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Name:	________________
Class:	12MTX________
Teacher:	________________

CHERRYBROOK TECHNOLOGY HIGH SCHOOL

2004

YEAR 12

AP4 EXAMINATION

MATHEMATICS EXTENSION 1

Time allowed - 2 HOURS
(Plus 5 minutes' reading time)

DIRECTIONS TO CANDIDATES:

- Attempt all questions.
- All questions are of equal value.
- Each question is to be commenced on a new page clearly marked Question 1, Question 2, etc on the top of the page. **
- Each question is to be returned in a separate bundle.
- All necessary working should be shown in every question. Full marks may not be awarded for careless or badly arranged work.
- Approved calculators may be used.

*****Each page must show your name and your class. *****

Question 1 (12 marks) **Marks**

(a) The point $(-6t, 9t^2)$, where t is a variable, lies on a curve. **2**
Find the Cartesian equation of the curve.

(b) Two of the roots of the equation $x^3 - 13kx^2 + 13kx - 1 = 0$
are k and $\frac{1}{k}$ where $k \neq 0$.

(i) Find the third root. **1**

(ii) Find the value(s) of k. **2**

(c) Solve: $\frac{x}{x-2} \geq 4, \quad x \neq 2$ **3**

(d) For the expansion of the expression $\left(x - \frac{3}{x}\right)^5$, find the coefficient of x. **2**

(e) Find $\frac{d}{dx}\left(2x^3 \tan^2 x\right)$. **2**

Question 2 (12 marks) *(Please start on a new page)* **Marks**

(a) Evaluate $\lim_{x \to 0} \frac{\sin 4x}{5x}$. **1**

(b) Find $\int \sin^2 3x \, dx$. **2**

(c) (i) Prove the identity **1**

$$\frac{\sin 2\theta}{2\sin\theta} - \cos\theta\cos 2\theta = 2\cos\theta\sin^2\theta$$

(ii) Hence solve the equation **2**

$$\frac{\sin 2\theta}{2\sin\theta} - \cos\theta\cos 2\theta = \cos\theta \text{ for } 0 \le \theta \le 2\pi.$$

(d) Use mathematical induction to prove that, for all positive integers n, **4**

$$\sum_{r=1}^{n} \frac{r^2}{(2r-1)(2r+1)} = \frac{n(n+1)}{2(2n+1)}$$

(e) Differentiate $\log_7 x^2$ with respect to x. **2**

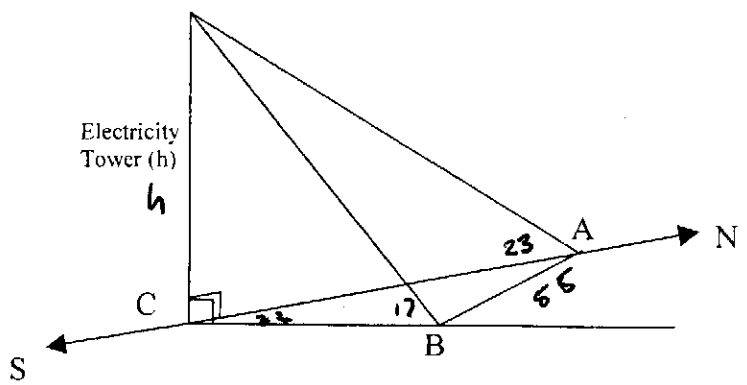
Question 3 (12 marks) *(Please start on a new page)* **Marks**

(a) Use the substitution $u = x - 2$ to evaluate $\int_3^4 \frac{x}{\sqrt{x-2}}\,dx$. **3**

(b) A person walks on level ground, in a northerly direction, away from an electricity tower. Whey they arrive at a point A, the angle of elevation to the top of the tower is 23^0. Another person walks on level ground on a bearing of 032^0T from the same tower, until they reach point B, and measures the angle of elevation to the top of the tower as 17^0.

It is known that the points A and B are 55 metres apart. Let h be the height of the tower and assume that the tower, with base C, is perpendicular to the ground.

(i) Copy the diagram below onto your page, clearly marking all information. **1**



NOT TO SCALE

(ii) Find the distances AC and BC, leaving your answers in terms of h. **2**

(iii) Hence, or otherwise, find the height h of the tower, correct to the nearest metre. **3**

(c) Consider the geometric series $\sin 2x + \sin 2x\cos 2x + \sin 2x\cos^2 2x + \ldots$ for $0 < x < \frac{\pi}{2}$. Show that the limiting sum S of the series exists and that $S = \cot x$. **3**

Question 4 (12 marks) *(Please start on a new page)* **Marks**

(a) (i) Write equations for the asymptotes of the curve $y = \ln(x-2)$. **2**

(ii) The inner surface of a bowl is of the shape formed by rotating About the y axis, the curve $y = \ln(x-2)$ between y = 0 and y = 2. The bowl is placed with its axis vertical and water is poured in. Show that the volume of water in the bowl when it is filled to a depth *h*, where h < 2, is given by **2**

$$\pi(4h - 4\frac{1}{2} + 4e^{h} + \frac{1}{2}e^{2h})\ unit^{3}.$$

(iii) If the bowl is filled at the rate of 60 $unit^3$ / s, find the rate at which the water level is rising when the depth of water is 1·25 *units.* Give your answer correct to 2 decimal places. **2**

(b) Corn cobs are cooked by immersing them in boiling water. On being removed, a corn cob cools in the air according to the equation $\frac{dT}{dt} = -k(T - T_O)$ where t is time in minutes, T is temperature in °C and T_O is the temperature of the air, while k is a positive constant.

(i) Verify that $T = T_O + Ae^{-kt}$ is a solution of the above equation where A is a constant. **2**

(ii) If the temperature of the boiling water is 100°C and that of the air is a constant $25^0 C$, find the values of A and k if a corn cob cools to 70°C in 3 minutes. **2**

(iii) How long should a person wait to enjoy the food at a temperature of 50°C? **2**

Question 5 (12 marks) *(Please start on a new page)* **Marks**

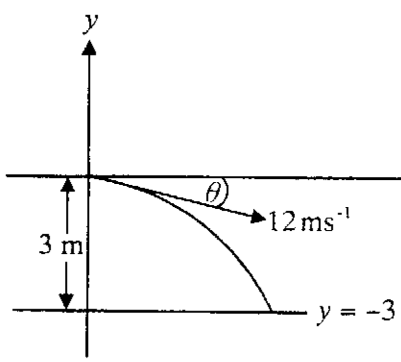
(a) A particle moves with a simple harmonic motion. It starts from rest at a point 6 cm from the centre of motion O. The particle has a speed of 10 *cm/s,* when it passes through O.

(i) Find the amplitude of the motion. **1**

(i) Find the period of motion. **2**

(ii) Find the acceleration after 3 seconds, correct to 2 significant figures. **3**

(b)



A child in a tree throws a ball and gives it a velocity of $12ms^{-1}$ at an angle of θ below the vertical. The height of the ball above the ground when it leaves the child's hand is 3 *m*. Take the origin to be the point where the ball leaves the child's hand. The position of the ball at time t seconds after the ball has been thrown and before it hits the ground is given by (x, y). The equations of motion for the ball are

$$\ddot{x} = 0 \text{ and } \ddot{y} = -10 .$$

(i) Show that **3**

$$x = 12t\cos\theta$$

$$\text{and } y = -12t\sin\theta - 5t^2$$

(ii) Given that $\tan\theta = \dfrac{5}{12}$, find the horizontal distance **3**
between the point where the ball leaves the child's hand

and the point where the ball hits the ground.

Question 6 (12 marks) *(Please start on a new page)* **Marks**

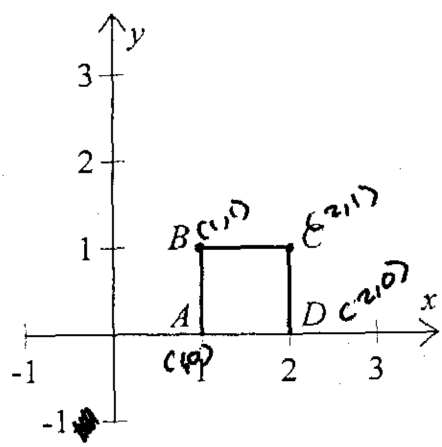
(a) The point $P(2ap, ap^2)$ lies on the parabola $x^2 = 4ay$ whose focus is at S. The tangent at P meets the Y-axis at Q.

(i) Find the coordinates of Q. **1**

(ii) Show that $\angle SPQ = \angle SQP$ **2**

(b) The acceleration of a particle is given by $a = -e^{-x}$. Initially $v = \sqrt{2}$ and $x = 0$. Find the velocity as a function of x. **2**

(c) The diagram shows a unit square $ABCD$, where A(1, 0), B(1, 1), C(2, 1) and D(2, 0).



Copy the diagram onto your answer sheet.

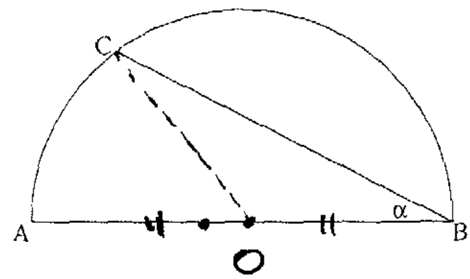
(i) A line l, passing through the origin with gradient m, cuts the sides AB and CD at P and Q respectively. Comment on the possible values of m. **1**

(ii) For what value(s) of m does the line l divide the area of the square in the ratio 2:1. **3**

(iii) Another line K passes through the origin with gradient n, and cuts the square through sides AB and BC at S and T respectively. Show that it is not possible for k to divide the area of the square in the ratio 2:1. **3**

Question 7 (12 marks) *(Please start on a new page)* **Marks**

(a) AB is the diameter of a semi-circle of unit radius with centre O and BC is a chord which makes an angle α with AB. The area of the semi-circle is bisected by this chord.



(i) Write an expression for $\angle BOC$ in terms of α. **1**

(ii) Show that the area of the segment is $\frac{1}{2}(\pi - 2\alpha - \sin 2\alpha)$ **1**

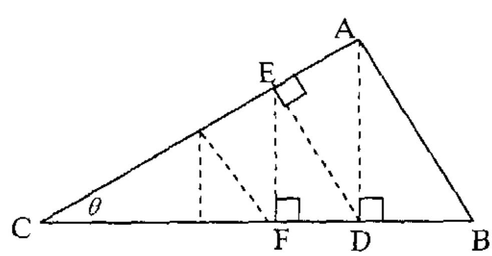
(iii) Hence show that $2\sin 2\alpha + 4\alpha - \pi = 0$ **2**

(iv) Prove that a root of this equation lies between $\alpha = 0.4$, and $\alpha = 0.5$. **2**

(v) By using the 'halving the interval' method, determine whether the root lies closer to 0.4 or 0.5. **1**

(b) In the triangle ABC, $\angle ACB = \theta$, where $0 < \theta < \frac{\pi}{2}$ and AC is of length d.

A fly starts at A, flies directly to the line CB, i.e. to the point D. It then flies directly to the line CA, ie to the point E. It then flies directly to the line CB and so on until it ultimately reaches C.



(i) Prove that $\angle ADE = \theta$. **1**

(ii) Show that the distance travelled by the fly when it reaches the point E is $d\sin\theta(1+\cos\theta)$. **2**

(iii) Show that the total distance travelled is given by **2**

$$s = \frac{d\sin\theta}{1-\cos\theta}.$$

END OF TEST

EXTENSION 1 AP1 2004

AMENDMENT

Question 4 (12 marks) *(Please start on a new page)* **Marks**

(a) (i) Sketch the curve $y = \ln(x - 2)$, showing its asymptote. **2**

EXTENSION 1 MATHEMATICS 2004 AP4 SOLUTIONS

QUESTION 1

(a) $x = -6t$

$t = \dfrac{-x}{6} \longrightarrow 1$

$y = 9t^2$

$y = 9\left(\dfrac{-x}{6}\right)^2$

$y = \dfrac{x^2}{4} \longrightarrow 1$

(b)(i) Let the third root be α.

$k \times \dfrac{1}{k} \times \alpha = 1$

$\alpha = 1 \longrightarrow 1$

(ii) $k + \dfrac{1}{k} + 1 = 13k$

$k^2 + 1 + k = 13k^2$

$12k^2 - k - 1 = 0 \longrightarrow 1$

$(4k+1)(3k-1) = 0$

$k = -\frac{1}{4} \longrightarrow \frac{1}{2}$

$k = \frac{1}{3} \longrightarrow \frac{1}{2}$

(c) $\dfrac{x}{x-2} \times (x-2)^2 \geq 4(x-2)^2$

$x^2 - 2x \geq 4x^2 - 16x + 16$

$3x^2 - 14x + 16 \leq 0 \longrightarrow 1$

$(x-2)(3x-8) \leq 0$

$2 \leq x \leq 2\frac{2}{3} \longrightarrow 1$

$2 < x \leq 2\frac{2}{3} \longrightarrow 1$

(d)

$x^5 - 5x^4 \cdot \dfrac{3}{x} + 10x^3\left(\dfrac{3}{x}\right)^2 - 10x^2\left(\dfrac{3}{x}\right)^3 + 5x\left(\dfrac{3}{x}\right)^4 - \left(\dfrac{3}{x}\right)^5 \longrightarrow 1$

$= \dots + 10x^3 \times \dfrac{9}{x^2} - \dots$

$= \dots + 90x - \dots$

$\therefore$ Coefficient of x is $90 \rightarrow 1$

(e) $\underbrace{6x^2\tan^2 x}_{\longrightarrow 1} + \underbrace{2x^3(2\tan x)\sec^2 x}_{\longrightarrow 1}$

$= 6x^2\tan^2 x + 4x^3\tan x\sec^2 x$

QUESTION 2

(a) $\lim_{x \to 0} \frac{\sin 4x}{4x} \times \frac{4}{5}$

$= 1 \times \frac{4}{5}$

$= \frac{4}{5} \longrightarrow 1$

(b) $\cos 2x = 1 - 2\sin^2 x$

$\cos 6x = 1 - 2\sin^2 3x$

$\sin^2 3x = \frac{1}{2} - \frac{1}{2}\cos 6x \rightarrow 1$

$\int \sin^2 3x \, dx = \int \frac{1}{2} - \frac{1}{2}\cos 6x \, dx$

$= \frac{1}{2}x - \frac{1}{12}\sin 6x + c \rightarrow 1$

(c)(i) $LHS = \frac{2\sin\theta\cos\theta}{2\sin\theta}$

$- \cos\theta(1 - 2\sin^2\theta)$

$= \cos\theta - \cos\theta + 2\cos\theta\sin^2\theta$

$= 2\cos\theta\sin^2\theta$

$= RHS \rightarrow 1$

(ii) $2\cos\theta\sin^2\theta = \cos\theta$

$\cos\theta(2\sin^2\theta - 1) = 0$

$\cos\theta = 0$

$\theta = \frac{\pi}{2}, \frac{3\pi}{2} \longrightarrow 1$

$2\sin^2\theta - 1 = 0$

$\sin^2\theta = \frac{1}{2}$

$\sin\theta = \pm\frac{1}{\sqrt{2}}$

$\theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4} \rightarrow 1$

(d) Prove true for $n=1$

$LHS = \frac{1}{1 \times 3} = \frac{1}{3}$

$RHS = \frac{1 \times 2}{2 \times 3} = \frac{1}{3}$

$LHS = RHS$

$\therefore$ true for $n=1 \rightarrow 1$

Assume true for $n=k$

ie $\sum_{r=1}^{k} \frac{r^2}{(2r-1)(2r+1)} = \frac{k(k+1)}{2(2k+1)}$

Hence prove true for $n=k+1$

ie: $\frac{k(k+1)}{2(2k+1)} + \frac{(k+1)^2}{(2k+1)(2k+3)} = \frac{(k+1)(k+2)}{2(2k+3)}$

$\longrightarrow 1$

$LHS = \frac{k(k+1)(2k+3) + 2(k+1)^2}{2(2k+1)(2k+3)}$

$= \frac{(k+1)[k(2k+3) + 2(k+1)]}{2(2k+1)(2k+3)}$

$= \frac{(k+1)[2k^2 + 3k + 2k + 2]}{2(2k+1)(2k+3)}$

$= \frac{(k+1)[2k^2 + 4k + k + 2]}{2(2k+1)(2k+3)} \rightarrow 1$?

$= \frac{(k+1)[2k(k+2) + 1(k+2)]}{2(2k+1)(2k+3)}$

$= \frac{(k+1)(k+2)(2k+1)}{2(2k+1)(2k+3)}$

$= \frac{(k+1)(k+2)}{2(2k+3)}$

$= RHS$

$\therefore$ If true for $n=k$ then true for $n=k+1$

QUESTION 2 (CONT.)

$\therefore$ if true for $n=1$ then true for $n=2$, etc

$\therefore$ true for all integers $n \geq 1$

$\longrightarrow$ 1

(e) $y = \log_7 x^2$

$= 2\log_7 x$

$= \dfrac{2\log_e x}{\log_e 7} \longrightarrow$ 1

$\dfrac{dy}{dx} = \dfrac{2}{\log_e 7} \cdot \dfrac{1}{x}$

$= \dfrac{2}{x\log_e 7} \longrightarrow$ 1

QUESTION 3

(a) $u = x - 2$

$du = dx$

when $x = 3,\ u = 1$

$x = 4,\ u = 2$

$$\therefore \int_3^4 \frac{x}{\sqrt{x-2}}\, dx$$

$$= \int_1^2 \frac{u+2}{\sqrt{u}}\, du \rightarrow ①$$

$$= \int_1^2 \left(u^{1/2} + 2u^{-1/2}\right) du$$

$$= \left[\frac{2}{3}u^{3/2} + 4u^{1/2}\right]_1^2 \longrightarrow ①$$

$$= \frac{4\sqrt{2}}{3} + 4\sqrt{2} - \left(\frac{2}{3} + 4\right)$$

$$= \frac{16\sqrt{2}}{3} - 4\tfrac{2}{3}$$

$$= \frac{16\sqrt{2} - 14}{3} \longrightarrow ①$$

OR

$$= \left[\frac{2}{3}(x-2)^{3/2} + 4(x-2)^{1/2}\right]_3^4$$

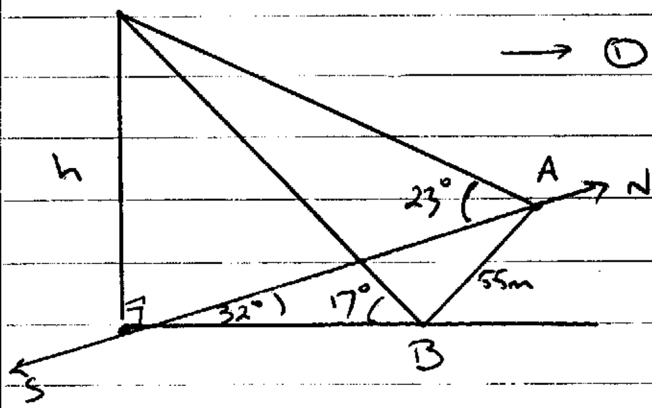
$$= \frac{4\sqrt{2}}{3} + 4\sqrt{2} - \left(\frac{2}{3} + 4\right)$$

$$= \frac{16\sqrt{2}}{3} - 4\tfrac{2}{3}$$

$$= \frac{16\sqrt{2} - 14}{3} \rightarrow ①$$

Penalize ① if boundaries do not match up with variable.

(b) (i)



$\longrightarrow ①$

(ii) $\tan 23^\circ = \dfrac{h}{AC}$

$$AC = \frac{h}{\tan 23^\circ} \longrightarrow ①$$

$\tan 17^\circ = \dfrac{h}{BC}$

$$BC = \frac{h}{\tan 17^\circ} \rightarrow ①$$

(iii) Using Cosine Rule:

$$55^2 = \left(\frac{h}{\tan 23^\circ}\right)^2 + \left(\frac{h}{\tan 17^\circ}\right)^2 - 2\left(\frac{h}{\tan 23^\circ}\right)\left(\frac{h}{\tan 17^\circ}\right)\cos 32^\circ \longrightarrow ①$$

$$3025 = h^2\left[\frac{1}{\tan^2 23^\circ} + \frac{1}{\tan^2 17^\circ} - \frac{2\cos 32^\circ}{\tan 23^\circ \tan 17^\circ}\right]$$

$$h^2 = \frac{3025}{\left\{\frac{1}{\tan^2 23} + \frac{1}{\tan^2 17} - \frac{2\cos 32}{\tan 23 \tan 17}\right.} \rightarrow ①$$

$h^2 = 951.5569\ldots$

$h = 31$ m (n.m.) $\longrightarrow ①$

(−① for incorrect accuracy)

QUESTION 3 (cont.).

(c) $r = \cos 2x \rightarrow$ 1

$0 < |\cos 2x| < 1$

for $0 < x < \frac{\pi}{2}$

$\therefore$ a limiting sum exists $\rightarrow$ 1

$$S = \frac{\sin 2x}{1 - \cos 2x}$$

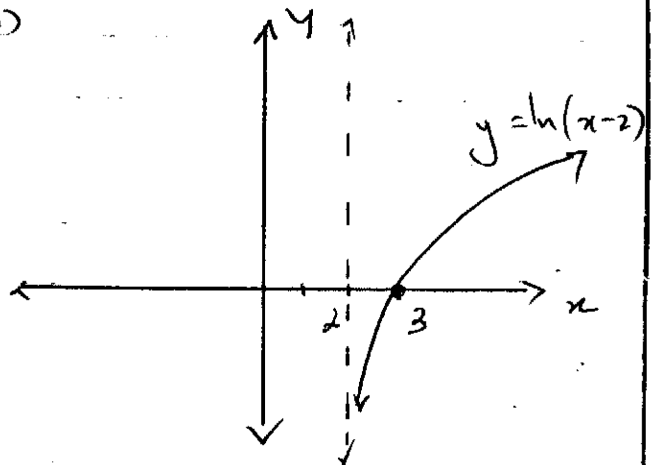
$$= \frac{2 \sin x \cos x}{2 \sin^2 x}$$

$$= \frac{\cos x}{\sin x}$$

$S = \cot x \longrightarrow$ 1

QUESTION 4

a) (i)



① for asymptote
① for x-intercept

(ii) $x = e^{y} + 2$

$$V = \pi \int_0^h (e^y + 2)^2 \, dy \longrightarrow ①$$

$$= \pi \int_0^h (e^{2y} + 4e^y + 4)\, dy$$

$$= \pi \left[\frac{e^{2y}}{2} + 4e^y + 4y \right]_0^h$$

$$= \pi \left[\tfrac{1}{2} e^{2h} + 4e^h + 4h - \tfrac{1}{2} - 4 \right]$$

$$= \pi \left[4h - 4\tfrac{1}{2} + 4e^h + \tfrac{1}{2} e^{2h} \right] \longrightarrow ①$$

(iii)

$$\frac{dV}{dt} = 60$$

$$\frac{dh}{dt} = \frac{dh}{dV} \cdot \frac{dV}{dt}$$

$$\frac{dV}{dh} = \pi (4 + 4e^h + e^{2h})$$

$$\therefore \frac{dh}{dV} = \frac{1}{\pi(4 + 4e^h + e^{2h})} \longrightarrow ①$$

$$\frac{dh}{dt} = \frac{1}{\pi(4 + 4e^{1.25} + e^{2.5})} \cdot 60$$

$$= 0.63358416\ldots$$

$$= 0.63 \text{ units/s} \longrightarrow ①$$

b) (i) $\text{LHS} = \dfrac{dT}{dt} = -Ake^{-kt} \longrightarrow ①$

$$\text{RHS} = -k(T - T_0)$$

$$= -k(T_0 + Ae^{-kt} - T_0)$$

$$= -Ake^{-kt}$$

$$= \text{LHS} \longrightarrow ①$$

(ii) $100 = 25 + Ae^0$

$$\therefore A = 75 \longrightarrow ①$$

$$70 = 25 + 75e^{-3k}$$

$$e^{-3k} = \frac{3}{5}$$

$$k = \frac{\log \frac{3}{5}}{-3}$$

$$= 0.170275207\ldots$$

$$\doteqdot 0.1703 \quad (4dp) \longrightarrow ①$$

(iii) $50 = 25 + 75e^{-0.1703t}$

$$e^{-0.1703t} = \frac{1}{3} \longrightarrow ①$$

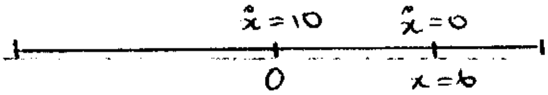
$$t = \frac{\log \frac{1}{3}}{-0.1703}$$

$$\doteqdot 6.45104\ldots$$

$$\doteqdot 6.45 \text{ mins. } (2dp) \longrightarrow ①$$

QUESTION 5

a)



(i) $v^2 = n^2(a^2 - x^2)$

$0 = n^2(a^2 - 36)$

$a^2 = 36$

$a = 6 \longrightarrow (1)$

(ii) $100 = n^2(36 - 0)$

$n = \frac{5}{3} \longrightarrow (1)$

$T = \frac{2\pi}{n} = 2\pi \times \frac{3}{5}$

$= \frac{6\pi}{5}$ s $\longrightarrow (1)$

(iii) $x = a\cos(nt + \alpha)$

$x = 6\cos\left(\frac{5t}{3} + \alpha\right)$

when $t=0$, $x=6$

$6 = 6\cos\alpha$

$\cos\alpha = 1$

$\alpha = 0$

$\therefore x = 6\cos\frac{5t}{3} \longrightarrow (1)$

$\dot{x} = -10\sin\frac{5t}{3}$

$\ddot{x} = -\frac{50}{3}\cos\frac{5t}{3}$

$= -\frac{50}{3}\cos 5$

$= -4.7277\ldots \longrightarrow (1)$

$= 4.73$ cm/s^2 in the negative direction

(Third mark for correct statement) $\longrightarrow (1)$

b) $\ddot{x} = 0$

$\dot{x} = c$

when $x = 0$, $\dot{x} = 12\cos(-\theta) \rightarrow ($

$= 12\cos\theta.$

$\therefore c = 12\cos\theta.$

$\dot{x} = 12\cos\theta$

$x = 12t\cos\theta + c$

when $t=0$, $x=0$ $\therefore c=0$

$\therefore x = 12t\cos\theta \longrightarrow (1)$

$\ddot{y} = -10$

$\dot{y} = -10t + c$

when $t=0$, $\dot{y} = 12\sin(-\theta)$

$= -12\sin\theta$

$\therefore c = -12\sin\theta.$

$\dot{y} = -10t - 12\sin\theta$

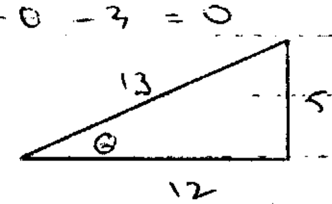
$y = -5t^2 - 12t\sin\theta + c$

when $t=0$, $y=0$ $\therefore c=0$

$\therefore y = -12t\sin\theta - 5t^2 \longrightarrow (1)$

(ii) $-3 = -12t\sin\theta - 5t^2$

$5t^2 + 12t\sin\theta - 3 = 0$



$\sin\theta = \frac{5}{13}$

$\cos\theta = \frac{12}{13}$

$5t^2 + \frac{60t}{13} - 3 = 0$

$65t^2 + 60t - 39 = 0 \longrightarrow (1)$

Qn. 5 (cont.)

$$t = \frac{-60 \pm \sqrt{60^2 - 4 \cdot 65 \cdot (-39)}}{130}$$

$$t = \frac{-60 \pm 117.2177461}{130}$$

$t = 0.4401$ s. or (negative answer which is not required)

$\longrightarrow$ (1)

$$x = 12t\cos\theta$$
$$= 12 \times 0.4401 \times \frac{12}{13}$$
$$= 4.875 \text{ m}$$

(3 dp) $\longrightarrow$ (1)

QUESTION 6

(a) (i) $m = p$

$y - ap^2 = p(x - 2ap)$

when $x = 0$, $y = -ap^2$

$Q(0, -ap^2) \rightarrow ①$

(ii) $SP^2 = (2ap)^2 + (ap^2 - a)^2$

$= 4a^2p^2 + a^2(p^2-1)^2$

$= a^2(4p^2 + p^4 - 2p^2 + 1)$

$= a^2(p^4 + 2p^2 + 1)$

$= a^2(p^2+1)^2$

$\therefore SP = a(p^2+1)$
$= ap^2 + a \rightarrow ①$

$SQ = a + ap^2$

$\therefore SP = SQ$

$\therefore \Delta_{SPQ}$ isosceles

$\therefore \angle SPQ = \angle SQP$

(equal angles of isos. Δ) $\rightarrow ①$

(b) $\ddot{x} = \frac{d}{dx}\left(\frac{1}{2}v^2\right) = -e^{-x}$

$\therefore \frac{1}{2}v^2 = \int -e^{-x}\,dx$

$\frac{1}{2}v^2 = e^{-x} + c \rightarrow ①$

when $x = 0$, $v = \sqrt{2}$

$1 = 1 + c$

$c = 0$

$\therefore \frac{1}{2}v^2 = e^{-x}$

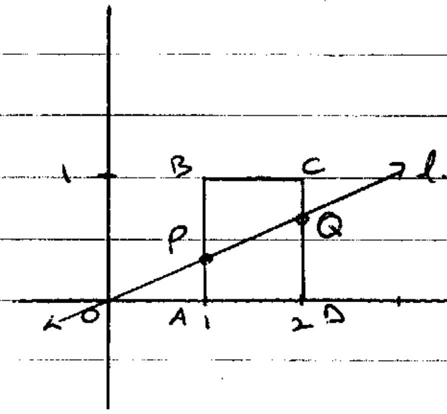
$v^2 = 2e^{-x}$

$v = \pm\sqrt{2}e^{-x/2}$

but $v = \sqrt{2}$ when $x = 0$

$\therefore v = \sqrt{2}e^{-x/2} \rightarrow ①$

(c)



(i) $m_{OD} = 0$

$m_{OC} = \frac{1}{2}$

$\therefore 0 \le m \le \frac{1}{2} \rightarrow ①$

(ii) $P(1, m)$ $Q(2, 2m)$

Area of trapezium $APQD = \frac{3m}{2}$

Area of trapezium $PBCQ$
$= 1 - \frac{3m}{2} \rightarrow ①$

$\therefore \dfrac{\frac{3m}{2}}{1 - \frac{3m}{2}} = \dfrac{2}{1}$

$\frac{3m}{2} = 2 - 3m$

$\frac{9m}{2} = 2$

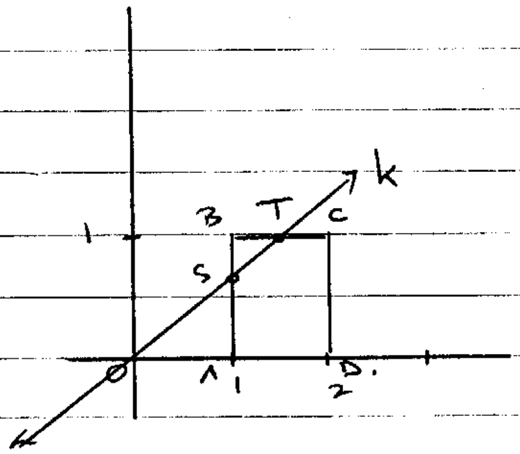
$m = \frac{4}{9} \rightarrow ①$

or $\dfrac{\frac{3m}{2}}{1 - \frac{3m}{2}} = \dfrac{1}{2}$

$\therefore m = \frac{2}{9} \rightarrow ①$

QUESTION 6 (cont.)

(ii)



$S(1, n) \quad T\left(\frac{1}{n}, 1\right)$

Area of $\Delta_{SBT} = \frac{1}{2}(1-n)\left(\frac{1}{n}-1\right)$

Area of $ASTCD = 1 - \frac{1}{2}(1-n)\left(\frac{1}{n}-1\right)$

Prove that $\dfrac{\text{Area of } \Delta_{SBT}}{\text{Area of } ASTCD} \neq \dfrac{1}{2}$

$$LHS = \frac{\frac{1}{2}(1-n)\left(\frac{1}{n}-1\right)}{1-\frac{1}{2}(1-n)\left(\frac{1}{n}-1\right)} \longrightarrow 1$$

$$= \frac{(1-n)\left(\frac{1}{n}-1\right)}{2-(1-n)\left(\frac{1}{n}-1\right)}$$

$$= \frac{\frac{1}{n}-1-1+n}{2-\left(\frac{1}{n}-1-1+n\right)}$$

$$= \frac{1-2n+n^2}{4n-1-n^2}$$

Let $\dfrac{1-2n+n^2}{4n-1-n^2} = \dfrac{1}{2} \longrightarrow 1$

$2-4n+2n^2 = 4n-1-n^2$

$3n^2 - 8n + 3 = 0$

$$n = \frac{8 \pm \sqrt{64-4.3.3}}{6}$$

$= 2.215\ldots$

or

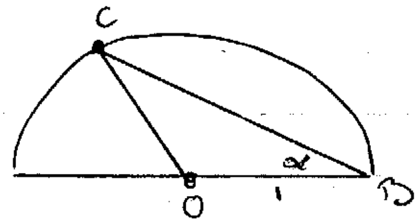
$0.4514\ldots$

but $\frac{1}{2} \leq n \leq 1 \longrightarrow 1$

$\therefore$ it is not possible for k to divide the area of the square in the ratio $2:1$

QUESTION 7

(a)



(i) $\angle BOC = \pi - 2\alpha$ (or $180 - 2\alpha$) $\rightarrow$ ①

(ii) $A = \frac{1}{2} r^2 (\theta - \sin\theta)$

$= \frac{1}{2}(\pi - 2\alpha - \sin(\pi - 2\alpha))$

$= \frac{1}{2}(\pi - 2\alpha - \sin 2\alpha)$ $\rightarrow$ ①

(iii) Area of segment $= \frac{1}{2} \times$ Area of semi-circle

$\frac{1}{2}(\pi - 2\alpha - \sin 2\alpha) = \frac{1}{4}\pi$ $\rightarrow$ ①

$2(\pi - 2\alpha - \sin 2\alpha) = \pi$

$2\pi - 4\alpha - 2\sin 2\alpha - \pi = 0$

$2\sin 2\alpha + 4\alpha - \pi = 0$ $\rightarrow$ ①

(iv) $f(\alpha) = 2\sin 2\alpha + 4\alpha - \pi$

$f(0.4) = 2\sin 0.8 + 4(0.4) - \pi$

$= -0.107 < 0$

$f(0.5) = 2\sin 1 + 2 - \pi$

$= 0.541 > 0$ $\rightarrow$ ①

Root lies between 0.4 and 0.5 as there is a change of sign. $\rightarrow$ ①

(v) $f(0.45) = 2\sin 0.9 + 4(0.45) - \pi$

$= 0.2251 > 0$

$\therefore$ Root lies closer to 0.4 than 0.5 $\rightarrow$ ①

(b) (i) In ΔACD and ΔADE,

$\angle ADC = \angle AED = 90°$ (given)

$\angle DAC = \angle EAD$ (common)

$\therefore \angle ADE = \angle ACD = \theta$

(angle sum of Δ) $\rightarrow$ ①

(ii) In ΔACD, $\sin\theta = \frac{AD}{AC} = \frac{AD}{d}$

$\therefore AD = d\sin\theta$ $\rightarrow$

In ΔDAE, $\cos\theta = \frac{DE}{AD}$

$DE = AD\cos\theta$

$= d\sin\theta\cos\theta$

$\therefore$ Distance $= d\sin\theta(1 + \cos\theta)$ $\rightarrow$ ①

(iii) $\angle DEF = \theta$

$\cos\theta = \frac{EF}{DE}$

$EF = DE\cos\theta$

$= d\sin\theta\cos^2\theta$

$\therefore$ Geometric series with $a = d\sin\theta$ and $r = \cos\theta$ $\rightarrow$ ①

Limiting sum as $0 < |r| < 1$

$S = \frac{a}{1-r}$

$= \frac{d\sin\theta}{1-\cos\theta}$ $\rightarrow$ ①