Student Name: ________________________

Student Number: ______________________

2016

YEAR 12

TRIAL HSC EXAMINATION

Mathematics Extension 1

General Instructions

- Reading time - 5 minutes
- Working time - 2 hours
- Write using black pen
- Board-approved calculators may be used
- A reference sheet is provided at the back of this paper
- Show relevant mathematical reasoning and/or calculations in Questions 11-14

Total marks - 70

Section I
10 marks

- Attempt Questions 1-10
- Allow about 15 minutes for this section

Section II
60 marks

- Attempt Questions 11-14
- Allow about 1 hour 45 minutes for this section

Section I

10 marks
Attempt Questions 1 - 10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1-10

---

1 What is the acute angle, to the nearest degree, between the lines $x-2y+1=0$ and $2x-y-1=0$?

(A) $37°$

(B) $45°$

(C) $90°$

(D) $143°$

2 What is the number of asymptotes on the graph of $y=\dfrac{1}{x^2-1}$?

(A) 1

(B) 2

(C) 3

(D) 4

3 Which of the following is the correct expression for $\int \dfrac{dx}{\sqrt{4-x^2}}$?

(A) $\cos^{-1}\dfrac{x}{2}+C$

(B) $\cos^{-1}2x+C$

(C) $\sin^{-1}\dfrac{x}{2}+C$

(D) $\sin^{-1}2x+C$

4 A curve has parametric equations $x=t-3$ and $y=t^2+2$. What is the Cartesian equation of this curve?

(A) $y=x^2-x-1$

(B) $y=x^2+x-1$

(C) $y=x^2-6x+11$

(D) $y=x^2+6x+11$

5 A particle is moving in a straight line with $v^2 = 36 - 4x^2$ and undergoing simple harmonic motion. If the particle is initially at the origin, which of the following is the correct equation for its displacement in terms of t?

(A) $x = 2\sin(3t)$

(B) $x = 3\sin(2t)$

(C) $x = 2\sin(9t)$

(D) $x = 3\sin(4t)$

6 Solve the inequality $\frac{x-4}{x} \geq 0$.

(A) $x < 0, x \geq 4$

(B) $x \leq 0, x \geq 4$

(C) $x < 0, x > 4$

(D) $x \leq 0, x < 4$

7 A rowing team consists of 8 rowers and a coxswain. The rowers are selected from 15 students in Year 9 and the coxswain is selected from 6 students in Year 8. In how many ways can the team be selected?

(A) ${}^{15}C_8 + {}^{6}C_1$

(B) ${}^{15}C_8 \times {}^{6}C_1$

(C) ${}^{15}P_8 + {}^{6}P_1$

(D) ${}^{15}P_8 \times {}^{6}P_1$

8 What is the value of k such that $\int_0^k \frac{1}{9+x^2} = \frac{\pi}{18}$?

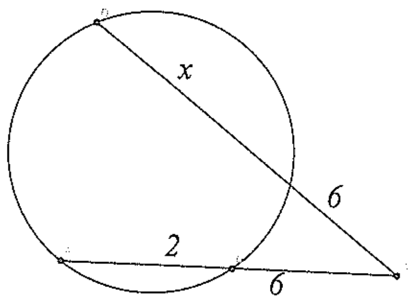
(A) $\sqrt{3}$

(B) $3\sqrt{3}$

(C) 1

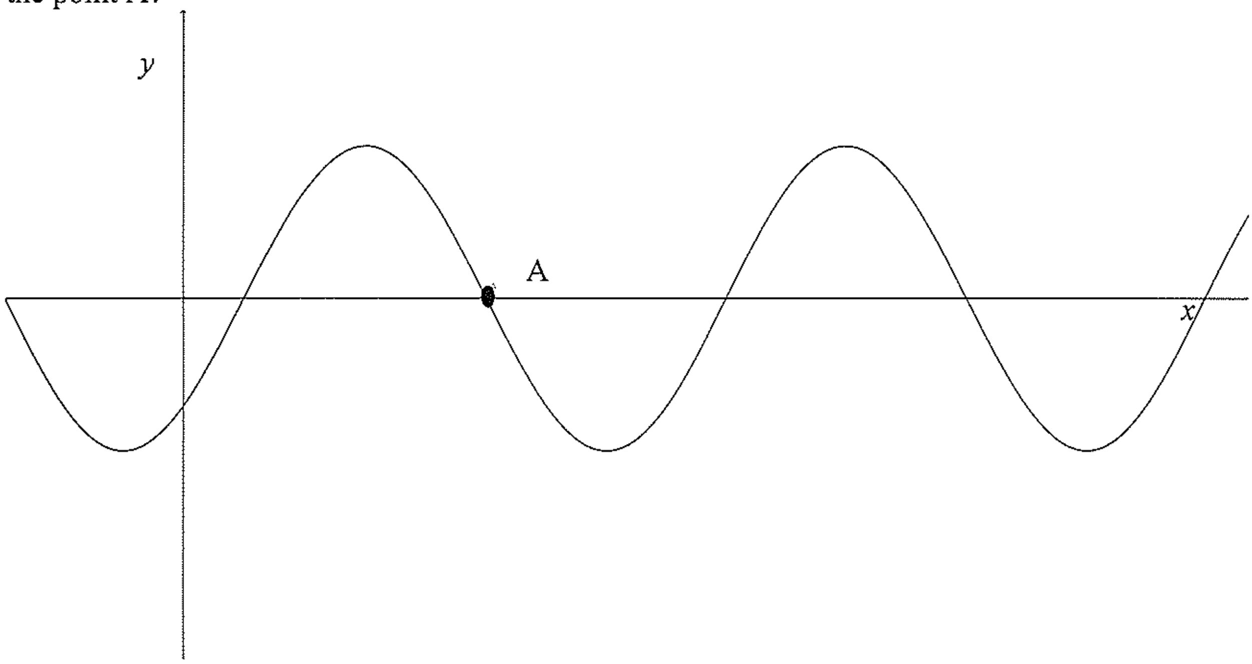
(D) 3

9 What is the value of x?



(A) -8
(B) 2
(C) 8
(D) -2

10. The graph of the function $y = \sin(4x - \frac{\pi}{3})$ is shown below. What is the x - coordinate of the point A?



(A) $-\frac{\pi}{6}$
(B) $\frac{\pi}{3}$
(C) $\frac{7\pi}{12}$
(D) $\frac{5\pi}{6}$

End of Section I

Section II

60 marks
Attempt Questions 11 – 14
Allow about 1 hour and 45 minutes for this section

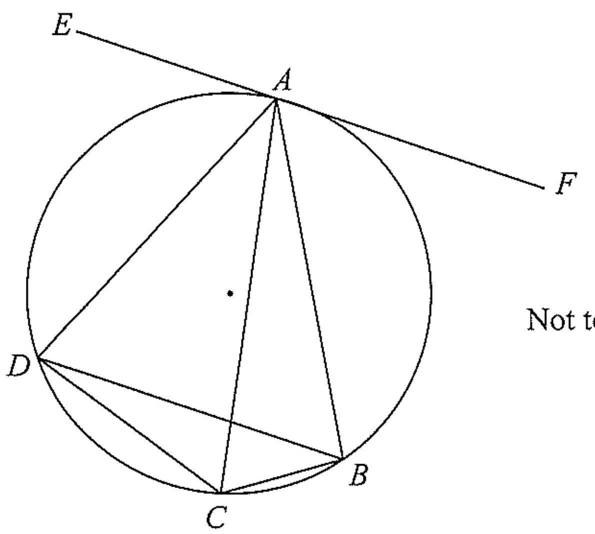
Answer each question in the appropriate writing booklet.

Your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) **Marks**

(a) Use Newton's method to find a second approximation to the positive root of $x - 2\sin x = 0$. Take $x = 1.6$ as the first approximation. **3**

(b) *ABCD* is a cyclic quadrilateral. *EAF* is a tangent at *A* to the circle. *CA* bisects $\angle BCD$. Copy the diagram into answer booklet. **3**



Not to scale

Show that EAF is parallel to DB.

(c) Factorise $2^{n+1} + 2^n$, and hence write $\frac{2^{1001}+2^{1000}}{3}$ as a power of 2. **2**

Question 11 continued on next page

(d) A particle moves in a straight line and its position at any time is given by:

$$x = 1 + \sqrt{3}\cos 4t + \sin 4t$$

(i) Prove the motion is simple harmonic. **2**

(ii) Express $\sqrt{3}\cos 4t + \sin 4t$ in the form $R\cos(4t - \alpha)$, where $R > 0$ and α is acute. Hence find the amplitude of the motion. **3**

(iii) When does the particle first reach maximum speed after time $t = 0$? **2**

End of Question 11

Question 12 (15 marks) **Marks**

(a) $P(2p, p^2)$ and $Q(2q, q^2)$ are two points on the parabola $x^2 = 4y$.
M is the midpoint of PQ.

(i) Show that $(p-q)^2 = 2(p^2+q^2)-(p+q)^2$. **1**

(ii) If P and Q move on the parabola so that $p-q=4$, show that the locus of M is the parabola $x^2 = 4y-16$. **2**

(iii) What is the focus of the locus of M? **1**

(b) What are the roots of the equation $x^3 + 6x^2 + 11x + 6 = 0$ given that the roots are consecutive. **3**

(c) The velocity V of a particle decreases according to the equation:

$$\frac{dV}{dt} = -k(V-P)$$

where t is the time in seconds and k is a positive constant. The initial velocity of the particle is 0 ms^{-1} and the terminal velocity or P is 60 ms^{-1}.

(i) Verify that $V = P + Ae^{-kt}$ is a solution of the above equation, where A is a constant **1**

(ii) What is the value of k if the velocity of the particle after 10 seconds is 35 ms^{-1}? Answer correct to two significant places. **3**

(d) What is the exact value of the definite integral $\int_0^{\frac{\pi}{3}} \sin^2 x \, dx$? **2**

(e) State the domain and range of $y = 4\cos^{-1}\left(\frac{3x}{2}\right)$. **2**

End of Question 12

Question 13 (15 marks) **Marks**

(a) (i) Use the substitution $y = \sqrt{x}$ to find

$$\int \frac{dx}{\sqrt{x(1-x)}}. \quad 3$$

(ii) Use the substitution $z = x - \frac{1}{2}$ to find another expression for

$$\int \frac{dx}{\sqrt{x(1-x)}}. \quad 3$$

(iii) Use the results of parts (i) and (ii) to express $\sin^{-1}(2x - 1)$ in terms of $\sin^{-1}\sqrt{x}$ for $0 < x < 1$. **1**

(b) Use the principle of mathematical induction to prove for $n \geq 1$ that **3**

$$\frac{1}{2!} + \frac{2}{3!} + \frac{3}{4!} + \ldots + \frac{n}{(n+1)!} = 1 - \frac{1}{(n+1)!}$$

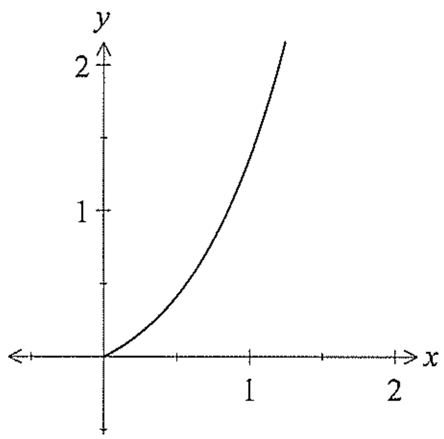
(c) Find $\lim_{x\to 0} \frac{\sin 5x}{3x}$. **2**

(d) Consider the function $f(x) = \frac{xe^x}{2}$ for $x \geq 0$.

(i) Show that $f'(x) > 0$ for all x in the domain. **1**

(ii) Explain why $f(x)$ has an inverse function $f^{-1}(x)$. **1**

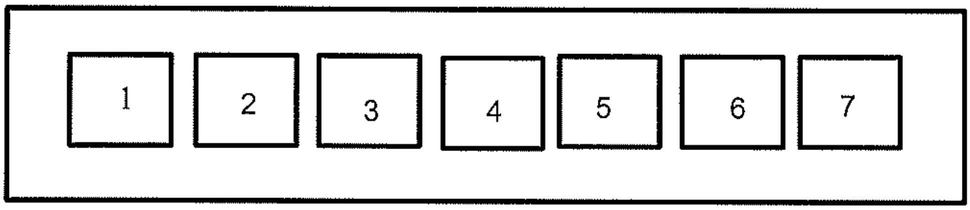
(iii) Copy the sketch of $y = f(x)$ below and insert a sketch of $y = f^{-1}(x)$. **1**



End of Question 13

Question 14 (15 marks) **Marks**

(a)



A security lock has 7 buttons as shown. Each person using the lock has a 4 number code.

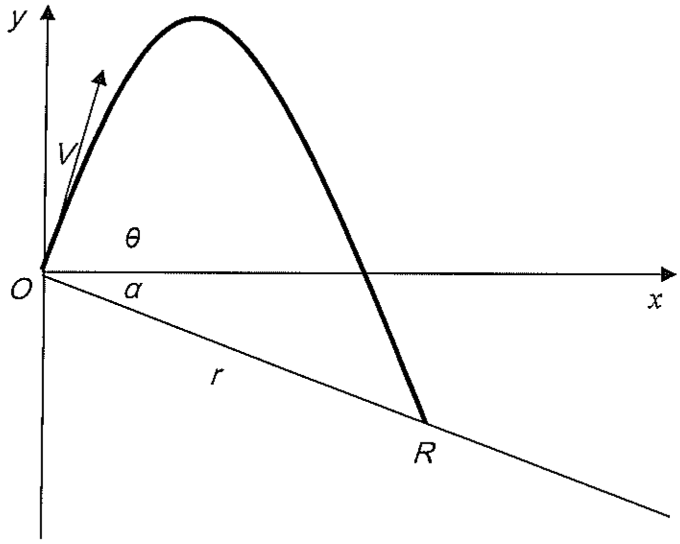
(i) How many codes are possible if numbers can be repeated and their order is important? **1**

(ii) How many codes are possible if numbers cannot be repeated and their order is important? **1**

(iii) Now suppose that the lock operates by holding 4 buttons down together, so that order is not important. How many different codes are possible? **1**

(b) Find all real x such that $|x-1| > 2\sqrt{1-x}$. **3**

Question 14 continued on next page



(c)

The diagram shows an inclined plane that makes an angle of α radians with the horizontal. A projectile is fired from O, at the top of the incline, with a speed of V ms^{-1} at an angle of elevation of θ to the horizontal as shown.

With above axes, you may assume that the position of the projectile is given by

$$x = Vt\cos\theta$$
$$y = Vt\sin\theta - \frac{1}{2}gt^2,$$

Where t is the time, in seconds, after firing and g is the acceleration due to gravity.

For ease of calculation let $K = \frac{g}{2V^2}$.

(i) Show that the path of the trajectory of the projectile is
$$y = x\tan\theta - Kx^2\sec^2\theta$$ **2**

(ii) Show that the coordinates of the point R, where the projectile hits the inclined plane are $\left(\frac{\sin(\theta+\alpha)\cos\theta}{K\cos\alpha}, \frac{-\sin(\theta+\alpha)\cos\theta\sin\alpha}{K\cos^2\alpha}\right)$ **4**

(iii) Find the range r of the projectile, where r = OR, down the inclined plane. **3**

End of paper

Name:
Student Number:

Teacher:

Mathematics Extension 1

Trial Higher School Certificate 2016

SECTION A - Multiple Choice

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample: $2 + 4 =$ (A) 2 (B) 6 (C) 8 (D) 9

A ○ B ● C ○ D ○

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ● B ⊗ C ○ D ○

If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word *correct* and drawing an arrow as follows.

A ⊗ B ⊗ (correct) C ○ D ○

Start Here →

1.	A ○	B ○	C ○	D ○
2.	A ○	B ○	C ○	D ○
3.	A ○	B ○	C ○	D ○
4.	A ○	B ○	C ○	D ○
5.	A ○	B ○	C ○	D ○
6.	A ○	B ○	C ○	D ○
7.	A ○	B ○	C ○	D ○
8.	A ○	B ○	C ○	D ○
9.	A ○	B ○	C ○	D ○
10.	A ○	B ○	C ○	D ○

Mathematics

Factorisation

$$a^2 - b^2 = (a+b)(a-b)$$
$$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$$
$$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$$

Angle sum of a polygon

$$S = (n-2) \times 180°$$

Equation of a circle

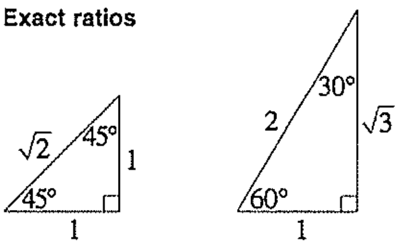
$$(x-h)^2 + (y-k)^2 = r^2$$

Trigonometric ratios and identities

$$\sin\theta = \frac{\text{opposite side}}{\text{hypotenuse}}$$
$$\cos\theta = \frac{\text{adjacent side}}{\text{hypotenuse}}$$
$$\tan\theta = \frac{\text{opposite side}}{\text{adjacent side}}$$

$$\operatorname{cosec}\theta = \frac{1}{\sin\theta}$$
$$\sec\theta = \frac{1}{\cos\theta}$$
$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$
$$\cot\theta = \frac{\cos\theta}{\sin\theta}$$
$$\sin^2\theta + \cos^2\theta = 1$$

Exact ratios



Sine rule

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Cosine rule

$$c^2 = a^2 + b^2 - 2ab\cos C$$

Area of a triangle

$$\text{Area} = \frac{1}{2}ab\sin C$$

Distance between two points

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Perpendicular distance of a point from a line

$$d = \frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$$

Slope (gradient) of a line

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Point-gradient form of the equation of a line

$$y - y_1 = m(x - x_1)$$

***n*th term of an arithmetic series**

$$T_n = a + (n-1)d$$

Sum to *n* terms of an arithmetic series

$$S_n = \frac{n}{2}[2a + (n-1)d] \quad \text{or} \quad S_n = \frac{n}{2}(a + l)$$

***n*th term of a geometric series**

$$T_n = ar^{n-1}$$

Sum to *n* terms of a geometric series

$$S_n = \frac{a(r^n - 1)}{r - 1} \quad \text{or} \quad S_n = \frac{a(1 - r^n)}{1 - r}$$

Limiting sum of a geometric series

$$S = \frac{a}{1-r}$$

Compound interest

$$A_n = P\left(1 + \frac{r}{100}\right)^n$$

Mathematics (continued)

Differentiation from first principles

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Derivatives

If $y = x^n$, then $\frac{dy}{dx} = nx^{n-1}$

If $y = uv$, then $\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$

If $y = \frac{u}{v}$, then $\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

If $y = F(u)$, then $\frac{dy}{dx} = F'(u)\frac{du}{dx}$

If $y = e^{f(x)}$, then $\frac{dy}{dx} = f'(x)e^{f(x)}$

If $y = \log_e f(x) = \ln f(x)$, then $\frac{dy}{dx} = \frac{f'(x)}{f(x)}$

If $y = \sin f(x)$, then $\frac{dy}{dx} = f'(x)\cos f(x)$

If $y = \cos f(x)$, then $\frac{dy}{dx} = -f'(x)\sin f(x)$

If $y = \tan f(x)$, then $\frac{dy}{dx} = f'(x)\sec^2 f(x)$

Solution of a quadratic equation

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Sum and product of roots of a quadratic equation

$\alpha + \beta = -\frac{b}{a}$ $\qquad$ $\alpha\beta = \frac{c}{a}$

Equation of a parabola

$$(x-h)^2 = \pm 4a(y-k)$$

Integrals

$$\int (ax+b)^n\,dx = \frac{(ax+b)^{n+1}}{a(n+1)} + C$$

$$\int e^{ax+b}\,dx = \frac{1}{a}e^{ax+b} + C$$

$$\int \frac{f'(x)}{f(x)}\,dx = \ln|f(x)| + C$$

$$\int \sin(ax+b)\,dx = -\frac{1}{a}\cos(ax+b) + C$$

$$\int \cos(ax+b)\,dx = \frac{1}{a}\sin(ax+b) + C$$

$$\int \sec^2(ax+b)\,dx = \frac{1}{a}\tan(ax+b) + C$$

Trapezoidal rule (one application)

$$\int_a^b f(x)\,dx \approx \frac{b-a}{2}\left[f(a) + f(b)\right]$$

Simpson's rule (one application)

$$\int_a^b f(x)\,dx \approx \frac{b-a}{6}\left[f(a) + 4f\left(\frac{a+b}{2}\right) + f(b)\right]$$

Logarithms – change of base

$$\log_a x = \frac{\log_b x}{\log_b a}$$

Angle measure

$180° = \pi$ radians

Length of an arc

$l = r\theta$

Area of a sector

$\text{Area} = \frac{1}{2}r^2\theta$

Mathematics Extension 1

Angle sum identities

$\sin(\theta+\phi) = \sin\theta\cos\phi + \cos\theta\sin\phi$

$\cos(\theta+\phi) = \cos\theta\cos\phi - \sin\theta\sin\phi$

$\tan(\theta+\phi) = \dfrac{\tan\theta + \tan\phi}{1-\tan\theta\tan\phi}$

***t* formulae**

If $t = \tan\dfrac{\theta}{2}$, then

$$\sin\theta = \frac{2t}{1+t^2}$$

$$\cos\theta = \frac{1-t^2}{1+t^2}$$

$$\tan\theta = \frac{2t}{1-t^2}$$

General solution of trigonometric equations

$\sin\theta = a, \quad \theta = n\pi + (-1)^n \sin^{-1} a$

$\cos\theta = a, \quad \theta = 2n\pi \pm \cos^{-1} a$

$\tan\theta = a, \quad \theta = n\pi + \tan^{-1} a$

Division of an interval in a given ratio

$$\left(\frac{mx_2+nx_1}{m+n}, \frac{my_2+ny_1}{m+n}\right)$$

Parametric representation of a parabola

For $x^2 = 4ay$,

$$x = 2at, \quad y = at^2$$

At $(2at, at^2)$,

tangent: $y = tx - at^2$

normal: $x + ty = at^3 + 2at$

At (x_1, y_1),

tangent: $xx_1 = 2a(y+y_1)$

normal: $y - y_1 = -\dfrac{2a}{x_1}(x-x_1)$

Chord of contact from (x_0, y_0): $xx_0 = 2a(y+y_0)$

Acceleration

$$\frac{d^2x}{dt^2} = \frac{dv}{dt} = v\frac{dv}{dx} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$$

Simple harmonic motion

$x = b + a\cos(nt+\alpha)$

$\ddot{x} = -n^2(x-b)$

Further integrals

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\frac{x}{a} + C$$

$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a} + C$$

Sum and product of roots of a cubic equation

$\alpha+\beta+\gamma = -\dfrac{b}{a}$

$\alpha\beta+\alpha\gamma+\beta\gamma = \dfrac{c}{a}$

$\alpha\beta\gamma = -\dfrac{d}{a}$

Estimation of roots of a polynomial equation

Newton's method

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

Binomial theorem

$$(a+b)^n = \sum_{k=0}^{n}\binom{n}{k}a^k b^{n-k} = \sum_{k=0}^{n}\binom{n}{k}a^{n-k}b^k$$