Name : ____________________
Class : 12 M X___

KW
FH
GF

CHERRYBROOK TECHNOLOGY HIGH SCHOOL

2001 AP4

YEAR 12 TRIAL HSC

MATHEMATICS EXTENSION I

[3 UNIT]

Time allowed - 2 hours
(plus 5 minutes reading time)

DIRECTIONS TO CANDIDATES:

- Attempt ALL questions.
- All questions are of equal value
- Standard Integrals are provided.
- Approved calculators may be used.
- All necessary working should be shown in every question. Full marks may not be awarded for careless or badly arranged work.
- Each question attempted is to be returned on a new page clearly marked Question 1, Question 2, etc on the top of the page.
- Each page must show your class and your name.

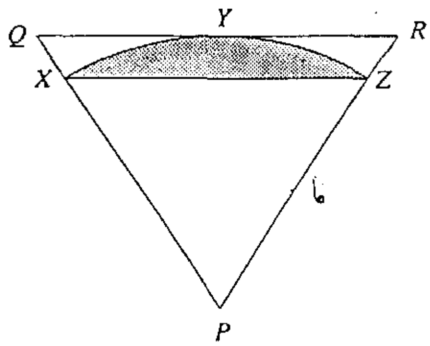
Students are advised that this is a school based Trial Examination *only* and cannot in any way guarantee the complete content nor format of the Higher School Certificate Examination.

Question 1: (12 Marks) **Marks**

(a) Find the co-ordinates of the point P that divides the interval joining $A(-1,1)$ and $B(3,5)$ externally in the ratio 3:1. **2**

(b) Find the exact value of $\int_1^{\sqrt{3}} \frac{dx}{\sqrt{4-x^2}}$ **3**

(c)



The diagram shows an equilateral triangle PQR with sides 6 centimetres in length, and XYZ is an arc of a circle with centre P. the point Y lies on QR.

(i) Show that the length of the radius PY is $3\sqrt{3}$ cm. **1**

(ii) Find the length of the arc XYZ. Leave your answer in exact form **1**

(iii) Find the area of the shaded segment XYZ. Leave your answer in exact form. **2**

(d) Given that $f(x) = \ln(2x+1) + e^{4x}$:

(i) Find $f'(0)$. **1**

(ii) Find $f''(0)$. **1**

(iii) Interpret these results geometrically at the point $(0,1)$. **1**

Question 2: (Start a New Page) (12 Marks)

(a) Use the substitution $u^2 = 4 + x$ to evaluate

$$\int_0^5 \frac{x}{\sqrt{4+x}}dx.$$ **4**

Leave your answer in exact form.

(b) (i) Sketch the graph of $y = \cos x$ in the domain $-\pi \le x \le \pi$.
Use this graph to show that $\cos x + x = 0$ has only one solution. **2**

(ii) Use Newton's Method with a first approximation of $x = -1$ to find a closer approximation to the root of $\cos x + x = 0$. **2**

Question 2 continued on Page 2

Question 2: (Continued) **Marks**

(c) There are 12 videotapes arranged in a row on a shelf in a video shop. There are 3 identical copies of *Gone With the Wind*, 4 of *Tootsie* and 5 of *Gladiator*.

(i) How many different arrangements of the videotapes are there? **1**

(ii) How many different arrangements are there if the videos of the same title are grouped together? **1**

(iii) The videos are arranged at random in a row on the shelf. Find the probability that the arrangement has a copy of *Gladiator* at one end and a copy of *Tootsie* at the other end of the row. Leave your answer as a simplified fraction. **2**

Question 3: (Start a New Page) (12 Marks)

(a) Evaluate $\lim_{x \to 0} \dfrac{\sin 3x}{5x}$. **1**

(b) (i) Using $t = \tan\dfrac{x}{2}$, write expressions for $\sin x$ and $\cos x$ in terms of t. **1**

(ii) Hence, or otherwise, solve $3\cos x + 5\sin x = 5$, in the domain $0^0 < x < 360^0$. Give your answer correct to the nearest degree. **3**

(c) Sketch $y = \sin^{-1} x$ and hence explain why $f(x) = \sin^{-1} x$ is an odd function. **2**

(d) An 8-person committee is to be formed from a group of 10 women and 15 men. In how many ways can the committee be formed if:

(i) the committee must contain 4 men and 4 women? **1**

(ii) there must be more women than men? **2**

(iii) there must be at least 2 women? **2**

Question 4: (Start a New Page) (12 Marks)

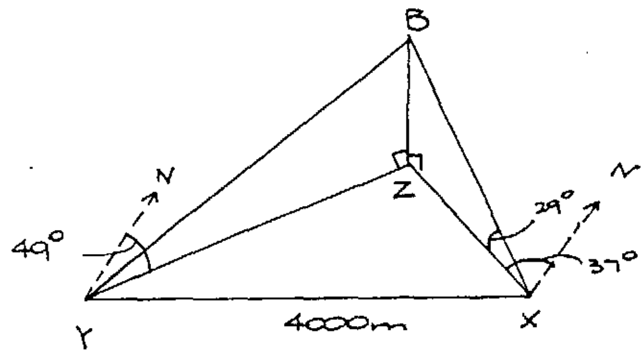
(a) Use the method of proof by Mathematical Induction to prove that **3**

$$\frac{1}{1\times 3} + \frac{1}{3\times 5} + \dots + \frac{1}{(2n-1)(2n+1)} = \frac{n}{2n+1}$$

(b) Find the maximum value of $3\cos x - 2\sin x$. **2**

Question 4 continued on Page 3

(c) 3



The angle of elevation of a hot air balloon (B) from point X is 29^0 and the bearing is $N37^0W$. From the point Y, 4000 metres due West of X, the bearing of the balloon is $N49^0E$. Show that the altitude, h metres, of the balloon is given by

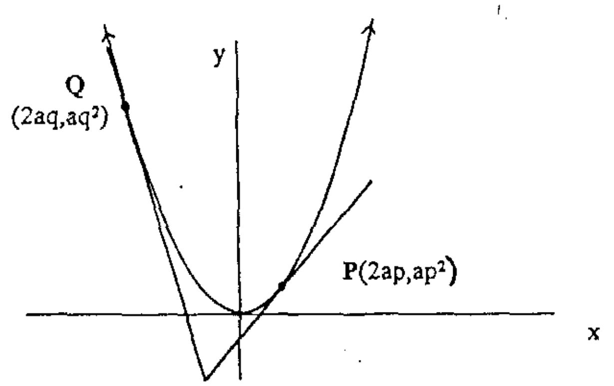
$$h = \frac{4000\sin 41^0 \tan 29^0}{\sin 86^0}.$$

(d) (i) If $f(x) = \log_e(x + \sqrt{x^2+1})$, find $f'(x)$. 2

(ii) Hence, evaluate $\int_0^1 \frac{dx}{\sqrt{x^2+1}}$. Leave answer in exact form. 2

Question 5: (Start a New Page) (12 Marks)

(a)



The points $P(2ap, ap^2)$ and $Q(2aq, aq^2)$ lie on the parabola $x^2 = 4ay$.

(i) Show that the equation of the tangent at P is given by $y = px - ap^2$. 2

(ii) If the tangent at P and the tangent at Q intersect at 45^0 show that $|p-q| = |1+pq|$. 1

(iii) If $q=2$, find p, using the above result. 2

Question 5 continued on Page 4

Question 5: (Continued) — Marks

(b) (i) Write an expression for $\cos 2A$ in terms of $\sin A$. **1**

(ii) Hence, show that $\sin^2 \frac{x}{2} = \frac{1}{2}(1 - \cos x)$. **1**

(iii) Sketch $y = \sin^2 \frac{x}{2}$ for $0 \le x \le 2\pi$. **2**

(iv) State the period and amplitude of $y = \sin^2 \frac{x}{2}$. **1**

(v) Find the exact area of the region bounded by the curve $y = \sin^2 \frac{x}{2}$ and the x-axis between $x = 0$ and $x = \frac{\pi}{3}$. **2**

Question 6: (Start a New Page) (12 Marks)

(a) Consider the function $f(x) = \frac{1}{x^2} - 1,\ x > 0$.

(i) Write down the equations of the horizontal and vertical asymptotes for $y = f(x)$. **1**

(ii) Show that $y = f(x)$ has no stationary points. **1**

(iii) Show that the curve of $y = f(x)$ is always concave up. **1**

(iv) Determine the inverse function $f^{-1}(x)$. **1**

(v) Sketch $y = f^{-1}(x)$. **1**

(b) Consider the expression $P(x) = x^3 + ax^2 + bx + 6$, where a and b are constants. When $P(x)$ is divided by $(x-2)(x+1)$ the remainder is $(4-4x)$, and the quotient is $(x+k)$.

(i) By using the division transformation*, or otherwise, find the values of a and b. **3**

*Division transformation: $P(x) = A(x) \times Q(x) + R(x)$.

(ii) Find the value of k for these values of a and b. **1**

(c) Find any values of m, which will make the expression **3**

$$(m+1)x^2 - 2(m-1)x + (2m-5)$$

a perfect square.

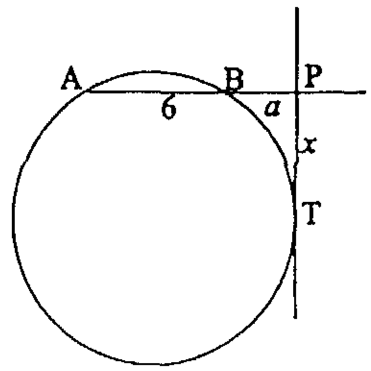
Question 7: (Start a New Page) (12 Marks) **Marks**

(a)

In the circle, the chord AB is 6 metres long. The chord is produced to the point P and BP is a metres.

A tangent to the circle cuts the chord at P. PT is x metres.

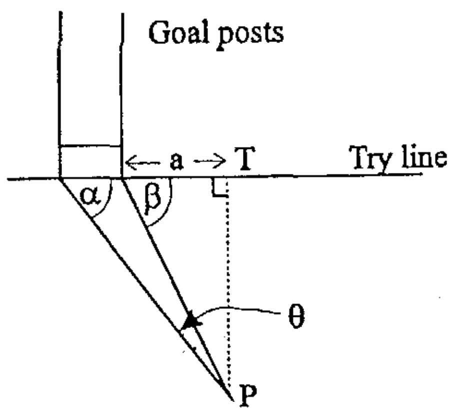
Show that $x = \sqrt{a(a+6)}$. **2**



(b)

In a rugby game, teams score points by *placing the ball over the try line at the end of* the field. A kicker may then take the ball back at rightangles from the try line and attempt to kick the ball between the goal posts.

In the diagram, a try has been scored a metres to the right of the goal posts. The kicker has brought the ball back to the point P to attempt his kick. The kicker wants to *maximise* θ, *his angle of view of the* goalposts.



Let PT be x metres and assume that the goal posts are 6 metres wide

(i) Show that $\tan\theta = \dfrac{6x}{a^2 + 6a + x^2}$. **3**

(ii) Letting $T = \tan\theta$, find the value of x for which T is a maximum. **2**

(iii) Hence show that the maximum angle, θ, is given by **2**

$$\theta = \tan^{-1}\left(\frac{3}{\sqrt{a^2 + 6a}}\right).$$

(iv) If a try is scored 10 metres to the right of the goal posts, find the maximum value of θ (to the nearest minute) and the corresponding value of x (to the nearest centimetre). **2**

(v) Explain why the goal kicker, to maximise his angle of view of the goal posts, should imagine himself at the point of contact of a tangent to the circle passing through the goal posts. **1**

End of Exam