 0 - 1 \times 6}{k-1}$ $\quad$ $8 = \frac{k \times 4 - 1 \times -4}{k-1}$ $-3(k-1) = -6$ $\quad$ $8(k-1) = 4k+4$ $-3k+3 = -6$ $\quad$ $8k-8 = 4k+4$ $-3k = -9$ $\quad$ $4k = 12$ $k = 3$ $\quad$ $k = 3$	2 Marks: Correct answer. 1 Mark: Uses the ratio formula with one correct value.
12(b) (i)	 To find the gradient of the normal $y = \frac{1}{4a}x^2$ and $\frac{dy}{dx} = \frac{1}{2a}x$ Gradient of the tangent at $P(2ap, ap^2)$ $\frac{dy}{dx} = \frac{1}{2a} \times 2ap = p$, Gradient of normal at $P(2ap, ap^2)$ $m = -\frac{1}{p}$ $(m_1 m_2 = -1)$ Equation of the normal at $P(2ap, ap^2)$ $y - y_1 = m(x - x_1)$ $y - ap^2 = -\frac{1}{p}(x - 2ap)$ $py - ap^3 = -x + 2ap$ $x + py = 2ap + ap^3$	2 Marks: Correct answer. 1 Mark: Finds the gradient of the tangent or shows some understanding of the problem.
12(b) (ii)	S is on the x-axis ($y = 0$) $x + p \times 0 = 2ap + ap^3$ $x = ap(2 + p^2)$ S is the point $(ap(2+p^2), 0)$ T is on the y-axis ($x = 0$) $0 + py = 2ap + ap^3$ $y = a(2 + p^2)$ T is the point $(0, a(2+p^2))$ $x = \frac{x_1 + x_2}{2}$ $= \pm\sqrt{2}$ $2ap = \frac{ap(2+p^2)+0}{2}$ or $ap^2 = \frac{0 + a(2+p^2)}{2}$	2 Marks: Correct answer.

12(b) (iii)	P is the midpoint of ST $x = \frac{x_1 + x_2}{2}$ or $y = \frac{y_1 + y_2}{2}$ $2ap = \frac{ap(2+p^2)+0}{2}$ $ap^2 = \frac{0+a(2+p^2)}{2}$ $4ap = 2ap + ap^3$ $2ap^2 = 2a + ap^2$ $ap(p^2 - 2) = 0$ $a(p^2 - 2) = 0$ $p = 0,\ p = \pm\sqrt{2}$ $\therefore p = \pm\sqrt{2}$ $\therefore p = \pm\sqrt{2}$	2 Marks: Correct answer.
12 (c) (i)	$\text{LHS} = \sin(x + \frac{\pi}{4})$ $= \sin x \cos\frac{\pi}{4} + \cos x \sin\frac{\pi}{4}$ $= \sin x \times \frac{1}{\sqrt{2}} + \cos x \times \frac{1}{\sqrt{2}}$ $= \frac{\sin x + \cos x}{\sqrt{2}}$ $= \text{RHS}$	2 Marks: Correct answer. 1 Mark: Uses the sum of angles formula or exact values.
12(c) (ii)	$\sin(x + \frac{\pi}{4}) = \frac{\sqrt{3}}{2}$ $x + \frac{\pi}{4} = \frac{\pi}{3}$ or $\frac{2\pi}{3}$ $x = \frac{\pi}{12}$ or $\frac{5\pi}{12}$	2 Marks: Correct answer. 1 Mark: Finds one solution or shows some understanding.
12(d) (i)	$N = 100 + Ae^{-0.09t}$ $\frac{dN}{dt} = -0.09 \times Ae^{-0.09t}$ $= -0.09(N - 100)$	1 Mark: Correct answer.

12(d) (ii)	When $t = 1$ then $N = 400$ $400 = 100 + Ae^{-0.09\times 1}$ $Ae^{-0.09\times 1} = 300$ $A = \frac{300}{e^{-0.09}} = 328.252285...$ We need to find t when $N = 200$ $200 = 100 + 328.25...e^{-0.09t}$ $e^{-0.09t} = \frac{100}{328.25...} = 0.3046...$ $t = \frac{\log_e 0.3064...}{-0.09}$ $= 13.206803...$ ≈ 13.2 years	2 Marks: Correct answer. 1 Mark: Finds the value of A or shows similar understanding of the problem.

	Solution	Criteria
13(a) (i)	$y = \dfrac{3x^2}{x^2+2}$ $\begin{array}{r} 3 \\ x^2+2\overline{)3x^2} \\ \underline{3x^2+6} \\ -6 \end{array}$ $y = 3 - \dfrac{6}{x^2+2}$ Therefore the horizontal asymptote is $y = 3$	1 Marks: Correct answer.
13(a) (ii)		2 Marks: Correct answer.
13(b) (i)		2 Marks: Correct answer. 1 Mark: States the angle in a semicircle is 90° or makes some progress towards the solution.
	$\angle PQR = 90^\circ$ (angle in a semicircle is 90°) $\angle SPQ + \angle PQR = 180^\circ$ (cointerior angles, $PS//QR$) $\therefore \angle SPQ = 90^\circ$ QS is a diameter ($\angle SPQ$ is an angle in a semicircle equal to 90°)	
13(b) (ii)	$\angle PQR = \angle TPQ = 90^\circ$ (alternate angles are equal, $PS//QR$) QT is a diameter ($\angle TPQ$ is an angle in a semicircle equal to 90°) If $TQ//PR$ then $PRQT$ is a parallelogram. $PR = QT$ (opposite sides of a parallelogram are equal) QT and PR are equal diameters. Therefore the two circles have equal radii.	2 Marks: Correct answer. 1 Mark: Shows some understanding.

13(c) (i)	$f(x) = \dfrac{2 - e^{-x}}{3}$ $f(x) = \dfrac{2}{3} - \dfrac{1}{3}e^{-x}$ Range: $y < \frac{2}{3}$	1 Marks: Correct answer.
13(c) (ii)	$f(x) = \dfrac{2 - e^{-x}}{3}$ By interchanging x and y $x = \dfrac{2 - e^{-y}}{3}$ $3x = 2 - e^{-y}$ $e^{-y} = 2 - 3x$ $e^{y} = \dfrac{1}{2 - 3x}$ $y = \log_e\left(\dfrac{1}{2 - 3x}\right)$ $y = -\log_e(2 - 3x), \ \ x < \frac{2}{3}$ $f^{-1}(x) = -\log_e(2 - 3x), \ \ x < \frac{2}{3}$	2 Marks: Correct answer. 1 Mark: Shows some understanding.
13(d) (i)	$x = 4\cos^2 t - 1$ $x = 2(\cos 2t + 1) - 1$ $\quad = 2\cos 2t + 1$ $\dot{x} = -4\sin 2t$ $\ddot{x} = -4 \times 2\cos 2t$ $\quad = -4(x - 1)$	2 Marks: Correct answer. 1 Mark: Shows some understanding.

13(d) (ii)	$x = 4\cos^2 t - 1$ or $x = 2\cos 2t + 1$ Stationary points $\dot{x} = 0$, $\therefore -4\sin 2t = 0$ or $t = 0, \frac{\pi}{2}, \pi, \ldots$ When $t = 0$ $x = 2\cos 2\times 0 + 1 = 3$ When $t = \pi$ $x = 2\cos 2\times \pi + 1 = 3$ When $t = \frac{\pi}{2}$ $x = 2\cos 2\times\frac{\pi}{2} + 1 = -1$ 	2 Marks: Correct answer. 1 Mark: Correct shape of the curve or shows some understanding.
(13)(d) (iii)	Velocity is increasing most rapidly when $\ddot{x}$ has the greatest positive value (or x takes the least value). Greatest value: $\ddot{x} = -8\cos 2t = 8$ when $t = \frac{\pi}{2}$ Velocity is increasing most rapidly at $\frac{\pi}{2}$ seconds.	1 Mark: Correct answer.

	Solution	Criteria
14(a)	Step 1: To prove the statement true for $n=2$ $3^n-2n-1=3^2-2\times2-1=4$ Divisible by 4 Result is true for $n=2$ Step 2: Assume the result true for $n=k$ $3^k-2k-1=4P$ (1) where P is an integer. To prove the result is true for $n=k+1$ $3^{k+1}-2(k+1)-1=4Q$ where Q is an integer. $\text{LHS}=3^{k+1}-2(k+1)-1$ $=3\times3^k-2k-2-1$ $=3\times(3^k-2k-1)+4k$ $=3(4P)+4k$ from (1) $=4(3P+k)$ $=4Q$ $=$ RHS Q is an integer as P and k are integers. Result is true for $n=k+1$ if true for $n=k$ Step 3: Result true by principle of mathematical induction.	3 Marks: Correct answer. 2 Marks: Proves the result true for $n=2$ and attempts to use the result of $n=k$ to prove the result for $n=k+1$. 1 Mark: Proves the result true for $n=2$.
14(b) (i)	$x=vt\cos\theta\rightarrow(1)$ $y=vt\sin\theta-\frac{gt^2}{2}\rightarrow(2)$ From (1) $y=\frac{x}{v\cos\theta}$ Substitute $\frac{x}{v\cos\theta}$ for t in (2) $y=v\left(\frac{x}{v\cos\theta}\right)\sin\theta-\frac{g}{2}\left(\frac{x}{v\cos\theta}\right)^2$ $y=x\tan\theta-\frac{gx^2}{2v^2}\sec^2\theta$ $y=x\tan\theta-\frac{gx^2}{2v^2}(1+\tan^2\theta)$ Since $v=3\sqrt{\frac{gh}{2}}$ $y=x\tan\theta-\frac{x^2}{9h}(1+\tan^2\theta)$	2 Marks: Correct answer. 1 Mark: Shows some understanding.

14(b) (ii)	$y = x\tan\theta - \frac{x^2}{9h}(1+\tan^2\theta)$ Substituting $x = 3h$ and $y = h$ $h = 3h\tan\theta - \frac{9h^2}{9h}(1+\tan^2\theta)$ $h = 3h\tan\theta - h(1+\tan^2\theta)$ $1 = 3\tan\theta - (1+\tan^2\theta)$ $\tan^2\theta - 3\tan\theta + 2 = 0$ $(\tan\theta - 2)(\tan\theta - 1) = 0$ $\tan\theta = 2$ or $\tan\theta = 1$	2 Marks: Correct answer.
14(b) (iii)	$y = x\tan\theta - \frac{x^2}{9h}(1+\tan^2\theta)$ $0 = x\tan\theta - \frac{x^2}{9h}(1+\tan^2\theta)$ $0 = x\left(\tan\theta - \frac{x}{9h}(1+\tan^2\theta)\right)$ $x = 0$ or $\tan\theta - \frac{x}{9h}(1+\tan^2\theta) = 0 \rightarrow (3)$ By substituting $\tan\theta = 2$ in (3) $2 - \frac{x}{9h}(1+4) = 0$ $2 - \frac{x}{9h} \times 5 = 0$ $\frac{5x}{9h} = 2$ $x = \frac{18h}{5}$	3 Marks: Correct answer.

14(c) (i)	$\sqrt{\frac{1-\sin 2\theta}{1+\sin 2\theta}} = \frac{1-\tan\theta}{1+\tan\theta}$ LHS= $\sqrt{\frac{1-\sin 2\theta}{1+\sin 2\theta}}$ $= \sqrt{\frac{1-\frac{2\tan\theta}{1+\tan^2\theta}}{1+\frac{2\tan\theta}{1+\tan^2\theta}}}$ $= \sqrt{\frac{1+\tan^2\theta-2\tan\theta}{1+\tan^2\theta+2\tan\theta}}$ $= \sqrt{\frac{(1-\tan\theta)^2}{(1+\tan\theta)^2}}$ $= \frac{1-\tan\theta}{1+\tan\theta}$ $=$ RHS	2 Marks: Correct answer. . 1 Mark: Shows some understanding.
14(c) (ii)	$\sqrt{\frac{1-\sin 2\theta}{1+\sin 2\theta}} = \frac{1-\tan\theta}{1+\tan\theta}$ $\theta = \frac{\pi}{12}$ $\sqrt{\frac{1-\sin\frac{\pi}{6}}{1+\sin\frac{\pi}{6}}} = \frac{1-\tan\frac{\pi}{12}}{1+\tan\frac{\pi}{12}}$ $\sqrt{\frac{1-\frac{1}{2}}{1+\frac{1}{2}}} = \frac{1-\tan\frac{\pi}{12}}{1+\tan\frac{\pi}{12}}$ $\sqrt{\frac{1}{3}} = \frac{1-\tan\frac{\pi}{12}}{1+\tan\frac{\pi}{12}}$	3 Marks: Correct answer. . 1 Mark: Shows some understanding.

	$\frac{1}{\sqrt{3}} = \frac{1-\tan\frac{\pi}{12}}{1+\tan\frac{\pi}{12}}$ $\sqrt{3}-\sqrt{3}\tan\frac{\pi}{12} = 1+\tan\frac{\pi}{12}$ $\sqrt{3}\tan\frac{\pi}{12}+\tan\frac{\pi}{12} = \sqrt{3}-1$ $\tan\frac{\pi}{12} = \frac{\sqrt{3}-1}{\sqrt{3}+1}$ By rationalising the denominator $\tan\frac{\pi}{12} = 2-\sqrt{3}$	

End of solutions