FH
JG
HK
KP

Name: ____________________

Class: 12MTX________

Teacher: ____________________

## CHERRYBROOK TECHNOLOGY HIGH SCHOOL

**2011 AP4**

**YEAR 12 TRIAL HSC EXAMINATION**

# MATHEMATICS EXTENSION 1

*Time allowed* - 2 HOURS
*(Plus 5 minutes reading time)*

## DIRECTIONS TO CANDIDATES:

- Attempt all questions.
- All questions are of equal value.
- Each question is to be commenced on a new page clearly marked Question 1, Question 2, etc on the top of the page and must show your name and class.
- All necessary working should be shown in every question. Full marks may not be awarded for careless or badly arranged work.
- Approved calculators may be used. Standard Integral Tables are provided.
- Write your name and class in the space provided at the top of this question paper.
- Your solutions will be collected in one bundle stapled in the top left corner. Please arrange them in order, Q1 to 7. The exam paper must be handed in with your solutions.

**Question 1 (12 Marks)** **Marks**

(a) Simplify $\frac{1+a^{-1}}{1+a^{-3}}$. **2**

(b) The acute angle between the lines $2x + y = 5$ and $y = mx - 3$ is $\frac{\pi}{4}$. **2**

Find the value of $m$.

(c) Find **2**

$$\lim_{x \to 0} \frac{1-cos2x}{x^2}$$

(d) Find the value of $k$ if $x + 3$ is a factor of $P(x) = 2x^3 - 5kx + 9$ **2**

(e) Solve for $x$ **2**

$$2\ln(x-2) = \ln(3x-2)$$

(f) Find $\int 2x\sqrt{1+3x^2}\,dx$ using the substitution $u = 1 + 3x^2$. **2**

**Question 2 ( 12 Marks) START A NEW PAGE** **Marks**

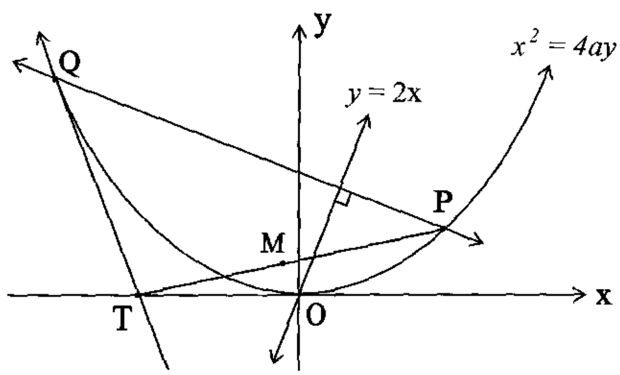
(a) $P(-2,5,), R(a,2)$ and $Q(4,-7)$ are collinear points.

(i) Find the ratio that $R$ divides $PQ$. **1**

(ii) Hence find the value of $a$ for the $x$ coordinate of $R$ **1**

(b) $P(2ap, ap^2)$ and $Q(2aq, aq^2)$ are two points on the parabola $x^2 = 4ay$ such that the line $PQ$ is perpendicular to the line $y = 2x$.

The tangent at $Q$ intersects the $x$ axis at $T$ and $M$ is the midpoint of $PT$.




(i) Show that $q = -p - 1$ **2**

(ii) Given the equation of the tangent at Q is $y = qx - aq^2$.
Show that the coordinates of $M$ the midpoint of $PT$ are **1**
$\left(\frac{a}{2}(p-1), \frac{a}{2}p^2\right)$

(iii) Show that the locus of $M$ is a parabola. **2**

(c) Find the values of $x$, where $0 \leq x \leq \pi$, for the series **2**
$1 + \sqrt{3}tanx + 3tan^2x + 3\sqrt{3}tan^3x + \ldots\ldots\ldots\ldots$ has a limiting sum.

(d) Solve $sin\theta + 4cos\theta = 1$, using $t = tan\frac{\theta}{2}$, for $0 \leq \theta \leq 2\pi$. **3**

**Question 3 ( 12 Marks) START A NEW PAGE** **Marks**

(a) Consider the function $f(x) = (x-2)^2$.

(i) Sketch $y = f(x)$. **1**

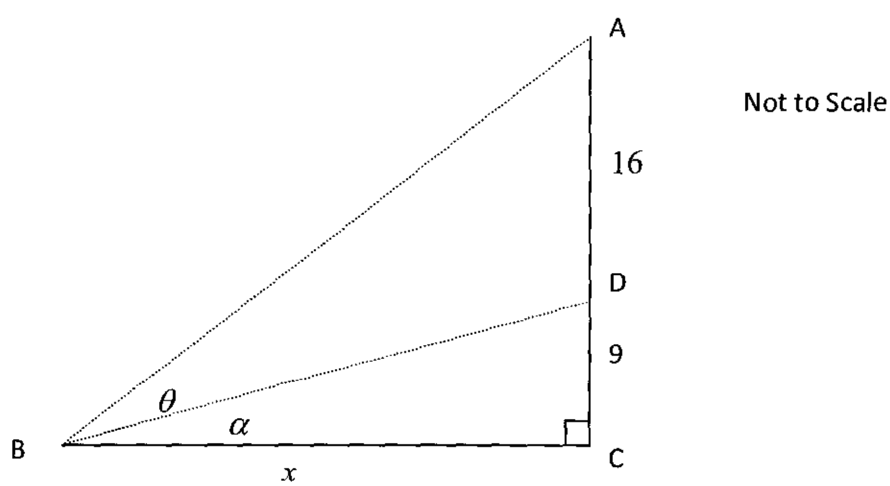
(ii) Explain why $f(x)$ does not have an inverse function for all $x$ in its domain? **1**

(iii) State a largest possible domain for which $f(x)$ has an inverse function $f^{-1}(x)$. **1**

(iv) Find the equation of the inverse function $f^{-1}(x)$ for your domain in part (iii). **1**

(v) Sketch $f^{-1}(x)$ onto your graph in part(i). **1**

(b)




$\angle ABD = \theta$, $\angle DBC = \alpha$, $DC = 9 metres, AD = 16 metres, BC = x\, metres$

(i) Show that $\theta = \tan^{-1}\left(\frac{25}{x}\right) - \tan^{-1}\left(\frac{9}{x}\right)$ **2**

(ii) Find the value of $x$ which maximizes the value of $\theta$. **4**

(c) Differentiate $cos^{-1}(x^2)$. **1**

**Question 4 ( 12 Marks) START A NEW PAGE** **Marks**

(a) Find the exact value of $\int_1^2 \frac{dx}{\sqrt{4-x^2}}$. **2**

(b) Find the general solution, in exact radian form, of the trigonometric equation **1**

$$cosec\,\theta = -\frac{2}{\sqrt{3}}.$$

(c) The region in the first quadrant bounded by the curve $y = \cos 4x$ and **3**

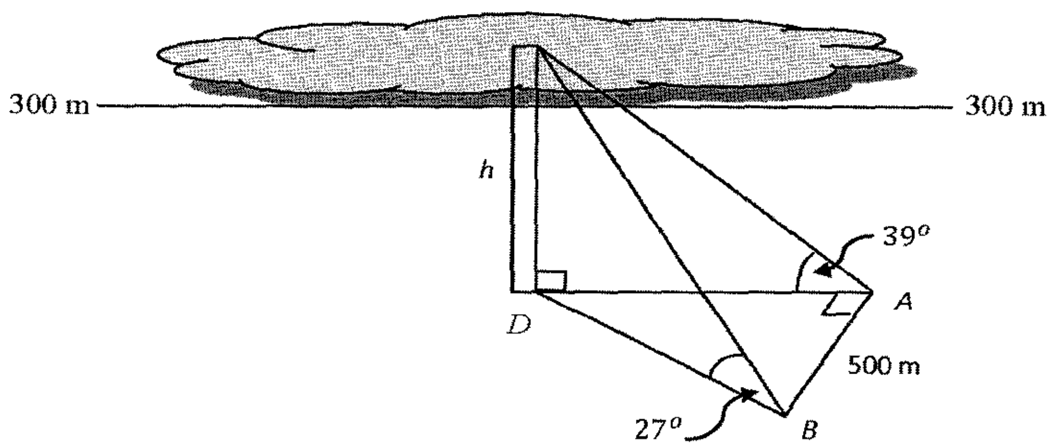
the $x$ axis between $x = 0$ and $x = \frac{\pi}{8}$ is rotated about the $x$ axis to form

a solid of revolution.

Calculate the volume of the solid, giving your answer in simplest exact form.

(d) Show that $\tan\frac{5\pi}{12} = \frac{\sqrt{3}+1}{\sqrt{3}-1}$ **2**

(e)



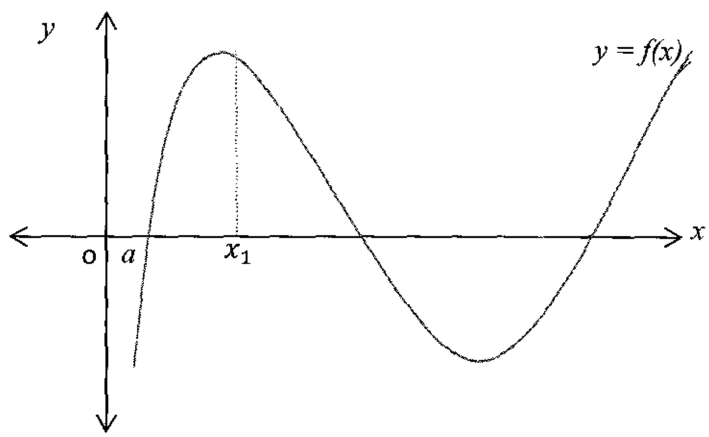

A vertical city tower is due west of a point $A$ on level ground. **4**
The angle of elevation to the top of the tower at $A$ is $39^o$. The point $B$ is 500 metres due south of $A$, and the angle of elevation to the top of the tower from $B$ is $27^o$.
Given that the base of a blanket of full cloud cover is at a constant 300 metres high, find the length, correct to 2 decimal places, of the part of the tower that is engulfed (surrounded) by cloud.

**Question 5 ( 12 Marks ) START A NEW PAGE** **Marks**

(a) Prove by mathematical induction that **3**

$$\frac{1}{2!}+\frac{2}{3!}+\frac{3}{4!}+\cdots+\frac{n}{(n+1)!}=1-\frac{1}{(n+1)!}\quad \text{for all integers } n\geq 1.$$

(b)




Copy the diagram onto your answer sheet.

A first approximation for the value of $a$, using Newton's method is $x_1$.

A further approximation $x_2$ is taken from $x_1$ using Newton's method.

(i) Illustrate this on your diagram, showing the position of $x_2$ clearly. **1**

(ii) Justify algebraically the location of $x_2$ using Newton's method. **1**

(c) Find the coefficient of the term independent of $x$ in the binomial expansion. **2**

$$\left(\frac{x^2}{2}+\frac{4}{x^4}\right)^{12}$$

(d) Show that $\binom{n}{r}+\binom{n}{r-1}=\binom{n+1}{r}$ **2**

(e) (i) Write down the binomial expansion of $(1+x)^{2n}$ in ascending powers of $x$ and differentiate both sides with respect to $x$. **1**

(ii) Hence show that **2**

$$2\,{}^{2n}C_2+4\,{}^{2n}C_4+6\,{}^{2n}C_6+\cdots\ldots\ldots+2n\,{}^{2n}C_{2n}=n2^{\,2n-1}$$

**Question 6 ( 12 Marks ) START A NEW PAGE** **Marks**

(a) An archer projects an arrow at an angle of $45°$ to the horizontal and the initial velocity of the arrow is $40ms^{-1}$. Take $g$ to be $10ms^{-2}$.

(i) Show that the horizontal speed of the arrow is constant at $\frac{40}{\sqrt{2}}ms^{-1}$ **1**

(ii) Show that the vertical distance $y$ of the arrow at any time $t$ is $y = -5t^2 + \frac{40}{\sqrt{2}}t$. **1**

(iii) Find the maximum height of the arrow. **1**

(iv) Calculate the distance away from the archer where the arrow lands. **2**

(b) A particle is moving in a straight line. At time $t$ seconds it has displacement $x$ metres from a fixed point $O$ on the line, velocity $v\ ms^{-1}$ given by $v = \frac{1}{x+1}$ and acceleration $a\ ms^{-2}$. Initially the particle is at the origin $O$.

(i) Express $a$ as a function of $x$. **1**

(ii) Express $x$ as a function of $t$. **2**

(c) The velocity, $v\ cm\,s^{-1}$, of a particle moving along the $x$-axis is given by

$v^2 = 40 + 32x - 8x^2$, where $x$ is in centimetres.

(i) Show that the motion is simple harmonic. **1**

(ii) Find the period and centre of motion. **1**

(iii) Find the amplitude of the motion. **2**

**Question 7 ( 12 Marks) START A NEW PAGE** **Marks**

(a) Let $T$ be the temperature inside a room at time $t$ and let $A$ be the temperature of its surroundings. Newton's Law of Cooling states that the rate of change of the temperature $T$ is proportional to $(T - A)$.

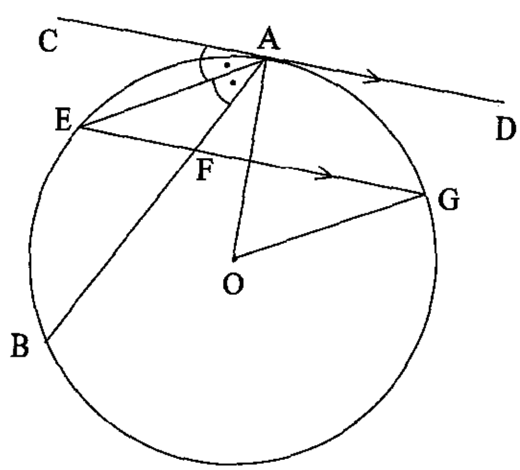
(i) Verify that $T = A + Be^{kt}$ (where $B$ and $k$ are constants) satisfies Newton's Law of Cooling. **1**

(ii) The constant temperature of the surrounding is $6^0 C$ and an air conditioning system causes the temperature inside a room to drop from $28^0 C$ to $18^0 C$ in 35 minutes. **2**

Find how long it takes for the inside room temperature to reach $10^0 C$.

(b) In the diagram, $AB$ is a chord of the circle with centre $O$. $CD$ is the tangent at $A$. The bisector of $\angle CAB$ meets the circle at $E$.

The line passing through $E$ and parallel to $CD$, crosses $AB$ at $F$ and meets the circle at $G$.




Copy the diagram onto your answer sheet.

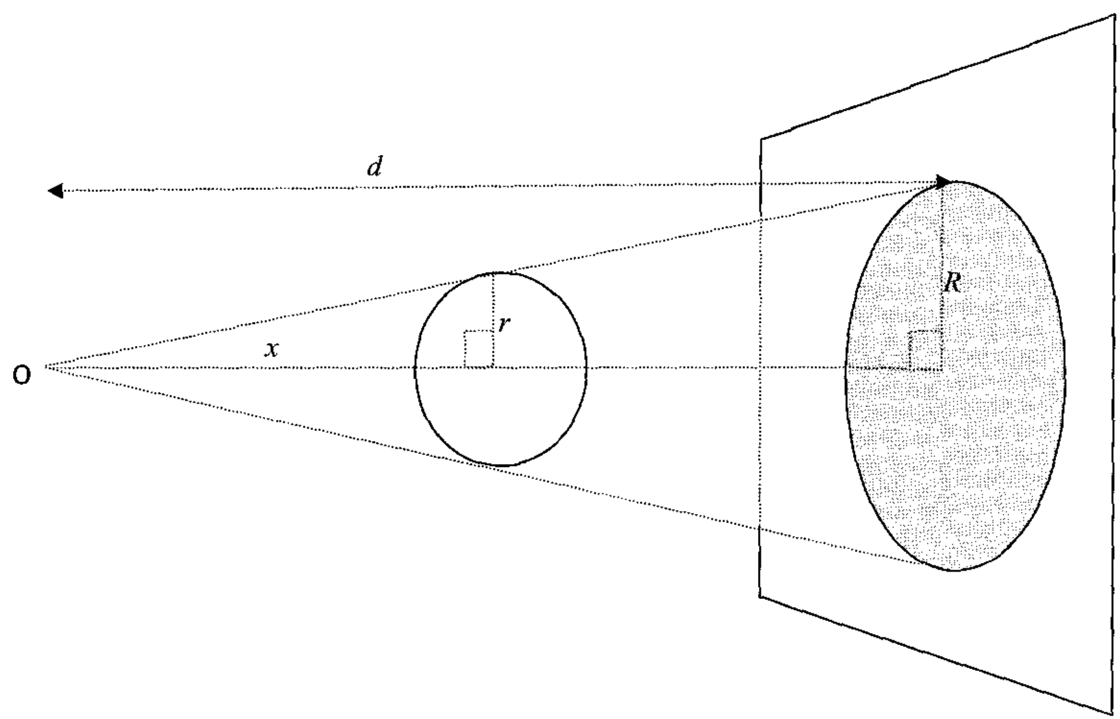
(i) Show that $\Delta AEF$ is isosceles. **2**

(ii) Show that $AFOG$ is a cyclic quadrilateral. **2**

(c) A light source $O$ is initially placed $d$ cm away from a screen.

A coin of radius $r$ is placed $x$ cm from the light source, such that its horizontal axis of symmetry passes through $O$.

The light source $O$ is moving horizontally towards the coin at a speed of *4cm* per second and casts a circular shadow of radius $R$ cm on the screen as shown.




(i) Show that the area of the shadow is $A = \dfrac{\pi d^2 r^2}{x^2}$ **2**

(ii) Find the rate of increase of the area of the shadow with respect to time when $x = \dfrac{d}{4}$. **3**

## End of Examination

# 2011 Extension 1 Trial Solutions



Q1 (a) $\dfrac{1+\frac{1}{a}}{1+\frac{1}{a^3}}$

$= \dfrac{a+1}{a} \div \dfrac{a^3+1}{a^3}$

$= \dfrac{a^2\cancel{(a+1)}}{\cancel{(a+1)}(a^2-a+1)}$

$= \dfrac{a^2}{a^2-a+1}$

(b) $m_1 = -2, \quad m_2 = m$

$\tan\frac{\pi}{4} = \left|\dfrac{m+2}{1-2m}\right|$

$\left|\dfrac{m-2}{1+2m}\right| = 1$

$|m-2| = |1+2m|$

$m+2 = 1-2m$

or

$m+2 = -1+2m$

$m = -\frac{1}{3}$ or $3$

(c) $\lim\limits_{x\to 0} \dfrac{2\sin^2 x}{x^2}$

$= 2 \times 1^2$

$= 2$

d) $P(-3) = 0$

$2(-3)^3 - 5k(-3) + 9 = 0$

$-54 + 15k + 9 = 0$

$15k = 45$

$k = 3$

(e) $(x-2)^2 = 3x-2$

$x^2 - 4x + 4 = 3x - 2$

$x^2 - 7x + 6 = 0$

$(x-6)(x-1) = 0$

$x = 6$ or $1$

when $x = 1$

$\ln(x-2) = \ln$ (negative).

$\therefore x \neq 1$

$x = 6$ only.

(f) $u = 1+3x^2$

$du = 6x\,dx$

$\int 2x\sqrt{1+3x^2}\,dx$

$\frac{1}{3}\int 6x\sqrt{1+3x^2}\,dx$

$\frac{1}{3}\int u^{\frac{1}{2}}\,du$

$\frac{1}{3}\times\frac{2}{3}u^{3/2} + c$

$\frac{2}{9}u^{3/2} + C$

$\frac{2}{9}\left(\sqrt{1+3x^2}\right)^3 + C$

Q2



a) i) $P(-2,5)$ $R(a,2)$ $Q(4,-7)$

$k : l$

$$2 = \frac{5l - 7k}{k+l}$$

$$2k + 2l = 5l - 7k$$

$$9k = 3l$$

$$\frac{9k}{9l} = \frac{3l}{9l}$$

$$\frac{k}{l} = \frac{3}{9}$$

$$\therefore k : l = 3 : 9$$

$$= 1 : 3$$

(ii) $a = \frac{3 \times -2 + 1 \times 4}{1+3}$

$$= \frac{-6+4}{4}$$

$$= -\frac{2}{4}$$

$$\therefore a = \frac{-1}{2}$$

b) i) $m_{PQ} = \frac{ap^2 - aq^2}{2ap - 2aq}$

$$= \frac{\cancel{a}\cancel{(p-q)}(p+q)}{2\cancel{a}\cancel{(p-q)}}$$

$$= \frac{p+q}{2}$$

grad of $PQ = -\frac{1}{2}$

$$\therefore \frac{p+q}{2} = -\frac{1}{2}$$

$$p+q = -1$$

$$\therefore q = -1-p.$$

ii) $y = qx - aq^2$

when $y=0$, $x = aq$

$\therefore T(aq, 0)$

midpt of $PT$

$$M\left(\frac{aq+2ap}{2}, \frac{0+ap^2}{2}\right)$$

sub $q = -1-p$

$$\left(\frac{-a-ap+2ap}{2}, \frac{ap^2}{2}\right)$$

$$\left(\frac{ap-a}{2}, \frac{ap^2}{2}\right)$$

$$\left(\frac{a}{2}(p-1), \frac{ap^2}{2}\right)$$

iii) $x = \frac{a}{2}(p-1)$

$$\frac{2x}{a} = p-1$$

$$\therefore p = \frac{2x}{a} + 1$$

Sub into $y = \frac{ap^2}{2}$

$$y = \frac{a}{2}\left(\frac{2x}{a}+1\right)^2$$

$$\frac{2y}{a} = \frac{4x^2}{a^2} + \frac{4x}{a} + 1$$

$(\times a^2)$

$$2ay = 4x^2 + 4ax + a^2$$

$(\div 4)$

$$\frac{1}{2}ay = x^2 + ax + \frac{a^2}{4}$$

$$\frac{1}{2}ay = \left(x + \frac{a}{2}\right)^2$$

$\therefore$ locus of $M$ is a parabola.

(c) $r = \sqrt{3}\tan x$

for limiting sum

$|\sqrt{3}\tan x| < 1$

$|\tan x| < \frac{1}{\sqrt{3}}$

$0 < x < \frac{\pi}{6}$

or

$\frac{5\pi}{6} < x < \pi$

d) i) $\sin\theta = \frac{2t}{1+t^2}$

$\cos\theta = \frac{1-t^2}{1+t^2}$

ii) $\left(\frac{2t}{1+t^2}\right) + 4\left(\frac{1-t^2}{1+t^2}\right) = 1$

$2t + 4 - 4t^2 = 1 + t^2$

$5t^2 - 2t - 3 = 0$

$(5t+3)(t-1) = 0$

$t = -\frac{3}{5}$ or $t = 1$

$\tan\frac{\theta}{2} = \frac{-3}{5}$ ($2^{nd}$ & $3^{rd}$ quad)

$0 \leq \frac{\theta}{2} \leq \pi$

$\frac{\theta}{2} = 2.601173153$

$\therefore \theta = 5.202346307$

$\frac{149\pi}{90}$

$\tan\frac{\theta}{2} = 1$

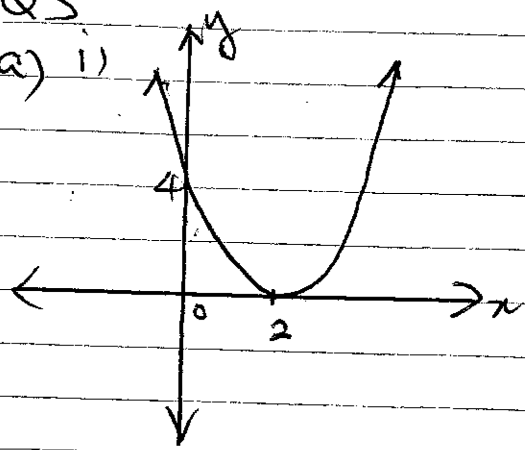
$\frac{\theta}{2} = \frac{\pi}{4}$

$\theta = \frac{\pi}{2}$

$\therefore \theta = 5.2^c$ or $\frac{\pi}{2}$

Q3

a) i)




④

ii) fails horizontal line test or for each y value there are 2 x values

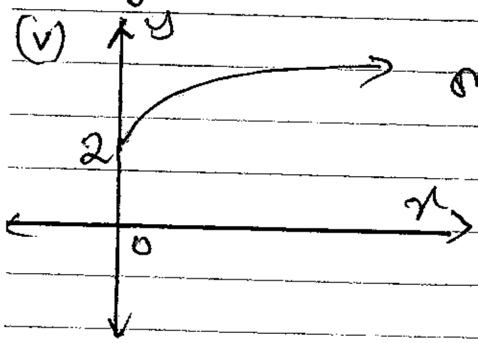
iii) $x \geq 2$ or $x \leq 2$

iv) $x = (y-2)^2$

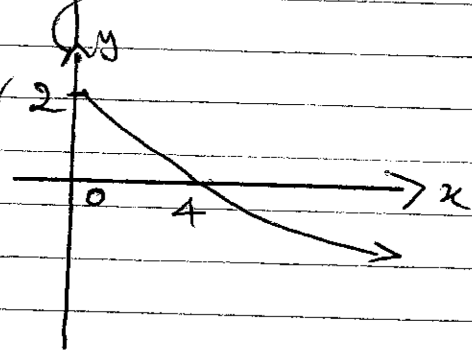
$\sqrt{x} = y-2$

$y = \sqrt{x} + 2$ or $y = -\sqrt{x}+2$

(v)




or




b) i) $\tan\alpha = \frac{9}{x}$

$\alpha = \tan^{-1}\frac{9}{x}$

$\tan(\theta+\alpha) = \frac{25}{x}$

$\theta + \alpha = \tan^{-1}\left(\frac{25}{x}\right)$

$\theta = \tan^{-1}\left(\frac{25}{x}\right) - \alpha$

$= \tan^{-1}\left(\frac{25}{x}\right) - \tan^{-1}\left(\frac{9}{x}\right)$

ii) $\frac{d\theta}{dx} = \frac{1}{1+\left(\frac{25}{x}\right)^2} \times \left(\frac{-25}{x^2}\right) - \frac{1}{1+\frac{81}{x^2}} \times \frac{-9}{x^2}$

$\frac{-25}{x^2+625} + \frac{9}{x^2+81} = 0$

$25(x^2+81) = 9(x^2+625)$

$25x^2 + 2025 = 9x^2 + 5625$

$16x^2 = 3600$

$x^2 = 225$

$x = \pm\sqrt{225}$

$= \pm 15$ since $x$ is length $x > 0$ $\therefore x = 15$

| $\theta$ | 14 | 15 | 16 |
|---|---|---|---|
| $\frac{d\theta}{dx}$ | 0.002 | 0 | -0.0017 |

$\therefore$ when $x = 15$, $\theta$ is maximum.

(c) $\frac{d}{dx}\left[\cos^{-1}(x^2)\right] = \frac{-2x}{\sqrt{1-(x^2)^2}}$

$= \frac{-2x}{\sqrt{1-x^4}}$

Q4.

(6)

a) $\int_1^2 \frac{dx}{\sqrt{4-x^2}}$

$= \left[\sin^{-1}\left(\frac{x}{2}\right)\right]_1^2$

$= \sin^{-1} 1 - \sin^{-1} \frac{1}{2}$

$= \frac{\pi}{2} - \frac{\pi}{6}$

$= \frac{\pi}{3}$

b) $\sin\theta = -\frac{\sqrt{3}}{2}$

$\theta = 2n\pi - \frac{\pi}{3}$ or $2n\pi + \frac{2\pi}{3}$

or

$\theta = m\pi - (-1)^m \frac{\pi}{3}$ or $m\pi + (-1)^{m+1} \frac{\pi}{3}$

m, n integers.

c) $V = \pi \int_0^{\pi/8} \cos^2 4x \, dx$

$= \frac{\pi}{2} \int_0^{\pi/8} (1 + \cos 8x) \, dx$

$= \frac{\pi}{2} \left[x + \frac{1}{8}\sin 8x\right]_0^{\pi/8}$

$= \frac{\pi}{2} \left[\left(\frac{\pi}{8} + \frac{1}{8}\sin\pi\right) - \left(0 + \frac{1}{8}\sin 0\right)\right]$

$= \frac{\pi}{2} \times \frac{\pi}{8}$

$= \frac{\pi^2}{16} \ u^3$

Q4



d) $\tan\left(\frac{\pi}{4}+\frac{\pi}{6}\right) = \dfrac{\tan\frac{\pi}{4}+\tan\frac{\pi}{6}}{1-\tan\frac{\pi}{4}\tan\frac{\pi}{6}}$

$$= \frac{1+\frac{1}{\sqrt{3}}}{1-1\times\frac{1}{\sqrt{3}}}$$

$$= \frac{\frac{\sqrt{3}+1}{\sqrt{3}}}{\frac{\sqrt{3}-1}{\sqrt{3}}}$$

$$= \frac{\sqrt{3}+1}{\sqrt{3}-1}$$

(e) $\tan 39° = \dfrac{h}{AD}$

$AD = h\cot 39°$

$\tan 27° = \dfrac{h}{BD}$

$BD = h\cot 27°$

$DB^2 = AD^2 + AB^2$

$h^2\cot^2 27° = h^2\cot^2 39° + 500^2$

$h^2(\cot^2 27° - \cot^2 39°) = 500^2$

$h^2 = \dfrac{500^2}{\cot^2 27° - \cot^2 39°}$

$h = 327.78\,m$

$\therefore$ part of tower surround by cloud $= 27.78\,m$

Q5

(8)

(a) When $n = 1$

$LHS = \frac{1}{2!}$

$= \frac{1}{2}$

$RHS = 1 - \frac{1}{2!}$

$= \frac{1}{2}$

$LHS = RHS$ $\therefore$ true when $n = 1$.

Assume true for $n = k$

i.e. $\frac{1}{2!} + \frac{2}{3!} + \dots + \frac{k}{(k+1)!} = 1 - \frac{1}{(k+1)!}$

Prove true for $n = k+1$.

i.e. $\frac{1}{2!} + \frac{2}{3!} + \dots \frac{k}{(k+1)!} + \frac{k+1}{(k+2)!} = 1 - \frac{1}{(k+2)!}$

$LHS = \left\{\frac{1}{2!} + \frac{2}{3!} + \dots \frac{k}{(k+1)!}\right\} + \frac{k+1}{(k+2)!}$

$= \left(1 - \frac{1}{(k+1)!}\right) + \frac{k+1}{(k+2)!}$ by assumption step.

$= 1 - \frac{(k+2) - (k+1)}{(k+2)!}$

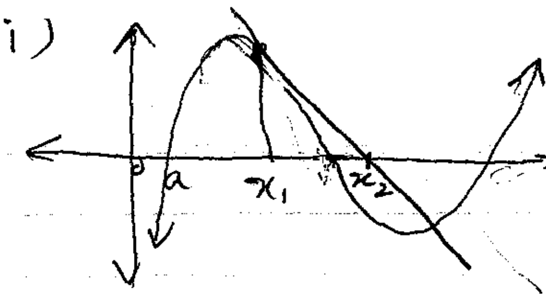
$= 1 - \frac{1}{(k+2)!}$

$= RHS$.

$\therefore$ True for all integers $n \geq 1$

by MI.

b.(i)




(b)(ii) $x_2 = x_1 - \dfrac{f(x_1)}{f'(x_1)}$

$f'(x_1) < 0, \quad f(x_1) > 0$

$\therefore x_2 = x_1 - \dfrac{(+)}{(-)}$

$= x_1 +$ positive quantity.

$\therefore x_2 > x_1$

.ie $x_2$ to the right of $x_1$

c) $T_{r+1} = {}^{12}C_r \left(\dfrac{x^2}{2}\right)^{12-r} \left(\dfrac{4}{x^4}\right)^r$

$= {}^{12}C_r \dfrac{x^{24-2r}}{2^{12-r}} \dfrac{2^{2r}}{x^{4r}}$

$= {}^{12}C_r \, 2^{3r-12} \, x^{24-6r}$

$\therefore 24 - 6r = 0$

$r = 4$.

Constant term

$= {}^{12}C_4 \, 2^0 = {}^{12}C_4$

$= 495$

d) prove: ${}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$.

$(1+x)^{n+1} = (1+x)(1+x)^n$

$= (1+x)^n + x(1+x)^n$

RHS coefficient $x^r = {}^{n+1}C_r$

RHS coefficient $x^r = {}^nC_r + {}^nC_{r-1}$

(10)

e (i) $(1+x)^{2n} = {}^{2n}C_0 + {}^{2n}C_1 x + {}^{2n}C_2 x^2 + \cdots + {}^{2n}C_{2n} x^n.$

$2n(1+x)^{2n-1} = {}^{2n}C_1 + 2\,{}^{2n}C_2 x + 3\,{}^{2n}C_3 x^2 + \cdots$
$+ r\,{}^{2n}C_r x^{r-1} + \cdots + n\,{}^{2n}C_{2n} x^{n-1}$

(ii) Sub $x = 1$

$2n\,2^{2n-1} = {}^{2n}C_1 + 2\,{}^{2n}C_2 + 3\,{}^{2n}C_3 + \cdots + r\,{}^{2n}C_r$
$+ \cdots + 2n\,{}^{2n}C_{2n}$

Sub $x = -1$

$0 = {}^{2n}C_1 - 2\,{}^{2n}C_2 + 3\,{}^{2n}C_3 + \cdots - (-1)^r\,{}^{2n}C_r$
$+ \cdots - 2n\,{}^{2n}C_{2n}$

Subtracting.

$2n \cdot 2^{2n-1} = 2\{2\,{}^{2n}C_2 + 4\,{}^{2n}C_4 + 6\,{}^{2n}C_6 + \cdots + 2n\,{}^{2n}C_{2n}\}$

$\therefore 2\,{}^{2n}C_2 + 4\,{}^{2n}C_4 + 6\,{}^{2n}C_6 + \cdots + 2n\,{}^{2n}C_{2n} = n\,2^{2n-1}$

$n \geq 1.$

(W)

26 a) (i) $\ddot{x} = 0$

$\dot{x} = c$

$\dot{x} = 40 \cos 45^\circ$

$= \frac{40}{\sqrt{2}} \text{ ms}^{-1}$.

(ii) $\ddot{y} = -10$

$\dot{y} = -10t + c_1$

$t = 0,\ \dot{y} = 40 \sin 45^\circ$

$\therefore c_1 = 40 \sin 45^\circ$

$= \frac{40}{\sqrt{2}}$

$\dot{y} = -10t + \frac{40}{\sqrt{2}}$

$y = \frac{-10t^2}{2} + \frac{40}{\sqrt{2}}t + c_2$

$t = 0,\ y = 0,\ c_2 = 0$.

$\therefore y = -5t^2 + \frac{40}{\sqrt{2}}t$

(iii) max height occurs when $\dot{y} = 0$.

$-10t + \frac{40}{\sqrt{2}} = 0$.

$t = \frac{4}{\sqrt{2}}$

$y = -5 \times \left(\frac{4}{\sqrt{2}}\right)^2 + \frac{40}{\sqrt{2}}$

$= -40 + 80$

$= 40$ m.

iv) Arrow lands when $y=0$

$$-5t^2 + \frac{40}{\sqrt{2}}t = 0$$

$$5t\left(-t + \frac{8}{\sqrt{2}}\right) = 0$$

$$\therefore t = 0 \text{ or } \frac{8}{\sqrt{2}}$$

when $t = \frac{8}{\sqrt{2}}$,

arrow land

$$\dot{x} = \frac{40}{\sqrt{2}}$$

$$x = \frac{40}{\sqrt{2}}t + C_4$$

$t=0$, $x=0$, $C_4=0$.

when $t = \frac{8}{\sqrt{2}}$

$$x = \frac{40}{\sqrt{2}} \times \frac{8}{\sqrt{2}}$$

$$= 160 \text{ m.}$$

b) (i) $V = \frac{1}{x+1}$

$$a = v\frac{dv}{dx}$$

$$= \frac{1}{x+1} \times \frac{-1}{(x+1)^2}$$

$$= \frac{-1}{(x+1)^3}$$

(ii) $\frac{dx}{dt} = \frac{1}{x+1}$

$$\frac{dt}{dx} = x+1$$

$$t = \int(x+1)\,dx$$

$$= \frac{x^2}{2} + x + C$$

$t=0$, $x=0$

$$C = 0$$

$$\therefore t = \frac{x^2}{2} + x$$

$$2t = x^2 + 2x$$

$$2t = (x^2 + 2x + 1) - 1$$

$$2t = (x+1)^2 - 1$$

$$x+1 = \pm\sqrt{2t+1}$$

$$x = -1 \pm \sqrt{2t+1}$$

Since $x=0$ when $t=0$

$$\therefore x = -1 + \sqrt{2t+1}$$

Q6 (13)

e)

(i) $v^2 = 40 + 32x - 8x^2$

$\frac{1}{2}v^2 = 20 + 16x - 4x^2$

$\frac{d}{dx}\left(\frac{1}{2}v^2\right) = 16 - 8x$

$= -8(x-2)$

$\therefore$ SHM.

(ii) period $= \frac{2\pi}{n} = \frac{2\pi}{\sqrt{8}}$

Centre at $x = 2$

(iii) $40 + 32x - 8x^2 = 0$

$-8(x^2 - 4x - 5) = 0.$

$-8(x-5)(x+1) = 0.$

particle stops at $x = -1$ and $x = 5$

$\therefore$ amplitude $= \frac{5 - -1}{2}$

$= 3$ cm

Q7.

(a)(i) $T = A + Be^{kt}$

$$\frac{dT}{dt} = kBe^{kt}$$
$$= k(A + Be^{kt} - A)$$
$$= k(T - A).$$

(ii) $A = 6$, $t = 0$, $T = 28$

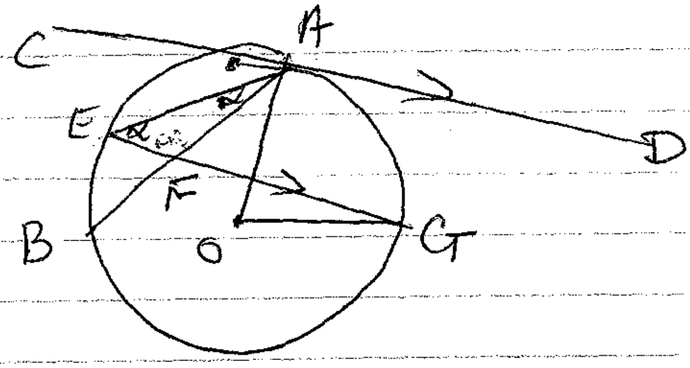
$$28 = 6 + Be^{0}$$
$$B = 22$$

$$T = 6 + 22e^{kt}$$

$T = 18$, $t = 35$

$$18 = 6 + 22e^{35k}$$
$$22e^{35k} = 12$$
$$e^{35k} = \frac{12}{22}$$
$$35k = \ln\left(\frac{12}{22}\right)$$
$$k = \frac{1}{35}\ln\left(\frac{12}{22}\right)$$
$$T = 6 + 22e^{\frac{1}{35}\ln\left(\frac{12}{22}\right)t}$$
$$10 = 6 + 22e^{\frac{1}{35}\ln\left(\frac{12}{22}\right)t}$$
$$e^{\frac{1}{35}\ln\left(\frac{12}{22}\right)t} = \frac{4}{22}$$
$$t = \frac{\ln\left(\frac{4}{22}\right)}{\frac{1}{35}\ln\left(\frac{12}{22}\right)} \approx 98.48 \text{ mins}$$

(b)




(i) $\angle EAF = \angle EAC$ (given)

$\angle AEF = \angle EAC$ (Alternate $\angle$'s, $CD \parallel EG$)

$\therefore \angle EAF = \angle AEF$ (both $= \angle EAC$)

$\therefore \triangle AEF$ is isosceles $\triangle$ (Base $\angle$'s equal)

(ii) Let $\angle EAF = \alpha$

$\therefore \angle AEF = \alpha$.

$\angle AFG = 2\alpha$ (Exterior $\angle$ of $\triangle AFE$ equals sum of two interior opposite $\angle$'s)

$\angle AOG = 2\alpha$ ($\angle$ at centre equal twice angle at the circumference subtented by arc $AG$).

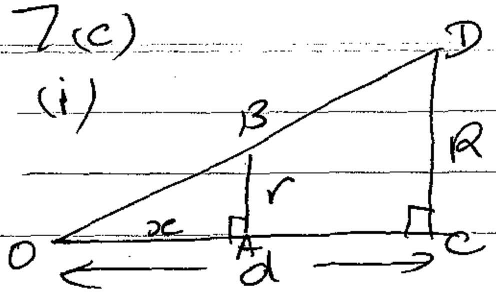
$\therefore \angle AOG = \angle AFG = 2\alpha$.

Equal angles $\angle AOG$ and $\angle AFG$ subtented by same arc $AG$ at the circumference at $O$ and $F$ are equals.

$\therefore$ $AFOG$ is cyclic quadrilateral.

7(c)

(i)




$\triangle OAB \parallel\!| \triangle OCD$ (equiangular)

$$\therefore \frac{r}{R} = \frac{x}{d}$$

$$R = \frac{rd}{x}$$

Area of shadow

$$A = \pi R^2$$
$$= \pi \left(\frac{rd}{x}\right)^2$$
$$= \frac{\pi r^2 d^2}{x^2}$$

(ii) $A = \pi r^2 d^2 x^{-2}$

$$\frac{dA}{dx} = \frac{-2\pi d^2 r^2}{x^3}$$

$$\frac{dA}{dt} = \frac{dA}{dx} \times \frac{dx}{dt}$$

$$\frac{dx}{dt} = -4$$ (negative for A of shadow, x is decreasing)

$$\therefore \frac{dA}{dt} = \frac{-2\pi d^2 r^2}{x^3} \times -4$$
$$= \frac{8\pi d^2 r^2}{x^3}$$

When $x = \frac{d}{4}$

$$\frac{dA}{dt} = \frac{8\pi d^2 r^2}{\left(\frac{d}{4}\right)^3}$$
$$= 8 \times 4^3 \pi d^2 r^2 = 512\pi r^2$$

Q7.

(a)(i) $T = A + Be^{kt}$

$$\frac{dT}{dt} = kBe^{kt}$$

$$= k\left(A + Be^{kt} - A\right)$$

$$= k(T - A).$$

(ii) $A = 6, \quad t = 0, \quad T = 28$

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$$10 = 6 + 22e^{\frac{1}{35}\ln\left(\frac{12}{22}\right)t}$$

$$e^{\frac{1}{35}\ln\left(\frac{12}{22}\right)t} = \frac{4}{22}$$

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