HB.
KW
AW.

Name: ____________________

Class: 12MTX________

Teacher: ____________________

CHERRYBROOK TECHNOLOGY HIGH SCHOOL

2003 AP4

YEAR 12 TRIAL HSC EXAMINATION

MATHEMATICS EXTENSION 1

Time allowed - 2 HOURS
(Plus 5 minutes reading time)

DIRECTIONS TO CANDIDATES:

- Attempt all questions.
- Each question is to be commenced on a new page clearly marked Question 1, Question 2, etc on the top of the page. **
- Each question is to be returned in a separate bundle.
- All necessary working should be shown in every question. Full marks may not be awarded for careless or badly arranged work.
- Approved calculators may be used.
- A Table of Standard Integrals is provided.

Each page must show your name and your class. *

QUESTION ONE. MARKS

(a) Solve for $x: 3^{x+1} = 2$ expressing the answer correct to two decimal places. 2

(b) Given that $\cos\alpha = \frac{5}{13}$, find the exact value of $\cos 2\alpha$ 2

(c) A teacher has 9 girls and 16 boys in her class. The teacher randomly chooses 4 students, one after the other, to form a clean up team to pick up papers in the playground. Assume that all students are present when the teacher makes her selection and that a student's selection in a team on one occasion is independent of their selection on another occasion.

(i) Find the probability that on one occasion the teacher chooses a team which contained only boys. 1

(ii) Find the probability that the youngest boy in the class is included in a team one day but the oldest girl is excluded. 1

(iii) During the course of the term, the teacher has to choose a clean up team on 9 separate occasions. What is the probability that on 3 of these occasions the team contains only boys? Express your answer correct to 4 decimal places. 2

(d) Find $\dfrac{d(e^x \tan^{-1} x)}{dx}$ 2

(e) The polynomial $P(x) = 2x^3 + kx^2 - 1$ is divided by $x+2$ and the remainder is 7. Find the value of k. 2

QUESTION TWO. **(START A NEW PAGE.)**

(a) Find the acute angle between the lines $y = x - 1$ and $\sqrt{2}y = x$. Give your answer correct to the nearest minute. 2

(b) Find $\int_0^2 2x\sqrt{1-\frac{x}{2}}\,dx$ using the substitution $u = 1 - \frac{x}{2}$. 3

(c) (i) Using the expansion $\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$, prove that $\sin 3A = 3\sin A - 4\sin^3 A$. 2

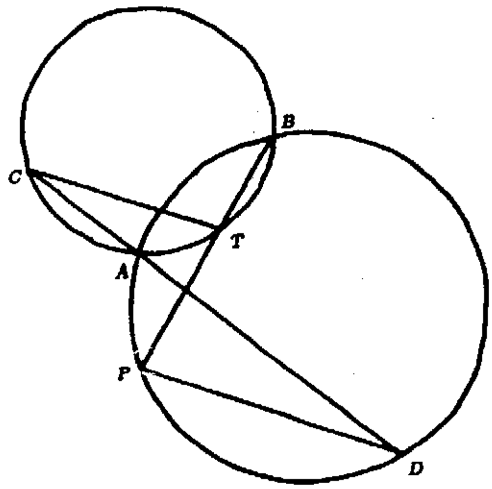
(ii) Hence, or otherwise, show that 2

$$\sin 3A = (2\cos A + 1)(\sin 2A - \sin A)$$

QUESTION 2 CONTINUED ON PAGE 2 ...

 MARKS

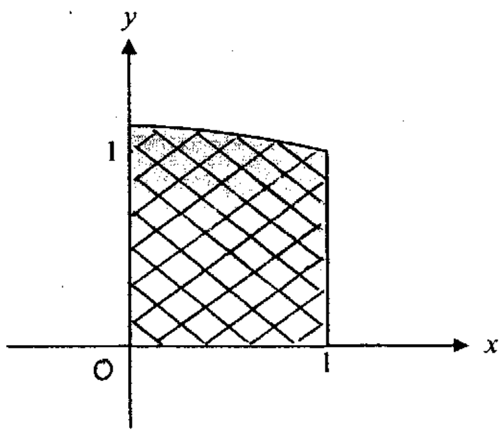
(d) Two circles meet in points A and B. CAD is a double chord and BTP is a chord of the larger circle. Prove that CT || PD. (Hint: Join AB) 3



QUESTION THREE. **(START A NEW PAGE)**

(a) The region bounded by the function $y = \dfrac{2}{\sqrt{x^2+3}}$, 3

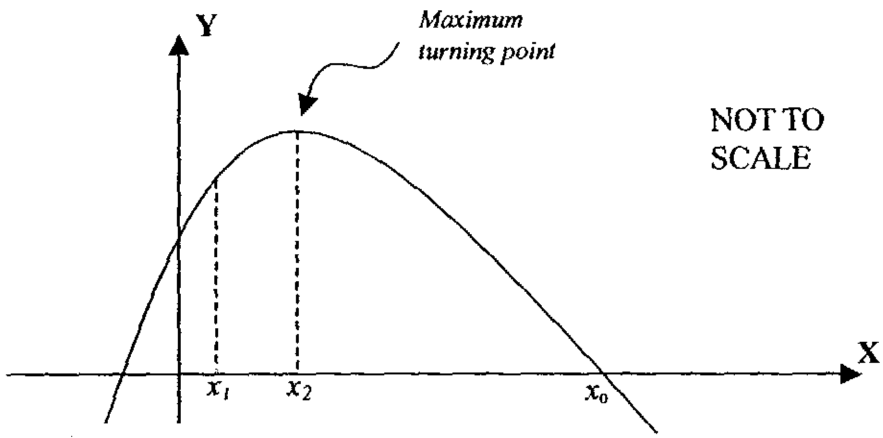
the $x-axis$ and the $y-axis$ and the line $x = 1$, is shaded in the diagram below. Find the volume of the solid formed when the region is rotated about the $x-axis$. Leave your answer in exact form.



QUESTION 3 CONTINUED ON PAGE 3 . . .

(b) (i) Use one application of Newton's method to estimate(to one decimal place) the root of $x^3 - 6x^2 + 24 = 0$ which lies near $x = 3$. 3

(ii) If x_0 is one of the roots of the function as indicated in the diagram below, explain briefly why Newton's method fails if the first approximation for x_0 is taken to be either x_1 and x_2. Copy the diagram below onto your paper and use it to help you with your explanation. 2

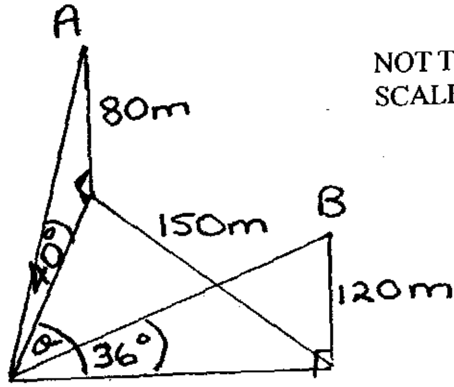


(c) Prove by mathematical induction, that for all positive integer values of n: 4

$$1 \times 5 + 2 \times 6 + 3 \times 7 + \dots\dots\dots + n(n+4) = \frac{1}{6}n(n+1)(2n+13)$$

QUESTION FOUR. **(START A NEW PAGE)**

(a) A surveyor observes two towers A and B. Tower A is due north with a height of 80m, and Tower B is on a bearing of $\theta\,(<90^0)$ with a height of 120m. The angles of elevation of the two towers are 40^0 and 36^0 respectively. The towers are 150m apart on a horizontal plane. Calculate the value of θ, to the nearest minute. 4



QUESTION 4 CONTINUED ON PAGE 4 . .

MARKS

(b) A softdrink can is placed in a fridge which maintains a constant temperature of $1\,^0$C. The temperature T of the can, measured in degrees Celsius, decreases according to the equation $\frac{dT}{dt} = -k(T-1)$ where k is a positive constant and t is the time in minutes.

(i) Show that $T = 1 + Ae^{-kt}$ is a solution to the equation. 1

(ii) If the initial temperature of the can is $40\,^0$C, and it cools to $20\,^0$C after 30 minutes, find the value of k. 2

(iii) How long will it take for the softdrink can to cool from its initial temperature of $40\,^0$C to $10\,^0$C? 2

(c) In a manufacturing process, 10% of the items made are faulty. If 20 items are selected at random, what is the probability (to 2 decimal places) of:

(i) no faulty items? 1

(ii) at least 2 faulty items? 2

QUESTION FIVE. **(START A NEW PAGE.)**

(a) (i) Show that the cartesian equation of the curve with parametric equations $x = 6t, y = 3t^2$ is $12y = x^2$. 1

(ii) Show that if the tangents at the points $P(6p,3p^2)$ and $Q(6q,3q^2)$, intersect on the $y-axis$, then $p^2 = q^2$. 3

(b) To promote the sale of Holden cars, a dealer offers a special deal in which no interest is charged for the first 3 months and then interest rates are left at 1% per month. Kevin buys a 6-cylinder car for \$30000, pays \$10000 in cash and agrees to pay the loan plus interest compounded monthly over 3 years.

(i) Write an equation to show the amount owing after 4 months. 1

QUESTION 5 CONTINUED ON PAGE 5 ...

(b) (ii) Find the amount of each monthly payment. 4

(c) Find the coefficient of the term x^{-4} in the expansion of 3

$$\left(2x - \frac{1}{3x^2}\right)^8$$

QUESTION SIX. (START A NEW PAGE.)

(a) (i) Sketch the graph of $y = 3\sin^{-1}\frac{x}{2}$, stating its domain and range. 2

(ii) Show that the area enclosed by the curve $y = 3\sin^{-1}\frac{x}{2}$, 3

the $x-axis$ and the line $x = 1$ is $\left(\frac{\pi}{2} + 3\sqrt{3} - 6\right)$ sq. units.

(b) (i) Find the general solution of the equation $\tan\alpha = -\frac{1}{\sqrt{3}}$ 2

expressing your answer in terms of π.

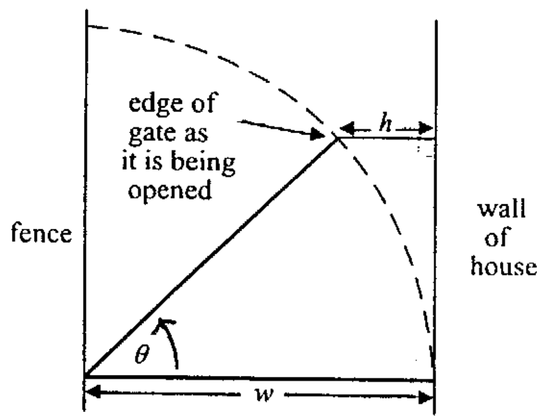
(ii) Hence, find a value of α such that 1

$$-\frac{3\pi}{2} < \alpha < -\pi$$

(c) Find $\frac{d}{dx}\left[\tan^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right)\right]$. 4

QUESTION SEVEN. **(START A NEW PAGE)** MARKS

(a) A gate w metres long, is closed when it is at right angles to a fence and the and the wall of a house. The fence and the wall are parallel. The gate opens out towards the fence. The horizontal opening, created as the gate is opened is the distance from the edge of the gate to the house so that it meets the house wall at right angles. This distance h, in metres, is shown in the diagram below. Let $\theta(t)$ radians be the angle of opening of the gate at time t seconds. The gate is initially shut, but is opened at a rate of $\frac{1}{\sqrt{1-t^2}}$ radians per second.



(i) Show that $h = w - w\cos\theta$. 1

(ii) Show that $\theta = \sin^{-1}t$. 1

(iii) Show that $\frac{dh}{dt} = \frac{wt}{\sqrt{1-t^2}}$. 2

(iv) Using the substitution $u = 1-t^2$, find an expression for h in terms of t. 2

(v) Hence, without using calculus, explain why $\int_0^1 \frac{t}{\sqrt{1-t^2}}dt = 1$. 2

(b) (i) Write an expression for the expansion of $(1+x)^n$ 1

(ii) Hence, show that

$$2\,{}^nC_2 + 6\,{}^nC_3 + 12\,{}^nC_4 + \ldots\ldots n(n-1)\,{}^nC_n = n(n-1)2^{n-2}$$ 3

END OF EXAM.

SOLUTIONS - YEAR 12 EXTENSION TRIAL 2003

QUESTION ONE

(a) $3^{x+1} = 2$

$\ln 3^{x+1} = \ln 2$

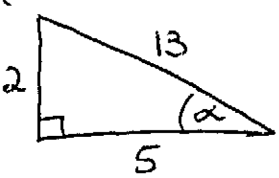
$(x+1)\ln 3 = \ln 2 \longleftarrow (1)$

$x+1 = \dfrac{\ln 2}{\ln 3}$

$x = \dfrac{\ln 2}{\ln 3} - 1$

$x = -0.37 \longleftarrow (1)$

(b) $\cos\alpha = \frac{5}{13}$



$\cos 2\alpha = \cos\alpha\cos\alpha - \sin\alpha\sin\alpha$

$= \left(\frac{5}{13}\right)^2 - \left(\frac{12}{13}\right)^2 \longleftarrow (1)$

$= \dfrac{-119}{169} \longleftarrow (1)$

OR $\cos 2\alpha = 2\cos^2\alpha - 1$

$= 2\left(\frac{5}{13}\right)^2 - 1$

$= \dfrac{-119}{169} \longleftarrow (1)$

(c)(i) $P(\text{all boys}) = \dfrac{{}^{16}C_4 \times {}^{9}C_0}{{}^{25}C_4}$

$= \dfrac{1820 \times 1}{12650}$

$= \dfrac{182}{1265} \longleftarrow (1)$

(ii) only choosing 3 people

P(youngest boy, eldest girl) =

$\dfrac{{}^{23}C_3}{{}^{25}C_4} = \dfrac{7}{50} \longleftarrow (1)$

(iii) only boys $\frac{182}{1265}$ from (i)

So required probability =

${}^{9}C_3\left(\frac{182}{1265}\right)^3\left(\frac{1083}{1265}\right)^6 \longleftarrow (1)$

$= 0.0985 \longleftarrow (1)$

(d) $\dfrac{d\, e^x \tan^{-1}x}{dx}$ let

$u = e^x \quad u' = e^x$

$\tan^{-1}x = v, \quad v' = \dfrac{1}{1+x^2}$ (1)

So $uv' + vu' \longleftarrow (1)$

$= e^x \dfrac{1}{1+x^2} + \tan^{-1}x\, e^x$

$= e^x\left[\dfrac{1}{1+x^2} + \tan^{-1}x\right]$

(f) $P(-2) = -16 + 4k - 1 = 7 \longleftarrow (1)$

So $7 = 4k - 17$

$\therefore k = 6 \longleftarrow (1)$

ie $2x^3 + 6x^2 - 1 = f(x)$

TOTAL 12 MARKS.

QUESTION TWO

(a) $y = x - 1 \quad m = 1$

$y = \dfrac{x}{\sqrt{2}} \quad m = \dfrac{1}{\sqrt{2}}$

So $\tan\theta = \dfrac{1 - \frac{1}{\sqrt{2}}}{1 + 1 \cdot \frac{1}{\sqrt{2}}} \longleftarrow (1)$

(a) $= \frac{\sqrt{2}-1}{\sqrt{2}} \div \frac{\sqrt{2}+1}{\sqrt{2}}$

$= \frac{\sqrt{2}-1}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}+1}$

$= \frac{\sqrt{2}-1}{\sqrt{2}+1}$

$= 9° 44' \longleftarrow 1$

(b) $\int_0^2 2x\sqrt{1-\frac{x}{2}}\,dx$

$u = 1 - x/2 \qquad \frac{du}{dx} = -\frac{1}{2}$

So $-\frac{x}{2} = u - 1$ Also

$\frac{x}{2} = 1-u \qquad x=0$ then $u=1$

$x = 2-2u \qquad x=2$ then $u=0$

1

So $\int_0^2 2x\sqrt{1-\frac{x}{2}}\,dx$ becomes 1

$\int_1^0 2.(2-2u)\sqrt{u}\,.-2\,du$

$= \int_1^0 -8(1-u)\sqrt{u}\,du$

$= 8\int_1^0 (u-1)\sqrt{u}\,du$

$= 8\int_1^0 u^{3/2} - u^{1/2}\,du$

$= 8\left[\frac{2}{5}u^{5/2} - \frac{2}{3}u^{3/2}\right]_1^0$

$= 8\left(\frac{2}{5} - 2/3\right) - 0$

$= \frac{32}{15} \longleftarrow 1$

(c) $\sin 3A = 3\sin A - 4\sin^3 A$

LHS: $\sin 3A = \sin(2A+A)$

$= \sin 2A\cos A + \cos 2A\sin A$

1 $\rightarrow = 2\sin A\cos A\cos A + (1-2\sin^2 A)\sin A$

$= 2\sin A(1-\sin^2 A) + \sin A - 2\sin^3 A$

$= 2\sin A - 2\sin^3 A + \sin A - 2\sin^3 A$

$= 3\sin A - 4\sin^3 A \longleftarrow 1$

$=$ RHS

(ii) RHS $= (2\cos A+1)(\sin 2A - \sin A)$

$= 2\cos A\sin 2A - 2\cos A\sin A + \sin 2A - \sin A$

1

$= 2\cos A(2\sin A\cos A) - \sin 2A + \sin 2A - \sin A$

$= 4\sin A(1-\sin^2 A) - \sin A$

$= 4\sin A - 4\sin^3 A - \sin A$

$= 3\sin A - 4\sin^3 A$ 1

$=$ LHS

(d) To Prove: $CT \parallel PD$

Given: CA, AD chords

BP chord.

Construction: Join AB

Proof: $\angle ACT = \angle ABT \longleftarrow 1$

($\angle$s on same arc are $=$)

But $\angle ABT = \angle ADP$ ($\angle$s on same arc are $=$)

$\therefore \angle ACT = \angle ADP \longleftarrow 1$

$\therefore CT \parallel PD$ (alt $\angle$s are $=$ only

QUESTION 3:

(a) $y = \dfrac{2}{\sqrt{x^3+3}}$

$V = \pi \int y^2\,dx$

$= \pi \int_0^1 \left(\dfrac{2}{\sqrt{x^3+3}}\right)^2 dx$ 1

$V = \pi \int_0^1 \dfrac{4}{x^2+3}\,dx$

$= \dfrac{4\pi}{\sqrt{3}} \int_0^1 \dfrac{\sqrt{3}}{x^2+(\sqrt{3})^2}\,dx$

$= \dfrac{4\pi}{\sqrt{3}} \left[\tan^{-1}\dfrac{x}{\sqrt{3}}\right]_0^1$ 1

$= \dfrac{4\pi}{\sqrt{3}} \left[\tan^{-1}\dfrac{1}{\sqrt{3}} - \tan^{-1} 0\right]$

$= \dfrac{4\pi}{\sqrt{3}} \left(\dfrac{\pi}{6} - 0\right)$

$= \dfrac{2\pi^2}{3\sqrt{3}}$ OR $\dfrac{2\sqrt{3}\pi^2}{9} u^3$ 1

(b)(i) $f(x) = x^3 - 6x^2 + 24$

$f'(x) = 3x^2 - 12x$

$f(3) = 27 - 54 + 24 = -3$ 1

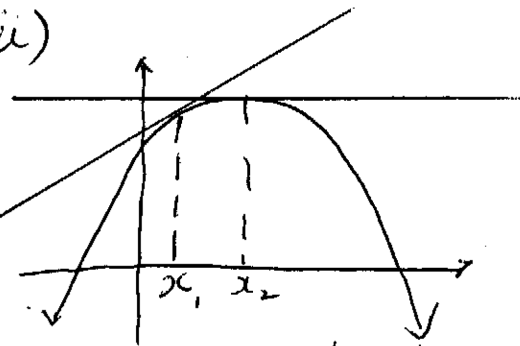
$f'(3) = 27 - 36 = -9$

$x_0 = x_1 - \dfrac{f(x_1)}{f'(x_1)}$ 1

$= 3 - \dfrac{-3}{-9}$

$= 2\tfrac{2}{3}$ or 2.7 1

(ii)



1

Both tangents drawn at x_1 and x_2 don't cut the axis near the root. The tangent at x_1 cuts further away from the root and at x_2 doesn't even cut the x-axis 1

(c) $1\times5 + 2\times6 + 3\times7 + \dots n(n+4) = \frac{1}{6}n(n+1)(2n+13)$

Step 1: When $n=1$

LHS: $1\times5 + 2\times6 + 3\times7 + \dots 1(1+4)$

$= 1\times5$

$= 5$

RHS $= \frac{1}{6}\cdot1\cdot2\cdot15 = 5$

$\therefore$ LHS = RHS. 1

$\therefore$ true for $n=1$.

Step 2: Let $n=k$

$1\times5 + 2\times6 + \dots k(k+4) = \frac{1}{6}k(k+1)(2k+13)$

Step 3: Prove true for $n=k+1$

ie $1\times5 + 2\times6 + \dots k(k+4) + (k+1)(k+5)$

$= \frac{1}{6}(k+1)(k+2)(2k+15)$ 1

Question 3 cont. (c)

LHS : $\frac{1}{6}k(k+1)(2k+13) + (k+1)(k+5)$ 1

$= \frac{1}{6}(k+1)\left[k(2k+13) + 6(k+5)\right]$

$= \frac{1}{6}(k+1)(2k^2+19k+30)$

$= \frac{1}{6}(k+1)(k+2)(2k+15)$ 1

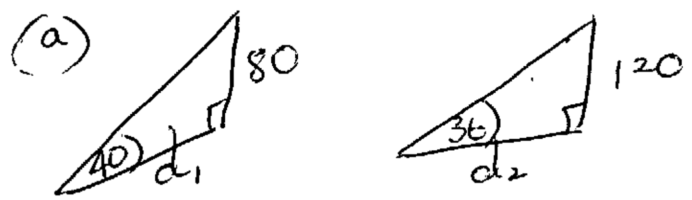
$=$ RHS.

Step 4: True for $n=1$, also true for $n=1+1$, ie $n=2$ also true for $n=2+1$, $n=3\ldots$

$\therefore$ true for all n.

QUESTION 4:

(a)



$\tan 40° = \frac{80}{d_1}$

$d_1 = \frac{80}{\tan 40}$ 1

$\tan 36 = \frac{120}{d_2}$

$d_2 = \frac{120}{\tan 36}$ 1

$$\therefore \cos\theta = \frac{\left(\frac{80}{\tan 40}\right)^2 + \left(\frac{120}{\tan 36}\right)^2 - 150^2}{2 \times \frac{80}{\tan 40} \times \frac{120}{\tan 36}}$$ 1

$\therefore \theta = 63°52'$ 1

(b) $T = 1 + Ae^{-kt}$

$\frac{dT}{dt} = -kAe^{-kt}$

$= -k\left[1 + Ae^{-kt} - 1\right]$

$= -k\left[T - 1\right]$

$\therefore T = 1 + Ae^{-kt}$ is a solution.

(ii) When $t=0$, $T=40$

$A = 39$

$\therefore T = 1 + 39e^{-kt}$

when $t = 30$ $T = 20$

$20 = 1 + 39e^{-30k}$

$\therefore e^{-30k} = \frac{19}{39}$

$-30k = \ln\frac{19}{39}$

$\therefore k = -\frac{1}{30}\ln\frac{19}{39}$

≈ 0.023970755

(iii) From (ii) $T = 1 + 39e^{\ldots}$

$\therefore e^{-0.0239t} = 9/39$

$-0.0239t = \ln 9/39$

$\therefore t = 61.17$

$t = 61$ mins 10 sec.

(c)(i) $P(\text{faulty}) = {}^{20}C_0(0.9)^{20}$

$= 0.12$

(ii) $P(\text{at least 2 faulty}) =$

$1 - P(\text{none} + 1 \text{ faul}\ldots$

$= 1 - (0.12\ldots + 0.27\ldots)$

QUESTION 5

(a)(i) $x = 6t \quad y = 3t^2$

$t = \frac{x}{6}$

$y = 3\left(\frac{x}{6}\right)^2$

$= \frac{x^2}{12} \longleftarrow ①$

$\therefore 12y = x^2$.

(ii) Tangents at P:

$y = px - 3p^2$ — ①

Tangent at Q

$y = qx - 3q^2$ — ②

Solve: $px - 3p^2 = qx - 3q^2$

$px - qx = -3q^2 + 3p^2$

$x(p-q) = 3(p^2 - q^2)$

$x = 3(p+q) \longleftarrow ①$

Since tangents meet on y axis, $x = 0$

$0 = 3(p+q)$

$p = -q$

$p^2 = q^2 \longleftarrow ①$

(b)(i) $A_1 = 20000 - P$

$A_2 = 20000 - 2P$

$A_3 = 20000 - 3P$

$A_4 = A_3(1.01) - P$ ①

$= (20000 - 3P)(1.01) - P$

(ii) $A_5 = [20000 - 3P(1.01) - P](1.01) - P$

$= 20000 - 3P(1.01)^2 - P(1 + 1.01)$ ①

$\vdots$

$A_{36} = (20000 - 3P)(1.01)^{33} - P(1 + 1.01 + 1.01^2 + \ldots 1.01^{32})$

$0 = (20000 - 3P)(1.01)^{33} - \frac{P(1.01^{33} - 1)}{1.01 - 1} \longleftarrow ①$

$P((1.01)^{33} - 1) = 0.01(20000 - 3P) \times (1.101)^{33}$

$P((1.01)^{33} - 1) = 0.01(1.01)^{33} 20000 - 3P(0.01)(1.01)^{33}$

$P[(1.01)^{33} - 1 + 3(0.01)(1.01)^{33}] = 0.01(1.01)^{33} \times 20000$ ①

$P = \frac{0.01(1.01)^{33} \times 20000}{1.01^{33} - 1 + 3(0.01)(1.01)^{33}}$

$= \frac{0.01(1.01)^{33} \times 20000}{1.01^{33}(1 + 0.03) - 1}$

$= \$645.38 \longleftarrow ①$

(c) $T_{k+1} = {}^nC_k\, a^{n-k} b^k$

So $T_{k+1} = {}^8C_k (2x)^{8-k} \left(\frac{-1}{3x^2}\right)^k$ ①

$= {}^8C_k\, 2^{8-k} x^{8-k} \left(\frac{-1}{3}\right)^k x^{-2k}$

$= {}^8C_k\, 2^{8-k} \left(\frac{-1}{3}\right)^k x^{8-k} x^{-2k}$

$\therefore T_{k+1} = K x^{8-3k}$.

So $x^{8-3k} = x^{-4}$

$\therefore k = 4 \longleftarrow ①$

QUESTION 5c CONTINUED

$\therefore$ Coefficient is

$T_{4+1} = {}^8C_4\, a^{8-4} b^4$

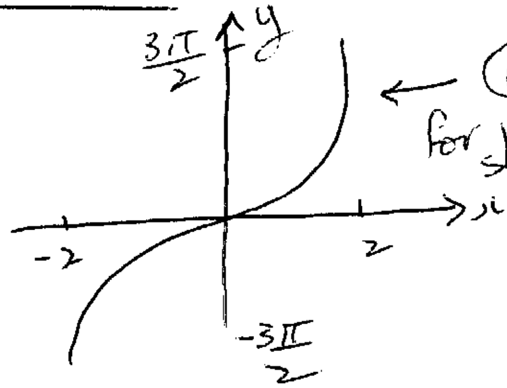
$= 70\,(16x^4)\frac{1}{81x^8}$

$= \frac{1120}{81}x^{-4}$

$= 13\frac{67}{81}$ ← 1

QUESTION 6:

(a)



← 1 for shape

Domain: $-2 \le x \le 2$ ← $\frac{1}{2}$

Range: $-\frac{3\pi}{2} \le y \le \frac{3\pi}{2}$ ← $\frac{1}{2}$

(ii) When $x = 1$, $y = 3\sin^{-1}\frac{1}{2}$

$= 3\times\frac{\pi}{6}$

$= \frac{\pi}{2}$ — 1

So $A = \frac{\pi}{2} - 2\int_0^{\frac{\pi}{2}} \sin\frac{y}{3}\,dy$

1 → $= \frac{\pi}{2} - 2\left[\frac{-\cos y/3}{1/3}\right]_0^{\pi/2}$

$= \frac{\pi}{2} - 6\left[-\cos\frac{\pi}{6}\right) - (-\cos 0)$

$= \frac{\pi}{2} + 3\sqrt{3} - 6$ units^2

↑ 1

b)(i) $\tan\alpha = -\frac{1}{\sqrt{3}}$

$\tan\alpha = \tan\left(-\frac{\pi}{6}\right)$

$\tan\alpha = \tan\left(\frac{5\pi}{6}\right)$

$\therefore \alpha = -\frac{\pi}{6} + n\pi$ OR

$\alpha = \frac{5\pi}{6} + n\pi$.

(ii) $\alpha = -\frac{\pi}{6} - \pi$ OR

$\alpha = \frac{5\pi}{6} - 2\pi$

$= -\frac{7\pi}{6}$ — 1

(c) $\frac{d}{dx}\tan^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right)$ 1

$= \dfrac{1}{1+\left(\frac{x}{\sqrt{1-x^2}}\right)^2} \times$

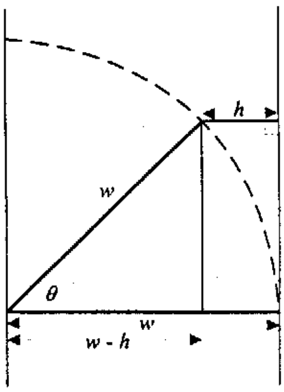
$\dfrac{\sqrt{1-x^2}.1 - \frac{x(-2x)\frac{1}{2}}{\sqrt{1-x^2}}}{1-x^2}$

$= \frac{1-x^2}{1} \times \frac{1-x^2+x^2}{(1-x^2)\sqrt{1-x^2}}$ ←

$= \frac{1}{\sqrt{1-x^2}}$ — 1

QUESTION SEVEN

a)



i) $\cos\theta = \dfrac{w-h}{w}$ 1

$w\cos\theta = w-h$

$h = w - w\cos\theta$.

ii) $\dfrac{d\theta}{dt} = \dfrac{1}{\sqrt{1-t^2}}$

$\theta = \int \dfrac{1}{\sqrt{1-t^2}}\,dt$. 1

$\theta = \sin^{-1}t + C$

$t=0 \quad \theta = 0 \quad \text{so } C = 0$

$\theta = \sin^{-1}t$.

iii) $\dfrac{d\theta}{dt} = \dfrac{1}{\sqrt{1-t^2}}$

Also $\dfrac{dh}{d\theta} = w\sin\theta$ — 1

So $\dfrac{dh}{dt} = \dfrac{dh}{d\theta}\cdot\dfrac{d\theta}{dt}$

$= w\sin\theta \cdot \dfrac{1}{\sqrt{1-t^2}}$

$= \dfrac{wt}{\sqrt{1-t^2}}$ 1 since $\theta = \sin^{-1}t$ & so $t = \sin\theta$

(iv) $\dfrac{dh}{dt} = \dfrac{wt}{\sqrt{1-t^2}}$

$h = w\int \dfrac{t}{\sqrt{1-t^2}}\,dt$.

Let $u = 1-t^2$, $\dfrac{du}{dt} = -2t$.

So $h = w\int -\dfrac{1}{2}\dfrac{du}{dt}\dfrac{1}{\sqrt{u}}\,dt$

$= -\dfrac{w}{2}\int u^{-1/2}\,du$ 1

$= -\dfrac{w}{2}u^{\frac{1}{2}} \times 2 + c$

$h = -w\sqrt{1-t^2} + c$

When $t=0 \quad h=0$

$0 = -w\sqrt{1} + c$ 1

$c = w$

$h = -w\sqrt{1-t^2} + w$.

(v) Now, $h = w - w\sqrt{1-t^2}$

The gate is fully open when $h = w$. 1

So $w = w - w\sqrt{1-t^2}$

$0 = -w\sqrt{1-t^2}$

$t = 1$ since $w = 0$ & $t \geq 0$

Question (a) Continued.

Also from (iii)

$$h = \omega \int \frac{t}{\sqrt{1-t^2}}\, dt$$

horizontal

The distance covered between $t=0$ & $t=1$ is ω. So $\omega = \omega \int_0^1 \frac{t}{\sqrt{1-t^2}}\, dt$

So $\int_0^1 \frac{t}{\sqrt{1-t^2}}\, dt = 1$ 1

(b)(i) $(1+x)^n = {}^nC_0 + {}^nC_1 x + \cdots {}^nC_n x$ 1

(ii) $n(1+x)^{n-1} = {}^nC_1 + 2\,{}^nC_2 x + 3\,{}^nC_3 x^2 \ldots + 4\,{}^nC_4 x^3 + \cdots n\,{}^nC$ 1

(by differentiation)

$n(n-1)(1+x)^{n-2} = 2\,{}^nC_2 + 6\,{}^nC_3 x + \cdots n(n-1)\,{}^nC_n x^{n}$

(by differentiating again)

By substituting $x=1$ we get

$2\,{}^nC_2 + 6\,{}^nC_3 + 12\,{}^nC_4 + \cdots n(n-1)\,{}^nC_n = n(n$

1

END AT LAST !!