AT KW KL JG

Name:	____________________
Class:	12MTX________
Teacher:	____________________

CHERRYBROOK TECHNOLOGY HIGH SCHOOL

2010 AP4

YEAR 12 TRIAL HSC EXAMINATION

MATHEMATICS EXTENSION 1

Time allowed - 2 HOURS
(Plus 5 minutes' reading time)

DIRECTIONS TO CANDIDATES:

- Attempt all questions.
- All questions are of equal value.
- Each question is to be commenced on a new page clearly marked Question 1, Question 2, etc on the top of the page. **
- All necessary working should be shown in every question. Full marks may not be awarded for careless or badly arranged work.
- Approved calculators may be used. Standard Integral Tables are provided
- Your solutions will be collected in one bundle stapled in the top left corner.

Please arrange them in order, Q1 to 7.

***Each page must show your name and your class.** **

Question 1 (12 Marks) **Marks**

(a) Two points *A* and *B* have coordinates $(-2,4)$ and $(2,1)$ respectively.

Find the point *P* which divides the interval *AB* externally in the ratio 3 : 2. **2**

(b) Find the acute angle between the lines $y=3x-1$ and $4x-2y=7$ **2**

(c) Solve the inequality $\frac{2x-1}{x+1} \geq 1$. **3**

(d) The equation $x^3-2x^2+4x-5=0$ has roots α, β, γ

(i) Write down the value of $\alpha\beta+\beta\gamma+\gamma\beta$ **1**

(ii) Find the value of $\alpha\beta\gamma$ **1**

(ii) Hence find the value of $\alpha^{-1}+\beta^{-1}+\gamma^{-1}$ **2**

(e) Find $\lim_{\theta \to 0} \frac{\tan 2\theta}{8\theta}$ **1**

End of Question 1.

Question 2 (12 Marks) Start a new page

(a) *P* $(2ap, ap^2)$ is a point on the parabola $x^2=4ay$.

(i) Show that the equation of the normal at *P* is **2**

$$x+py=2ap+ap^3$$

(ii) *Q* is the point with coordinates $(0, 2a+ap^2)$, find the coordinates of *M*, the midpoint of *PQ*. **1**

(iii) Find the equation of the locus of *M*, **1**

Question 2 continued...... **Marks**

(b) Use the substitution $u = 1 - x$ to evaluate $\int_{-15}^{0} \frac{x\,dx}{\sqrt{1-x}}$. **3**

(c) Find $\int \sin^2 6x\,dx$ **2**

(d) (i) Sketch the graph $y = |x-2|$ **1**

(ii) Hence solve for x

$$|x-2| < \tfrac{1}{2}x$$ **2**

End of Question 2.

Question 3 (12 Marks) Start a new page

(a) Solve the equation $\sin 2x + \cos x = 0$ for $0 \leq x \leq 2\pi$. **3**

(b) (i) Use the substitution $t = \tan\frac{\theta}{2}$ to show that

$$\operatorname{cosec}\theta + \cot\theta = \cot\frac{\theta}{2}$$ **2**

(ii) Hence show that

8

$$\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} (\operatorname{cosec}\theta + \cot\theta)\,d\theta = \ln 2$$ **3**

(c) A tower is 828 metres tall. At a point due east of the tower, the angle of elevation is 53 degrees. At another point due south of P, the angle of elevation is θ degrees. The distance from P to Q is 840 metres.

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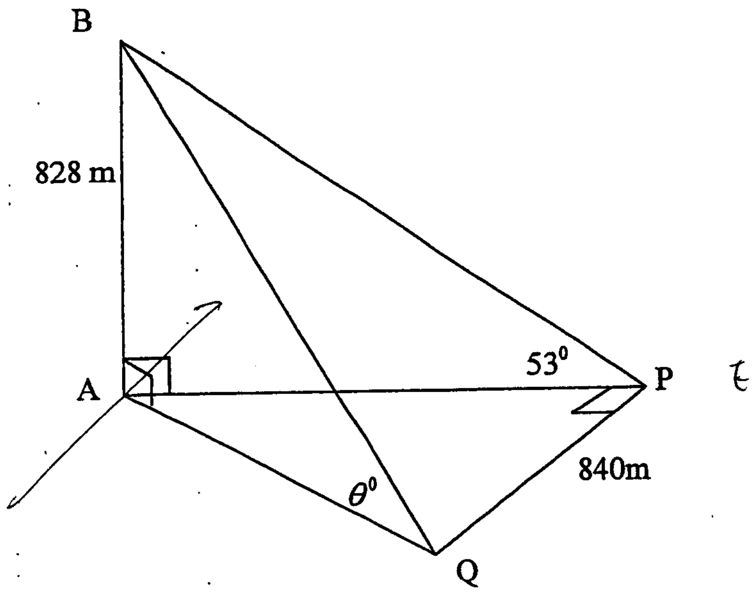

(i) Prove that $\cot\theta = \sqrt{\dfrac{828^2\cot^2 53^\circ + 840^2}{828^2}}$ **3**

(ii) Find θ, correct to the nearest degree. **1**

End of Question 3.

Question 4 (12 Marks) Start a new page **Marks**

(a) The circles intersect at *J* and *K*, *LKM* is a straight line.

TL and *TM* are tangents. Let $\angle LMT = \alpha$ and $\angle MLT = \beta$.

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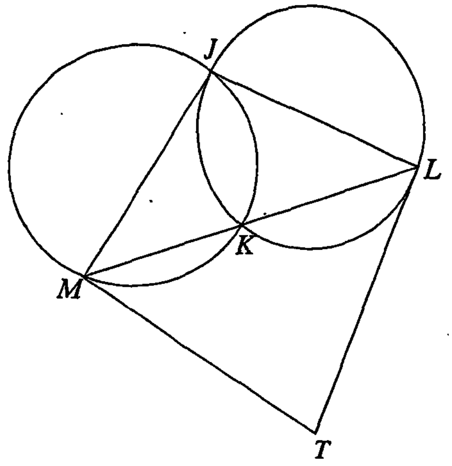

Copy the diagram onto your answer sheet

Hence prove that *TMJL* is a cyclic quadrilateral. **3**

(b) A particle is moving along a straight line with acceleration $\ddot{x} = 8x^3$,

where x is the displacement from the origin *O* in metres. Intially the particle

is 1 metre to the right of the origin moving with velocity $v = 2\,ms^{-1}$.

(i) Show that $\ddot{x} = \dfrac{d(\frac{1}{2}v^2)}{dx}$. **1**

(ii) Show that $v^2 = 4x^4$. **2**

(iii) Explain why the velocity v cannot be negative. **1**

(iv) Find the time taken for the particle to travel to a point 2 metres

to the right of the origin. **2**

Question 4 continued............ **Marks**

(c) Let T be the temperature of a cup of tea at time t minutes after it has been brought into a room of temperature A. The Newton's Law of Cooling states that the rate of change of temperature T is proportional to $(T - A)$, that is $\frac{dT}{dt} = -k(T - A)$, where k is a positive constant.

(i) Show that $T = A + Be^{-kt}$, where B is a constant, satisfies Newton's Law of Cooling. **1**

(ii) A cup of tea, with an initial temperature of $95^\circ C$, is brought into a room of temperature $25^\circ C$. After 10 minutes, the temperature of the tea drops to $60^\circ C$. Find the temperature of the tea after another 5 minutes. **2**

End of Question 4.

Question 5 (12 Marks) Start a new page

(a) A particle is moving in a straight line so that its displacement x metres from a fixed point on the line at any time t seconds is given by

$$x = \frac{3}{2}\sin 2t + 2\cos 2t$$

(i) Show that $\ddot{x} = -n^2 x$, describe the motion of the particle and state the period of the motion **2**

(ii) Find the maximum displacement of the particle. **2**

Question 5 continued........ **Marks**

(b)

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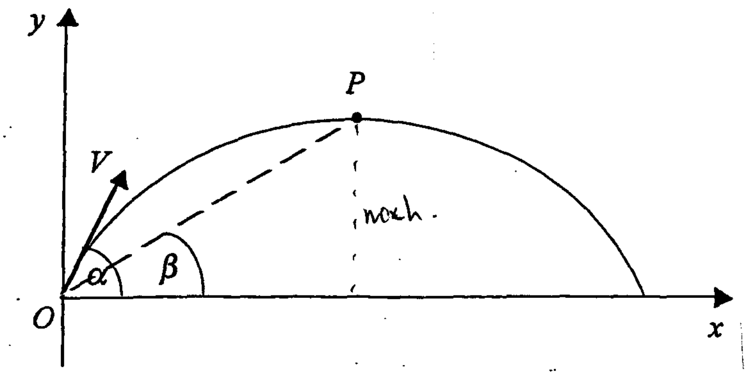


A particle is projected from a point *O* with speed $V\,ms^{-1}$ at an angle α radians above the horizontal, where $0 \le \alpha \le \frac{\pi}{2}$. It moves in a vertical plane subject to gravity where the acceleration due to gravity is $10\,ms^{-2}$. At time t seconds, it has horizontal and vertical displacements x metres and y metres respectively from *O*. At point *P* where it attains its greatest height the angle of elevation of the particle from *O* is β radians.

(i) Use integration to show that $x = Vt\cos\alpha$ and $y = Vt\sin\alpha - 5t^2$ **2**

(ii) Show that $\tan\beta = \frac{1}{2}\tan\alpha$. **3**

(iii) If the particle has greatest height 80 metres above *O* at a horizontal distance 120 metres from *O*, find the exact values of α and *V*. **3**

End of Question 5.

Question 6 (12 Marks) Start a new page **Marks**

(a) Find $\frac{d}{dx}\left(e^x \tan^{-1} x\right)$ **2**

(b) Find the exact value of $\int_1^{\sqrt{3}} \frac{2\,dx}{\sqrt{4-x^2}}$ **3**

(c) (i) State the domain and range for $y=\cos^{-1} 4x$ **1**

(ii) Sketch the graph of $y=\cos^{-1} 4x$ **1**

(iii) Find the equation of the tangent to $y=\cos^{-1} 4x$ at $x=-\frac{1}{8}$.

Leave answer in exact form. **3**

(d) (i) Sketch the graphs of $y=\sin^{-1} x$,for $0 \le x \le 1$

and $y=\sin x$, for $0 \le x \le \frac{\pi}{2}$ on separate diagrams. **1**

(ii) By considering your graphs drawn in part (i), find the exact value of

$\int_0^1 \sin^{-1} x\,dx + \int_0^{\frac{\pi}{2}} \sin x\,dx$ **1**

End of Question 6.

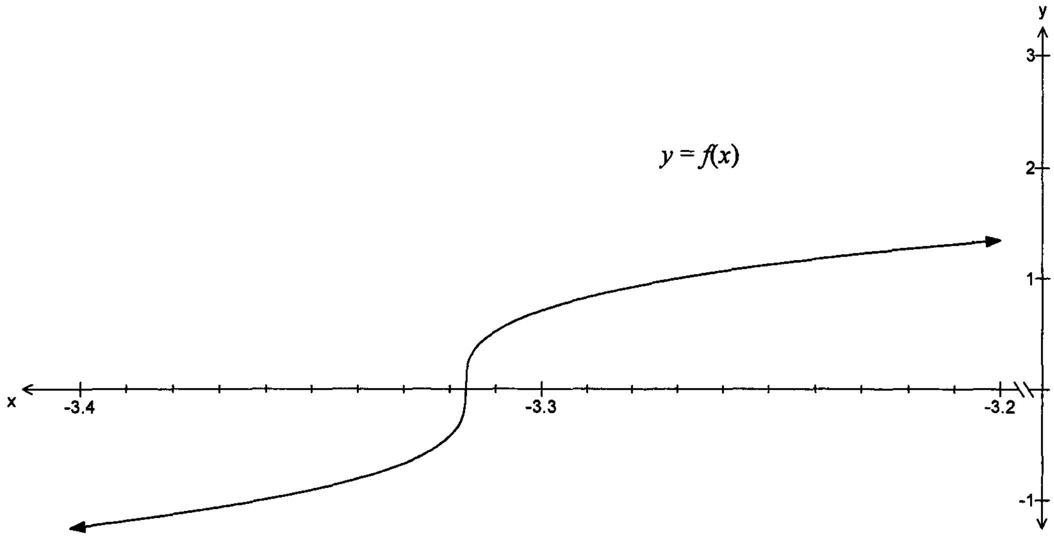
Question 7 (12 Marks) Start a new page

(a) Prove, using the method of mathematical induction,

that $4^n + 14$ is divisible by 6 for all $n \ge 1$. **3**

Question 7 continued................ **Marks**

(b) The curve $f(x) = (x^3 - 12x)^{\frac{1}{3}}$ is shown below.



(i) Find $f'(x)$. (No need to simplify your answer.) **1**

(ii) Taking an initial estimate of $x_1 = -3 \cdot 3$,

use one application of Newtons' Method to obtain another

approximation to the root of $f(x) = 0$. **1**

(iii) Explain why using $x_1 = -3 \cdot 3$ does not produce a better approximation

to the root than the original estimate. **1**

(c) Find the term independent of x in the expansion of $\left(x^2 - \dfrac{2}{x}\right)^9$ **2**

Question 7 Continued.............. **Marks**

(d) (i) Using the expansion of $(1+x)^{n-1}$ show that

$$\binom{n-1}{1}+\binom{n-1}{2}+\ldots\binom{n-1}{n-2}=2^{n-1}-2.$$ **2**

(ii) Find the least positive integer n, such that

$$\binom{n-1}{1}+\binom{n-1}{2}+\ldots\binom{n-1}{n-2}>1\,000$$ **2**

END OF EXAM

Ext 1 TRIAL 2010

(a) $3 : -2$

$P = \left(\dfrac{3\times 2 + -2\times -2}{3+(-2)}, \dfrac{3\times 1 + -2\times 4}{3+(-2)}\right)$

$= (10, -5)$

(b) $y = 3x - 1$ $\quad$ $4x - 2y = 7$

$\therefore m_1 = 3$ $\quad$ $2y = 4x - 7$

$y = 2x - \frac{7}{2}$

$\therefore m_2 = 2$

$\tan\theta = \left|\dfrac{3-2}{1+3\times 2}\right|$

$= \frac{1}{7}$

$\theta = 8^\circ 8'$

(c) $(x+1)(2x-1) \geq (x+1)^2$

$(x+1)(2x-1) - (x+1)^2 \geq 1$

$(x+1)(2x-1-x-1) \geq 1$

$(x+1)(x-2) \geq 1$

$x \neq -1$ $\quad$ -1 $\quad$ 2

$x < -1$ or $x \geq 2$

(d) (i) $\alpha\beta + \beta\gamma + \gamma\alpha = \frac{c}{a}$

$= 4.$

(ii) $\alpha\beta\gamma = \frac{-d}{a}$

$= 5$

(iii) $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma}$

$\dfrac{\alpha\beta + \beta\gamma + \gamma\alpha}{\alpha\beta\gamma} = \dfrac{4}{5}$

(e) $\frac{1}{4}\lim\limits_{\theta \to 0} \dfrac{\tan 2\theta}{2\theta} = \frac{1}{4}$

Q2 a) $y = \dfrac{x^2}{4a}$

(i) $\dfrac{dy}{dx} = \dfrac{x}{2a}$

at $x = 2ap$

$\dfrac{dy}{dx} = p$

$\therefore$ grad of normal $= -\frac{1}{p}$

Equation of normal

$y - ap^2 = -\frac{1}{p}(x - 2ap)$

$py - ap^3 = -x + 2ap$

$x + py = 2ap + ap^3.$

(ii) $M = \left(\dfrac{2ap}{2}, \dfrac{ap^2 + 2a + ap^2}{2}\right)$

$= (ap, a + ap^2)$

(iii) $x = ap$

$p = \frac{x}{a}$

$y = a(1 + p^2)$

$= a\left(1 + \frac{x^2}{a^2}\right)$

$y = a + \frac{x^2}{a}$

$x^2 = ay - a^2$

$x^2 = a(y - a)$

Q2

b) $u = 1 - x$

$du = -dx$

When $x = -15$, $u = 16$

$x = 0$, $u = 1$

$$\int_{-15}^{0} \frac{x\,dx}{\sqrt{1-x}} = -\int_{16}^{1} \frac{1-u}{\sqrt{u}}\,du$$

$$= \int_{1}^{16} u^{-\frac{1}{2}} - u^{\frac{1}{2}}\,du$$

$$= \left[2u^{\frac{1}{2}} - \frac{2}{3}u^{3/2}\right]_1^{16}$$

$$= \left(8 - \frac{128}{3}\right) - \left(2 - \frac{2}{3}\right)$$

$$= -36.$$

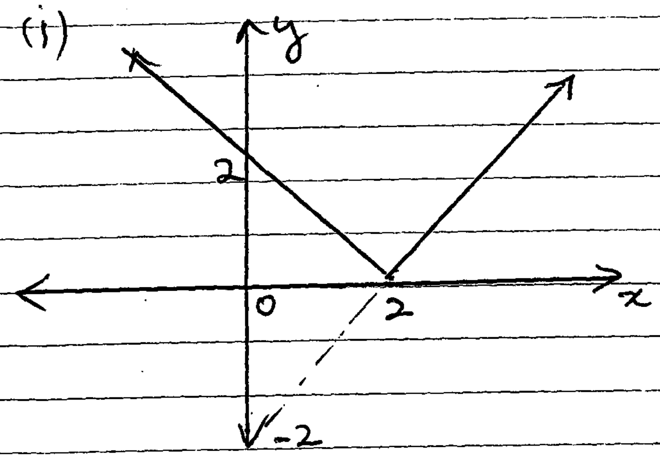
c) $\int \sin^2 6x\,dx$

$$= \frac{1}{2}\int (1 - \cos 12x)\,dx$$

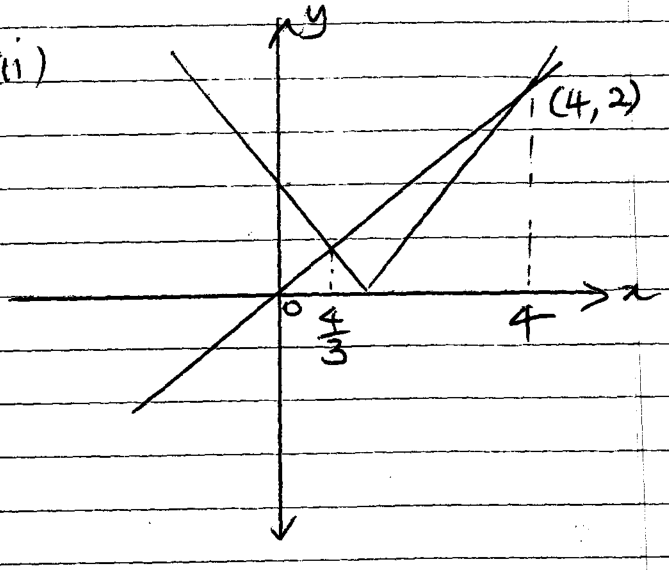
$$= \frac{1}{2}\left[x - \frac{\sin 2x}{12}\right] + C$$

$$= \frac{x}{2} - \frac{\sin 2x}{24} + C$$

d) (i)



(ii)



$x - 2 = \frac{1}{2}x$

$\frac{1}{2}x = 2$

$x = 4$

$-x + 2 = \frac{1}{2}x$

$\frac{3}{2}x = 2$

$x = \frac{4}{3}$

$\therefore \frac{4}{3} < x < 4$

Q3

(a) $\sin 2x + \cos x = 0$

$2\sin x \cos x + \cos x = 0$

$\cos x(2\sin x + 1) = 0$

$\cos x = 0$ or $\sin x = -\frac{1}{2}$

$x = \frac{\pi}{2}, \frac{3\pi}{2}$ $\quad$ $x = \frac{7\pi}{6}, \frac{11\pi}{6}$

b) (i) $t = \tan\frac{\theta}{2}$

$\operatorname{cosec}\theta = \frac{1+t^2}{2t}$

$\cot\theta = \frac{1-t^2}{2t}$

$LHS = \operatorname{cosec}\theta + \cot\theta$

$= \frac{1+t^2+1-t^2}{2t}$

$= \frac{2}{2t}$

$= \frac{1}{t}$

$= \cot\frac{\theta}{2} = RHS$.

ii) $\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} (\operatorname{cosec} x + \cot x)\,dx$

$= \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \cot\frac{x}{2}\,dx$

$= 2\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \frac{\frac{1}{2}\cos\frac{x}{2}}{\sin\frac{x}{2}}\,dx$

$= 2\left[\ln\left(\sin\frac{x}{2}\right)\right]_{\frac{\pi}{3}}^{\frac{\pi}{2}}$

$= 2\left[\ln\left(\sin\frac{\pi}{4}\right) - \ln\left(\sin\frac{\pi}{6}\right)\right]$

$= 2\left[\ln\frac{1}{\sqrt{2}} - \ln\frac{1}{2}\right]$

$= 2\ln\left(2^{-\frac{1}{2}} \div 2^{-1}\right)$

$= 2\ln 2^{\frac{1}{2}}$

$= \ln 2$

c) (i) In ΔPAB

$\tan 53^\circ = \frac{828}{AP}$

$AP = 828\cot 53^\circ$

In ΔABQ

$\tan\theta = \frac{828}{AQ}$

$AQ = 828\cot\theta$

In ΔAPQ.

$AP^2 + 840^2 = AQ^2$

$828^2\cot^2 53 + 840^2 = 828^2\cot^2\theta$

$\cot^2\theta = \frac{828^2\cot^2 53^\circ + 840^2}{828^2}$

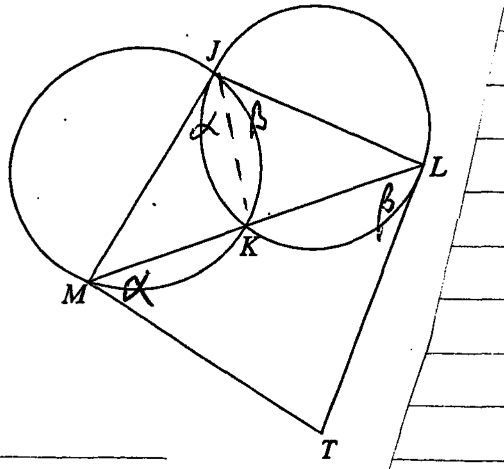
$\cot\theta = \sqrt{\frac{828^2\cot^2 53 + 840^2}{828^2}}$

(ii) $\theta = 38.35^\circ$

$= 38^\circ$.

Q4

a)



$\angle TMK = \angle MJK$

$= \alpha$ (alternate segment theorem) 1

$\angle KLJ = \angle KJL$

$= \beta$ (alternate segment theorem)

In ΔLMT

$\angle LTM = 180 - (\alpha+\beta)$ ($\angle$ sum of Δ) 1

$\angle MJL = \alpha + \beta$ (adjacent $\angle$'s)

$\therefore \angle MJL + \angle MJL$

$= (\alpha+\beta) + 180 - (\alpha+\beta)$

$= 180°$

$\therefore$ opposite $\angle$'s supplementary

$\therefore$ TMJL is cyclic quadrilateral

b) (i) $\dfrac{d}{dx}(\tfrac{1}{2}v^2) = \dfrac{d(\tfrac{1}{2}v^2)}{dv} \cdot \dfrac{dv}{dx}$

$= v\dfrac{dv}{dx}$

$= \dfrac{dx}{dt} \cdot \dfrac{dv}{dx}$ 1

$= \dfrac{dv}{dt}$

$= \ddot{x}$

OR

$\ddot{x} = \dfrac{dv}{dt}$

$= \dfrac{dx}{dt} \times \dfrac{dv}{dx}$

$= v \times \dfrac{dv}{dx}$

$= \dfrac{d}{dv}(\tfrac{1}{2}v^2)\dfrac{dv}{dx}$

$= \dfrac{d}{dx}(\tfrac{1}{2}v^2)$

ii) $\dfrac{d}{dx}(\tfrac{1}{2}v^2) = 8x^3$

$\tfrac{1}{2}v^2 = \int 8x^3\,dx$

$= 2x^4 + c$ 1

when $x=1$, $v=2$, $\therefore c = 0$.

$\tfrac{1}{2}v^2 = 2x^4$

$v^2 = 4x^4$. 1

iii) since $x \geq 1$, $\ddot{x}$ acceleration $8x^3 > 0$ ~~, velocity $v^2 = 4x^4$ which > 0~~ and initial $v > 0$ 1

$\therefore$ v cannot be negative.

iv) $v > 0 \quad \therefore v = 2x^2$

$\dfrac{dx}{dt} = 2x^2$

$\dfrac{dt}{dx} = \dfrac{1}{2x^2}$

$t = \tfrac{1}{2}\int x^{-2}\,dx$ 1

$= -\tfrac{1}{2}x^{-1} + c$

When $t=0$, $x=1$, $c = \tfrac{1}{2}$

$t = -\tfrac{1}{2}x^{-1} + \tfrac{1}{2}$

when $x = 2$

$t = -\tfrac{1}{2} \times 2^{-1} + \tfrac{1}{2}$

$= \tfrac{1}{4}$ seconds. 1

OR. $t = \displaystyle\int_1^2 \frac{1}{2x^2}\,dx$

$= \tfrac{1}{2}\left[-x^{-1}\right]_1^2$

$= \tfrac{1}{2}\left[-\tfrac{1}{2} + 1\right]$

$= \tfrac{1}{4}$ seconds

a better answer.

Initially $v>0$ and $x=1$. To change direction $v=0$, but $v=0$ when $x=0$ $\therefore$ impossible

c) i) $T = A + Be^{-kt}$

$\frac{dT}{dt} = Be^{-kt} \times -k$

$= -kBe^{-kt}$

$= -k(A + Be^{-k} - A)$ |

$= -k(T - A)$

ii) $A = 25$

$\therefore T = 25 + Be^{-kt}$

when $t = 0$, $T = 95$

$95 = 25 + B$

$\therefore B = 70$

$T = 25 + 70e^{-kt}$

when $t = 10$, $T = 60$

$60 = 25 + 70e^{-10k}$

$70e^{-10k} = 35$

$e^{-10k} = \frac{1}{2}$

$k = -\frac{1}{10}\ln\frac{1}{2}$ |

$T = 25 + 70e^{(\frac{1}{10}\ln\frac{1}{2})t}$

when $t = 15$

$T = 25 + 70e^{(\frac{1}{10}\ln\frac{1}{2}) \times 15}$

$= 49 \cdot 75°C$ |

Q5.

a) $x = \frac{3}{2}\sin 2t + 2\cos 2t$

$\dot{x} = 3\cos 2t - 4\sin 2t$

$\ddot{x} = -6\sin 2t - 8\cos 2t$ |

$= -4\left[\frac{3}{2}\sin 2t + 2\cos 2t\right]$

$= -4x$

$\therefore$ Motion is SHM.

oscillating about origin |

period $= \pi$ seconds.

OR.

$x = R\sin(2t + \alpha)$.

$R = \sqrt{(\frac{3}{2})^2 + 2^2} = \frac{5}{2}$

$x = \frac{5}{2}\sin(2t + \alpha)$

$\dot{x} = 2 \times \frac{5}{2}\cos(2t + \alpha)$

$= 5\cos(2t + \alpha)$

$\ddot{x} = -10\sin(2t + \alpha)$

$= -4(\frac{5}{2}\sin(2t + \alpha))$

$= -4x$

$\therefore$ Motion is SHM with centre $x = 0$ and period $T = \frac{2\pi}{2}$

$= \pi$.

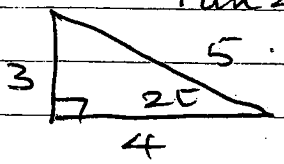
ii) Max displacement when $\dot{x}=0$

$3\cos 2t - 4\sin 2t = 0$.

$4\sin 2t = 3\cos 2t$

$\frac{\sin 2t}{\cos 2t} = \frac{3}{4}$

$\tan 2t = \frac{3}{4}$ 1



$\therefore$ Max displacement

$x = \frac{3}{2} \times \frac{3}{5} + 2 \times \frac{4}{5}$

$= 2.5\text{ m}$ 1

OR

$x = \frac{5}{2}\sin(2t+\alpha)$.

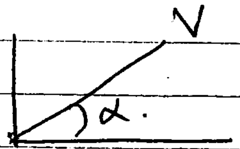
Since $-1 \le \sin(2t+\alpha) \le 1$

max displacement when $\sin(2t+\alpha) = 1$.

$\therefore x = \frac{5}{2}$

$= 2.5\text{ m}$.

b)



$\dot{y} = V\sin\alpha$

$\dot{x} = V\cos\alpha$

$x = \int V\cos\alpha \, dt$

$= Vt\cos\alpha + C_1$ 1

$t=0,\ x=0 \quad \therefore C_1 = 0$.

$\therefore x = Vt\cos\alpha$.

$\ddot{y} = -10$

$\dot{y} = -10t + C_2$

$t=0,\ \dot{y} = V\sin\alpha,\ C_2 = V\sin\alpha$

$\dot{y} = -10t + V\sin\alpha$

$y = \frac{-10t^2}{2} + Vt\sin\alpha + C_3$. 1

$t=0,\ y=0,\ \therefore C_3 = 0$.

$\therefore y = -5t^2 + Vt\sin\alpha$.

$y = Vt\sin\alpha - 5t^2$

ii) $\dot{y} = 0$ at greatest height

$-10t + V\sin\alpha = 0$

$t = \frac{V\sin\alpha}{10}$ 1

Sub t into eqn of x.

$x = \frac{V^2\sin\alpha\cos\alpha}{10}$

Sub t into eqn of y

$y = \frac{-5V^2\sin^2\alpha}{100} + \frac{V^2\sin^2\alpha}{10}$

$= \frac{5V^2\sin^2\alpha}{100}$ 1

$\tan\beta = \frac{y}{x}$

$= \frac{\cancel{5}\cancel{V^2}\sin^{\cancel{2}}\alpha}{{}_2\cancel{100}} \times \frac{\cancel{10}}{\cancel{V^2}\cancel{\sin\alpha}\cos\alpha}$

$= \frac{1}{2}\frac{\sin\alpha}{\cos\alpha}$ 1

$\therefore \tan\beta = \frac{1}{2}\tan\alpha$.

ii) $P(120, 80)$

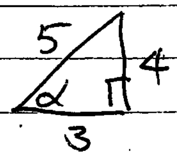
$\tan\beta = \frac{80}{120}$

$= \frac{2}{3}$

from (ii) $\tan\beta = \frac{1}{2}\tan\alpha$

$\therefore \frac{1}{2}\tan\alpha = \frac{2}{3}$

$\tan\alpha = \frac{4}{3}$ 1



$y = \frac{v^2 \sin^2\alpha}{20}$

$80 = \frac{V^2 \times \left(\frac{4}{5}\right)^2}{20}$ 1

$80 = \frac{1}{20} \times V^2 \times \frac{16}{25}$

$V^2 = 8 \times 20 \times \frac{25}{16}$

$= 2500$

$V = 50$ 1

Q6

(a) $\frac{d}{dx}\left[e^x \tan^{-1} x\right]$

$= e^x \tan^{-1} x + \frac{e^x}{1+x^2}$ 1

b) $\int_1^{\sqrt{3}} \frac{2}{\sqrt{4-x^2}}\, dx$

$= \int_1^{\sqrt{3}} \frac{2}{\sqrt{2^2 - x^2}}\, dx$

$= 2\left[\sin^{-1}\frac{x}{2}\right]_1^{\sqrt{3}}$ 1

$= 2\left[\sin^{-1}\frac{\sqrt{3}}{2} - \sin^{-1}\frac{1}{2}\right]$

$= 2\left[\frac{\pi}{3} - \frac{\pi}{6}\right]$ 1

$= 2 \times \frac{\pi}{6}$

$= \frac{\pi}{3}$. 1

c) i) Domain:

$-1 \le 4x \le 1$

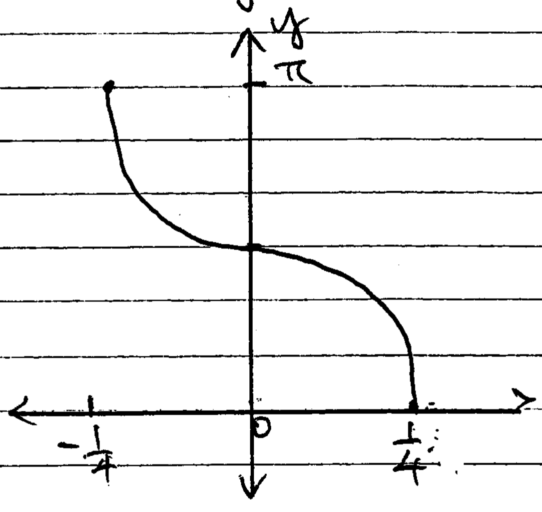
$-\frac{1}{4} \le x \le \frac{1}{4}$

Range:

$0 \le y \le \pi$

1

ii)



1

ii) $y = \cos^{-1} 4x$

$$\frac{dy}{dx} = \frac{-4}{\sqrt{1-16x^2}} \quad 1$$

at $x = -\frac{1}{8}$.

$$\frac{dy}{dx} = \frac{-4}{\sqrt{1-\frac{1}{4}}}$$

$$= -\frac{8}{\sqrt{3}}. \quad 1$$

at $x = -\frac{1}{8}$, $y = \cos^{-1}\left(-\frac{1}{2}\right)$

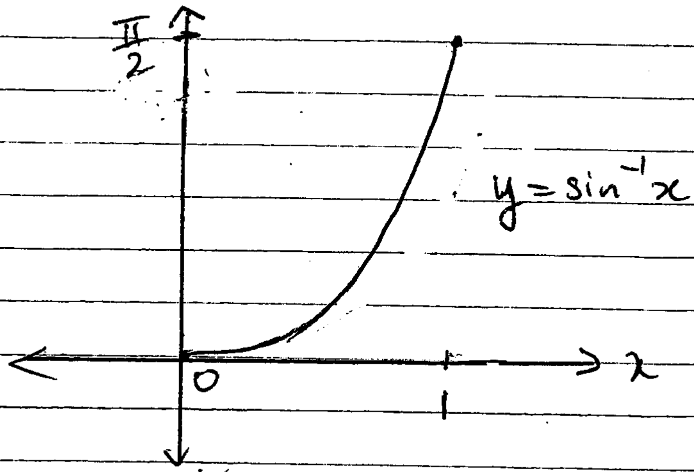
$$= \frac{2\pi}{3}.$$

Equation of tangent

$$y - \frac{2\pi}{3} = \frac{-8}{\sqrt{3}}\left(x + \frac{1}{8}\right) \rightarrow y - \frac{2\pi}{3} = \frac{-8x}{\sqrt{3}} - \frac{1}{\sqrt{3}} \quad 1$$

$$y = -\frac{8}{\sqrt{3}}x - \frac{1}{\sqrt{3}} + \frac{2\pi}{3}. \rightarrow 24x + 3\sqrt{3}y + 3 - 2\sqrt{3}\pi$$

d)
i)



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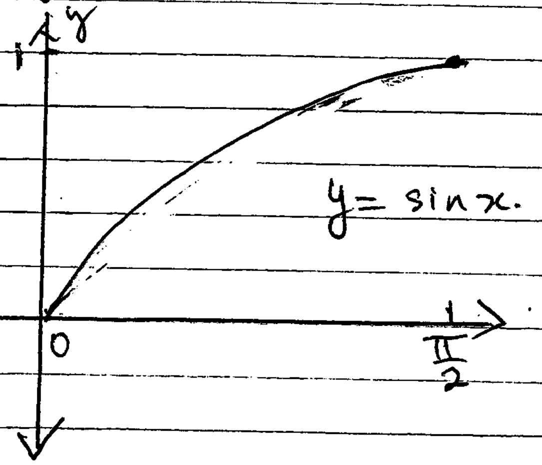

(ii) $\int_0^1 \sin^{-1}x\,dx + \int_0^{\frac{\pi}{2}} \sin x\,dx$

$$= \frac{\pi}{2} \times 1 \quad 1$$

$$= \frac{\pi}{2}$$

Q7.

a) For $n=1$

$4^1+14=18$

$=6\times 3$

$\therefore$ Statement is true for $n=1$.

Assume statement is true for $n=k$.

$4^k+14=6p$, for p some integer. 1

Prove that statement is true for $n=k+1$.

$$4^{k+1}+14 = 4\times 4^k+14$$
$$= 4\times 4^k + 4\times 14 - 3\times 14$$
$$= 4(4^k+14) - 6\times 7 \quad 1$$
$$= 4\times 6p - 6\times 7$$
$$= 6(4p-7) \quad 1$$

$\therefore$ $4^{k+1}+14$ is divisible by 6.

Since statement is true for $n=1$ and assume true for $n=k$ and proved true for $n=k+1$, then it is true for $n=2,3,4\ldots$ ie all positive integral values of n.

b) (i) $f'(x)=\frac{1}{3}(x^3-12x)^{-\frac{2}{3}}(3x^2-12)$ or $(x^2-4)(x^3-12x)^{-\frac{2}{3}}$ 1

ii) $x_1=-3.3$.

$$f(x_1) = (-3.3^3-12\times -3.3)^{\frac{1}{3}}$$
$$\approx 1.54$$

$$f'(x_1) = \frac{1}{3}(-3.3^3-12\times -3.3)^{-\frac{2}{3}}(3\times(-3.3)^2-12)$$
$$\approx 2.90.$$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$
$$= -3.3 - \frac{1.54}{2.90}$$
$$= -3.83$$

(iii) Since graph at $x = -3\cdot3$ is almost flat, then the tangent at $x = -3\cdot3$ cuts the x axis further away from the actual x intercept ie root of $f(x)$. 1

(c) $T_{k+1} = {}^9C_k \, (x^2)^{9-k} \left(\frac{-2}{x}\right)^k$

$= {}^9C_k \, x^{18-2k} (-2)^k \, x^{-k}$

$= {}^9C_k \, (-2)^k \, x^{18k-3k}$. 1

Term independent of x is when

$18 - 3k = 0$

$k = 6$.

$T_7 = {}^9C_6 \, (-2)^6 \, x^0$

$= 84 \times 64$

$= 5376$ 1

d) $(1+x)^{n-1} = \binom{n-1}{0} 1^{n-1} + \binom{n-1}{1} 1^{n-2} x + \dots\dots + \binom{n-1}{n-2} x^{n-2} + \binom{n-1}{n-1} x^{n-1}$ 1

(i) Let $x = 1$

$2^{n-1} = 1 + \binom{n-1}{1} + \dots \binom{n-1}{n-2} + 1$. 1

$\therefore 2^{n-1} - 2 = \binom{n-1}{1} + \binom{n-2}{2} + \dots + \binom{n-1}{n-2}$

(ii) $2^{n-1} - 2 > 1000$

$2^{n-1} > 1002$

$(n-1)\ln 2 > \ln 1002$ 1

$n-1 > \dfrac{\ln 1002}{\ln 2}$

$n > \dfrac{\ln 1002}{\ln 2} + 1$

$n > 10.9$.

$\therefore$ least positive integer is $n = 11$. 1